

常曲率大深宽比河湾拟序扰动结构与 床面响应关系探讨*

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摘要: 蜿蜒河流床面形态既是其复杂动力结构响应的结果,同时也是决定河流进一步演化方向的重要因素.以蜿蜒河流中一种典型的大深宽比河湾为背景,探索其动力结构与床面响应的关系,将黏性不可压缩流体方程、泥沙输移方程和床面变形方程耦合,通过摄动方法求解床面响应,分析床面形态变化特性.研究成果显示在水流二维扰动作用下,河道中浅滩深槽呈现规则响应.当弯曲度等于0时,床面响应形态围绕河道中轴线基本呈反对称分布;当弯曲度不等于0时,床面响应形态呈不对称分布,中轴线向凹岸偏移.该文给出了由 Reynolds(雷诺)数、扰动波数、床面形态增减率等构成的床面响应发展趋势稳定关系的判别方法.

关键词: 蜿蜒河流; 大深宽比河湾; 扰动波; 床面动力响应

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引 言

河流床面形态是其复杂动力因素影响的结果,其主导因素与床面响应之间的关系一直被各国学者所重视.近几十年来,通过流体动力学家和地貌学家相互的合作,构建了定量描述多种河床地貌过程理论模型.理论分析和试验表明,大多数观察到的地貌形态与水动力不稳定机制相关,这种不稳定机制是流体与易侵蚀边界之间的可移动界面不稳定.Johannesson 和 Parker^[1-2]建立了弯曲河流易侵蚀河床数学模型,在线性方程中加入小扰动,将水流、推移质和床面进行耦合,计算水流和床面形态,反映了水流塑造河床形态的机理.Seminara 等^[3-4]构建了线性模型和弱非线性模型,研究弯曲河流床面形态的不稳定响应;Martin 等^[5]对不稳定流量作用下的水槽实验进行观察,得到砂质床面的响应状况,定量描述了流量增减与床面响应增减的关系.Nelson 等^[6]针对大洪水情况下的河道,对均匀河道宽度、非均匀河道宽度和实际河流进行动态地貌平衡计算.对于尺度较小的流体与床面响应微观动力过程,著名河流动力学家 Yalin^[7]认为局部扰动造成流动失稳诱发沙纹的形成,但他们未给出理论模型解释;之后白玉川

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和罗纪生^[8]基于这一观点,对明渠层流失稳与沙纹成因机理进行研究,认为当床面边界附近扰动波或拟序结构以及水流自身所产生的床面底部切应力大于 Shields(希尔兹)切应力后,床面即产生响应,形成沙纹.Xu 和 Bai(徐海珏和白玉川)^[9]基于这一观点,构建了扰动与床面作用的三波共振非线性理论模型,分析了顺直河流沙纹动力过程及演变规律;Xu 和 Bai(徐海珏和白玉川)^[10]建立了曲率变化的窄深型弯道水流理论模型,使用 Muller 法和 Q-R 方法求解特征值,研究弯道层流流动动力不稳定特征,探讨了曲率、波数和波频率这些参数与水流动力不稳定特征的变化关系.许栋和白玉川等^[11]还进行了固定边壁的动床弯道模型实验,总体表明:微弯河道中浅滩、深槽分布规则;当河道弯曲超过一定程度时,床面形态响应变得复杂;随着弯曲度的增大,河道的输沙能力有减小趋势.白玉川和冀自清等^[12]以常曲率窄深型河流为背景,对水流动力稳定性特征进行研究,得到弯道流速分布的理论计算公式和临界 Reynolds 数计算公式;与顺直河道相比,其稳定中性曲线沿坐标前移,失稳 Reynolds 数增加,扰动波数的响应范围减小,流动状态更易保持.Bai 和 Ji(白玉川和冀自清)等^[13]从理论上分析常曲率河湾紊流拟序结构的稳定性与自适应特征,将紊流拟序涡体结构看做一种扰动,计算不同弯曲度下河湾拟序涡体的扰动增长率和扰动波数响应范围,得到不同尺度紊流结构对河湾参数的响应情况,解释了河湾得以保持的动力机理.在上述研究的基础上,本文针对深宽比 (H/b) 大于 10 的近真实河流,采用以河道中心线为横轴、断面方向为纵轴的曲线坐标系,通过无量纲化选定河流控制参数,耦合计算水流、输沙、床面变形等方程,使用 Muller 方法求解特征值,追赶法求解床面响应,探讨了扰动与河床形态响应之间的关系。

1 理论模式

1.1 坐标变换

以河道中心 $y = y(x)$ 为横坐标 s ,以垂直于 s 坐标为纵坐标 n ,以垂直于河流地面的方向为垂向坐标 z ,建立跟踪河道走向的正交曲线坐标系,如图 1。

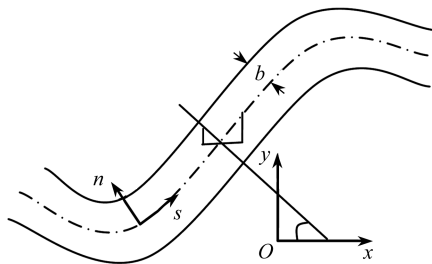


图 1 河道正交曲线坐标系

Fig. 1 The orthogonal curvilinear coordinate system for the channel

曲线坐标系与直角坐标系之间的换算关系为

$$x = x_0 - n \frac{dy_0}{ds}, \quad y = y_0 + n \frac{dx_0}{ds};$$

曲率半径

$$r = r(s) = \left(\frac{dx_0}{ds} \frac{d^2y_0}{ds^2} - \frac{d^2x_0}{ds^2} \frac{dy_0}{ds} \right)^{-1};$$

Lamé(拉梅)系数 $h_s = 1 - N, h_n = 1$, 其中 $N = n/r(s)$; $C(s) = 1/r(s)$ 为弯道曲率. 当弯道接近正余弦函数形状, 则曲率等参数有如下形式:

$$\frac{1}{r} = C(s) e^{i(\alpha_r s - \omega_g t)}, \quad N = \frac{n}{r} = C(s) n e^{i(\alpha_r s - \omega_g t)},$$

其中, α_r 为弯道的波数, ω_g 为弯道蠕动频率; 对于固定弯道, $\omega_g = 0$; 对于常曲率 $\alpha_r = 0$.

1.2 控制方程

在正交曲线坐标系 (s, n) 下, 连续性方程和水流运动方程变为:

连续性方程

$$\frac{1}{1-N} \frac{\partial u_s}{\partial s} - \frac{u_n}{(1-N)r} + \frac{\partial u_n}{\partial n} + \frac{\partial u_z}{\partial z} = 0, \quad (1)$$

s 方向运动方程

$$\begin{aligned} \frac{\partial u_s}{\partial t} + \frac{u_s}{1-N} \frac{\partial u_s}{\partial s} + u_n \frac{\partial u_s}{\partial n} + u_z \frac{\partial u_s}{\partial z} - \frac{u_s u_n}{(1-N)r} = \\ f_s - \frac{1}{\rho(1-N)} \frac{\partial P}{\partial s} + \nu \Delta u_s + \\ \nu \left\{ \left[-\frac{2}{(1-N)} \frac{\partial u_n}{\partial s} - \frac{\partial u_s}{\partial n} - \frac{u_s}{(1-N)r} \right] \frac{1}{(1-N)r} \right\}, \end{aligned} \quad (2)$$

n 方向运动方程

$$\begin{aligned} \frac{\partial u_n}{\partial t} + \frac{u_s}{1-N} \frac{\partial u_n}{\partial s} + u_n \frac{\partial u_n}{\partial n} + u_z \frac{\partial u_n}{\partial z} - \frac{u_s^2}{(1-N)r} = \\ f_n - \frac{1}{\rho} \frac{\partial P}{\partial n} + \nu \Delta u_n + \nu \left\{ \left[\frac{2}{(1-N)} \frac{\partial u_n}{\partial s} - \frac{\partial u_n}{\partial n} - \frac{u_n}{(1-N)r} \right] \frac{1}{(1-N)r} \right\}, \end{aligned} \quad (3)$$

z 方向运动方程

$$\frac{\partial u_z}{\partial t} + \frac{u_s}{1-N} \frac{\partial u_z}{\partial s} + u_n \frac{\partial u_z}{\partial n} + u_z \frac{\partial u_z}{\partial z} = f_z - \frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \Delta u_z - \frac{\nu}{(1-N)r} \frac{\partial u_z}{\partial n}, \quad (4)$$

其中, $u_i (i = s, n, z)$ 分别为 s, n 和 z 方向的速度; $f_i (i = s, n, z)$ 分别为 s, n 和 z 方向上的质量力, P 为压力; $\Delta = \frac{1}{(1-N)^2} \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2} + \frac{\partial^2}{\partial z^2}$; ν 为分子粘滞和紊动粘滞系数之和.

河床变形方程

$$(1-p') \frac{\partial z_b}{\partial t} + \frac{1}{1+N} \left\{ \frac{\partial q_s}{\partial s} + \frac{\partial}{\partial n} [1+N] q_n \right\} = 0; \quad (5)$$

沿流向的泥沙输沙率

$$q_s = q_{s0} (u_s)^M; \quad (6)$$

沿横向的泥沙输沙率

$$\frac{q_n}{q_s} = \frac{u_n}{u_s} + \frac{u_n(0)}{u_s T(0)} - \frac{\beta}{\gamma} \frac{\partial z_b}{\partial n}. \quad (7)$$

采用 Engelund-Hansen 的速度指数型输沙率公式, z_b 为床面高程, q_s, q_n 为 s, n 方向的单宽体积输沙率, $u_n(0)$ 为初始时刻的横向流速, $T(\xi)$ 为无量纲的流速分布形状函数, $\xi = \tilde{z}/\tilde{h}$, γ 为

半宽深比, q_{s0} , M , β 为经验系数.

1.3 方程无量纲化及参数化

采用 3 个尺度对空间进行无量纲化, 即: 横向尺度用半河宽 b , 垂向尺度用水深 H , 河道曲率半径用河道的最小曲率半径 r_m , 分别进行无量纲化. 在常曲率河道中, r_m 即为河道中心线曲率半径. 速度尺度用河道内平均流速 U 无量纲化, 时间尺度用 b/U 无量纲化, 即

$$\text{空间量 } s = b \cdot \tilde{s}, \quad n = b \cdot \tilde{n}, \quad z = H \cdot \tilde{z}, \quad z_b = H \cdot \tilde{z}_b, \quad r = r_m \cdot \tilde{R};$$

$$\text{速度量 } u_s = U \cdot \tilde{u}_s, \quad u_n = U \cdot \tilde{u}_n, \quad u_z = U \cdot \tilde{u}_z;$$

$$\text{压力 } P = \rho U^2 \cdot \tilde{p};$$

$$\text{输沙率 } q_s = \tilde{q}_s \cdot \sqrt{R_q g D_s D_s}, \quad q_n = \tilde{q}_n \cdot \sqrt{R_q g D_s D_s};$$

$$\text{时间量 } t = \frac{b}{U} \cdot \tilde{t};$$

$$\text{参数 } N = \frac{n}{r} = \frac{b \cdot \tilde{n}}{r_m \cdot \tilde{R}} = \frac{b}{r_m} \cdot \tilde{N}, \quad Q_0 = \frac{\sqrt{R_q g D_s D_s}}{(1-p') U h}, \quad \delta = \frac{H}{b}, \quad \psi = \frac{b}{r_m},$$

其中, D_s 为床沙中值粒径, $D_s = 0.5 \text{ mm}$, R_q 为泥沙水下比重, $R_q = 2650 \text{ kg/m}^3$, p' 为泥沙孔隙率, $p' = 0.4$ ^[14], 则得无量纲连续性方程、运动方程:

$$\frac{1}{(1-\psi\tilde{N})} \frac{\partial \tilde{u}_s}{\partial \tilde{s}} - \frac{\psi}{1-\psi\tilde{N}} \frac{\tilde{u}_n}{\tilde{R}} + \frac{\partial \tilde{u}_n}{\partial \tilde{n}} + \frac{1}{\delta} \frac{\partial \tilde{u}_z}{\partial \tilde{z}} = 0, \quad (8)$$

$$\begin{aligned} \frac{\partial \tilde{u}_s}{\partial \tilde{t}} + \frac{\tilde{u}_s}{(1-\psi\tilde{N})} \frac{\partial \tilde{u}_s}{\partial \tilde{s}} + \tilde{u}_n \frac{\partial \tilde{u}_s}{\partial \tilde{n}} + \frac{1}{\delta} \tilde{u}_z \frac{\partial \tilde{u}_s}{\partial \tilde{z}} - \frac{\psi \tilde{u}_s \tilde{u}_n}{(1-\psi\tilde{N}) \tilde{R}} = \\ f_s \cdot \frac{b}{U^2} - \frac{1}{(1-\psi\tilde{N})} \frac{\partial \tilde{p}}{\partial \tilde{s}} + \frac{1}{Re} \bar{\Delta} \tilde{u}_s + \\ \frac{\psi}{Re} \left\{ -\frac{2}{(1-\psi\tilde{N})^2} \frac{1}{\tilde{R}} \frac{\partial \tilde{u}_n}{\partial \tilde{s}} - \frac{1}{(1-\psi\tilde{N})} \frac{1}{\tilde{R}} \frac{\partial \tilde{u}_s}{\partial \tilde{n}} + \frac{\psi \tilde{u}_s}{(1-\psi\tilde{N})^2 \tilde{R}^2} \right\}, \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial \tilde{u}_n}{\partial \tilde{t}} + \frac{\tilde{u}_s}{(1-\psi\tilde{N})} \frac{\partial \tilde{u}_n}{\partial \tilde{s}} + \tilde{u}_n \frac{\partial \tilde{u}_s}{\partial \tilde{n}} + \frac{1}{\delta} \tilde{u}_z \frac{\partial \tilde{u}_n}{\partial \tilde{z}} + \frac{\psi \tilde{u}_s^2}{(1-\psi\tilde{N}) \tilde{R}} = \\ f_n \cdot \frac{b}{U^2} - \frac{\partial \tilde{p}}{\partial \tilde{n}} + \frac{1}{Re} \bar{\Delta} \tilde{u}_n + \\ \frac{\psi}{Re} \left\{ \frac{1}{(1-\psi\tilde{N})} \frac{1}{\tilde{R}} \left[\frac{2}{(1-\psi\tilde{N})} \frac{\partial \tilde{u}_s}{\partial \tilde{s}} - \frac{\psi \tilde{u}_n}{(1-\psi\tilde{N}) \tilde{R}} - \frac{\partial \tilde{u}_n}{\partial \tilde{n}} \right] \right\}, \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{\partial \tilde{u}_z}{\partial \tilde{t}} + \frac{\tilde{u}_s}{(1-\psi\tilde{N})} \frac{\partial \tilde{u}_z}{\partial \tilde{s}} + \tilde{u}_n \frac{\partial \tilde{u}_z}{\partial \tilde{n}} + \frac{1}{\delta} \tilde{u}_z \frac{\partial \tilde{u}_z}{\partial \tilde{z}} = \\ f_z \cdot \frac{b}{U^2} - \frac{1}{\delta} \frac{\partial \tilde{p}}{\partial \tilde{z}} + \frac{1}{Re} \bar{\Delta} \tilde{u}_z - \frac{\psi}{Re} \frac{1}{(1-\psi\tilde{N}) \tilde{R}} \frac{\partial \tilde{u}_z}{\partial \tilde{n}}, \end{aligned} \quad (11)$$

式中, $\bar{\Delta} = \frac{1}{(1-\psi\tilde{N})^2} \frac{\partial^2}{\partial \tilde{s}^2} + \frac{\partial^2}{\partial \tilde{n}^2} + \frac{1}{\delta^2} \frac{\partial^2}{\partial \tilde{z}^2}$, Reynolds 数 $Re = Ub/\nu$.

无量纲床面响应及输沙方程:

$$\frac{\partial \tilde{z}_b}{\partial \tilde{t}} + \frac{\sqrt{R_q g D_s D_s}}{(1-p') H U} \frac{1}{1+\psi\tilde{N}} \left[\frac{\partial \tilde{q}_s}{\partial \tilde{s}} + (1+\psi\tilde{N}) \frac{\partial \tilde{q}_s}{\partial \tilde{n}} + \frac{\psi}{\tilde{R}} \cdot \tilde{q}_n \right] = 0, \quad (12)$$

$$\tilde{q}_s = (\tilde{u}_s)^M, \quad (13)$$

$$\frac{\tilde{q}_n}{\tilde{q}_s} = \frac{\tilde{u}_n}{\tilde{u}_s} + \frac{\tilde{u}_n(0)}{\tilde{u}_s T(0)} - \frac{\beta}{\gamma} \frac{\partial \tilde{z}_b}{\partial \tilde{n}}, \quad (14)$$

式中,经验系数 $M = 5$ (平整床面), $\gamma = b/H = 0.1$. 初始时刻 $\tilde{u}_n(0) = 0$, 采用 Johannesson, Parker^[2] 的横纵向输沙表达关系式,

$$\beta = \frac{1 + \alpha_* \mu}{f_* \mu} \sqrt{\frac{\tau_C^*}{\tau_C^*}} = 1.16,$$

其中, τ_C^* 为泥沙颗粒的 Shields 应力, τ_C^* 是临界 Shields 应力, $\tau_C^* = \tau^*$ (平整床面),

$$\tau^* = \frac{\tilde{\tau}_s}{\rho R_q g D_s}, \quad \tilde{\tau}_s = C_f \sqrt{\tilde{u}_s^2 + \tilde{u}_{nT}^2} \tilde{u}_s.$$

1.4 摄动分析

摄动分析是存在一个系统,这个系统 S_ε 中包含一个难以精确确定或作缓慢变化的参数 ε , 可以令 $\varepsilon = 0$, 使系统 S_ε 退化为 S_0 , 而把 S_ε 看作是 S_0 受到摄动(由于 $\varepsilon \neq 0$ 而引起的)而形成的受扰系统. 这种方法最早应用于天体力学, 后来广泛应用于物理学和力学理论研究. 在数学和力学研究领域, 研究者对数学方程进行摄动求解, 如汪维刚、莫嘉琪等^[15] 对一类双参数非线性高阶反应扩散方程进行摄动求解, 得到具有便于求解和精度高的各次近似解. 随后, 冯依虎、莫嘉琪等^[16] 对一类广义非线性强阻尼扰动发展方程的行波解进行小参数方法和引入伸长变量得到了渐近解. Saghafian, Rajabi 等^[17] 使用摄动方法对小 Poiseuille(泊肃叶)流 N-S 方程中的流速、压力和温度场进行摄动展开, 得到方程的不同阶数, 最终求解方程的每一项. 本文考虑河宽小于曲率半径情况下, ψ 为一小参数, 与其相应的项为弯曲形状引起的修正量, 是对顺直河道修正, 代表弯道二次、三次及高次流的部分. 从方程(8)~(14)可以看出, 控制方程中存在小参数 $\psi (= b/r_m)$, 本文采用摄动展开法对控制方程进行求解. 河湾中水流结构均为紊流结构, 按照流体力学中模拟紊流结构的理论方法^[18-20], 对河湾中水流结构进行摄动展开, 取

$$\begin{bmatrix} \tilde{u}_s \\ \tilde{u}_n \\ \tilde{u}_z \\ \tilde{p} \end{bmatrix} = \sum_{i=0}^{\infty} \begin{bmatrix} \tilde{u}_{s\psi i} \\ \tilde{u}_{n\psi i} \\ \tilde{u}_{z\psi i} \\ \tilde{p}_{\psi i} \end{bmatrix} \psi^i + \varepsilon_T \begin{bmatrix} \tilde{u}_{sT} \\ \tilde{u}_{nT} \\ \tilde{u}_{zT} \\ \tilde{p}_T \end{bmatrix}, \quad (15)$$

式中, ε_T 为拟序扰动参数, 其对应的量代表拟序扰动量.

1.4.1 基本量与弯曲修正量分析

1) ψ^0 项求解

河岸边界条件

$$n = \pm 1; u_{s\psi 0}(\pm 1, z) = 0;$$

水面条件

$$z = \tilde{\eta}, \left. \frac{\partial \tilde{u}_{s0}}{\partial \tilde{z}} \right|_{\tilde{\eta}} = 0;$$

河底条件

$$z = -\tilde{h}, u_{s0}(-\tilde{h}) = 0.$$

在大深宽比情况下

$$\delta = \frac{H}{b} \gg 1, \frac{1}{\delta} \ll 1, \frac{1}{\delta^2} \ll 1,$$

转化为二维问题,最终得到

$$\tilde{p}_{\psi 0} - \tilde{p}_a = -\frac{1}{Fr^2}(-\tilde{\eta} + \tilde{z}), \quad (16)$$

$$\tilde{u}_{s\psi 0} = \frac{1}{2} \frac{Re}{Fr^2} \left[\frac{\partial \tilde{\eta}}{\partial \tilde{s}} - \frac{J}{\delta} \right] (\tilde{n}^2 - 1), \quad (17)$$

式中, \tilde{p}_a 为标准大气压.

2) ψ^1 项,即弯曲修正量

由边界条件 $\hat{u}_{s\psi 1}(\pm 1) = 0, \hat{u}_{n\psi 1}(\pm 1) = 0$ 得

$$\hat{u}_{s\psi 1} = -\frac{1}{6} \frac{Re}{Fr^2} \left(\frac{2 \cdot \partial \tilde{\eta}}{\partial \tilde{s}} - \frac{J}{\delta} \right) C(s) \cdot (\tilde{n} - \tilde{n}^3). \quad (18)$$

1.4.2 拟序扰动量稳定特征

由于河道的弯曲,河湾水流结构既受水动力自身不稳定的影响,同时也受河湾弯曲边界的影响.河道弯曲形状及形状的波动特征,可进一步诱发流动的不稳定性特征.考虑大深宽比情况, $\tilde{u}_{zT} = 0$, 拟序扰动方程变为二维情况,则

扰动量函数的形式为

$$\begin{bmatrix} \tilde{u}_{sT} \\ \tilde{u}_{nT} \\ \tilde{p}_T \end{bmatrix} = \begin{bmatrix} \hat{u}_{sT}(\tilde{n}) \\ \hat{u}_{nT}(\tilde{n}) \\ \hat{p}_T(\tilde{n}) \end{bmatrix} e^{i[\alpha \cdot \tilde{s} - \omega \tau]}. \quad (19)$$

将式(19)代入式(8)~(11),得到扰动项满足的方程:

$$F_0(\psi^0) + F_1(\psi^1) + F_2(\psi^2) = 0. \quad (20)$$

采用时间不稳定模式, α 是扰动波的波数, $\omega = \alpha \cdot c$ (c 代表波速) 是扰动波的频率(复数),其虚部代表由 (α, c) 表达的扰动量的增长率或衰减率.方程(20)的特征值满足

$$F\left(\tilde{\alpha}, Re, \omega, \tilde{R}, \psi, \frac{J}{\delta}\right) = 0. \quad (21)$$

方程(21)代表了弯曲河道流动性特征的 Orr-Sommerfeld 方程.该方程显示出,由于河流的弯曲,有附加参数如河道曲率半径 \tilde{R} 、弯曲系数 ψ 等进入了水流稳定性判别方程表达式,从而影响弯曲水流稳定性特征.

1.5 床面响应方程求解

由方程(8)~(14),得床面变形方程:

$$\frac{\partial \tilde{z}_b}{\partial \tilde{t}} = -\frac{Q_0}{1 + \psi \tilde{N}} \frac{\partial \tilde{q}_s}{\partial \tilde{s}} - Q_0 \frac{\partial \tilde{q}_s}{\partial \tilde{n}} - \frac{Q_0 \psi}{\tilde{R}(1 + \psi \tilde{N})} \cdot \tilde{q}_n. \quad (22)$$

积分最终化简,得

$$\begin{aligned} \hat{z}_b = & \frac{Q_0}{1 + \psi \tilde{N}} \frac{\varepsilon_T \alpha}{\omega} M U^{M-1} \hat{u}_{sT} + \left[Q_0 \varepsilon_T (M-1) U^{M-2} \frac{\partial U}{\partial \tilde{n}} \left(\frac{1}{i\omega} \right) + \right. \\ & \left. \frac{Q_0 \psi \varepsilon_T U^{M-1}}{\tilde{R}(1 + \psi \tilde{N})} \left(\frac{1}{i\omega} \right) \right] \hat{u}_{nT} + Q_0 \varepsilon_T U^{M-1} \left(\frac{1}{i\omega} \right) \frac{\partial \hat{u}_{nT}}{\partial \tilde{n}} - \end{aligned}$$

$$\left[Q_0 \frac{\beta \left(\frac{1}{i\omega} \right)}{\gamma} M U^{M-1} \frac{\partial U}{\partial \tilde{n}} + \frac{Q_0 \psi}{\tilde{R}(1 + \psi \tilde{N})} \frac{\beta \left(\frac{1}{i\omega} \right)}{\gamma} U^M \right] \frac{\partial \hat{\tilde{z}}_b}{\partial \tilde{n}} - \frac{Q_0 \psi}{\tilde{R}(1 + \psi \tilde{N})} \frac{\beta}{\gamma} U^M \left(\frac{1}{i\omega} \right) \frac{\partial^2 \hat{\tilde{z}}_b}{\partial \tilde{n}^2}, \tag{23}$$

$$\tilde{z}_b = \hat{\tilde{z}}_b \cdot \exp^{i(\alpha_T \tilde{s} - \omega_T \tilde{t})}, \tag{24}$$

$$Q_0 = \frac{\sqrt{R_q g D_s D_s}}{(1 - p') U}, \tag{25}$$

其中, α 为扰动波的波数, ω 为扰动波的频率, ε_T 为内部扰动参数, ε_T 为扰动函数振幅与平均流速的比值 ($\varepsilon_T =$ 扰动函数振幅 / U)。

2 计算结果及讨论

2.1 流场计算

不考虑流速沿水深方向的分布, 因此流速在水深方向上近似为一恒定值, 图 2 为基本流流速沿横向的分布图。图 2 中基本流 \tilde{U}_s 在 n 方向呈抛物线分布 (已进行归一化处理), 且与 s 无关。在 $n = 0$ 处, 流速达到最大值。在 $n = \pm 1$ 处, 流速为 0, 符合无滑移边界条件规律。

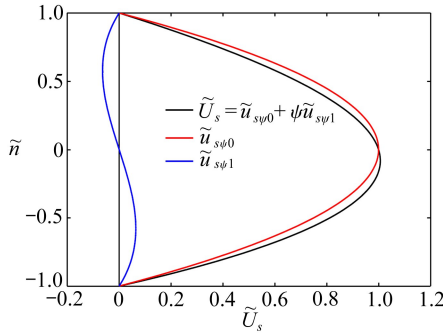
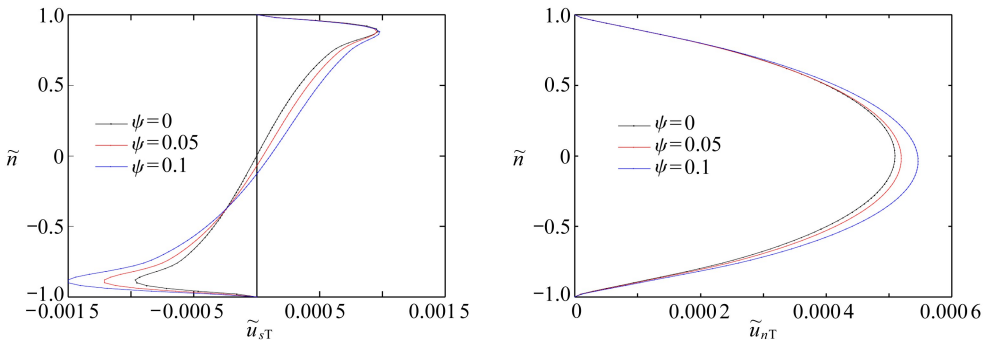


图 2 基本流及二次流沿横向分布图

Fig. 2 The transverse distributions of the basic flow and secondary flow velocities



(a) 纵向扰动函数

(b) 横向扰动函数

(a) The streamwise disturbing function

(b) The transverse disturbing function

图 3 扰动函数形状函数分布图

Fig. 3 Distributions of the disturbing function shapes

在此基础上引入横向和流向的小扰动波,并探讨其对大深宽比河流河床形态响应.图 3 为不同弯曲度下紊流拟序扰动流速形状函数分布图,从图中看出,当弯曲度接近 0 ($\psi = 0$),即顺直河道的情况,其纵向扰动流速形状函数关于河道轴线呈反对称分布;而横向扰动流速形状函数则呈对称分布.随着弯曲程度的增大 ($\psi = 0, 0.05, 0.1$),其纵向扰动流速形状函数发生偏斜,凹岸一侧较凸岸增大明显;横向扰动流速增大,基本呈对称分布.

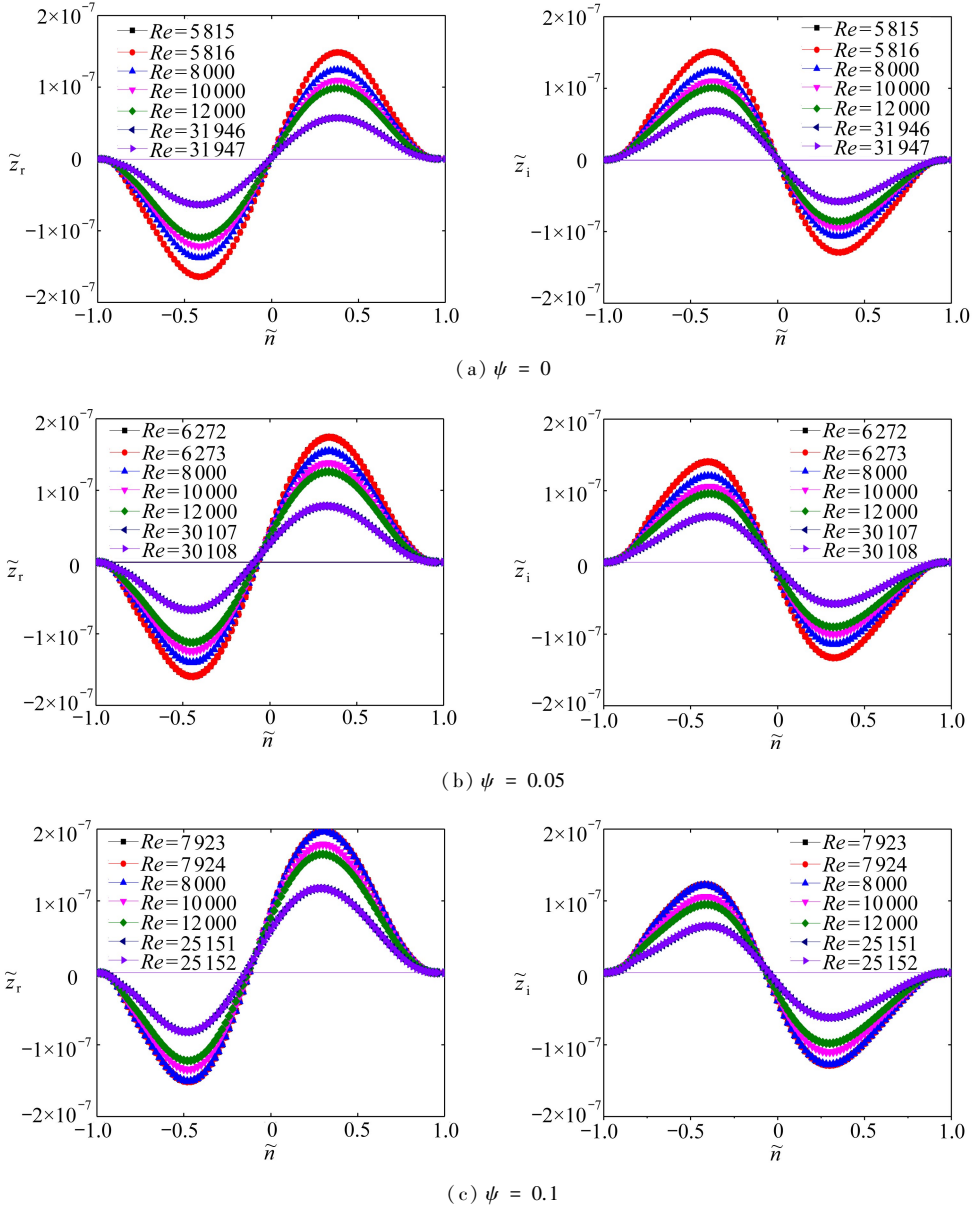


图 4 河流横向床面高程分布图 (z_r 为实部, z_i 为虚部)

Fig. 4 The transverse distributions of the bed elevation in the channel (z_r for real part, z_i for imaginary part)

2.2 床面形态响应计算

图 4 为不同弯曲度和 Reynolds 数情况下,沿横向的床面形态响应形状函数,即式 (23), z_i = $z_r + iz_i$, z_r 表示形状函数的实部, z_i 表示形状函数的虚部.从图中看出,床面响应的床面高程

近似呈正弦形式分布,且凸岸为正,凹岸为负;在一定范围内,横向高程的幅值随 Reynolds 数的增大而减小,随着弯曲度的增大而增大.这是由于随着河流弯曲度的增大,河流横向水力梯度增大,水流对凹岸的冲刷和凸岸的淤积均变强烈,所以床断面的高程会随着弯曲度的增大而增大.

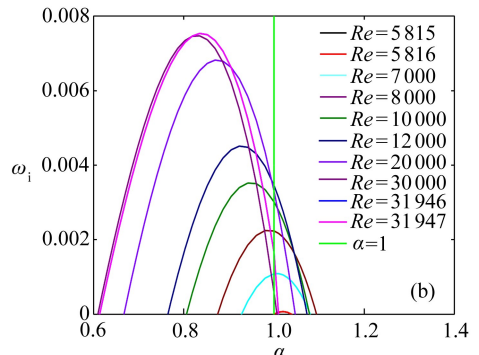
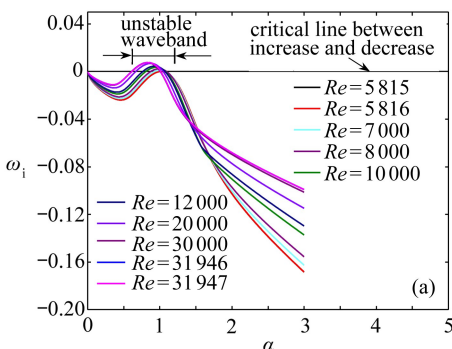
Pittaluga, Seminara 等^[4]的研究表明,在某一弯曲扰动波数下,增长率存在最大值,蜿蜒河流中存在典型非线性扰动波,其被选波数具有依赖于初始扰动振幅的特点.受此结果启发,为了探究蜿蜒河流河床响应的不稳定特征,选取围绕特征波数 $\alpha = 1$ 和 $\alpha = 1.05$ 的水流不稳定波段进行研究,即 $\omega_i > 0$,本文计算了围绕特征波数为 $\alpha = 1$ 和 $\alpha = 1.05$ 拟序扰动波周围的可能响应情况(见图 5~8).在不同 Reynolds 数围绕特征波数为 $\alpha = 1$ 的拟序扰动波群,随弯曲度的不同($\psi = 0, 0.05, 0.1$),其波数-增长率特征谱见图 5(a)、6(a)和 7(a);围绕特征波数为 $\alpha = 1.05$ 的拟序扰动波群,在弯曲度 $\psi = 0$ 情况下其波数-增长率特征谱见图 8(a).相应于以上特征谱,紊流拟序扰动对床面产生影响,图 5(b)、6(b)、7(b)和 8(b)为可以增长发展的扰动量,其对应于图 5(a)、6(a)、7(a)和 8(a)中的不稳定波段,具体扰动频率如表 1 所示.在表 1 中,考虑不同的弯曲度($\psi = 0, 0.05, 0.1$),对比其中 $Re = 8\ 000$, $Re = 10\ 000$ 和 $Re = 12\ 000$ 的扰动频率,可以得出随弯曲度增大,扰动频率呈减小趋势.从表 1 中还可以看出,特征谱中扰动量频率 ω 的虚部 ω_i 存在最大值,即存在增长率最大的扰动结构.

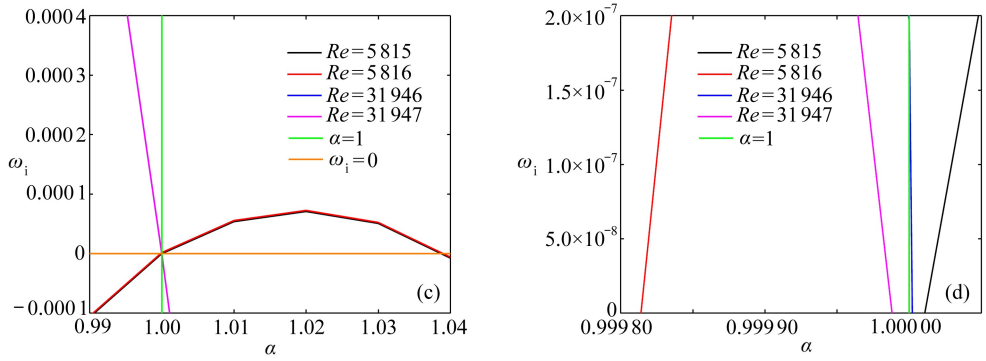
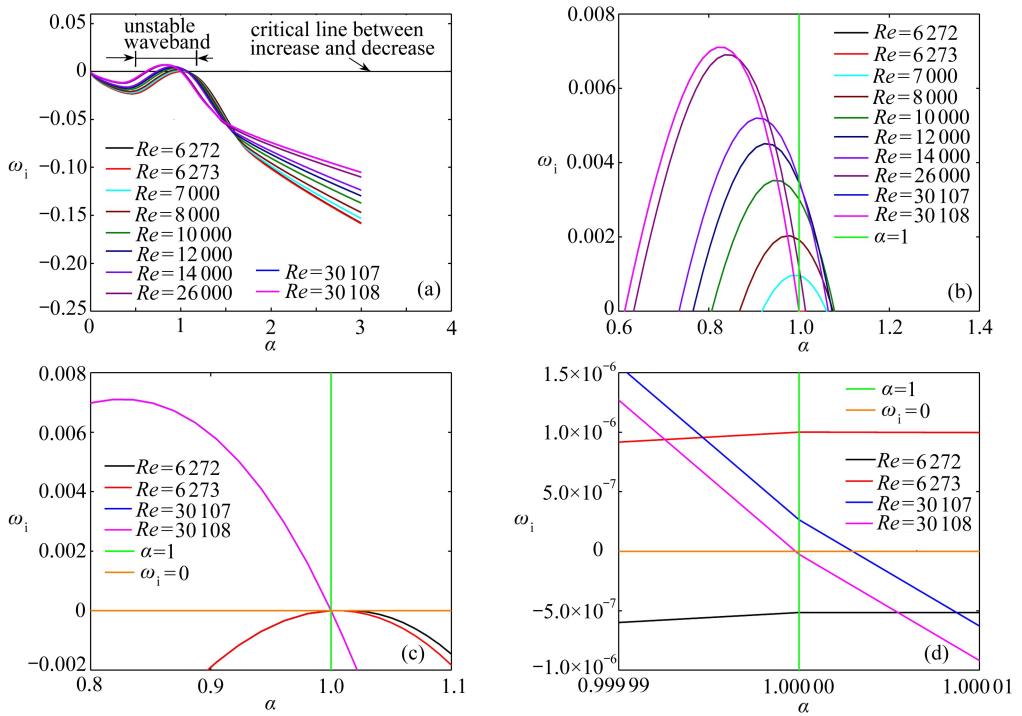
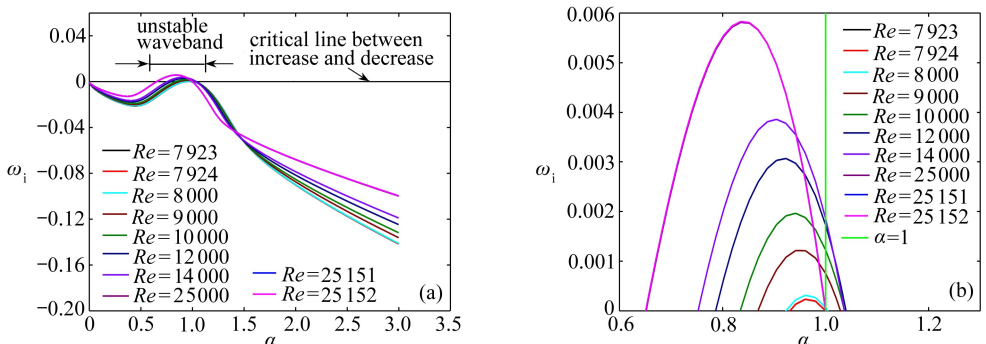
表 1 不同弯曲度下的扰动频率

Table 1 Perturbation frequencies for different curvatures

$\psi = 0$		$\psi = 0.05$		$\psi = 0.1$	
Reynolds	perturbation frequency ω	Reynolds	perturbation frequency ω	Reynolds	perturbation frequency ω
$Re = 5\ 815$	(0.261, -0.000 000 057)	$Re = 6\ 272$	(0.256, -0.000 000 517)	$Re = 7\ 923$	(0.242, -0.000 000 673)
$Re = 5\ 816$	(0.261, 0.000 001 755)	$Re = 6\ 273$	(0.256, 0.000 001 001)	$Re = 7\ 924$	(0.242, 0.000 000 173)
$Re = 6\ 000$	(0.260, 0.000 322 7)	$Re = 8\ 000$	(0.246, 0.001 921 239)	$Re = 8\ 000$	(0.242, 0.000 063 58)
$Re = 8\ 000$	(0.247, 0.002 663 802)	$Re = 10\ 000$	(0.236, 0.003 014 2)	$Re = 10\ 000$	(0.232, 0.001 223 97)
$Re = 10\ 000$	(0.238, 0.003 738 852)	$Re = 12\ 000$	(0.229, 0.003 456 5)	$Re = 12\ 000$	(0.225, 0.001 729 216)
$Re = 12\ 000$	(0.230, 0.004 162 772)	$Re = 25\ 000$	(0.200, 0.001 433 844)	$Re = 20\ 000$	(0.205, 0.001 16)
$Re = 20\ 000$	(0.210, 0.003 304 555)	$Re = 30\ 107$	(0.193, 0.000 000 264)	$Re = 25\ 000$	(0.196, 0.000 037 6)
$Re = 28\ 000$	(0.197, 0.001 168 97)	$Re = 30\ 108$	(0.193, -0.000 000 25)	$Re = 25\ 151$	(0.196, 0.000 000 104)
$Re = 31\ 946$	(0.192, 0.000 000 199)			$Re = 25\ 152$	(0.196, -0.000 000 15)
$Re = 31\ 947$	(0.192, -0.000 000 103)				

从图 5~8 可以看出,随着弯曲度的增加,弯道或河湾内拟序扰动波增长率明显小于直道,且随着 Reynolds 数的增加,拟序扰动波的增长率逐渐增大.这与 Reynolds 数越大紊动越强烈,水流结构越不稳定相一致,且增长率较大的扰动波数响应范围朝着波数较小的方向移动.



图 5 扰动波数-增长率特征谱图 ($\psi = 0$)Fig. 5 The perturbation wave number-growth rates ($\psi = 0$)图 6 扰动波数-增长率特征谱图 ($\psi = 0.05$)Fig. 6 The perturbation wave number-growth rates ($\psi = 0.05$)

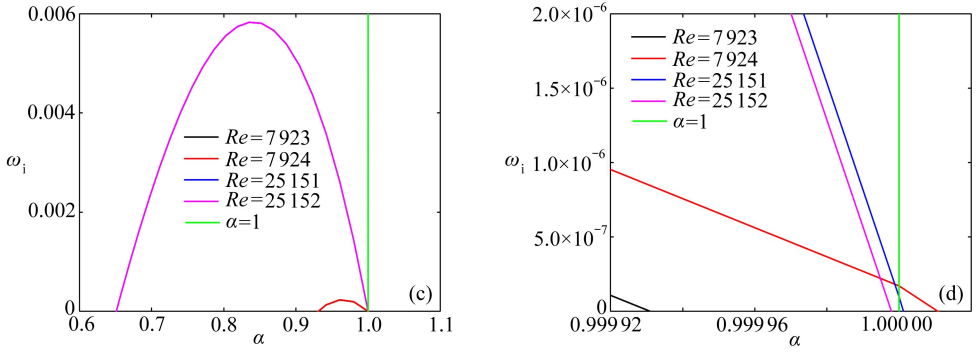


图 7 扰动波数-增长率特征谱图 ($\psi = 0.1$)

Fig. 7 The perturbation wave number-growth rates ($\psi = 0.1$)

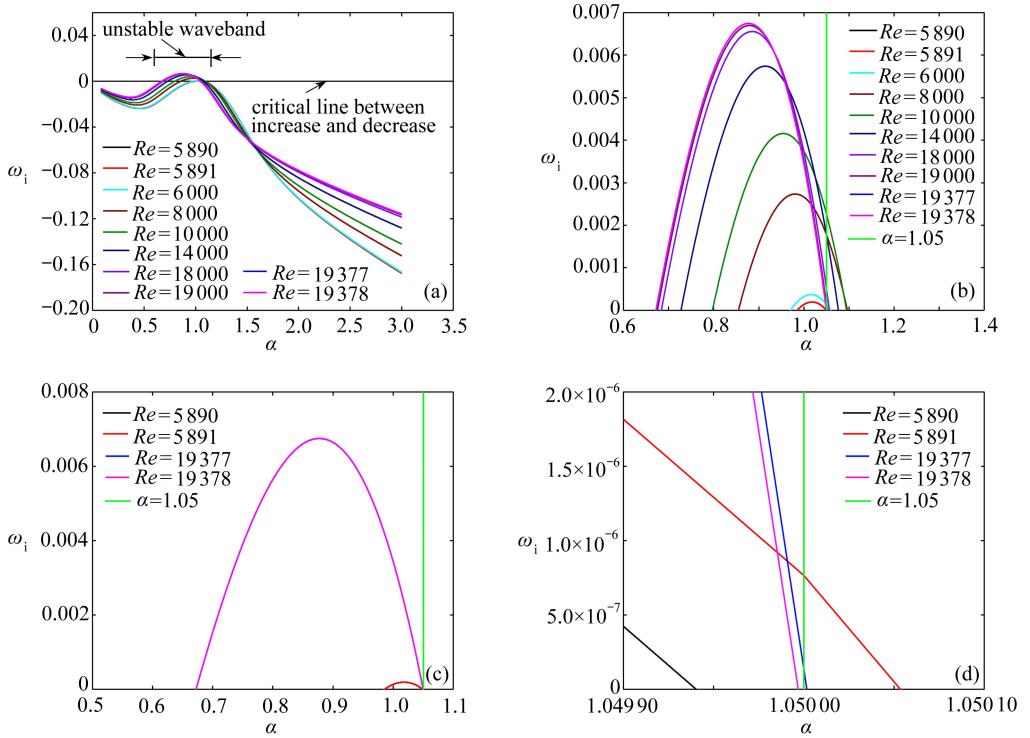
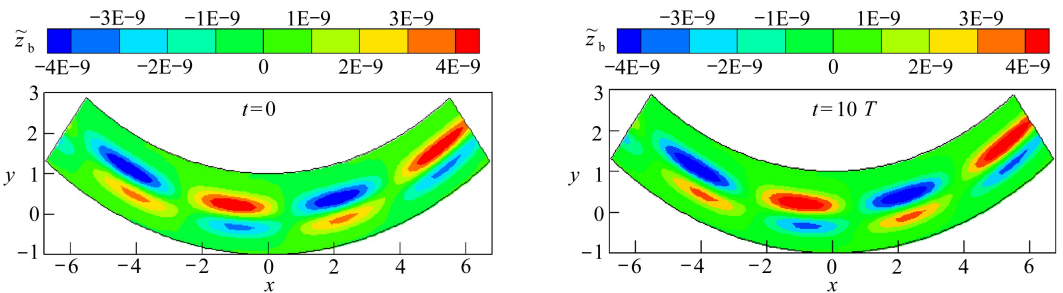
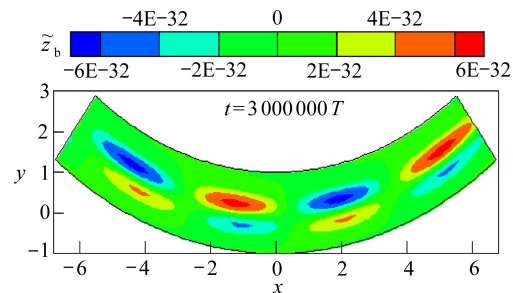
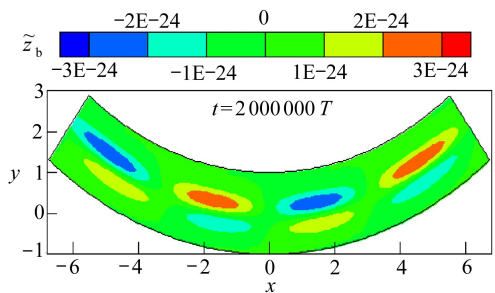
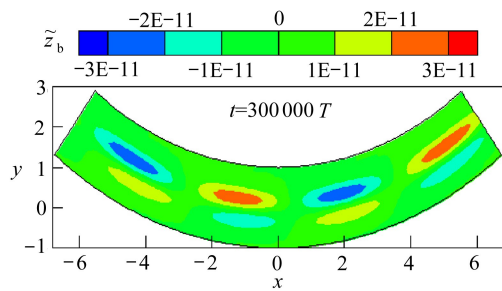
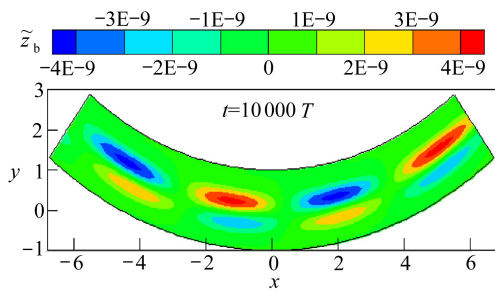
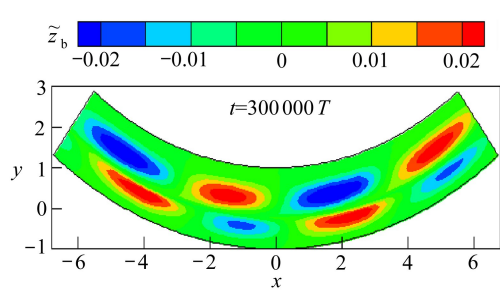
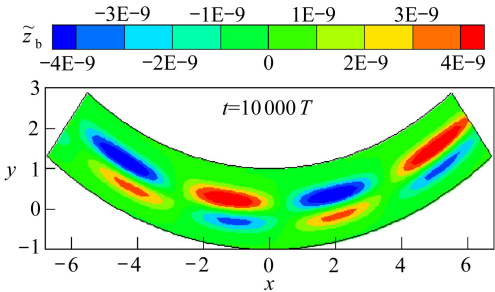
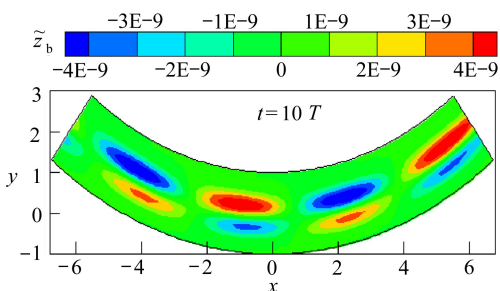
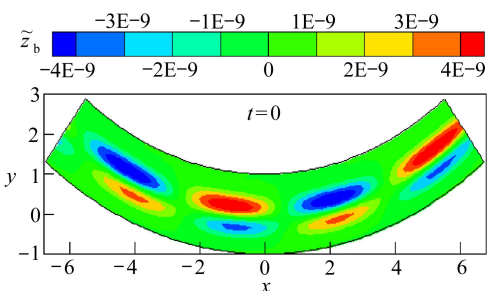
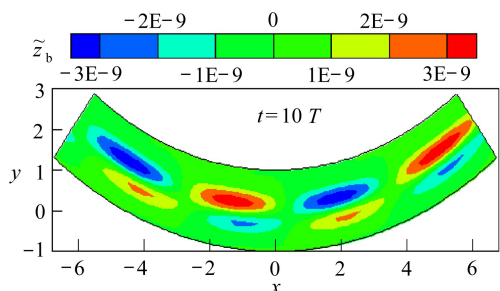
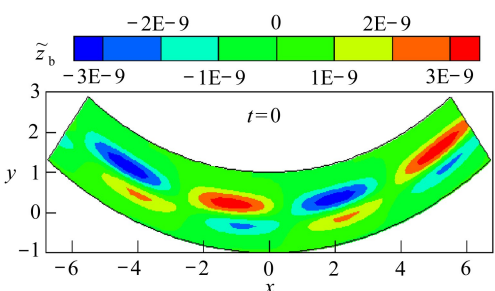
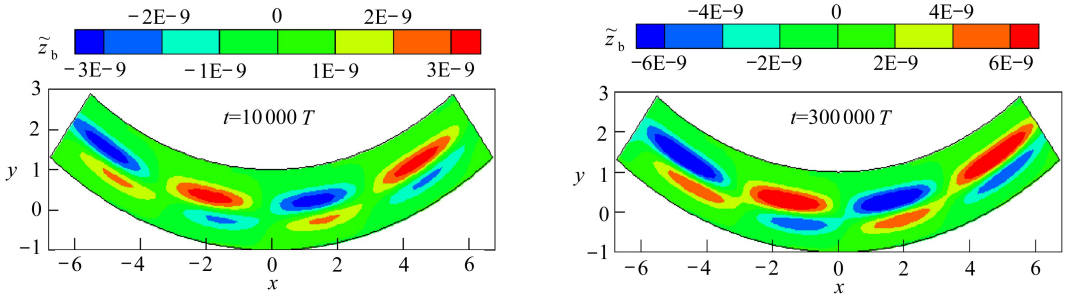
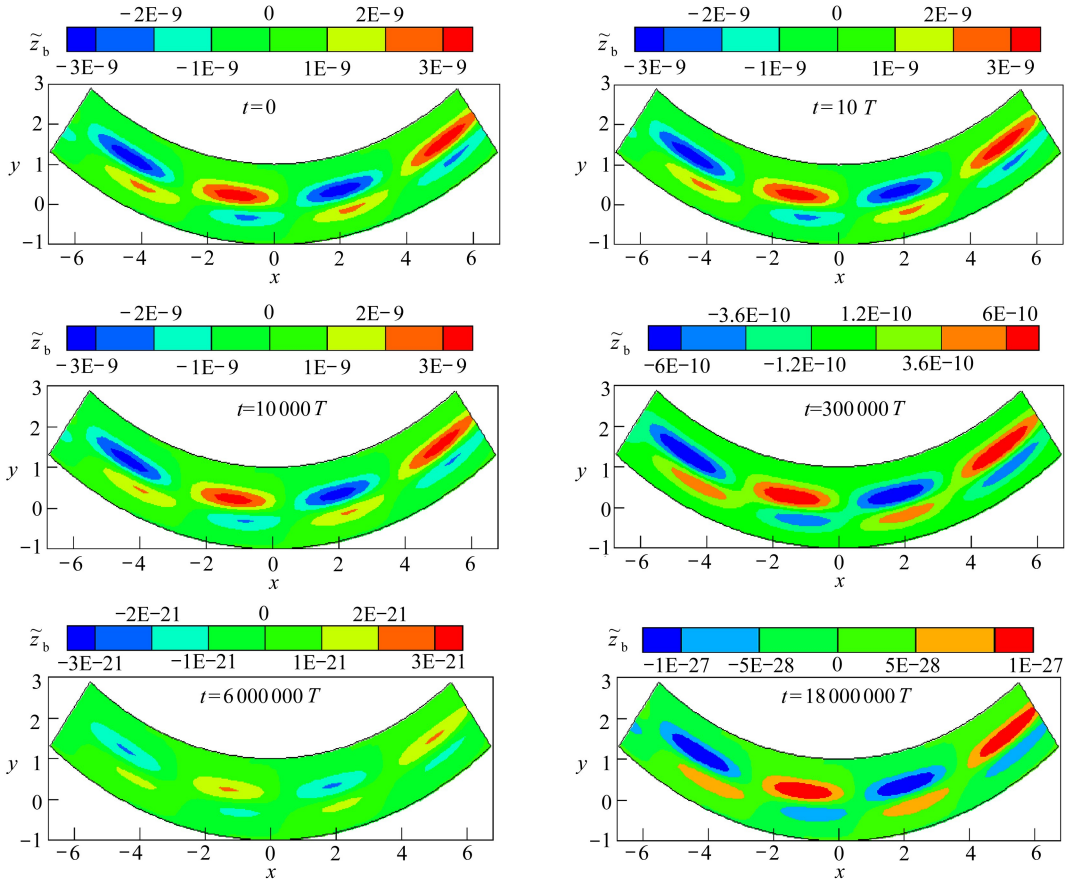


图 8 扰动波数-增长率特征谱图 ($\psi = 0$)

Fig. 8 The perturbation wave number-growth rates ($\psi = 0$)



(a) 衰减 ($Re = 7923$)(a) Reduction ($Re = 7923$)(b) 增长 ($Re = 7924$)(b) Increase ($Re = 7924$)

(c) 增长 ($Re = 25\ 151$)(c) Increase ($Re = 25\ 151$)(d) 衰减 ($Re = 25\ 152$)(d) Reduction ($Re = 25\ 152$)图 9 两个波长范围内床面响应随时间变化图 ($\psi = 0.1$)Fig. 9 Variation diagrams of the bed response vs. time within 2 wavelengths ($\psi = 0.1$)

通过寻找模态和分析扰动波数-增长率特征谱,得到不同 Reynolds 数的不稳定范围,如图 5(b)、6(b)、7(b)和 8(b).为了更好地探究床面响应情况,取图 5(b)、6(b)、7(b)、8(b)中的一部分放大,得到图 5(c)、6(c)、7(c)、8(c)和图 5(d)、6(d)、7(d)、8(d),且其中 $\alpha = 1$ 和 $\alpha = 1.05$ 为特征波数, $\omega_i = 0$ 为增长率为 0 的基本线.与图 7(d)相对应的两个波长范围内床面响应随时间变化情况如图 9 所示,其中, T 为行走一个波长所用的时间.图 9(a)、(d)为床面衰减情

况,图 9(b)、(c)为床面增长情况。从图 9 的床面响应随时间变化图,并结合相应的图例可以看出,随着时间的增加,床面形态开始变化,对于 $Re = 7\ 923$ 和 $Re = 25\ 152$ 的情况,床面响应衰减,随着时间的增加最终消失;对于 $Re = 7\ 924$ 和 $Re = 25\ 151$ 的情况,在一定的范围内持续增长。图 9(a)、(b),即 $Re = 7\ 923$ 和 $Re = 7\ 924$, 对应于床面响应的衰减和增长,说明这种方法能够精确判定床面响应增长与否。这就为河流床面形态的响应判别提供了一种新方法。

根据图 5~8 的结果,进一步计算了特征波数分别为 $\alpha = 1$ 和 $\alpha = 1.05$, 弯曲度分别为 $\psi = 0, 0.05, 0.1$ 的床面形态。

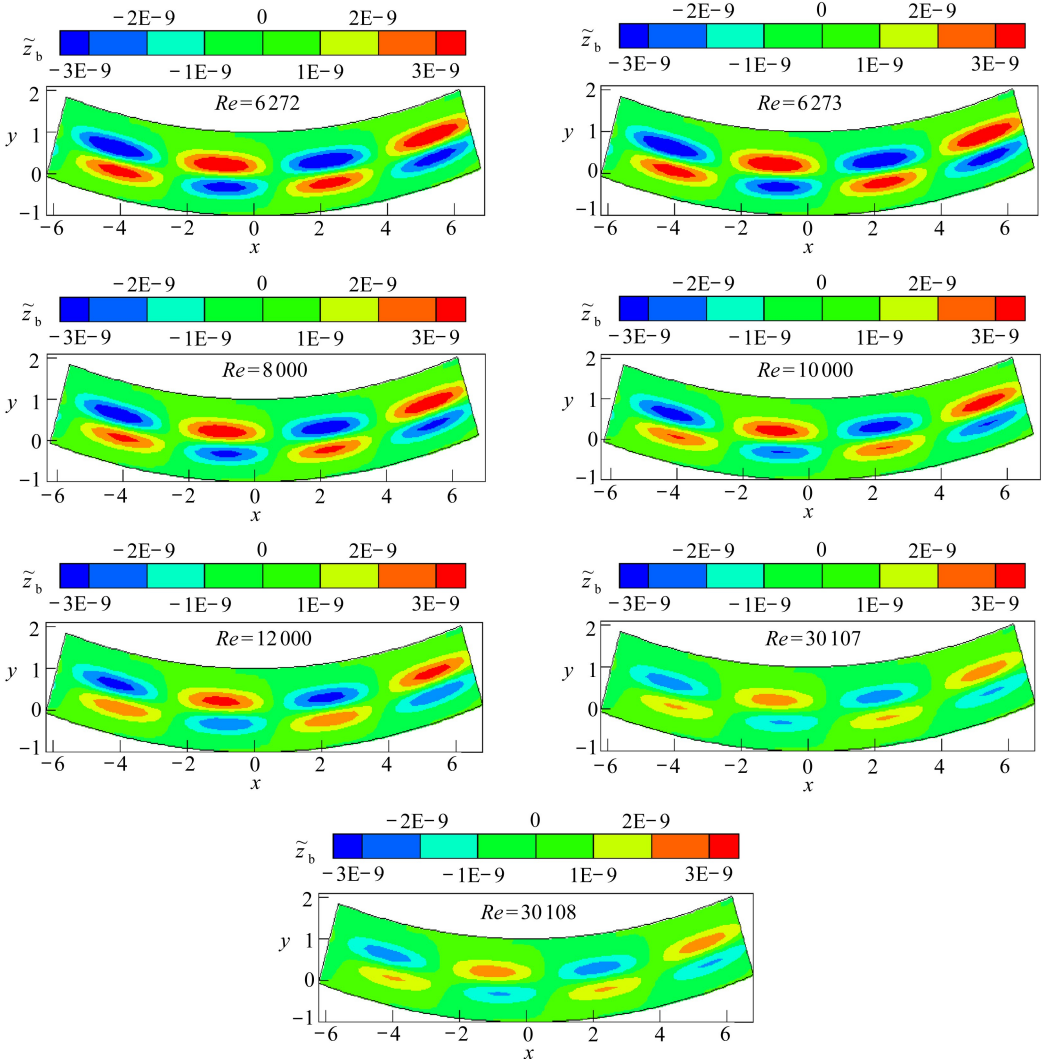


图 10 两个波长范围床面高程的形状函数云图 ($\psi = 0.05$)

Fig. 10 Nephograms of the bed elevation shape functions within 2 wavelengths ($\psi = 0.05$)

以特征波数 $a = 1$ 和弯曲度 $\psi = 0.05, 0.1$ 为例分析床面响应的形状函数特性。图 10、图 11 分别为两个波长范围内微弯河道床面高程形状函数的云图。从图中可以看出,随着 Reynolds 数的增大,床面高程的幅值逐渐减小,这是由于随着 Reynolds 数的增大,拟序扰动不稳定性增加,很难形成大的紊流涡体结构,从而导致床面形态响应的幅值减小。同时,床面响应幅值形状

函数表现为关于河道中心线反对称分布的波状形态,随着弯曲度的增大,床面高程的幅值呈增大趋势。

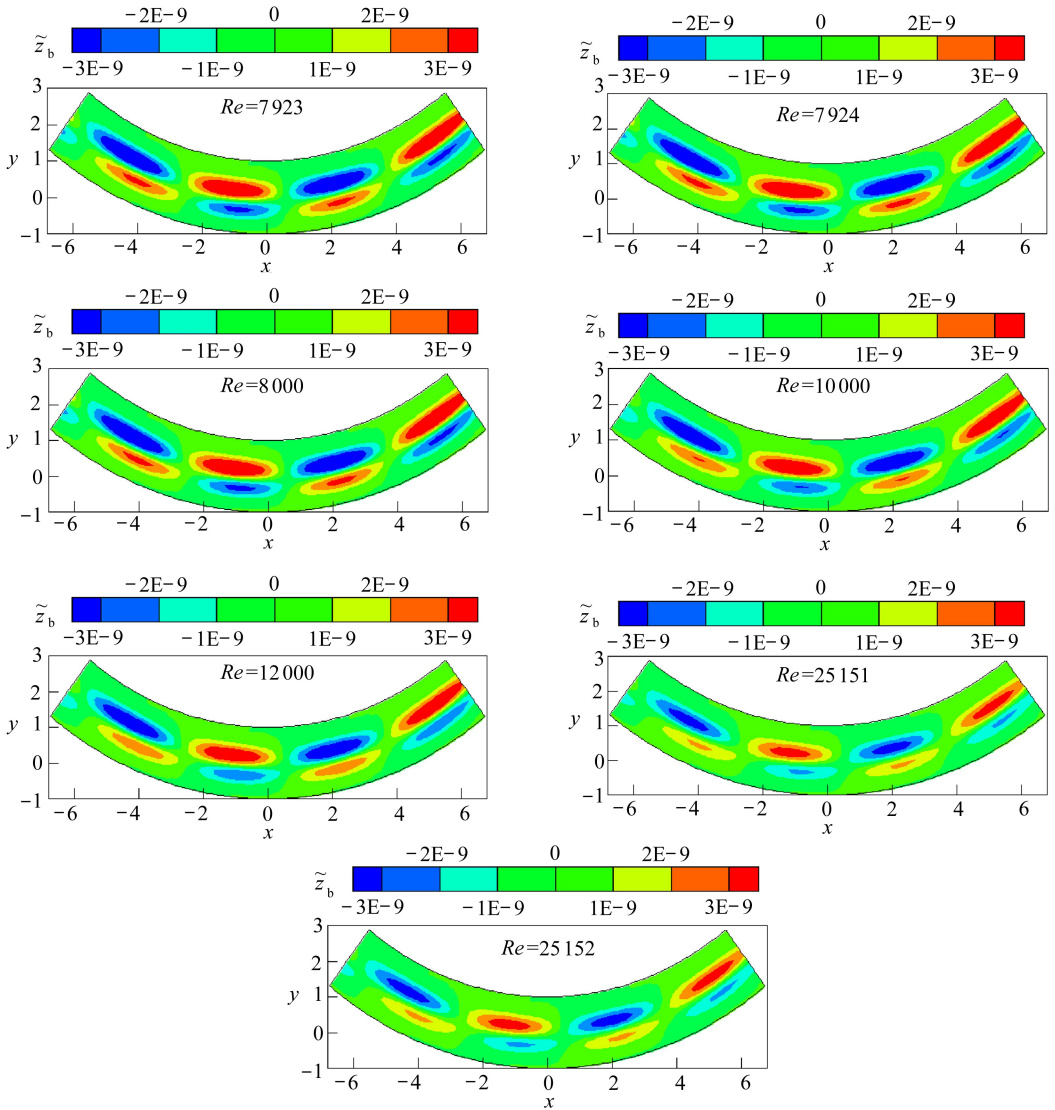


图 11 两个波长范围床面高程的形状函数云图 ($\psi = 0.1$)

Fig. 11 Nephograms of the bed elevation shape functions within 2 wavelengths ($\psi = 0.1$)

3 结 论

本文以常曲率大深宽比河湾为背景,讨论了弯曲河流的床面响应.通过寻找模态和分析扰动波数-增长率特征谱,得到 Reynolds 数与床面响应不稳定范围,确定了一种能够精确判定床面响应增长与否的判别方法:

1) 河道弯曲的出现,影响了扰动波的发展变化.随着河道弯曲度增大,扰动波增长率明显小于顺直河道,这与一定情况下的天然河流总是向一定弯曲度形态发展,并趋于相对稳定性的现象相一致。

2) 不同弯曲度和 Reynolds 数情况下,沿横向的床面形态响应形状函数呈正弦形式分布,且凸岸为正,凹岸为负;随着 Reynolds 数的增大,拟序扰动不稳定性增加,很难形成大的紊流涡体结构,从而导致床面形态响应的幅值减小,床面形态向稳定方向发展。

3) 床面响应幅值形状函数表现为关于河道中心线反对称分布。随着弯曲度的增大,床面高程的幅值呈增大趋势;随 Reynolds 数的增大而呈减小趋势。在弯曲度相同的情况下,Reynolds 数的增加会使扰动频率的虚部变大,Reynolds 数越大紊动越强烈,水流越不稳定;弯道内横向泥沙输移也对水流稳定和河床形态发展变化产生一定的影响。

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Coherent Disturbance Structures and Bed Topography Responses in Large Depth-to-Width Ratio River Bends With Constant Curvatures

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Abstract: The bed topography is the result of the dynamic response of a complex meandering river system, and is an important factor influencing the further river development. Based on meandering rivers characterized by large depth-to-width ratios, the relation between the hydraulic structure and the bed topography was explored. The flow characteristics and bed topography responses were discussed through coupling of the N-S equations, the sediment transport equations as well as the bed deformation equations, and with the perturbation method. Research results show that shallows and deep grooves present regular responses under the effects of 2D flow disturbances. For a zero curvature, the bed topography shows an anti-symmetric distribution about the channel centerline; while for a non-zero curvature, the channel centerline deviates toward the concave bank. Finally, the criteria for the judgement on the stability of the bed topography influenced by the Reynolds number, the disturbance wave number and the decay ratio of the bed topography, are given.

Key words: meandering river; large depth-to-width ratio river bend; coherent disturbance wave; dynamic bed topography response

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