

双材料接合半无限体三维矩形界面 裂纹应力强度因子分析*

徐春晖¹, 秦太验¹, 野田尚昭²

(1. 中国农业大学 理学院, 北京 100083;
2. 九州工业大学 工学部, 日本北九州市 804-8550)

(王银邦推荐)

摘要: 基于体积力法, 研究了双材料接合半无限体三维矩形界面裂纹的应力强度因子问题. 在数值计算中, 未知的体积力密度采用基本密度函数和多项式乘积的形式来近似, 其中基本密度函数是根据界面裂纹应力的振荡奇异性来选取的. 计算结果表明, 基于本算法得到的数值结果其收敛精度和计算误差都是令人满意的. 算例中, 给出了应力强度因子随矩形形状及双材料参数的变化规律.

关键词: 应力强度因子; 体积力法; 界面裂纹; 复合材料; 断裂力学; 奇异积分方程

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引 言

随着复合材料的广泛应用, 界面裂纹问题越来越受到广大学者的关注^[1-10]. 但是, 有关双材料界面裂纹的研究大多局限于二维问题, 由于数学上的复杂性, 三维界面裂纹问题无法得到解析解, 只有特殊形状的界面裂纹问题得到了一些数值结果. 文献[11-15]给出了双材料界面上存在环形裂纹时应力强度因子的数值结果. Noda 等人^[16]基于体积力法研究了扭矩和拉力作用下轴对称界面裂纹问题, 其未知的体积力密度采用的是基本密度函数和多项式乘积的形式. 类似的方法, 也被 Wang 等人^[17]应用于研究三维无限体中存在矩形裂纹的问题.

Chen 等人^[18]对双材料接合半无限体存在界面裂纹问题进行了断裂力学分析, 给出了相应的超奇异微积分方程. 由于裂纹尖端位移和应力存在振荡奇异性, 方程的求解要比一般的裂纹问题复杂得多. 本文基于此方程, 对矩形界面裂纹进行了研究, 基本密度函数的选取充分考虑了裂纹尖端位移和应力的振荡奇异性. 结果表明, 计算的收敛性和精度都是令人满意的.

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作者简介: 徐春晖(1971—), 女, 副教授, 博士(Tel: + 86-10-62736992; E-mail: xuchunhui_cau@163.com);
秦太验(联系人, Tel: + 86-10-62736992; E-mail: tyqin@cau.edu.cn).

1 平片裂纹的超奇异微积分方程

假设两弹性半空间沿 $x-y$ 平面理想化地粘结在一起(见图1), 上、下半平面的弹性常数分别为 (μ_1, ν_1) 和 (μ_2, ν_2) , 这里 μ_1, μ_2 是材料的剪切模量, ν_1, ν_2 是材料的 Poisson 比. 假设裂纹位于双材料的界面上, 单位拉应力 σ_z^∞ 作用在距裂纹无穷远处.

Chen 等人^[18] 给出了一般三维界面裂纹问题的超奇异微积分方程, 如式(1)至(5)所示:

$$\begin{aligned} & \mu_1(\Lambda_2 - \Lambda_1) \frac{\partial \Delta u_z(x, y)}{\partial x} + \\ & \mu_1 \frac{2(\Lambda - \Lambda_1 - \Lambda_2)}{2\pi} \times \\ & \int_S \frac{1}{r^3} \Delta u_x(\xi, \eta) dS(\xi, \eta) + \\ & 3\mu_1 \frac{\Lambda_1 + \Lambda_2 - \Lambda}{2\pi} \times \\ & \left\{ \int_S \frac{(x - \xi)^2}{r^5} \Delta u_x(\xi, \eta) dS(\xi, \eta) + \right. \\ & \left. \int_S \frac{(x - \xi)(y - \eta)}{r^5} \Delta u_y(\xi, \eta) dS(\xi, \eta) \right\} = 0, \end{aligned} \quad (1)$$

$$\begin{aligned} & \mu_1(\Lambda_2 - \Lambda_1) \frac{\partial \Delta u_z(x, y)}{\partial y} + \mu_1 \frac{2\Lambda - \Lambda_1 - \Lambda_2}{2\pi} \int_S \frac{1}{r^3} \Delta u_y(\xi, \eta) dS(\xi, \eta) + \\ & 3\mu_1 \frac{\Lambda_1 + \Lambda_2 - \Lambda}{2\pi} \left\{ \int_S \frac{(x - \xi)(y - \eta)}{r^5} \Delta u_x(\xi, \eta) dS(\xi, \eta) + \right. \\ & \left. \int_S \frac{(y - \eta)^2}{r^5} \Delta u_y(\xi, \eta) dS(\xi, \eta) \right\} = 0, \end{aligned} \quad (2)$$

$$\begin{aligned} & \mu_1(\Lambda_1 - \Lambda_2) \left[\frac{\partial \Delta u_x(x, y)}{\partial x} + \frac{\partial \Delta u_y(x, y)}{\partial y} \right] + \\ & \mu_1 \frac{\Lambda_1 + \Lambda_2}{2\pi} \int_S \frac{1}{r^3} \Delta u_z(\xi, \eta) dS(\xi, \eta) = -\sigma_z^\infty, \quad (x, y) \in S, \end{aligned} \quad (3)$$

$$\begin{cases} \Lambda = \frac{\mu_2}{\mu_1 + \mu_2}, \quad \Lambda_1 = \frac{\mu_2}{\mu_1 + \kappa_1 \mu_2}, \quad \Lambda_2 = \frac{\mu_2}{\mu_1 + \kappa_2 \mu_1}, \\ \kappa_1 = 3 - 4\nu_1, \quad \kappa_2 = 3 - 4\nu_2, \quad r^2 = (x - \xi)^2 + (y - \eta)^2, \end{cases} \quad (4)$$

$$\Delta u_i(x, y) = u_i(x, y, 0^+) - u_i(x, y, 0^-), \quad i = x, y, z. \quad (5)$$

在式(1)至式(5)中, 未知函数是由式(5)所定义的裂纹开口位移 $\Delta u_x, \Delta u_y, \Delta u_z$. 这里 σ_z^∞ 是 σ_z 在无穷远处的拉应力. 此处区域 S 上的积分需采用有限部积分的概念.

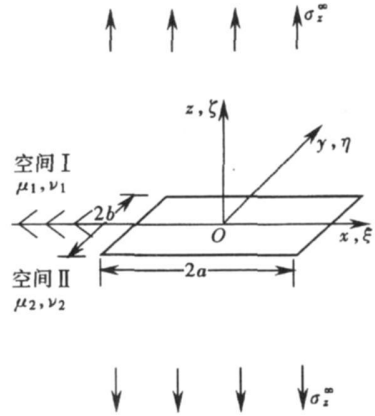


图1 计算模型示意图

2 奇异微积分方程的数值解法

假设双材料界面上存在矩形界面裂纹, 考虑距裂纹无穷远处有拉应力作用的问题. 对于二维问题, 根据裂纹尖端应力的奇异性, 基本密度函数与裂纹开口位移之间的关系采用如下的形式(Rice 和 Sih, 1965^[4]):

$$\begin{cases} \Delta u_z + i\Delta u_x = \sum_{l=1}^2 \left\{ \frac{\kappa_l - 1}{\mu_l(1 + \kappa_l)} w_z(\xi) + i \frac{1}{\mu_l} w_x(\xi) \right\} (\sigma_0 + i\tau_0) = \\ \sum_{l=1}^2 \frac{1 + \kappa_l}{4\mu_l \cosh \pi \varepsilon} \sqrt{a^2 - \xi^2} \left(\frac{a - \xi}{a + \xi} \right)^{i\varepsilon}, \\ \Delta u_y = \sum_{l=1}^2 \frac{1}{\mu_l} w_y(\xi) \tau_0 = \sum_{l=1}^2 \frac{1 + \kappa_l}{\mu_l} \frac{1}{4} \sqrt{a^2 - \xi^2} \tau_0 \end{cases} \quad (6)$$

这里, σ_0 、 τ_0 分别是距裂纹无穷远处的拉应力和剪应力, 双材料参数 ε 和参数 κ 的定义如下:

$$\varepsilon = \frac{1}{2\pi} \ln \left(\frac{\mu_2 \kappa_1 + \mu_1}{\mu_1 \kappa_2 + \mu_2} \right), \quad \kappa_l = \begin{cases} \frac{3 - \nu_l}{1 + \nu_l}, & \text{平面应力,} \\ 3 - 4\nu_l, & \text{平面应变.} \end{cases} \quad (7)$$

在本文算法中, 未知函数采用基本密度函数和多项式的乘积形式. 首先, 假设

$$\Delta u_i(\xi, \eta) = w_i(\xi, \eta) F_i(\xi, \eta), \quad i = x, y, z. \quad (8)$$

对于三维界面裂纹问题, 考虑到裂纹尖端应力和位移的振荡奇异性, 基本密度函数可假设为

$$\begin{cases} w_x(\xi, \eta) = \sum_{l=1}^2 \frac{1 + k_l}{4\mu_l \cosh \pi \varepsilon} \sqrt{a^2 - \xi^2} \sqrt{b^2 - \eta^2} \sin \left(\varepsilon \ln \left(\frac{a - \xi}{a + \xi} \right) \right), \\ w_y(\xi, \eta) = \sum_{l=1}^2 \frac{1 + k_l}{4\mu_l \cosh \pi \varepsilon} \sqrt{a^2 - \xi^2} \sqrt{b^2 - \eta^2} \sin \left(\varepsilon \ln \left(\frac{b - \eta}{b + \eta} \right) \right), \\ w_z(\xi, \eta) = \sum_{l=1}^2 \frac{1 + k_l}{4\mu_l \cosh \pi \varepsilon} \sqrt{a^2 - \xi^2} \sqrt{b^2 - \eta^2} \times \\ \cos \left(\varepsilon \ln \left(\frac{a - \xi}{a + \xi} \right) \right) \cos \left(\varepsilon \ln \left(\frac{b - \eta}{b + \eta} \right) \right). \end{cases} \quad (9)$$

多项式采用如下形式, 未知量为多项式的系数 α 、 β 、 γ :

$$\begin{cases} F_x(\xi, \eta) = \alpha_0 + \alpha_1 \eta + \dots + \alpha_{n-1} \eta^{(n-1)} + \alpha_n \eta^n + \alpha_{n+1} \xi + \alpha_{n+2} \xi \eta + \dots + \\ \alpha_{2n} \xi \eta^n + \dots + \alpha_{l-n-1} \xi^m + \alpha_{l-n} \xi^m \eta + \dots + \alpha_{l-1} \xi^m \eta^l = \\ \sum_{i=0}^{l-1} \alpha_i G_i(\xi, \eta), \quad l = (m+1)(n+1), \\ F_y(\xi, \eta) = \beta_0 + \beta_1 \eta + \dots + \beta_{n-1} \eta^{(n-1)} + \beta_n \eta^n + \beta_{n+1} \xi + \beta_{n+2} \xi \eta + \dots + \\ \beta_{2n} \xi \eta^n + \dots + \beta_{l-n-1} \xi^m + \beta_{l-n} \xi^m \eta + \dots + \beta_{l-1} \xi^m \eta^l = \\ \sum_{i=0}^{l-1} \beta_i G_i(\xi, \eta), \quad l = (m+1)(n+1), \\ F_z(\xi, \eta) = \gamma_0 + \gamma_1 \eta + \dots + \gamma_{n-1} \eta^{(n-1)} + \gamma_n \eta^n + \gamma_{n+1} \xi + \gamma_{n+2} \xi \eta + \dots + \\ \gamma_{2n} \xi \eta^n + \dots + \gamma_{l-n-1} \xi^m + \gamma_{l-n} \xi^m \eta + \dots + \gamma_{l-1} \xi^m \eta^l = \\ \sum_{i=0}^{l-1} \gamma_i G_i(\xi, \eta), \quad l = (m+1)(n+1), \\ G_0(\xi, \eta) = 1, G_1(\xi, \eta) = \eta, \dots, G_{n+1}(\xi, \eta) = \xi, \dots, G_{l-1}(\xi, \eta) = \xi^m \eta^l. \end{cases} \quad (10)$$

3 数值结果和讨论

3.1 无量纲应力强度因子的定义

考虑距裂纹无穷远处有单位拉应力 $\sigma_z^\infty = 1$ 作用时, 三维矩形界面裂纹的问题. 根据式 (9) 和文献[19] 中关于一般界面裂纹应力强度因子的表达式, 可定义无量纲应力强度因子 F_I 、 F_{II} 和 F_{III} 的表达式如下:

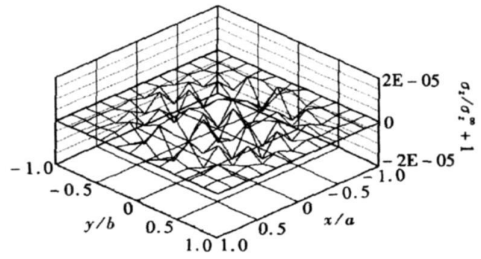
$$\left\{ \begin{aligned} F_I + iF_{II} &= \frac{K_I(x, y)|_{x=x, y=\pm b} + iK_{II}(x, y)|_{x=x, y=\pm b}}{\sigma_z^\infty \sqrt{\pi b}} = \\ &= \frac{\sqrt{a^2 - x^2} \left[\cos \left(\varepsilon \ln \left(\frac{a-x}{a+x} \right) \right) F_z(x, y)|_{x=x, y=\pm b} + 2i \varepsilon F_y(x, y)|_{x=x, y=\pm b} \right]}{\sigma_z^\infty \sqrt{\pi b}}, \\ F_{III} &= \frac{K_{III}(x, y)|_{x=x, y=\pm b}}{\sigma_z^\infty \sqrt{\pi b}} = \\ &= \sum_{j=1}^2 \frac{1 + K_j}{4 \mu_j \cosh \pi \varepsilon} \frac{1}{(1/\mu_1 + 1/\mu_2)} \sqrt{a^2 - x^2} \sin \left(\varepsilon \ln \left(\frac{a-x}{a+x} \right) \right) F_x|_{x=x, y=\pm b}. \end{aligned} \right. \quad (11)$$

3.2 计算精度和收敛性

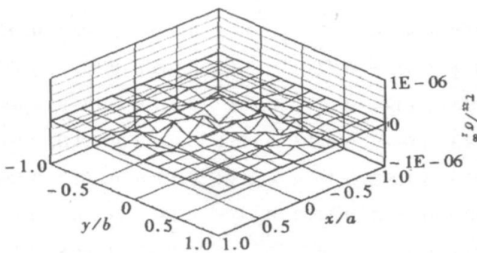
图 2(a) 至(c) 给出了 $a/b = 1$, $\nu_1 = \nu_2 = 0$, $\varepsilon = 0.02$ 时裂纹表面的计算残余误差, 这里配置点数为 $100(10 \times 10)$, 多项式次数为 $m = n = 8$. 结果表明应力残余误差

$$(\sigma_z/\sigma_z^\infty + 1), \tau_x/\sigma_z^\infty, \tau_y/\sigma_z^\infty < 1.5 \times 10^{-6}.$$

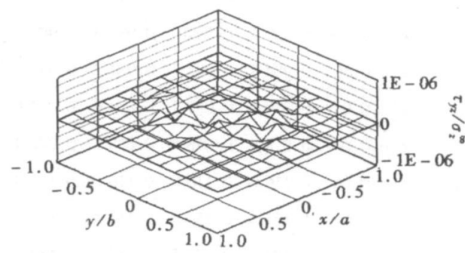
对于各同性材料, 无量纲应力强度因子随多项式次数的变化情况见表 1(配置点数 20×20 , $a/b = 1$, $\nu_1 = \nu_2 = 0.3$), 表 1 同时给出了与 Qin 等人^[19] 和 Wang 等人^[17] 的结果比较. 表 2 给出了



(a) $(\sigma_z/\sigma_z^\infty + 1) \approx 0$



(b) $\tau_x/\sigma_z^\infty \approx 0$



(c) $\tau_y/\sigma_z^\infty \approx 0$

图 2 残余应力的变化

$\varepsilon = 0.02$ (配置点数 10×10 , $a/b = 1$, $\nu_1 = \nu_2 = 0.3$) 的计算结果. 计算表明, 结果收敛性很好.

3.3 与二维问题结果的比较

如果矩形裂纹细长, 即 $a/b \rightarrow \infty$, 三维问题就退化成二维裂纹问题, 应力强度因子的理论结果为 $F_I = 1$ 、 $F_{II} = 2\varepsilon$ 、 $F_{III} = 0$. 本文计算中取 $a/b = 8$, 多项式次数取为 $m = n = 8$, 配置点数 10×10 , 应力强度因子的值见表 3. 从表中可以看出, 当 $a/b = 8$ 时, 三维问题的计算

结果与二维时的理论结果符合得很好.

表 1 应力强度因子 F_I 的收敛性 ($\varepsilon = 0, a/b = 1, y = b$) (配置点数 20×20)

x/a	0/11	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11
$m = n = 4$	0.752 1	0.750 7	0.746 2	0.737 9	0.725 0	0.706 6	0.682 1	0.650 9	0.610 8	0.553 8	0.449 7
$m = n = 6$	0.753 8	0.752 0	0.746 7	0.737 7	0.724 8	0.707 2	0.683 6	0.652 0	0.609 4	0.548 2	0.442 3
$m = n = 8$	0.753 4	0.751 6	0.746 3	0.737 3	0.724 3	0.706 3	0.682 1	0.650 0	0.608 1	0.551 3	0.454 3
Qin ^[19]	0.753 4	0.751 2	0.746 2	0.737 9	0.725 5	0.707 2	0.682 1	0.649 7	0.609 0	0.552 1	0.446 4
Wang ^[17]	0.753 4	0.751 7	0.746 5	0.737 6	0.724 5	0.706 6	0.682 8	0.651 2	0.608 6	0.549 2	0.453 6

表 2 应力强度因子的收敛性 ($\varepsilon = 0.02, a/b = 1, y = b, \nu_1 = 0.3, \nu_2 = 0.3$)

x/a	0/11	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	
F_I	$m = n = 4$	0.753 1	0.751 2	0.745 7	0.736 4	0.723 3	0.705 9	0.682 8	0.651 7	0.607 3	0.538 5	0.417 7
	$m = n = 6$	0.752 4	0.750 7	0.745 6	0.736 7	0.723 7	0.706 0	0.682 6	0.651 9	0.609 8	0.546 5	0.432 9
	$m = n = 8$	0.752 8	0.751 1	0.745 9	0.736 9	0.723 8	0.705 8	0.682 2	0.651 4	0.609 9	0.549 0	0.440 0
F_{II}	$m = n = 4$	0.027 2	0.027 1	0.026 8	0.026 4	0.025 7	0.024 8	0.023 6	0.022 1	0.020 0	0.017 1	0.012 7
	$m = n = 6$	0.027 3	0.027 2	0.027 0	0.026 5	0.025 9	0.025 0	0.023 8	0.022 3	0.020 3	0.017 4	0.013 1
	$m = n = 8$	0.027 4	0.027 3	0.027 1	0.026 6	0.026 0	0.025 1	0.023 9	0.022 4	0.020 3	0.017 6	0.013 3
F_{III}	$m = n = 4$	0	0.001 0	0.002 1	0.003 1	0.004 2	0.005 3	0.006 5	0.007 9	0.009 4	0.010 9	0.012 0
	$m = n = 6$	0	0.001 0	0.002 0	0.003 1	0.004 1	0.005 2	0.006 4	0.007 6	0.009 1	0.010 6	0.012 0
	$m = n = 8$	0	0.001 0	0.002 0	0.003 1	0.004 1	0.005 1	0.006 3	0.007 5	0.008 9	0.010 5	0.012 0

表 3 $y = b$ 时的无量纲应力强度因子 ($\nu_1 = 0.3, \nu_2 = 0.3, a/b = 8$)

x/a	0/11	1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11	
F_I	$\varepsilon = 0.02$	0.994 7	0.994 6	0.994 2	0.993 3	0.991 7	0.988 8	0.983 8	0.975 0	0.958 0	0.917 5	0.795 4
	$\varepsilon = 0.04$	0.993 8	0.993 7	0.993 2	0.992 3	0.990 7	0.987 8	0.982 8	0.973 9	0.956 8	0.916 0	0.793 1
	$\varepsilon = 0.06$	0.992 0	0.991 9	0.991 4	0.990 5	0.988 9	0.986 0	0.980 9	0.971 9	0.954 5	0.913 4	0.789 2
	$\varepsilon = 0.08$	0.989 1	0.989 0	0.988 5	0.987 5	0.985 9	0.983 0	0.977 9	0.968 7	0.950 9	0.909 2	0.783 6
	$\varepsilon = 0.10$	0.984 8	0.984 7	0.984 2	0.983 3	0.981 6	0.978 6	0.973 3	0.964 0	0.946 1	0.903 7	0.775 5
F_{II}	$\varepsilon = 0.02$	0.039 7	0.039 7	0.039 6	0.039 6	0.039 5	0.039 4	0.039 1	0.038 7	0.037 8	0.035 8	0.030 4
	$\varepsilon = 0.04$	0.078 6	0.078 6	0.078 5	0.078 4	0.078 3	0.078 0	0.077 5	0.076 6	0.074 9	0.071 0	0.060 1
	$\varepsilon = 0.06$	0.116 0	0.116 0	0.116 0	0.115 8	0.115 6	0.115 2	0.114 4	0.113 1	0.110 6	0.104 7	0.088 5
	$\varepsilon = 0.08$	0.151 5	0.151 5	0.151 4	0.151 2	0.150 9	0.150 3	0.149 3	0.147 6	0.144 2	0.136 4	0.115 1
	$\varepsilon = 0.10$	0.184 5	0.184 5	0.184 4	0.184 2	0.183 8	0.183 1	0.181 9	0.179 7	0.175 5	0.165 8	0.139 4
$F_{III} \times 10^2$	$\varepsilon = 0.02$	0	0.060 9	0.121 7	0.182 3	0.242 3	0.301 3	0.358 4	0.411 8	0.457 9	0.479 3	0.488 7
	$\varepsilon = 0.04$	0	0.108 6	0.237 1	0.355 1	0.472 1	0.587 2	0.698 5	0.802 8	0.893 2	0.937 0	0.954 0
	$\varepsilon = 0.06$	0	0.170 4	0.340 6	0.510 1	0.678 4	0.843 9	1.004	1.155	1.286	1.354	1.375
	$\varepsilon = 0.08$	0	0.214 1	0.427 9	0.641 1	0.856 2	1.061	1.263	1.453	1.620	1.714	1.736
$\varepsilon = 0.10$	0	0.248 4	0.496 5	0.743 9	0.989 7	1.232	1.467	1.690	1.886	2.009	2.026	

3.4 一般情况的解

对于一般情况, 以下的计算中, 多项式的次数取 $m = n = 8$, 配置点取为 10×10 . 表 4 给出了无量纲应力强度因子 F_I 和 F_{II} 随双材料参数 ε 变化的情况. 由表 4 可以看出, F_I 的值

随参数 ε 的增大而减小, 而 F_{II} 的值随参数 ε 的增加而增加. 本文中并没有具体给出 F_{III} 的结果, 因为 F_{III} 的值比 F_I 和 F_{II} 要小得多, 分别满足 $F_{III} \leq 0.01 \times F_I$ 和 $F_{III} \leq 0.5 \times F_{II}$.

表 4 点 $(0, b)$ 处无量纲应力强度因子 F_I 和 F_{II} ($\nu_1 = 0.3, \nu_2 = 0.3$)

	F_I				F_{II}			
	$a/b = 1$	$a/b = 2$	$a/b = 4$	$a/b = 8$	$a/b = 1$	$a/b = 2$	$a/b = 4$	$a/b = 8$
$\varepsilon = 0.02$	0.752 8	0.905 2	0.976 0	0.994 7	0.027 4	0.035 2	0.038 8	0.039 7
$\varepsilon = 0.04$	0.750 9	0.903 8	0.975 0	0.993 8	0.054 2	0.069 6	0.076 8	0.078 6
$\varepsilon = 0.06$	0.747 8	0.901 3	0.973 0	0.992 0	0.079 8	0.102 7	0.113 4	0.116 0
$\varepsilon = 0.08$	0.743 3	0.897 5	0.969 9	0.989 1	0.104 0	0.133 8	0.147 9	0.151 5
$\varepsilon = 0.10$	0.737 3	0.892 1	0.965 4	0.984 8	0.126 3	0.162 7	0.180 1	0.184 5

4 结 论

基于体积力法, 研究了距裂纹无穷远处有单位拉应力作用时, 双材料接合面上存在平片矩形界面裂纹的问题, 结论如下:

- 1) 根据界面裂纹应力和位移的振荡奇异性, 对未知函数采用基本密度函数和多项式乘积的形式进行近似. 数值结果表明, 本文的计算方法是成功的, 结果也是令人满意的.
- 2) 在数值计算中, 当 $a/b \geq 8$ 时, 三维问题的计算结果与二维时的理论结果符合得很好.
- 3) 对界面裂纹问题, 3 种应力强度因子同时存在, 所以比一般的情况更危险. F_I 的值随参数 ε 的增大而减小, 而 F_{II} 的值随参数 ε 的增加而增加. F_{III} 的值比 F_I 和 F_{II} 要小得多, 分别满足 $F_{III} \leq 0.01 \times F_I$ 和 $F_{III} \leq 0.5 \times F_{II}$.

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Numerical Solutions of Singular Integral Equations for Planar Rectangular Interfacial Crack in Three Dimensional Bimetals

XU Chun-hui¹, QIN Tai-yan¹, NODA Nao-Aki²

1. College of Science, China Agricultural University, Beijing 100083, P. R. China;

2. Department of Mechanical Engineering, Kyushu Institute of Technology,
Kitakyushu, 804-8550, Japan)

Abstract: Stress intensity factors for a three dimensional rectangular interfacial crack are considered on the idea of the body force method. In the numerical calculations, unknown body force densities were approximated by the products of the fundamental densities and power series. Here the fundamental densities were chosen to express singular stress fields due to an interface crack exactly. The calculation shows that the present method gives rapidly converging numerical solutions and highly satisfied boundary conditions. The stress intensity factors for a rectangular interface crack are indicated accurately with varying the aspect ratio and bimaterial parameter.

Key words: stress intensity factor; body force method; interface crack; composite material; fracture mechanics; singular integral equation