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统计能量分析中参数不确定性分析*

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摘要: 统计能量分析方法是计算结构高频振动噪声的有效方法之一,内损耗因子和耦合损耗因子是其中重要的参数但不易测量,测量误差通常比较大,导致计算得到的子系统振动能量和真实值之间存在偏差.为解决上述问题,该文采用了 4 种不同的区间分析方法:区间矩阵摄动法、基于区间变量特性法、仿射算法和仿射逆矩阵法,从理论上计算了统计能量分析子系统的振动能量区间,该区间结果充分考虑了内损耗因子和耦合损耗因子的测量误差对计算结果的影响,对传统的统计能量分析理论进行了完善.然后,通过算例比较了每种方法所求子系统总能量区间的优劣.

关 键 词: 统计能量分析; 测量误差; 损耗因子; 区间分析方法

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引 言

统计能量分析(SEA),Lyon 提出其能较好地处理复杂系统在高频段的耦合动力学问题[1-4].在稳态统计能量分析中,一个复杂系统可分为若干耦合的子系统,它们输入的功率等于阻尼损耗功率和传输功率之和.由自身阻尼造成的功率损失与内损耗因子和响应能量乘积相关,子系统相互间的传递功率与耦合损耗因子和响应能量乘积相关,而最终产生的子系统总能量是关于输入功率和损耗因子的函数.

利用计算机技术,可以求解复杂的功率流平衡方程以获得每个子系统的总能量.但在正常情况下,内损耗因子与耦合损耗因子这两个参数是非常小的量,想要准确测得十分困难,并且测量值通常存在误差,那么经过计算得到的子系统总能量的结果就会和真实值产生一定量的偏差,而这个偏差的大小与结构的可靠性问题密切相关.本文尝试用区间分析方法来求解这个偏差大小.

近几十年来涌现了一些不错的区间分析方法.Qiu 等所给出的区间矩阵摄动法与子区间摄动法,在参数小变化的假设下,将此方法用在结构特征值[5]和静力位移[6-7]的求解上.郭书祥等[8]从线性方程组的分量形式入手,将区间变量的不确定性用离差与均值来表征,提出了基于区间变量特性的解法.Comba 等[9]将传统区间算法进行综合和改进后提出了仿射算法,该算法的优点在于能够绕开区间扩张问题.随着仿射算法更深入的研究,Manson[10]将其用在不确定结构的分析上.Degrauwe 等[11]给出了仿射逆矩阵的求解公式.Staudt 等[12]利用仿射逆矩阵

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对相位稳定性问题进行了探讨.

本文采用4种不同的区间分析方法:区间矩阵摄动法、基于区间变量特性法、仿射基本算法以及仿射逆矩阵法,计算了统计能量分析子系统的振动能量区间,然后通过算例比较了每种方法所求子系统振动能量区间的优劣.

1 统计能量分析基本原理

对简单振子系统来说,其有下列关系式:

$$P_{\mathrm{d}} = C\dot{x}^2 = 2\zeta\omega_n M\dot{x}^2 = 2\zeta\omega_n E = \omega_n \eta E, \tag{1}$$

其中 P_{d} 是损耗功率, C 和 E 是振子系统的阻尼系数和能量, $\dot{x}$ 和 M 是振子的速度和质量, η 和 ω_n 是振子系统的内损耗因子以及振子固有频率, ζ 是阻尼比.

对于统计能量分析中某个子系统 i, 其内损耗功率参照简单振子可写为

$$P_{i\mathrm{d}} = \omega\eta_i E_i, \tag{2}$$

其中 ω 是所计算带宽 $\Delta\omega$ 的中心频率, η_i, E_i 是子系统 i 的内损耗因子以及能量.

记 $P'_{ij}(P'_{ji})$ 是由子系统 $i(j)$ 传给 $j(i)$ 的单向功率流,则保守耦合情况下,类似式(2)有

$$P'_{ij} = \omega\eta_{ij}E_i, \ P'_{ji} = \omega\eta_{ji}E_j, \tag{3}$$

其中 $\eta_{ij}(\eta_{ij})$ 是子系统 $i(j)$ 向子系统 $j(i)$ 传递能量时的耦合损耗因子.因此,从子系统 i 向子系统 j 所传递的纯功率流为

$$P_{ij} = P'_{ij} - P'_{ji} = \omega\eta_{ij}E_i - \omega\eta_{ji}E_j. \tag{4}$$

假定复杂系统由 N 个子系统组成,由运动方程得子系统 i 的功率流平衡式为

$$P_{i,\mathrm{in}} = \dot{E}_i + P_{i\mathrm{d}} + \sum_{j=1,\, j\neq i}^{N} P_{ij},$$

式中 $P_{i,\mathrm{in}}$ 是外界向子系统 i 的输入功率.当为稳态振动时, $\dot{E}_i = 0$.当各子系统的激励互不相关以及保守耦合时可得

$$P_{i,\mathrm{in}} = \omega\eta_i E_i + \sum_{j=1,\, j\neq i}^{N} (\omega\eta_{ij}E_i - \omega\eta_{ji}E_j). \tag{5}$$

2 采用4种不同区间方法计算子系统的振动能量区间

2.1 基于区间矩阵摄动法计算子系统的振动能量区间

由式(5)可得,两个子系统在稳态下的功率流平衡方程为

$$\begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \boldsymbol{\eta}\boldsymbol{E} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \boldsymbol{P}, \tag{6}$$

其中 $P_1 = p_1/\omega$, $P_2 = p_2/\omega$, p_1 和 p_2 均为实际输入功率, ω 为中心频率.当损耗因子矩阵 $\boldsymbol{\eta}$ 和输入功率向量 $\boldsymbol{P}$ 分别具有摄动量 $\delta\boldsymbol{\eta}$ 和 $\delta\boldsymbol{P}$ 时,由矩阵摄动理论知摄动平衡方程为

$$(\boldsymbol{\eta} + \delta\boldsymbol{\eta})(\boldsymbol{E} + \delta\boldsymbol{E}) = (\boldsymbol{P} + \delta\boldsymbol{P}), \tag{7}$$

$$\boldsymbol{E} + \delta\boldsymbol{E} = (\boldsymbol{\eta} + \delta\boldsymbol{\eta})^{-1}(\boldsymbol{P} + \delta\boldsymbol{P}). \tag{8}$$

将带有测量误差的内损耗因子和输入功率用区间变量表示如下:

$$\begin{cases} \eta_1^{\mathrm{I}} = [\eta_1^{\mathrm{l}}, \eta_1^{\mathrm{u}}], \ \eta_2^{\mathrm{I}} = [\eta_2^{\mathrm{l}}, \eta_2^{\mathrm{u}}], \ \eta_{12}^{\mathrm{I}} = [\eta_{12}^{\mathrm{l}}, \eta_{12}^{\mathrm{u}}], \\ \eta_{21}^{\mathrm{I}} = [\eta_{21}^{\mathrm{l}}, \eta_{21}^{\mathrm{u}}], \ P_1^{\mathrm{I}} = [P_1^{\mathrm{l}}, P_1^{\mathrm{u}}], \ P_2^{\mathrm{I}} = [P_2^{\mathrm{l}}, P_2^{\mathrm{u}}], \end{cases} \tag{9}$$

其中上标“I”代表参数的一个区间,“l”代表区间下限,“u”代表区间上限.方程(9)中的内损耗因子区间和输入功率区间已经将测量误差充分地考虑进去.

将式(9)代入到式(6)中,可得区间功率流平衡方程:

$$\begin{bmatrix} \eta_1^{\mathrm{I}}+\eta_{12}^{\mathrm{I}} & -\eta_{21}^{\mathrm{I}} \\ -\eta_{12}^{\mathrm{I}} & \eta_2^{\mathrm{I}}+\eta_{21}^{\mathrm{I}} \end{bmatrix}\begin{bmatrix} E_1^{\mathrm{I}} \\ E_2^{\mathrm{I}} \end{bmatrix}=\begin{bmatrix} P_1^{\mathrm{I}} \\ P_2^{\mathrm{I}} \end{bmatrix}, \quad \text{or} \quad \boldsymbol{\eta}^{\mathrm{I}}\boldsymbol{E}^{\mathrm{I}}=\boldsymbol{P}^{\mathrm{I}}. \tag{10}$$

采用一阶摄动方法[13]求解区间功率流平衡方程(10),可得子系统 1 和子系统 2 的总能量区间为

$$\begin{bmatrix} E_1^{\mathrm{c}} \\ E_2^{\mathrm{c}} \end{bmatrix}=\frac{1}{\eta_1^{\mathrm{c}}\eta_2^{\mathrm{c}}+\eta_1^{\mathrm{c}}\eta_{21}^{\mathrm{c}}+\eta_2^{\mathrm{c}}\eta_{12}^{\mathrm{c}}}\begin{bmatrix} \eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}} & \eta_{21}^{\mathrm{c}} \\ \eta_{12}^{\mathrm{c}} & \eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}} \end{bmatrix}\begin{bmatrix} P_1^{\mathrm{c}} \\ P_2^{\mathrm{c}} \end{bmatrix}=$$
$$\frac{1}{T}\begin{bmatrix} (\eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}})P_1^{\mathrm{c}}+\eta_{21}^{\mathrm{c}}P_2^{\mathrm{c}} \\ \eta_{12}^{\mathrm{c}}P_1^{\mathrm{c}}+(\eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}})P_2^{\mathrm{c}} \end{bmatrix}, \tag{11}$$

$$\begin{bmatrix} \Delta E_1^{\mathrm{I}} \\ \Delta E_2^{\mathrm{I}} \end{bmatrix}=\frac{1}{T}\begin{bmatrix} \eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}} & \eta_{21}^{\mathrm{c}} \\ \eta_{12}^{\mathrm{c}} & \eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}} \end{bmatrix}\left\{\begin{bmatrix} \Delta P_1^{\mathrm{I}} \\ \Delta P_2^{\mathrm{I}} \end{bmatrix}-\begin{bmatrix} \Delta\eta_1^{\mathrm{I}}+\Delta\eta_{12}^{\mathrm{I}} & \Delta\eta_{21}^{\mathrm{I}} \\ \Delta\eta_{12}^{\mathrm{I}} & \Delta\eta_2^{\mathrm{I}}+\Delta\eta_{21}^{\mathrm{I}} \end{bmatrix}\begin{bmatrix} E_1^{\mathrm{c}} \\ E_2^{\mathrm{c}} \end{bmatrix}\right\}=$$
$$\frac{1}{T}\begin{bmatrix} (\eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}})[\Delta P_1^{\mathrm{I}}-(\Delta\eta_1^{\mathrm{I}}+\Delta\eta_{12}^{\mathrm{I}})E_1^{\mathrm{c}}-\Delta\eta_{21}^{\mathrm{I}}E_2^{\mathrm{c}}]+ \\ \eta_{21}^{\mathrm{c}}[\Delta P_2^{\mathrm{I}}-(\Delta\eta_2^{\mathrm{I}}+\Delta\eta_{21}^{\mathrm{I}})E_2^{\mathrm{c}}-\Delta\eta_{12}^{\mathrm{I}}E_1^{\mathrm{c}}] \\ \eta_{12}^{\mathrm{c}}[\Delta P_1^{\mathrm{I}}-(\Delta\eta_1^{\mathrm{I}}+\Delta\eta_{12}^{\mathrm{I}})E_1^{\mathrm{c}}-\Delta\eta_{21}^{\mathrm{I}}E_2^{\mathrm{c}}]+ \\ (\eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}})[\Delta P_2^{\mathrm{I}}-(\Delta\eta_2^{\mathrm{I}}+\Delta\eta_{21}^{\mathrm{I}})E_2^{\mathrm{c}}-\Delta\eta_{12}^{\mathrm{I}}E_1^{\mathrm{c}}] \end{bmatrix}, \tag{12}$$

其中 $T=\eta_1^{\mathrm{c}}\eta_2^{\mathrm{c}}+\eta_1^{\mathrm{c}}\eta_{21}^{\mathrm{c}}+\eta_2^{\mathrm{c}}\eta_{12}^{\mathrm{c}}$, 上标“c”代表区间均值,如 $\eta^{\mathrm{c}}=(\eta^{\mathrm{l}}+\eta^{\mathrm{u}})/2$, “Δ”代表不确定区间,如 $\Delta\eta^{\mathrm{I}}=[(\eta^{\mathrm{l}}-\eta^{\mathrm{u}})/2,(\eta^{\mathrm{u}}-\eta^{\mathrm{l}})/2]$.

由式(11)和(12)可求得两个子系统的振动能量区间,即 $E_1^{\mathrm{I}}=E_1^{\mathrm{c}}+\Delta E_1^{\mathrm{I}}=[E_1^{\mathrm{l}},E_1^{\mathrm{u}}]$ 和 $E_2^{\mathrm{I}}=E_2^{\mathrm{c}}+\Delta E_2^{\mathrm{I}}=[E_2^{\mathrm{l}},E_2^{\mathrm{u}}]$, 所得的振动能量区间 E_1^{I} 和 E_2^{I} 充分考虑了内损耗因子和外部输入功率的测量误差对统计能量分析计算结果的影响;而且根据区间数学的基本原理可知该能量区间包含了振动能量的真实值,在此区间内进行工程设计和分析是安全可靠的,后文中采用不同的区间分析方法所得的振动能量区间具有相同的意义.

2.2 基于区间变量特性法计算子系统振动能量区间

设有区间数 $X^{\mathrm{I}}=[x^{\mathrm{l}},x^{\mathrm{u}}]$, X^{I} 的均值表示为 $X^{\mathrm{c}}=(x^{\mathrm{l}}+x^{\mathrm{u}})/2$, X^{I} 的离差表示为 $X^{\mathrm{r}}=(x^{\mathrm{u}}-x^{\mathrm{l}})/2$, 采用基于区间变量特性法[6]将区间功率流平衡方程(10)写成分量形式:

$$\begin{cases} P_1^*=(\eta_1^*+\eta_{12}^*)E_1^*-\eta_{21}^*E_2^*, \\ P_2^*=-\eta_{12}^*E_1^*+(\eta_2^*+\eta_{21}^*)E_2^*, \end{cases} \tag{13}$$

其中

$$\eta_1^*\in\eta_1^{\mathrm{I}}=[\eta_1^{\mathrm{l}},\eta_1^{\mathrm{u}}],\ \eta_2^*\in\eta_2^{\mathrm{I}}=[\eta_2^{\mathrm{l}},\eta_2^{\mathrm{u}}],\ \eta_{12}^*\in\eta_{12}^{\mathrm{I}}=[\eta_{12}^{\mathrm{l}},\eta_{12}^{\mathrm{u}}],$$
$$\eta_{21}^*\in\eta_{21}^{\mathrm{I}}=[\eta_{21}^{\mathrm{l}},\eta_{21}^{\mathrm{u}}],\ P_1^*\in P_1^{\mathrm{I}}=[P_1^{\mathrm{l}},P_1^{\mathrm{u}}],\ P_2^*\in P_2^{\mathrm{I}}=[P_2^{\mathrm{l}},P_2^{\mathrm{u}}].$$

用区间均值和区间离差表示式(13),可得如下方程:

$$\begin{cases} P_1^{\mathrm{c}}+P_1^{\mathrm{r}}=-(\eta_{21}^{\mathrm{c}}+\eta_{21}^{\mathrm{r}})(E_2^{\mathrm{c}}+E_2^{\mathrm{r}})+[(\eta_1+\eta_{12})^{\mathrm{c}}-(\eta_1+\eta_{12})^{\mathrm{r}}](E_1^{\mathrm{c}}+E_1^{\mathrm{r}}), \\ P_1^{\mathrm{c}}-P_1^{\mathrm{r}}=-(\eta_{21}^{\mathrm{c}}-\eta_{21}^{\mathrm{r}})(E_2^{\mathrm{c}}-E_2^{\mathrm{r}})+[(\eta_1+\eta_{12})^{\mathrm{c}}+(\eta_1+\eta_{12})^{\mathrm{r}}](E_1^{\mathrm{c}}-E_1^{\mathrm{r}}), \\ P_2^{\mathrm{c}}+P_2^{\mathrm{r}}=-(\eta_{12}^{\mathrm{c}}+\eta_{12}^{\mathrm{r}})(E_1^{\mathrm{c}}+E_1^{\mathrm{r}})+[(\eta_2+\eta_{21})^{\mathrm{c}}-(\eta_2+\eta_{21})^{\mathrm{r}}](E_2^{\mathrm{c}}+E_2^{\mathrm{r}}), \\ P_2^{\mathrm{c}}-P_2^{\mathrm{r}}=-(\eta_{12}^{\mathrm{c}}-\eta_{12}^{\mathrm{r}})(E_1^{\mathrm{c}}-E_1^{\mathrm{r}})+[(\eta_2+\eta_{21})^{\mathrm{c}}+(\eta_2+\eta_{21})^{\mathrm{r}}](E_2^{\mathrm{c}}-E_2^{\mathrm{r}}). \end{cases} \tag{14}$$

写成矩阵形式为

$$\begin{bmatrix} \eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}} & -\eta_{21}^{\mathrm{c}} & -(\eta_1^{\mathrm{r}}+\eta_{12}^{\mathrm{r}}) & -\eta_{21}^{\mathrm{r}} \\ -\eta_{12}^{\mathrm{c}} & \eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}} & -\eta_{12}^{\mathrm{r}} & -(\eta_2^{\mathrm{r}}+\eta_{21}^{\mathrm{r}}) \\ -(\eta_1^{\mathrm{r}}+\eta_{12}^{\mathrm{r}}) & -\eta_{21}^{\mathrm{r}} & \eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}} & -\eta_{21}^{\mathrm{c}} \\ -\eta_{12}^{\mathrm{r}} & -(\eta_2^{\mathrm{r}}+\eta_{21}^{\mathrm{r}}) & -\eta_{12}^{\mathrm{c}} & \eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}} \end{bmatrix} \begin{bmatrix} E_1^{\mathrm{c}} \\ E_2^{\mathrm{c}} \\ E_1^{\mathrm{r}} \\ E_2^{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} P_1^{\mathrm{c}} \\ P_2^{\mathrm{c}} \\ P_1^{\mathrm{r}} \\ P_2^{\mathrm{r}} \end{bmatrix}. \tag{15}$$

根据 Cramer(克莱姆)法则即可求得两个子系统的振动能量区间为

$$E_1^{\mathrm{I}}=[E_1^{\mathrm{l}},E_1^{\mathrm{u}}]=[E_1^{\mathrm{c}}-E_1^{\mathrm{r}},E_1^{\mathrm{c}}+E_1^{\mathrm{r}}],\ E_2^{\mathrm{I}}=[E_2^{\mathrm{l}},E_2^{\mathrm{u}}]=[E_2^{\mathrm{c}}-E_2^{\mathrm{r}},E_2^{\mathrm{c}}+E_2^{\mathrm{r}}]. \tag{16}$$

2.3 基于仿射算法计算子系统振动能量区间

仿射算法可以解决经典区间方法中的区间扩张问题,文献[10]给出了仿射算法的一些基本公式,将区间变量 $x^{\mathrm{I}}=[x^{\mathrm{l}},x^{\mathrm{u}}]$ 和 $y^{\mathrm{I}}=[y^{\mathrm{l}},y^{\mathrm{u}}]$ 表示为

$$\begin{cases} \langle x\rangle = x_0+x_1[\varepsilon_1]+\cdots+x_n[\varepsilon_n]+x_{\mathrm{e}}[\varepsilon_{\mathrm{e}}], \\ \langle y\rangle = y_0+y_1[\varepsilon_1]+\cdots+y_n[\varepsilon_n]+y_{\mathrm{e}}[\varepsilon_{\mathrm{e}}], \end{cases} \tag{17}$$

其中 $[\varepsilon_i]=[-1,1],\quad i=1,2,\cdots,n,$

$[\varepsilon_{\mathrm{e}}]=[-1,1]$.

由功率流平衡方程(6)可得

$$E_1=\frac{P_1(\eta_2+\eta_{21})+P_2\eta_{21}}{\eta_1\eta_2+\eta_2\eta_{12}+\eta_1\eta_{21}},\ E_2=\frac{P_2(\eta_1+\eta_{12})+P_1\eta_{12}}{\eta_1\eta_2+\eta_2\eta_{12}+\eta_1\eta_{21}}. \tag{18}$$

将输入功率区间、内损耗因子区间以及耦合损耗因子区间代入式(18)可得

$$E_1^{\mathrm{I}}=\frac{P_1^{\mathrm{I}}(\eta_2^{\mathrm{I}}+\eta_{21}^{\mathrm{I}})+P_2^{\mathrm{I}}\eta_{21}^{\mathrm{I}}}{\eta_1^{\mathrm{I}}\eta_2^{\mathrm{I}}+\eta_2^{\mathrm{I}}\eta_{12}^{\mathrm{I}}+\eta_1^{\mathrm{I}}\eta_{21}^{\mathrm{I}}},\ E_2^{\mathrm{I}}=\frac{P_2^{\mathrm{I}}(\eta_1^{\mathrm{I}}+\eta_{12}^{\mathrm{I}})+P_1^{\mathrm{I}}\eta_{12}^{\mathrm{I}}}{\eta_1^{\mathrm{I}}\eta_2^{\mathrm{I}}+\eta_2^{\mathrm{I}}\eta_{12}^{\mathrm{I}}+\eta_1^{\mathrm{I}}\eta_{21}^{\mathrm{I}}}. \tag{19}$$

利用仿射算法[9-10]对式(19)中的分子和分母进行计算得

$$\begin{aligned} P_1^{\mathrm{I}}(\eta_2^{\mathrm{I}}+\eta_{21}^{\mathrm{I}})+P_2^{\mathrm{I}}\eta_{21}^{\mathrm{I}} ={}& (P_1^{\mathrm{c}}+P_1^{\mathrm{r}}[\varepsilon_1])(\eta_2^{\mathrm{c}}+\eta_2^{\mathrm{r}}[\varepsilon_1]+\eta_{21}^{\mathrm{c}}+\eta_{21}^{\mathrm{r}}[\varepsilon_1])+ \\ & (P_2^{\mathrm{c}}+P_2^{\mathrm{r}}[\varepsilon_1])(\eta_{21}^{\mathrm{c}}+\eta_{21}^{\mathrm{r}}[\varepsilon_1])= \\ & P_1^{\mathrm{c}}(\eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}})+P_2^{\mathrm{c}}\eta_{21}^{\mathrm{c}}+\frac{1}{2}P_2^{\mathrm{r}}\eta_{21}^{\mathrm{r}}+\frac{1}{2}P_1^{\mathrm{r}}(\eta_2^{\mathrm{r}}+\eta_{21}^{\mathrm{r}})+ \\ & [P_1^{\mathrm{c}}(\eta_2^{\mathrm{r}}+\eta_{21}^{\mathrm{r}})+P_2^{\mathrm{c}}\eta_{21}^{\mathrm{r}}+P_1^{\mathrm{r}}(\eta_2^{\mathrm{c}}+\eta_{21}^{\mathrm{c}})+P_2^{\mathrm{r}}\eta_{21}^{\mathrm{c}}][\varepsilon_1]+ \\ & \frac{1}{2}[P_2^{\mathrm{r}}\eta_{21}^{\mathrm{r}}+P_1^{\mathrm{r}}(\eta_2^{\mathrm{r}}+\eta_{21}^{\mathrm{r}})][\varepsilon_{\mathrm{e}}], \end{aligned} \tag{20}$$

$$\begin{aligned} P_2^{\mathrm{I}}(\eta_1^{\mathrm{I}}+\eta_{12}^{\mathrm{I}})+P_1^{\mathrm{I}}\eta_{12}^{\mathrm{I}} ={}& (P_2^{\mathrm{c}}+P_2^{\mathrm{r}}[\varepsilon_1])(\eta_1^{\mathrm{c}}+\eta_1^{\mathrm{r}}[\varepsilon_1]+\eta_{12}^{\mathrm{c}}+\eta_{12}^{\mathrm{r}}[\varepsilon_1])+ \\ & (P_1^{\mathrm{c}}+P_1^{\mathrm{r}}[\varepsilon_1])(\eta_{12}^{\mathrm{c}}+\eta_{12}^{\mathrm{r}}[\varepsilon_1])= \\ & P_2^{\mathrm{c}}(\eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}})+P_1^{\mathrm{c}}\eta_{12}^{\mathrm{c}}+\frac{1}{2}P_1^{\mathrm{r}}\eta_{12}^{\mathrm{r}}+\frac{1}{2}P_2^{\mathrm{r}}(\eta_1^{\mathrm{r}}+\eta_{12}^{\mathrm{r}})+ \\ & [P_2^{\mathrm{c}}(\eta_1^{\mathrm{r}}+\eta_{12}^{\mathrm{r}})+P_1^{\mathrm{c}}\eta_{12}^{\mathrm{r}}+P_2^{\mathrm{r}}(\eta_1^{\mathrm{c}}+\eta_{12}^{\mathrm{c}})+P_1^{\mathrm{r}}\eta_{12}^{\mathrm{c}}][\varepsilon_1]+ \\ & \frac{1}{2}[P_1^{\mathrm{r}}\eta_{12}^{\mathrm{r}}+P_2^{\mathrm{r}}(\eta_1^{\mathrm{r}}+\eta_{12}^{\mathrm{r}})][\varepsilon_{\mathrm{e}}], \end{aligned} \tag{21}$$

$$\begin{aligned} \eta_1^{\mathrm{I}}\eta_2^{\mathrm{I}}+\eta_2^{\mathrm{I}}\eta_{12}^{\mathrm{I}}+\eta_1^{\mathrm{I}}\eta_{21}^{\mathrm{I}} ={}& (\eta_1^{\mathrm{c}}+\eta_1^{\mathrm{r}}[\varepsilon_1])(\eta_2^{\mathrm{c}}+\eta_2^{\mathrm{r}}[\varepsilon_1])+ \\ & (\eta_2^{\mathrm{c}}+\eta_2^{\mathrm{r}}[\varepsilon_1])(\eta_{12}^{\mathrm{c}}+\eta_{12}^{\mathrm{r}}[\varepsilon_1])+ \\ & (\eta_1^{\mathrm{c}}+\eta_1^{\mathrm{r}}[\varepsilon_1])(\eta_{21}^{\mathrm{c}}+\eta_{21}^{\mathrm{r}}[\varepsilon_1])= \\ & \eta_1^{\mathrm{c}}\eta_2^{\mathrm{c}}+\eta_2^{\mathrm{c}}\eta_{12}^{\mathrm{c}}+\eta_1^{\mathrm{c}}\eta_{21}^{\mathrm{c}}+\frac{1}{2}(\eta_1^{\mathrm{r}}\eta_2^{\mathrm{r}}+\eta_2^{\mathrm{r}}\eta_{12}^{\mathrm{r}}+\eta_1^{\mathrm{r}}\eta_{21}^{\mathrm{r}})+ \end{aligned}$$

$$(\eta_1^r\eta_2^c+\eta_1^c\eta_2^r+\eta_2^r\eta_{12}^c+\eta_2^c\eta_{12}^r+\eta_1^r\eta_{21}^c+\eta_1^c\eta_{21}^r)[\varepsilon_1]+$$
$$\frac{1}{2}(\eta_1^r\eta_2^r+\eta_2^r\eta_{12}^r+\eta_1^r\eta_{21}^r)[\varepsilon_e].\tag{22}$$

利用式(19)～(22)可以得到两个子系统的总能量区间 E_1^{I} 和 E_2^{I}.

2.4 基于仿射逆矩阵计算子系统振动能量区间

根据文献[11],仿射矩阵 $\langle \boldsymbol{x}\rangle$ 和 $\langle \boldsymbol{y}\rangle$ 可以表述为

$$\langle \boldsymbol{x}\rangle=\begin{bmatrix}\langle x_{11}\rangle & \cdots & \langle x_{1k}\rangle\\ \vdots & \ddots & \vdots\\ \langle x_{m1}\rangle & \cdots & \langle x_{mk}\rangle\end{bmatrix},\ \langle \boldsymbol{y}\rangle=\begin{bmatrix}\langle y_{11}\rangle & \cdots & \langle y_{1k}\rangle\\ \vdots & \ddots & \vdots\\ \langle y_{m1}\rangle & \cdots & \langle y_{mk}\rangle\end{bmatrix},\tag{23}$$

其中 $\langle \boldsymbol{x}\rangle$ 和 $\langle \boldsymbol{y}\rangle$ 也可表述为

$$\begin{cases}\langle \boldsymbol{x}\rangle=\boldsymbol{x}_0+\boldsymbol{x}_1[\varepsilon_1]+\cdots+\boldsymbol{x}_n[\varepsilon_n]+\boldsymbol{x}_e[\varepsilon_e],\\ \langle \boldsymbol{y}\rangle=\boldsymbol{y}_0+\boldsymbol{y}_1[\varepsilon_1]+\cdots+\boldsymbol{y}_n[\varepsilon_n]+\boldsymbol{y}_e[\varepsilon_e].\end{cases}\tag{24}$$

仿射矩阵 $(\boldsymbol{I}+\boldsymbol{x}_1[\varepsilon_1])$ 的逆可以表示为

$$(\boldsymbol{I}+\boldsymbol{x}_1[\varepsilon_1])^{-1}=(\boldsymbol{I}+\boldsymbol{x}_1^2/2)-\boldsymbol{x}_1[\varepsilon_1]+[|\boldsymbol{x}_1^2|/2+\boldsymbol{Z}^3(\boldsymbol{I}-\boldsymbol{Z})^{-1}][\varepsilon_e],\tag{25}$$

其中 $\boldsymbol{Z}=|\boldsymbol{x}_1|$,$\boldsymbol{I}$ 代表单位矩阵,$[\varepsilon_i]=[-1,1]$,$[\varepsilon_e]=[-1,1]$.基于保守估计,利用式(24)和(25),方程(10)可以重新写为

$$\left\{\begin{bmatrix}\eta_1^c+\eta_{12}^c & -\eta_{21}^c\\ -\eta_{12}^c & \eta_2^c+\eta_{21}^c\end{bmatrix}+\begin{bmatrix}\eta_1^r+\eta_{12}^r & -\eta_{21}^r\\ -\eta_{12}^r & \eta_2^r+\eta_{21}^r\end{bmatrix}[\varepsilon_1]\right\}\begin{bmatrix}E_1^{\mathrm{I}}\\ E_2^{\mathrm{I}}\end{bmatrix}=\begin{bmatrix}P_1^{\mathrm{I}}\\ P_2^{\mathrm{I}}\end{bmatrix}.\tag{26}$$

式(26)两边同乘 $\begin{bmatrix}\eta_1^c+\eta_{12}^c & -\eta_{21}^c\\ -\eta_{12}^c & \eta_2^c+\eta_{21}^c\end{bmatrix}^{-1}$,并将式(25)代入,可得子系统振动能量区间为

$$\begin{aligned}\begin{bmatrix}E_1^{\mathrm{I}}\\ E_2^{\mathrm{I}}\end{bmatrix}=&\left\{\left(\boldsymbol{I}+\frac{1}{2}\boldsymbol{A}_1^2\right)-\boldsymbol{A}_1[\varepsilon_1]+\right.\\ &\left.\left[\frac{1}{2}|\boldsymbol{A}_1^2|+\boldsymbol{Z}^3(\boldsymbol{I}-\boldsymbol{Z})^{-1}\right][\varepsilon_e]\right\}(\boldsymbol{\eta}^c)^{-1}(\boldsymbol{P}^c+\boldsymbol{P}^r[\varepsilon_1])=\\ &\left\{\left(\boldsymbol{I}+\frac{1}{2}\boldsymbol{A}_1^2\right)(\boldsymbol{\eta}^c)^{-1}-\boldsymbol{A}_1(\boldsymbol{\eta}^c)^{-1}[\varepsilon_1]+\right.\\ &\left.\left[\frac{1}{2}|\boldsymbol{A}_1^2|+\boldsymbol{Z}^3(\boldsymbol{I}-\boldsymbol{Z})^{-1}\right]|(\boldsymbol{\eta}^c)^{-1}|[\varepsilon_e]\right\}(\boldsymbol{P}^c+\boldsymbol{P}^r[\varepsilon_1])=\\ &\left(\boldsymbol{I}+\frac{1}{2}\boldsymbol{A}_1^2\right)(\boldsymbol{\eta}^c)^{-1}\boldsymbol{P}^c-\frac{1}{2}\boldsymbol{A}_1(\boldsymbol{\eta}^c)^{-1}\boldsymbol{P}^r+\\ &\left[\left(\boldsymbol{I}+\frac{1}{2}\boldsymbol{A}_1^2\right)(\boldsymbol{\eta}^c)^{-1}\boldsymbol{P}^r-\boldsymbol{A}_1(\boldsymbol{\eta}^c)^{-1}\boldsymbol{P}^c\right][\varepsilon_1]+\\ &\left\{\left[\frac{1}{2}|\boldsymbol{A}_1^2|+\boldsymbol{Z}^3(\boldsymbol{I}-\boldsymbol{Z})^{-1}\right]|(\boldsymbol{\eta}^c)^{-1}||\boldsymbol{P}^c|+\right.\\ &\left.\left[\frac{1}{2}|\boldsymbol{A}_1^2|+\boldsymbol{Z}^3(\boldsymbol{I}-\boldsymbol{Z})^{-1}\right]|(\boldsymbol{\eta}^c)^{-1}|\boldsymbol{P}^r+\frac{1}{2}|\boldsymbol{A}_1(\boldsymbol{\eta}^c)^{-1}\boldsymbol{P}^r|\right\}[\varepsilon_e],\end{aligned}\tag{27}$$

式中

$$\boldsymbol{\eta}^c=\begin{bmatrix}\eta_1^c+\eta_{12}^c & -\eta_{21}^c\\ -\eta_{12}^c & \eta_2^c+\eta_{21}^c\end{bmatrix},$$

$$\boldsymbol{A}_1 = \begin{bmatrix} \eta_1^c + \eta_{12}^c & -\eta_{21}^c \\ -\eta_{12}^c & \eta_2^c + \eta_{21}^c \end{bmatrix}^{-1} \begin{bmatrix} \eta_1^r + \eta_{12}^r & -\eta_{21}^r \\ -\eta_{12}^r & \eta_2^r + \eta_{21}^r \end{bmatrix}, \ \boldsymbol{Z} = |\boldsymbol{A}_1| .$$

上面采用4种不同的区间分析方法计算了两耦合统计能量分析子系统的振动能量区间,子系统振动能量区间都包含了振动能量的真实值,而且所得区间充分考虑了内损耗因子和耦合损耗因子的测量误差对振动能量计算结果的影响,从理论上对传统的统计能量分析方法进行了完善.下面将在统计能量分析的框架下,通过实例来选取最优的区间分析方法.

3　算例分析

图1为两块相互耦合的板子系统,假设两块板的内损耗因子和耦合损耗因子的测量误差都控制在5%以内.将带有测量误差的内损耗因子、耦合损耗因子和输入功率都用区间变量来表示,则子系统1的内损耗因子为 $\eta_1^I = [0.07, 0.08]$, 耦合损耗因子为 $\eta_{12}^I = [0.008, 0.009]$; 子系统2的内损耗因子为 $\eta_2^I = [0.05, 0.06]$, 耦合损耗因子为 $\eta_{21}^I = [0.002, 0.003]$; 子系统1输入功率为 $P_1^I = [1\,000, 1\,050]$ W;子系统2输入功率为 $P_2^I = [800, 850]$ W;分析中心频率为 $\omega = 1\,000$ Hz .

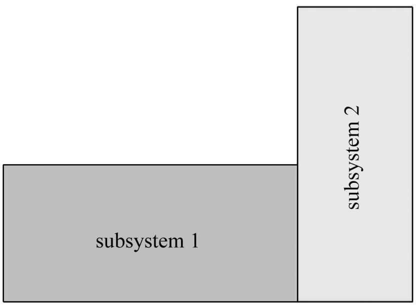

图1　两振子耦合系统

Fig. 1　The SEA model for the 2 subsystems

根据内损耗因子、耦合损耗因子和输入功率区间可得相应的区间均值和不确定区间为

$$\eta_1^c = 0.075,\ \eta_2^c = 0.055,\ \eta_{12}^c = 0.008\,5,\ \eta_{21}^c = 0.002\,5,$$
$$P_1^c = 1.025\ \text{W},\ P_2^c = 0.825\ \text{W},$$
$$\Delta\eta_1^I = [-0.005, 0.005],\ \Delta\eta_2^I = [-0.005, 0.005],$$
$$\Delta\eta_{12}^I = [-0.000\,5, 0.000\,5],\ \Delta\eta_{21}^I = [-0.000\,5, 0.000\,5],$$
$$\Delta P_1^I = [-0.025, 0.025]\ \text{W},\ \Delta P_2^I = [-0.025, 0.025]\ \text{W} .$$

采用第2节的理论可分别获得子系统1和子系统2的振动能量区间,如表1所示.

表1　振动能量结果对比

Table 1　Comparative results of the total energy intervals

method	E_1^I /J	E_2^I /J
interval perturbation approach	[11.46, 14.04]	[13.94, 18.48]
properties of interval variables	[11.55, 14.17]	[14.17, 18.79]
affine arithmetic	[11.96, 13.60]	[14.84, 17.74]
inverse affine matrix	[12.25, 13.29]	[15.09, 17.47]

为了比较表1中4种区间分析方法所得振动能量区间结果的优劣,现利用Monte-Carlo(蒙

特-卡罗)方法来获得子系统 1 和子系统 2 能量区间的精确解,其 MATLAB 主程序如下:

```
ROUND = 10E4;
h1 = 0.075+0.005 * rand(ROUND,1);
h2 = 0.055+0.005 * rand(ROUND,1);
h12 = 0.008 5+0.000 5 * rand(ROUND,1);
h21 = 0.002 5+0.000 5 * rand(ROUND,1);
P1 = 1.025+0.025 * rand(ROUND,1);
P2 = 0.825+0.025 * rand(ROUND,1);
E1 = (P2. * h21+P1. * h2+P1. * h21)./(h1. * h2+h1. * h21+h2. * h12);
E2 = (P1. * h12+P2. * h1+P2. * h12)./(h1. * h2+h1. * h21+h2. * h12).
```

其中,ROUND 为循环次数, h1 为子系统 1 的内损耗因子, h2 为子系统 2 的内损耗因子, h12 和 h21 为子系统 1 和子系统 2 之间的耦合损耗因子, P1 为子系统 1 的输入功率, P2 为子系统 2 的输入功率.经计算,子系统 1 振动能量的最小值为 11.96 J,子系统 1 振动能量的最大值为 13.13 J,子系统 2 振动能量的最小值为 14.80 J,子系统 2 振动能量的最大值为 16.76 J,即子系统 1 振动能量区间的精确解为[11.96,13.13] J,子系统 2 振动能量区间的精确解为[14.80,16.76] J.

在工程实际问题中,针对结构和设备振动破坏方面的研究,一般更加关注结构振动能量的最大值,将 Monte-Carlo 方法得到的振动能量区间精确解和表 1 中 4 种区间方法所得到的结果进行对比发现,采用逆矩阵法更加接近真实解.

4 结　　论

在统计能量分析理论中,由于耦合损耗因子和内损耗因子测量值通常存在误差,导致由传统的统计能量分析计算得到的振动能量和真实值存在偏差.为了解决这个问题,本文从功率流平衡方程入手,通过 4 种不同的区间分析方法:区间矩阵摄动法、基于区间变量特性法、仿射基本算法以及仿射逆矩阵法,计算了统计能量分析子系统的振动能量区间,并详细给出了每种算法的求解公式.

通过 Monte-Carlo 方法得到了不同子系统振动能量区间的精确解,经过对比分析得出采用逆矩阵法来研究统计能量分析方法中参数和外载荷不确定性要优于区间摄动法、基于区间变量特性法和仿射算法.

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Parameter Uncertainty in Statistical Energy Analysis

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Abstract: The statistical energy analysis (SEA) is an effective method to calculate the vibration and noise, where the damping loss factor and the coupling loss factor have very small values and usually are difficult to accurately measure. Then large measurement errors result in significant deviation between the calculated value and the true value of the total energy. To tackle this problem, 4 kinds of different energy interval analysis methods: the interval matrix perturbation approach, the method based on the properties of interval variables, the affine arithmetic and the inverse affine matrix, were used to calculate the steady-state SEA subsystems, where the effects of measurement errors of the damping loss factor and the coupling loss factor on the calculation results were fully considered. Two numerical examples with different errors of loss factors were provided, and the total energy intervals based on different methods were compared. The work improves the existent SEA theory and proves the superiority of the inverse affine matrix over other methods.

Key words: statistical energy analysis; measurement error; damping loss factor; interval analysis method

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