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具有泄漏时滞和混合加性时变时滞复数神经网络的状态估计*

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摘要: 研究了具有泄漏时滞、加性离散时变时滞、加性分布时变时滞复数神经网络的状态估计问题.在复数神经网络不分解条件下, 通过构造合适的 Lyapunov-Krasovskii 泛函, 并应用自由权矩阵、矩阵不等式和倒数凸组合法等方法, 通过可观测的输出测量来估计神经元状态, 给出了判断误差状态模型全局渐近稳定的与时滞相关的复数线性矩阵不等式.最后, 通过一个数值仿真算例验证了理论分析的有效性.

关 键 词: 泄漏时滞; 加性时变时滞; 复数神经网络; 线性矩阵不等式; 状态估计
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引 言

神经网络技术是 20 世纪末迅速发展起来的一门高新技术.由于神经网络具有良好的非线性映射能力、自学习适应能力, 为解决不确定非线性系统的建模和控制问题提供了一条新的思路, 吸引了国内外的众多学者和工程技术人员从事神经网络控制的研究.因此, 神经网络控制逐步发展成为智能控制的一个重要分支.

根据神经网络处理数据的类型, 可将其分为实数神经网络(RVNNs)和复数神经网络(CVNNs).虽然 RVNNs 已在诸多领域得到了应用, 但其在实现几何变换(例如二维(2D)仿射变换)和对称问题的检测和排除 OR(XOR)方面表现较差[1-3].学者们发现在这些实际应用中需要处理更高维度的数据.作为 RVNNs 的通用扩展, CVNNs 具有更复杂的特性, 因此 CVNNs 被提出[4].目前, CVNNs 因其在计算机视觉与遥感、量子装置、生理神经装置和系统的时空分析中的综合应用而受到广泛关注[5-7].同时, 当神经网络被用于系统硬件实现时, 由于放大器转换速度的限制, 神经网络中其当前状态不可避免地受到过去状态的影响.即当前状态的变化率不仅与当前时刻的状态有关, 而且也依赖于过去某时刻或某段时间的状态.这种时间延迟经常是导致模型不稳定性、振荡或其他不良性能的主要来源之一[8-10].因此, 具有时间延迟的神经网络的动力学行为引起了许多学者的关注, 并且已经得到了许多有意义的成果[11-13].然而,这些文献大多数研究的是具有单个延迟的模型.通常, 在实际情况中, 从一个点传输到

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另一个点的信号可能会经历一些网络段，由于可变的网络传输条件，可能会导致不同属性的时间延迟.例如,在状态反馈网络控制中，物理设备、控制器、传感器和执行器位于不同的位置，信号从一个设备传输到另一个设备时，将发生两个附加的时变延迟：一个是从传感器到控制器、另一个是从控制器到执行器[14].因此,研究具有加性时变时滞的 CVNNs 动力学行为具有重要意义.

一般而言，由于神经网络的复杂性和技术限制，完全获得有关网络输出中神经元状态的信息是不可行的[15].特别是在规模较大的神经网络中，通常获得的只是神经元状态的部分信息.为了充分利用神经元的状态以实现一些重要的性能，有必要通过可用的输出测量来估计神经元状态.目前，对于神经网络的状态估计问题，已经报道了大量结果[15-21].文献[17]假定不确定性参数是范数有界的，基于一种新的边界技术，研究了一类具有时变时滞的不确定神经网络的鲁棒状态估计问题.文献[18-19]通过构造合适的 Lyapunov-Krasovskii 泛函，利用 Newton-Leibniz 公式和线性矩阵不等式(LMI)技术，讨论了具有离散区间时变时滞和分布时变时滞的神经网络的状态估计问题.文献[20]研究了具有多次衰落测量的离散时间忆阻神经网络的 H_∞ 状态估计器的设计问题.文献[21]通过选择适当的 Lyapunov-Krasovskii 泛函并利用倒数凸组合法和矩阵不等式技术，研究了具有两个加性离散时变时滞的复数神经网络的状态估计问题.鉴于此，本文拟将研究具有泄漏时滞、加性离散时变时滞、加性分布时变时滞的复数神经网络的状态估计问题，并给出网络状态估计的不等式判据.相比已有文献，本文的贡献主要体现在 3 个方面：

1）本文推广了文献[21]的结果,当本文的泄漏时滞为 0，并且不考虑加性分布时变时滞时,就可以得到文献[21]中的结果；

2）文中考虑的模型是在不要求 CVNNs 的激活函数分解为实部与虚部的条件下研究的，其得到的结果以复数线性矩阵不等式(CVLMI)的形式给出；

3）利用泄漏时滞、加性离散时变时滞、加性分布时变时滞的全部信息构造适当的 Lyapunov-Krasovskii 泛函.

本文的结构如下：第 1 节介绍了相关的预备知识，包括神经网络模型、相关的假设及引理；第 2 节给出了具有泄漏时滞、混合加性时变时滞复数神经网络全局渐近稳定的充分性判据；第 3 节通过一个数值仿真实例验证了提出理论的有效性；最后总结了全文所做的工作.

1 预 备 知 识

本文中做如下符号说明：$\mathcal{I}$表示相应维度的单位矩阵；$\mathbf{R}$，$\mathbf{C}$ 分别表示实数域、复数域；R^n，C^n 分别表示由 n 维实数、复数向量构成的空间；$R^{n\times m}$，$C^{n\times m}$ 分别表示由 $n\times m$ 维实数、复数矩阵构成的集合.$\boldsymbol{X}\geqslant\boldsymbol{Y}(\boldsymbol{X}>\boldsymbol{Y})$ 表示 $\boldsymbol{X}-\boldsymbol{Y}$ 是半正定的(正定的)；“T”“ * ” 分别表示矩阵的转置、矩阵的共轭转置.

1.1 模型及基本引理介绍

考虑如下具有泄漏时滞和混合加性时变时滞的 CVNNs：

$$\begin{aligned}\dot{\boldsymbol{w}}(t)=&-\boldsymbol{D}\boldsymbol{w}(t-\delta)+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{w}(t))+\boldsymbol{B}\boldsymbol{f}(\boldsymbol{w}(t-d_1(t)-d_2(t)))+\\&\boldsymbol{C}\int_{t-\beta_1(t)-\beta_2(t)}^{t}\boldsymbol{f}(\boldsymbol{w}(s))\mathrm{d}s+\boldsymbol{J}(t),\end{aligned}\tag{1}$$

式中 $t\geqslant 0$，$\boldsymbol{w}(t)=(w_1(t),w_2(t),\cdots,w_n(t))^{\mathrm{T}}\in C^n$ 表示在 t 时刻的 n 个神经元状态向量；自反馈连接权矩阵 $\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\cdots,d_n)\in R^{n\times n}$ 且 $d_i>0\ (i=1,2,\cdots,n)$；$\boldsymbol{A}=(a_{ij})_{n\times n}\in C^{n\times n}$，

$\boldsymbol{B}=(b_{ij})_{n\times n}\in C^{n\times n}$,$\boldsymbol{C}=(c_{ij})_{n\times n}\in C^{n\times n}(i,j=1,2,\cdots,n)$ 分别为连接权矩阵、离散时变时滞连接权矩阵和分布时变时滞连接权矩阵；外部输入 $\boldsymbol{J}=(J_1,J_2,\cdots,J_n)^{\mathrm{T}}\in C^n$；$\boldsymbol{f}(\boldsymbol{w}(t))=(f_1(w_1(t)),f_2(w_2(t)),\cdots,f_n(w_n(t)))^{\mathrm{T}}\in C^n$ 表示神经元激活函数;泄漏时滞 $\delta>0$, 加性离散时变时滞、分布时变时滞分别为 $d_i(t)$, $\beta_i(t)(i=1,2)$, 满足 $0\leqslant d_i(t)\leqslant d_i$,$0\leqslant\beta_i(t)\leqslant\beta_i(i=1,2)$, 其中 d_i, β_i是连续的实数,并且 $d(t)=d_1(t)+d_2(t)$, $d=d_1+d_2$,$\beta(t)=\beta_1(t)+\beta_2(t)$, $\beta=\beta_1+\beta_2$.CVNNs(1)的初值条件为

$$\boldsymbol{w}(s)=\boldsymbol{\phi}(s),\qquad s\in[-h,0],$$

其中 $\boldsymbol{\phi}\in C^n$ 在$[-h,0]$上是连续的, 并且 $h=\max\{d,\beta,\delta\}$.

众所周知, 神经元状态的信息通常不完全来自应用中的网络测量(输出), 并且网络测量受到非线性干扰.因此假设网络测量满足

$$\boldsymbol{m}(t)=\boldsymbol{E}\boldsymbol{w}(t)+\boldsymbol{F}\boldsymbol{z}(t,\boldsymbol{w}(t)),\tag{2}$$

其中 $\boldsymbol{m}(t)\in C^m$ 表示测量输出,$\boldsymbol{E}\in C^{m\times n}$, $\boldsymbol{F}\in C^{m\times m}$ 是已知的连接权矩阵, $\boldsymbol{z}=(z_1,z_2,\cdots,z_m)^{\mathrm{T}}$: $\mathbf{R}\times C^n\rightarrow C^m$ 表示网络输出上的神经元非线性干扰.

建立 CVNNs(1)相应的状态估计模型如下:

$$\dot{\boldsymbol{v}}(t)=-\boldsymbol{D}\boldsymbol{v}(t-\delta)+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{v}(t))+\boldsymbol{B}\boldsymbol{f}(\boldsymbol{v}(t-d(t)))+$$
$$\boldsymbol{C}\int_{t-\beta(t)}^{t}\boldsymbol{f}(\boldsymbol{v}(s))\mathrm{d}s+\boldsymbol{J}(t)+\boldsymbol{K}(\boldsymbol{m}(t)-\boldsymbol{n}(t)),\tag{3}$$

$$\boldsymbol{n}(t)=\boldsymbol{E}\boldsymbol{v}(t)+\boldsymbol{F}\boldsymbol{z}(t,\boldsymbol{v}(t)),\tag{4}$$

其中 $\boldsymbol{v}(t)$ 是模型(1) 的状态估计量, 并且 $\boldsymbol{K}\in C^{n\times m}$ 表示估计量的增益矩阵.

令 $\boldsymbol{h}(t)=\boldsymbol{w}(t)-\boldsymbol{v}(t)$, 则误差状态模型为

$$\dot{\boldsymbol{h}}(t)=-\boldsymbol{K}\boldsymbol{E}\boldsymbol{h}(t)-\boldsymbol{D}\boldsymbol{h}(t-\delta)+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{h}(t))+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{h}(t-d(t)))+$$
$$\boldsymbol{C}\int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s-\boldsymbol{K}\boldsymbol{F}\tilde{\boldsymbol{z}}(t,\boldsymbol{h}(t)),\tag{5}$$

其中 $\boldsymbol{g}(\boldsymbol{h}(t))=\boldsymbol{f}(\boldsymbol{w}(t))-\boldsymbol{f}(\boldsymbol{v}(t))$, $\tilde{\boldsymbol{z}}(t,\boldsymbol{h}(t))=\boldsymbol{z}(t,\boldsymbol{w}(t))-\boldsymbol{z}(t,\boldsymbol{v}(t))$.

下文将讨论增益矩阵 $\boldsymbol{K}$ 的存在问题, 使得误差状态模型(5)全局渐近稳定.

为了获得主要结果, 本文需要如下假设.

假设 1 激活函数$f_i(\cdot)$ 是连续的,并且对任意 $\alpha_1,\alpha_2\in\mathbf{C}$,存在 $\gamma_i\in\mathbf{R}$,$i=1,2,\cdots,n$, 有

$$|f_i(\alpha_1)-f_i(\alpha_2)|\leqslant\gamma_i|\alpha_1-\alpha_2|,$$

令 $\boldsymbol{\Gamma}=\mathrm{diag}(\gamma_1,\gamma_2,\cdots,\gamma_n)$.

假设 2 对任意的$\boldsymbol{\mu}_1$,$\boldsymbol{\mu}_2\in C^n$, 存在一个常数矩阵 $\boldsymbol{L}=(\boldsymbol{L}_1,\boldsymbol{L}_2,\cdots,\boldsymbol{L}_m)\in R^{n\times m}$, $i=1,2,\cdots,m$, 满足

$$|z_i(t,\boldsymbol{\mu}_1)-z_i(t,\boldsymbol{\mu}_2)|\leqslant|\boldsymbol{L}_i^{\mathrm{T}}(\boldsymbol{\mu}_1-\boldsymbol{\mu}_2)|,\qquad t\geqslant 0.$$

引理 1[21] $\boldsymbol{X}\in C^{n\times m}$ 是一个正定的 Hermite 矩阵,常数函数 $\boldsymbol{\varrho}$:$[a,b]\rightarrow C^n$,并且 $a\leqslant b$, 则有

$$\left(\int_a^b\boldsymbol{\varrho}(s)\mathrm{d}s\right)^*\boldsymbol{X}\left(\int_a^b\boldsymbol{\varrho}(s)\mathrm{d}s\right)\leqslant(b-a)\int_a^b\boldsymbol{\varrho}^*(s)\boldsymbol{X}\boldsymbol{\varrho}(s)\mathrm{d}s.$$

引理 2[21] 对任意的向量$\boldsymbol{\zeta}\in C^m$, $\alpha\in(0,1)$, 正定矩阵$\boldsymbol{Q}\in C^{n\times n}$, 矩阵$\boldsymbol{W}_1$, $\boldsymbol{W}_2\in C^{n\times m}$, $\boldsymbol{Y}\in C^{n\times n}$, 如果 $\boldsymbol{Y}$ 满足条件

$$\begin{pmatrix}\boldsymbol{Q} & \boldsymbol{Y}\\ \boldsymbol{Y}^* & \boldsymbol{Q}\end{pmatrix}>\mathbf{0},$$

则下列不等式成立：

$$\Phi(\alpha,\boldsymbol{Q}) = \frac{1}{\alpha}\boldsymbol{\zeta}^*\boldsymbol{W}_1^*\boldsymbol{Q}\boldsymbol{W}_1\boldsymbol{\zeta} + \frac{1}{1-\alpha}\boldsymbol{\zeta}^*\boldsymbol{W}_2^*\boldsymbol{Q}\boldsymbol{W}_2\boldsymbol{\zeta} \geqslant \begin{pmatrix}\boldsymbol{W}_1\boldsymbol{\zeta}\\ \boldsymbol{W}_2\boldsymbol{\zeta}\end{pmatrix}^*\begin{pmatrix}\boldsymbol{Q} & \boldsymbol{Y}\\ \boldsymbol{Y}^* & \boldsymbol{Q}\end{pmatrix}\begin{pmatrix}\boldsymbol{W}_1\boldsymbol{\zeta}\\ \boldsymbol{W}_2\boldsymbol{\zeta}\end{pmatrix}.$$

2　主要结果

定理 1　在假设 1 和假设 2 成立的条件下，如果存在正定的对角矩阵 $\boldsymbol{M}_i(i=1,2,3,4)$，$\boldsymbol{G}\in R^{n\times n}$，正定的 Hermite 矩阵 $\boldsymbol{P}_i\in C^{n\times n}(i=1,2,\cdots,10)$ 以及复数矩阵 $\boldsymbol{R}_{11}$，$\boldsymbol{R}_{12}$，$\boldsymbol{R}_{13}$，$\boldsymbol{R}_{22}$，$\boldsymbol{R}_{23}$，$\boldsymbol{R}_{33}$，$\boldsymbol{S}_{11}$，$\boldsymbol{S}_{12}$，$\boldsymbol{S}_{22}$，$\boldsymbol{T}_{11}$，$\boldsymbol{T}_{12}$，$\boldsymbol{T}_{22}$，$\boldsymbol{W}_1$，$\boldsymbol{W}_2$，$\boldsymbol{W}_3$，$\boldsymbol{W}_4$，$\boldsymbol{Q}$，使得以下 LMIs 满足

$$\boldsymbol{R} = \begin{pmatrix}\boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \boldsymbol{R}_{13}\\ & \boldsymbol{R}_{22} & \boldsymbol{R}_{23}\\ * & & \boldsymbol{R}_{33}\end{pmatrix} > \boldsymbol{0}, \tag{6}$$

$$\boldsymbol{S} = \begin{pmatrix}\boldsymbol{S}_{11} & \boldsymbol{S}_{12}\\ * & \boldsymbol{S}_{22}\end{pmatrix} > \boldsymbol{0}, \tag{7}$$

$$\boldsymbol{T} = \begin{pmatrix}\boldsymbol{T}_{11} & \boldsymbol{T}_{12}\\ * & \boldsymbol{T}_{22}\end{pmatrix} > \boldsymbol{0}, \tag{8}$$

$$\boldsymbol{X}_1 = \begin{pmatrix}\boldsymbol{P}_6 & \boldsymbol{W}_1\\ * & \boldsymbol{P}_6\end{pmatrix} > \boldsymbol{0}, \tag{9}$$

$$\boldsymbol{X}_2 = \begin{pmatrix}\boldsymbol{P}_7 & \boldsymbol{W}_2\\ * & \boldsymbol{P}_7\end{pmatrix} > \boldsymbol{0}, \tag{10}$$

$$\boldsymbol{X}_3 = \begin{pmatrix}\boldsymbol{P}_9 & \boldsymbol{W}_3\\ * & \boldsymbol{P}_9\end{pmatrix} > \boldsymbol{0}, \tag{11}$$

$$\boldsymbol{X}_4 = \begin{pmatrix}\boldsymbol{P}_{10} & \boldsymbol{W}_4\\ * & \boldsymbol{P}_{10}\end{pmatrix} > \boldsymbol{0}, \tag{12}$$

$$\boldsymbol{\Omega} = (\boldsymbol{\Omega}_{ij})_{20\times 20} < \boldsymbol{0}, \tag{13}$$

其中

$$\begin{aligned}
&\boldsymbol{\Omega}_{ji} = \boldsymbol{\Omega}_{ij}^*,\ \boldsymbol{\Omega}_{11} = d_1^2\boldsymbol{P}_6 + d^2\boldsymbol{P}_7 + \beta_1^2\boldsymbol{P}_9 + \beta^2\boldsymbol{P}_{10} - \boldsymbol{Q} - \boldsymbol{Q}^*,\\
&\boldsymbol{\Omega}_{12} = \boldsymbol{R}_{11} - \boldsymbol{Q}^* - \boldsymbol{UE} + \boldsymbol{P}_1,\ \boldsymbol{\Omega}_{13} = -\boldsymbol{QD},\ \boldsymbol{\Omega}_{1,12} = \boldsymbol{QA},\ \boldsymbol{\Omega}_{1,15} = \boldsymbol{QB},\ \boldsymbol{\Omega}_{1,16} = \boldsymbol{R}_{12},\\
&\boldsymbol{\Omega}_{1,17} = \boldsymbol{R}_{13},\ \boldsymbol{\Omega}_{1,18} = \boldsymbol{QC},\ \boldsymbol{\Omega}_{1,19} = -\boldsymbol{P}_1\boldsymbol{D},\ \boldsymbol{\Omega}_{1,20} = -\boldsymbol{UF},\\
&\boldsymbol{\Omega}_{22} = -\boldsymbol{D}^*\boldsymbol{P}_1 - \boldsymbol{P}_1\boldsymbol{D} + \boldsymbol{P}_2 + \delta^2\boldsymbol{P}_3 + \boldsymbol{R}_{12}^* + \boldsymbol{R}_{13}^* + \boldsymbol{R}_{12} + \boldsymbol{R}_{13} + \boldsymbol{S}_{11} +\\
&\qquad d_1^2\boldsymbol{P}_6 + d^2\boldsymbol{P}_7 - \boldsymbol{P}_6 - \boldsymbol{P}_7 - \boldsymbol{P}_9 - \boldsymbol{P}_{10} - \boldsymbol{UE} - \boldsymbol{E}^*\boldsymbol{U}^* + \boldsymbol{\Gamma}^*\boldsymbol{M}_1\boldsymbol{\Gamma} + \boldsymbol{L}^*\boldsymbol{GL},\\
&\boldsymbol{\Omega}_{23} = -\boldsymbol{QD} + \boldsymbol{P}_1\boldsymbol{D},\ \boldsymbol{\Omega}_{24} = -\boldsymbol{R}_{12} + \boldsymbol{W}_1^*,\ \boldsymbol{\Omega}_{25} = -\boldsymbol{R}_{13} + \boldsymbol{W}_2^*,\ \boldsymbol{\Omega}_{26} = \boldsymbol{P}_6 - \boldsymbol{W}_1^*,\\
&\boldsymbol{\Omega}_{27} = \boldsymbol{P}_7 - \boldsymbol{W}_2^*,\ \boldsymbol{\Omega}_{28} = \boldsymbol{W}_3^*,\ \boldsymbol{\Omega}_{29} = \boldsymbol{W}_4^*,\ \boldsymbol{\Omega}_{2,10} = \boldsymbol{P}_9 - \boldsymbol{W}_3^*,\ \boldsymbol{\Omega}_{2,11} = \boldsymbol{P}_{10} - \boldsymbol{W}_4^*,\\
&\boldsymbol{\Omega}_{2,12} = \boldsymbol{S}_{12} + \boldsymbol{QA},\ \boldsymbol{\Omega}_{2,15} = \boldsymbol{QB},\ \boldsymbol{\Omega}_{2,16} = \boldsymbol{R}_{22} + \boldsymbol{R}_{23}^*,\ \boldsymbol{\Omega}_{2,17} = \boldsymbol{R}_{23} + \boldsymbol{R}_{33}^*,\ \boldsymbol{\Omega}_{2,18} = \boldsymbol{QC},\\
&\boldsymbol{\Omega}_{2,19} = \boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{D},\ \boldsymbol{\Omega}_{2,20} = -\boldsymbol{UF},\ \boldsymbol{\Omega}_{33} = -\boldsymbol{P}_2,\ \boldsymbol{\Omega}_{3,19} = -\boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{D},\\
&\boldsymbol{\Omega}_{44} = \boldsymbol{T}_{11} - \boldsymbol{S}_{11} - \boldsymbol{P}_6 + \boldsymbol{\Gamma}^*\boldsymbol{M}_2\boldsymbol{\Gamma},\ \boldsymbol{\Omega}_{46} = \boldsymbol{P}_6 - \boldsymbol{W}_1,\ \boldsymbol{\Omega}_{4,13} = \boldsymbol{T}_{12} - \boldsymbol{S}_{12},
\end{aligned}$$

$$\boldsymbol{\Omega}_{4,16} = -\boldsymbol{R}_{22},\ \boldsymbol{\Omega}_{4,17} = -\boldsymbol{R}_{23},\ \boldsymbol{\Omega}_{55} = -\boldsymbol{T}_{11} - \boldsymbol{P}_7 + \boldsymbol{\Gamma}^* \boldsymbol{M}_3 \boldsymbol{\Gamma},\ \boldsymbol{\Omega}_{57} = \boldsymbol{P}_7 - \boldsymbol{W}_2,$$
$$\boldsymbol{\Omega}_{5,14} = -\boldsymbol{T}_{12},\ \boldsymbol{\Omega}_{5,16} = -\boldsymbol{R}_{23}^*,\ \boldsymbol{\Omega}_{5,17} = -\boldsymbol{R}_{33},\ \boldsymbol{\Omega}_{66} = -2\boldsymbol{P}_6 + \boldsymbol{W}_1 + \boldsymbol{W}_1^*,$$
$$\boldsymbol{\Omega}_{77} = -2\boldsymbol{P}_7 + \boldsymbol{W}_2 + \boldsymbol{W}_2^* + \boldsymbol{\Gamma}^* \boldsymbol{M}_4 \boldsymbol{\Gamma},\ \boldsymbol{\Omega}_{88} = -\boldsymbol{P}_9,\ \boldsymbol{\Omega}_{8,10} = \boldsymbol{P}_9 - \boldsymbol{W}_3,\ \boldsymbol{\Omega}_{99} = -\boldsymbol{P}_{10},$$
$$\boldsymbol{\Omega}_{9,11} = \boldsymbol{P}_{10} - \boldsymbol{W}_4,\ \boldsymbol{\Omega}_{10,10} = -2\boldsymbol{P}_9 + \boldsymbol{W}_3 + \boldsymbol{W}_3^*,\ \boldsymbol{\Omega}_{11,11} = -2\boldsymbol{P}_{10} + \boldsymbol{W}_4 + \boldsymbol{W}_4^*,$$
$$\boldsymbol{\Omega}_{12,12} = \boldsymbol{S}_{22} + \beta^2 \boldsymbol{P}_8 - \boldsymbol{M}_1,\ \boldsymbol{\Omega}_{13,13} = \boldsymbol{T}_{22} - \boldsymbol{S}_{22} - \boldsymbol{M}_2,\ \boldsymbol{\Omega}_{14,14} = -\boldsymbol{T}_{22} - \boldsymbol{M}_3,$$
$$\boldsymbol{\Omega}_{15,15} = -\boldsymbol{M}_4,\ \boldsymbol{\Omega}_{16,16} = -\boldsymbol{P}_4,\ \boldsymbol{\Omega}_{17,17} = -\boldsymbol{P}_5,\ \boldsymbol{\Omega}_{18,18} = -\boldsymbol{P}_8,$$
$$\boldsymbol{\Omega}_{19,19} = -\boldsymbol{P}_3,\ \boldsymbol{\Omega}_{20,20} = -\boldsymbol{G},$$

其他的 $\boldsymbol{\Omega}_{ij} = \boldsymbol{0}$, 那么就说 CVNNs(1)的误差状态模型(5)是全局渐近稳定的.并且估计量的增益矩阵 $\boldsymbol{K}$ 定义为 $\boldsymbol{K} = \boldsymbol{Q}^{-1}\boldsymbol{U}$.

证明 令 $\boldsymbol{\xi}(t) = \left(\boldsymbol{h}(t), \int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s, \int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^*$, $\boldsymbol{\eta}(t) = (\boldsymbol{h}(t), \boldsymbol{g}(\boldsymbol{h}(t)))^*$, 构造如下的 Lyapunov-Krasovskii 函数:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t) + V_8(t), \tag{14}$$

其中

$$V_1(t) = \left(\boldsymbol{h}(t) - \boldsymbol{D}\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^* \boldsymbol{P}_1 \left(\boldsymbol{h}(t) - \boldsymbol{D}\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right),$$
$$V_2(t) = \int_{t-\delta}^{t} \boldsymbol{h}^*(s)\boldsymbol{P}_2\boldsymbol{h}(s)\mathrm{d}s + \delta\int_{-\delta}^{0}\int_{t+\mu}^{t} \boldsymbol{h}^*(s)\boldsymbol{P}_3\boldsymbol{h}(s)\mathrm{d}s\mathrm{d}u,$$
$$V_3(t) = \boldsymbol{\xi}^*(t)\boldsymbol{R}\boldsymbol{\xi}(t),$$
$$V_4(t) = \int_{t-d_1}^{t} \boldsymbol{\eta}^*(s)\boldsymbol{S}\boldsymbol{\eta}(s)\mathrm{d}s + \int_{t-d}^{t} \boldsymbol{\eta}^*(s)\boldsymbol{T}\boldsymbol{\eta}(s)\mathrm{d}s,$$
$$V_5(t) = d_1\int_{-d_1}^{0}\int_{t+\theta}^{t} \boldsymbol{h}^*(s)\boldsymbol{P}_4\boldsymbol{h}(s)\mathrm{d}s\mathrm{d}\theta + d\int_{-d}^{0}\int_{t+\theta}^{t} \boldsymbol{h}^*(s)\boldsymbol{P}_5\boldsymbol{h}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_6(t) = d_1\int_{-d_1}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_6\dot{\boldsymbol{h}}(s)\mathrm{d}s\mathrm{d}\theta + d\int_{-d}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_7\dot{\boldsymbol{h}}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_7(t) = \beta\int_{-\beta}^{0}\int_{t+\epsilon}^{t} \boldsymbol{g}^*(\boldsymbol{h}(s))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s\mathrm{d}\epsilon,$$
$$V_8(t) = \beta_1\int_{-\beta_1}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_9\dot{\boldsymbol{h}}(s)\mathrm{d}s\mathrm{d}\theta + \beta\int_{-\beta}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_{10}\dot{\boldsymbol{h}}(s)\mathrm{d}s\mathrm{d}\theta.$$

沿着模型(5)求 $V(t)$ 的导数, 根据引理 1,有

$$\begin{aligned}
\dot{V}_1(t) = {} & [\dot{\boldsymbol{h}}(t) - \boldsymbol{D}\boldsymbol{h}(t) + \boldsymbol{D}\boldsymbol{h}(t-\delta)]^* \boldsymbol{P}_1 \left(\boldsymbol{h}(t) - \boldsymbol{D}\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \\
& \left(\boldsymbol{h}(t) - \boldsymbol{D}\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^* \boldsymbol{P}_1 [\dot{\boldsymbol{h}}(t) - \boldsymbol{D}\boldsymbol{h}(t) + \boldsymbol{D}\boldsymbol{h}(t-\delta)] = \\
& \dot{\boldsymbol{h}}^*(t)\boldsymbol{P}_1\boldsymbol{h}(t) + \boldsymbol{h}^*(t)\boldsymbol{P}_1\dot{\boldsymbol{h}}(t) + \boldsymbol{h}^*(t)(-\boldsymbol{D}^*\boldsymbol{P}_1 - \boldsymbol{P}_1\boldsymbol{D})\boldsymbol{h}(t) + \\
& \boldsymbol{h}^*(t-\delta)\boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{h}(t) + \boldsymbol{h}^*(t)\boldsymbol{P}_1\boldsymbol{D}\boldsymbol{h}(t-\delta) + \\
& \boldsymbol{h}^*(t)\boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{D}\left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^* \boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{D}\boldsymbol{h}(t) + \\
& \dot{\boldsymbol{h}}^*(t)(-\boldsymbol{P}_1\boldsymbol{D})\left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^* (-\boldsymbol{D}^*\boldsymbol{P}_1)\dot{\boldsymbol{h}}(t) + \\
& \boldsymbol{h}^*(t-\delta)(-\boldsymbol{D}^*\boldsymbol{P}_1\boldsymbol{D})\left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) +
\end{aligned}$$

$$\left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}(-\boldsymbol{D}^{*}\boldsymbol{P}_1\boldsymbol{D})\boldsymbol{h}(t-\delta), \tag{15}$$

$$\begin{aligned}
\dot{V}_2(t) = {} & \boldsymbol{h}^{*}(t)\boldsymbol{P}_2\boldsymbol{h}(t) - \boldsymbol{h}^{*}(t-\delta)\boldsymbol{P}_2\boldsymbol{h}(t-\delta) + \\
& \delta\int_0^{\delta}[\boldsymbol{h}^{*}(t)\boldsymbol{P}_3\boldsymbol{h}(t) - \boldsymbol{h}^{*}(t-u)\boldsymbol{P}_3\boldsymbol{h}(t-u)]\mathrm{d}u \leqslant \\
& \boldsymbol{h}^{*}(t)(\boldsymbol{P}_2 + \delta^2\boldsymbol{P}_3)\boldsymbol{h}(t) + \boldsymbol{h}^{*}(t-\delta)(-\boldsymbol{P}_2)\boldsymbol{h}(t-\delta) + \\
& \left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}(-\boldsymbol{P}_3)\left(\int_{t-\delta}^{t} \boldsymbol{h}(s)\mathrm{d}s\right),
\end{aligned} \tag{16}$$

$$\begin{aligned}
\dot{V}_3(t) = {} & \dot{\boldsymbol{\xi}}^{*}(t)\boldsymbol{R}\boldsymbol{\xi}(t) + \boldsymbol{\xi}^{*}(t)\boldsymbol{R}\dot{\boldsymbol{\xi}}(t) = \\
& \begin{pmatrix} \dot{\boldsymbol{h}}(t) \\ \boldsymbol{h}(t) - \boldsymbol{h}(t-d_1) \\ \boldsymbol{h}(t) - \boldsymbol{h}(t-d) \end{pmatrix}^{*} \begin{pmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \boldsymbol{R}_{13} \\ & \boldsymbol{R}_{22} & \boldsymbol{R}_{23} \\ * & & \boldsymbol{R}_{33} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{h}}(t) \\ \int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s \\ \int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s \end{pmatrix} + \\
& \begin{pmatrix} \dot{\boldsymbol{h}}(t) \\ \int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s \\ \int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s \end{pmatrix}^{*} \begin{pmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \boldsymbol{R}_{13} \\ & \boldsymbol{R}_{22} & \boldsymbol{R}_{23} \\ * & & \boldsymbol{R}_{33} \end{pmatrix} \begin{pmatrix} \dot{\boldsymbol{h}}(t) \\ \boldsymbol{h}(t) - \boldsymbol{h}(t-d_1) \\ \boldsymbol{h}(t) - \boldsymbol{h}(t-d) \end{pmatrix} = \\
& \dot{\boldsymbol{h}}^{*}(t)\boldsymbol{R}_{11}\boldsymbol{h}(t) + \boldsymbol{h}^{*}(t)\boldsymbol{R}_{11}\dot{\boldsymbol{h}}(t) + \boldsymbol{h}^{*}(t)(\boldsymbol{R}_{12}^{*} + \boldsymbol{R}_{13}^{*} + \boldsymbol{R}_{12} + \boldsymbol{R}_{13})\boldsymbol{h}(t) + \\
& \dot{\boldsymbol{h}}^{*}(t)\boldsymbol{R}_{12}\left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{12}^{*}\dot{\boldsymbol{h}}(t) + \dot{\boldsymbol{h}}^{*}(t)\boldsymbol{R}_{13}\left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \\
& \left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{13}^{*}\dot{\boldsymbol{h}}(t) + \boldsymbol{h}^{*}(t)(\boldsymbol{R}_{22} + \boldsymbol{R}_{23}^{*})\left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \\
& \left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}(\boldsymbol{R}_{22} + \boldsymbol{R}_{23})\boldsymbol{h}(t) + \boldsymbol{h}^{*}(t)(\boldsymbol{R}_{23} + \boldsymbol{R}_{33})\left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) + \\
& \left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}(\boldsymbol{R}_{23}^{*} + \boldsymbol{R}_{33})\boldsymbol{h}(t) - \boldsymbol{h}^{*}(t-d_1)\boldsymbol{R}_{12}^{*}\boldsymbol{h}(t) - \boldsymbol{h}^{*}(t)\boldsymbol{R}_{12}\boldsymbol{h}(t-d_1) - \\
& \boldsymbol{h}^{*}(t-d)\boldsymbol{R}_{13}^{*}h(t) - \boldsymbol{h}^{*}(t)\boldsymbol{R}_{13}\boldsymbol{h}(t-d) - \boldsymbol{h}^{*}(t-d_1)\boldsymbol{R}_{22}\left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) - \\
& \left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{22}\boldsymbol{h}(t-d_1) - \boldsymbol{h}^{*}(t-d)\boldsymbol{R}_{23}^{*}\left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) - \\
& \left(\int_{t-d_1}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{23}\boldsymbol{h}(t-d) - \boldsymbol{h}^{*}(t-d_1)\boldsymbol{R}_{23}\left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) - \\
& \left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{23}^{*}\boldsymbol{h}(t-d_1) - \\
& \boldsymbol{h}^{*}(t-d)\boldsymbol{R}_{33}\left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right) - \left(\int_{t-d}^{t} \boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{R}_{33}\boldsymbol{h}(t-d),
\end{aligned} \tag{17}$$

$$\begin{aligned}
\dot{V}_4(t) = {} & \boldsymbol{\eta}^{*}(t)\boldsymbol{S}\boldsymbol{\eta}(t) + \boldsymbol{\eta}^{*}(t-d_1)(\boldsymbol{T}-\boldsymbol{S})\boldsymbol{\eta}(t-d_1) - \boldsymbol{\eta}^{*}(t-d)\boldsymbol{T}\boldsymbol{\eta}(t-d) = \\
& \boldsymbol{h}^{*}(t)\boldsymbol{S}_{11}\boldsymbol{h}(t) + \boldsymbol{g}^{*}(\boldsymbol{h}(t))\boldsymbol{S}_{12}^{*}\boldsymbol{h}(t) + \boldsymbol{h}^{*}(t)\boldsymbol{S}_{12}\boldsymbol{g}(\boldsymbol{h}(t)) + \\
& \boldsymbol{g}^{*}(\boldsymbol{h}(t))\boldsymbol{S}_{22}\boldsymbol{g}(\boldsymbol{h}(t)) + \boldsymbol{h}^{*}(t-d_1)(\boldsymbol{T}_{11} - \boldsymbol{S}_{11})\boldsymbol{h}(t-d_1) + \\
& \boldsymbol{g}^{*}(\boldsymbol{h}(t-d_1))(\boldsymbol{T}_{12}^{*} - \boldsymbol{S}_{12}^{*})\boldsymbol{h}(t-d_1) + \boldsymbol{h}(t-d_1)^{*}(\boldsymbol{T}_{12} - \boldsymbol{S}_{12})\boldsymbol{g}(\boldsymbol{h}(t-d_1)) + \\
& \boldsymbol{g}^{*}(\boldsymbol{h}(t-d_1))(\boldsymbol{T}_{22} - \boldsymbol{S}_{22})\boldsymbol{g}(\boldsymbol{h}(t-d_1)) - \boldsymbol{h}^{*}(t-d)\boldsymbol{T}_{11}\boldsymbol{h}(t-d) -
\end{aligned}$$

$$\boldsymbol{g}^{*}(\boldsymbol{h}(t-d))\boldsymbol{T}_{12}^{*}\boldsymbol{h}(t-d)-\boldsymbol{h}^{*}(t-d)\boldsymbol{T}_{12}\boldsymbol{g}(\boldsymbol{h}(t-d))-\boldsymbol{g}^{*}(\boldsymbol{h}(t-d))\boldsymbol{T}_{22}\boldsymbol{g}(\boldsymbol{h}(t-d)), \tag{18}$$

$$\dot{V}_5(t)=\boldsymbol{h}^{*}(t)(d_1^2\boldsymbol{P}_4+d^2\boldsymbol{P}_5)\boldsymbol{h}(t)-d_1\int_{t-d_1}^{t}\boldsymbol{h}^{*}(s)\boldsymbol{P}_4\boldsymbol{h}(s)\mathrm{d}s-d\int_{t-d}^{t}\boldsymbol{h}^{*}(s)\boldsymbol{P}_5\boldsymbol{h}(s)\mathrm{d}s\leqslant \boldsymbol{h}^{*}(t)(d_1^2\boldsymbol{P}_4+d^2\boldsymbol{P}_5)\boldsymbol{h}(t)-\left(\int_{t-d_1}^{t}\boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{P}_4\left(\int_{t-d_1}^{t}\boldsymbol{h}(s)\mathrm{d}s\right)-\left(\int_{t-d}^{t}\boldsymbol{h}(s)\mathrm{d}s\right)^{*}\boldsymbol{P}_5\left(\int_{t-d}^{t}\boldsymbol{h}(s)\mathrm{d}s\right), \tag{19}$$

$$\dot{V}_6(t)=\dot{\boldsymbol{h}}^{*}(t)(d_1^2\boldsymbol{P}_6+d^2\boldsymbol{P}_7)\dot{\boldsymbol{h}}(t)-d_1\int_{t-d_1}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_6\dot{\boldsymbol{h}}(s)\mathrm{d}s-d\int_{t-d}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_7\dot{\boldsymbol{h}}(s)\mathrm{d}s, \tag{20}$$

$$\dot{V}_7(t)=\beta^2\boldsymbol{g}^{*}(\boldsymbol{h}(t))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(t))-\beta\int_{t-\beta}^{t}\boldsymbol{g}^{*}(\boldsymbol{h}(s))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s\leqslant \beta^2\boldsymbol{g}^{*}(\boldsymbol{h}(t))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(t))-\beta(t)\int_{t-\beta(t)}^{t}\boldsymbol{g}^{*}(\boldsymbol{h}(s))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s\leqslant \beta^2\boldsymbol{g}^{*}(\boldsymbol{h}(t))\boldsymbol{P}_8\boldsymbol{g}(\boldsymbol{h}(t))-\left(\int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s\right)^{*}\boldsymbol{P}_8\left(\int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s\right), \tag{21}$$

$$\dot{V}_8(t)=\dot{\boldsymbol{h}}^{*}(t)(\beta_1^2\boldsymbol{P}_9+\beta^2\boldsymbol{P}_{10})\dot{\boldsymbol{h}}(t)-\beta_1\int_{t-\beta_1}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_9\dot{\boldsymbol{h}}(s)\mathrm{d}s-\beta\int_{t-\beta}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_{10}\dot{\boldsymbol{h}}(s)\mathrm{d}s. \tag{22}$$

基于引理 1 和引理 2，有

$$-d_1\int_{t-d_1}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_6\dot{\boldsymbol{h}}(s)\mathrm{d}s=-d_1\int_{t-d_1}^{t-d_1(t)}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_6\dot{\boldsymbol{h}}(s)\mathrm{d}s-d_1\int_{t-d_1(t)}^{t}\dot{\boldsymbol{h}}^{*}(s)\boldsymbol{P}_6\dot{\boldsymbol{h}}(s)\mathrm{d}s\leqslant$$

$$-\frac{d_1}{d_1-d_1(t)}\left(\int_{t-d_1}^{t-d_1(t)}\dot{\boldsymbol{h}}(s)\mathrm{d}s\right)^{*}\boldsymbol{P}_6\left(\int_{t-d_1}^{t-d_1(t)}\dot{\boldsymbol{h}}(s)\mathrm{d}s\right)-\frac{d_1}{d_1(t)}\left(\int_{t-d_1(t)}^{t}\dot{\boldsymbol{h}}(s)\mathrm{d}s\right)^{*}\boldsymbol{P}_6\left(\int_{t-d_1(t)}^{t}\dot{\boldsymbol{h}}(s)\mathrm{d}s\right)=$$

$$-\frac{d_1}{d_1-d_1(t)}\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}^{*}\begin{pmatrix}\boldsymbol{0}\\ -\mathcal{I}\\ \mathcal{I}\end{pmatrix}\boldsymbol{P}_6\begin{pmatrix}\boldsymbol{0}\\ -\mathcal{I}\\ \mathcal{I}\end{pmatrix}^{*}\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}-\frac{d_1}{d_1(t)}\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}^{*}\begin{pmatrix}\mathcal{I}\\ \boldsymbol{0}\\ -\mathcal{I}\end{pmatrix}\boldsymbol{P}_6\begin{pmatrix}\mathcal{I}\\ \boldsymbol{0}\\ -\mathcal{I}\end{pmatrix}^{*}\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}\leqslant$$

$$-\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}^{*}\begin{pmatrix}\boldsymbol{0}&\mathcal{I}\\ -\mathcal{I}&\boldsymbol{0}\\ \mathcal{I}&-\mathcal{I}\end{pmatrix}\begin{pmatrix}\boldsymbol{P}_6&\boldsymbol{W}_1\\ *&\boldsymbol{P}_6\end{pmatrix}\begin{pmatrix}\boldsymbol{h}(t)\\ \boldsymbol{h}(t-d_1)\\ \boldsymbol{h}(t-d_1(t))\end{pmatrix}=$$

$$
-\begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-d_1) \\ \boldsymbol{h}(t-d_1(t)) \end{pmatrix}^* \begin{pmatrix} \boldsymbol{P}_6 & -\boldsymbol{W}_1^* & -\boldsymbol{P}_6+\boldsymbol{W}_1^* \\ -\boldsymbol{W}_1 & \boldsymbol{P}_6 & -\boldsymbol{P}_6+\boldsymbol{W}_1 \\ -\boldsymbol{P}_6+\boldsymbol{W}_1 & -\boldsymbol{P}_6+\boldsymbol{W}_1^* & 2\boldsymbol{P}_6-\boldsymbol{W}_1-\boldsymbol{W}_1^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-d_1) \\ \boldsymbol{h}(t-d_1(t)) \end{pmatrix}. \tag{23}
$$

同理可得

$$
-d\int_{t-d}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_7\dot{\boldsymbol{h}}(s)\,\mathrm{d}s \leqslant
$$

$$
-\begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-d) \\ \boldsymbol{h}(t-d(t)) \end{pmatrix}^* \begin{pmatrix} \boldsymbol{P}_7 & -\boldsymbol{W}_2^* & -\boldsymbol{P}_7+\boldsymbol{W}_2^* \\ -\boldsymbol{W}_2 & \boldsymbol{P}_7 & -\boldsymbol{P}_7+\boldsymbol{W}_2 \\ -\boldsymbol{P}_7+\boldsymbol{W}_2 & -\boldsymbol{P}_7+\boldsymbol{W}_2^* & 2\boldsymbol{P}_7-\boldsymbol{W}_2-\boldsymbol{W}_2^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-d) \\ \boldsymbol{h}(t-d(t)) \end{pmatrix}, \tag{24}
$$

$$
-\beta_1\int_{t-\beta_1}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_9\dot{\boldsymbol{h}}(s)\,\mathrm{d}s \leqslant
$$

$$
-\begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-\beta_1) \\ \boldsymbol{h}(t-\beta_1(t)) \end{pmatrix}^* \begin{pmatrix} \boldsymbol{P}_9 & -\boldsymbol{W}_3^* & -\boldsymbol{P}_9+\boldsymbol{W}_3^* \\ -\boldsymbol{W}_3^* & \boldsymbol{P}_9 & -\boldsymbol{P}_9+\boldsymbol{W}_3 \\ -\boldsymbol{P}_9+\boldsymbol{W}_3^* & -\boldsymbol{P}_9+\boldsymbol{W}_3^* & 2\boldsymbol{P}_9-\boldsymbol{W}_3-\boldsymbol{W}_3^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-\beta_1) \\ \boldsymbol{h}(t-\beta_1(t)) \end{pmatrix}, \tag{25}
$$

$$
-\beta\int_{t-\beta}^{t} \dot{\boldsymbol{h}}^*(s)\boldsymbol{P}_{10}\dot{\boldsymbol{h}}(s)\,\mathrm{d}s \leqslant
$$

$$
-\begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-\beta) \\ \boldsymbol{h}(t-\beta(t)) \end{pmatrix}^* \begin{pmatrix} \boldsymbol{P}_{10} & -\boldsymbol{W}_4^* & -\boldsymbol{P}_{10}+\boldsymbol{W}_4^* \\ -\boldsymbol{W}_4 & \boldsymbol{P}_{10} & -\boldsymbol{P}_{10}+\boldsymbol{W}_4 \\ -\boldsymbol{P}_{10}+\boldsymbol{W}_4 & -\boldsymbol{P}_{10}+\boldsymbol{W}_4^* & 2\boldsymbol{P}_{10}-\boldsymbol{W}_4-\boldsymbol{W}_4^* \end{pmatrix} \begin{pmatrix} \boldsymbol{h}(t) \\ \boldsymbol{h}(t-\beta) \\ \boldsymbol{h}(t-\beta(t)) \end{pmatrix}. \tag{26}
$$

另外，由假设 1 和假设 2 可得

$$
0 \leqslant \boldsymbol{h}^*(t)\boldsymbol{\Gamma}^*\boldsymbol{M}_1\boldsymbol{\Gamma}\boldsymbol{h}(t) - \boldsymbol{g}^*(\boldsymbol{h}(t))\boldsymbol{M}_1\boldsymbol{g}(\boldsymbol{h}(t)), \tag{27}
$$

$$
0 \leqslant \boldsymbol{h}^*(t-d_1)\boldsymbol{\Gamma}^*\boldsymbol{M}_2\boldsymbol{\Gamma}\boldsymbol{h}(t-d_1) - \boldsymbol{g}^*(\boldsymbol{h}(t-d_1))\boldsymbol{M}_2\boldsymbol{g}(\boldsymbol{h}(t-d_1)), \tag{28}
$$

$$
0 \leqslant \boldsymbol{h}^*(t-d)\boldsymbol{\Gamma}^*\boldsymbol{M}_3\boldsymbol{\Gamma}\boldsymbol{h}(t-d) - \boldsymbol{g}^*(\boldsymbol{h}(t-d))\boldsymbol{M}_3\boldsymbol{g}(\boldsymbol{h}(t-d)), \tag{29}
$$

$$
0 \leqslant \boldsymbol{h}^*(t-d(t))\boldsymbol{\Gamma}^*\boldsymbol{M}_4\boldsymbol{\Gamma}\boldsymbol{h}(t-d(t)) - \boldsymbol{g}^*(\boldsymbol{h}(t-d(t)))\boldsymbol{M}_4\boldsymbol{g}(\boldsymbol{h}(t-d(t))), \tag{30}
$$

$$
0 \leqslant \boldsymbol{h}^*(t)\boldsymbol{L}^*\boldsymbol{G}\boldsymbol{L}\boldsymbol{h}(t) - \tilde{\boldsymbol{z}}^*(t,\boldsymbol{h}(t))\boldsymbol{G}\tilde{\boldsymbol{z}}(t,\boldsymbol{h}(t)). \tag{31}
$$

采用自由权矩阵得到

$$
\begin{aligned}
0 = {} & [\boldsymbol{h}(t)+\dot{\boldsymbol{h}}(t)]^*\boldsymbol{Q}\Big[-\dot{\boldsymbol{h}}(t)-\boldsymbol{KE}\boldsymbol{h}(t)-\boldsymbol{D}\boldsymbol{h}(t-\delta)+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{h}(t))+ \\
& \boldsymbol{B}\boldsymbol{g}(\boldsymbol{h}(t-d(t)))+\boldsymbol{C}\int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\,\mathrm{d}s-\boldsymbol{KF}\tilde{\boldsymbol{z}}(t,\boldsymbol{h}(t))\Big]+ \\
& \Big[-\dot{\boldsymbol{h}}(t)-\boldsymbol{KE}\boldsymbol{h}(t)-\boldsymbol{D}\boldsymbol{h}(t-\delta)+\boldsymbol{A}\boldsymbol{g}(\boldsymbol{h}(t))+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{h}(t-d(t)))+ \\
& \boldsymbol{C}\int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\,\mathrm{d}s-\boldsymbol{KF}\tilde{\boldsymbol{z}}(t,\boldsymbol{h}(t))\Big]^*\boldsymbol{Q}^*[\boldsymbol{h}(t)+\dot{\boldsymbol{h}}(t)].
\end{aligned} \tag{32}
$$

由式(15)~(32)相加可得

$$\dot{V}(t) \leqslant \boldsymbol{\chi}^{*}(t)\boldsymbol{\Omega}\boldsymbol{\chi}(t), \tag{33}$$

其中

$$\begin{aligned}\boldsymbol{\chi}(t) = \Big(&\dot{\boldsymbol{h}}(t), \boldsymbol{h}(t), \boldsymbol{h}(t-\delta), \boldsymbol{h}(t-d_1), \boldsymbol{h}(t-d), \boldsymbol{h}(t-d_1(t)),\\ &\boldsymbol{h}(t-d(t)), \boldsymbol{h}(t-\beta_1), \boldsymbol{h}(t-\beta), \boldsymbol{h}(t-\beta_1(t)),\\ &\boldsymbol{h}(t-\beta(t)), \boldsymbol{g}(\boldsymbol{h}(t)), \boldsymbol{g}(\boldsymbol{h}(t-d_1)), \boldsymbol{g}(\boldsymbol{h}(t-d)),\\ &\boldsymbol{g}(\boldsymbol{h}(t-d(t))), \int_{t-d_1}^{t}\boldsymbol{h}(s)\mathrm{d}s, \int_{t-d}^{t}\boldsymbol{h}(s)\mathrm{d}s, \int_{t-\beta(t)}^{t}\boldsymbol{g}(\boldsymbol{h}(s))\mathrm{d}s,\\ &\int_{t-\delta}^{t}\boldsymbol{h}(s)\mathrm{d}s, \tilde{\boldsymbol{z}}(t, \boldsymbol{h}(t))\Big)^{*}.\end{aligned}$$

因此，由式(13)和(33)可得

$$\dot{V}(t) \leqslant 0, \qquad t \geqslant 0, \tag{34}$$

所以，误差状态模型(5)是全局渐近稳定的，证毕. □

3 数值仿真

考虑以下具有二维神经元的 CVNNs(1)模型:

$$\boldsymbol{A} = \begin{bmatrix} -2.5-3\mathrm{i} & -1.5+0.3\mathrm{i} \\ 2+0.9\mathrm{i} & 0.5-1.8\mathrm{i} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 1-1.7\mathrm{i} & -0.9-2.5\mathrm{i} \\ -0.2+0.3\mathrm{i} & 1.0-0.2\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} -0.2+0.4\mathrm{i} & 0.1+0.2\mathrm{i} \\ -0.1+1.4\mathrm{i} & 0.5+1.2\mathrm{i} \end{bmatrix}, \boldsymbol{J} = \begin{bmatrix} 3\cos(0.8t)+(8\sin t+3)\mathrm{i} \\ 2\sin t-(3\cos(0.9t)-2)\mathrm{i} \end{bmatrix},$$

$$\delta = 0.3, \boldsymbol{D} = \mathrm{diag}(0.1, 0.2), \boldsymbol{f}_1(\boldsymbol{w}) = \boldsymbol{f}_2(\boldsymbol{w}) = 0.3\tanh(\boldsymbol{w}),$$

$$d_1(t) = 0.2|\sin(2t)|, d_2(t) = 0.2|\cos(4t)|,$$

$$\beta_1(t) = 0.1|\sin(3t)|, \beta_2(t) = 0.2|\cos(2t)|.$$

因此，当 $\boldsymbol{\Gamma} = \mathrm{diag}(0.3,0.3)$, $d_1 = 0.2, d = 0.4, \beta_1 = 0.1, \beta = 0.3$ 时，满足假设 1 中的条件.

网络输出测量(2)考虑如下:

$$\boldsymbol{E} = \begin{bmatrix} 0.2+1.2\mathrm{i} & -2-1.5\mathrm{i} \\ -1-\mathrm{i} & -1-0.2\mathrm{i} \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} -1+2\mathrm{i} & 1-0.5\mathrm{i} \\ -1.5+2\mathrm{i} & 1-\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{z}_1(t,\boldsymbol{w}(t)) = t+0.1\boldsymbol{w}_1(t)+0.2\boldsymbol{w}_2(t), \boldsymbol{z}_2(t,\boldsymbol{w}(t)) = t-0.1\boldsymbol{w}_1(t)+0.3\boldsymbol{w}_2(t),$$

当 $\boldsymbol{L} = \begin{bmatrix} 0.1 & -0.1 \\ 0.2 & 0.3 \end{bmatrix}$ 时，容易验证假设 2 满足.运用 MATLAB 工具箱 YALMIP，可求解 LMI (13)，得

$$\boldsymbol{P}_1 = \begin{bmatrix} 32.18+0.00\mathrm{i} & -8.97+6.64\mathrm{i} \\ -8.97-6.64\mathrm{i} & 33.12+0.00\mathrm{i} \end{bmatrix}, \boldsymbol{P}_2 = \begin{bmatrix} 16.65+0.00\mathrm{i} & -10.04+5.50\mathrm{i} \\ -10.04-5.50\mathrm{i} & 16.18+0.00\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{P}_3 = \begin{bmatrix} 283.35+0.00\mathrm{i} & -132.54+7.92\mathrm{i} \\ -132.54-7.92\mathrm{i} & 183.41+0.00\mathrm{i} \end{bmatrix}, \boldsymbol{P}_4 = \begin{bmatrix} 86.55+0.00\mathrm{i} & 0.37-0.37\mathrm{i} \\ 0.37+0.37\mathrm{i} & 86.44+0.00\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{P}_5 = \begin{bmatrix} 91.02+0.00\mathrm{i} & 2.45-2.08\mathrm{i} \\ 2.45+2.08\mathrm{i} & 90.54+0.00\mathrm{i} \end{bmatrix}, \boldsymbol{P}_6 = \begin{bmatrix} 12.23+0.00\mathrm{i} & 2.69+5.30\mathrm{i} \\ 2.69-5.30\mathrm{i} & 19.79+0.00\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{P}_7 = \begin{bmatrix} 3.32+0.00\mathrm{i} & 1.79+2.73\mathrm{i} \\ 1.79-2.73\mathrm{i} & 7.48+0.00\mathrm{i} \end{bmatrix}, \boldsymbol{P}_8 = \begin{bmatrix} 108.34+0.00\mathrm{i} & 38.85-19.51\mathrm{i} \\ 38.85+19.51\mathrm{i} & 93.48+0.00\mathrm{i} \end{bmatrix},$$

$$\boldsymbol{P}_9 = \begin{bmatrix} 18.57+0.00\mathrm{i} & 0.07+3.33\mathrm{i} \\ 0.07-3.33\mathrm{i} & 22.12+0.00\mathrm{i} \end{bmatrix}, \boldsymbol{P}_{10} = \begin{bmatrix} 5.77+0.00\mathrm{i} & 0.88+3.91\mathrm{i} \\ 0.88-3.91\mathrm{i} & 10.51+0.00\mathrm{i} \end{bmatrix},$$

$$R_{11}=\begin{bmatrix}24.17+0.00i & -10.48+5.26i\\ -10.48-5.26i & 15.95+0.00i\end{bmatrix},$$

$$R_{12}=\begin{bmatrix}-0.96+0.04i & -0.22-0.08i\\ -0.20+0.14i & -1.19-0.06i\end{bmatrix},$$

$$R_{13}=\begin{bmatrix}-2.35-0.30i & -1.16-0.49i\\ -1.45+0.33i & -3.69+0.25i\end{bmatrix},\ R_{22}=\begin{bmatrix}17.14+0.00i & -0.71+3.12i\\ -0.71-3.12i & 19.82+0.00i\end{bmatrix},$$

$$R_{23}=\begin{bmatrix}-2.79+0.05i & 1.40-0.02i\\ 1.45+0.06i & -1.76-0.06i\end{bmatrix},\ R_{33}=\begin{bmatrix}14.09+0.00i & -2.01+3.58i\\ -2.01-3.58i & 16.27+0.00i\end{bmatrix},$$

$$S_{11}=\begin{bmatrix}40.25+0.00i & -17.31+9.76i\\ -17.31-9.76i & 36.08+0.00i\end{bmatrix},\ S_{12}=\begin{bmatrix}4.03+4.42i & -3.35+0.42i\\ 0.32-4.00i & 2.00+7.62i\end{bmatrix},$$

$$S_{22}=\begin{bmatrix}43.57+0.00i & -5.09+15.58i\\ -5.09-15.58i & 44.39+0.00i\end{bmatrix},$$

$$T_{11}=\begin{bmatrix}26.62+0.00i & -10.73+6.00i\\ -10.73-6.00i & 23.77+0.00i\end{bmatrix},$$

$$T_{12}=\begin{bmatrix}2.29+2.44i & -1.72+0.36i\\ 0.11-2.14i & 1.03+4.42i\end{bmatrix},\ T_{22}=\begin{bmatrix}40.70+0.00i & -2.56+7.95i\\ -2.56-7.95i & 40.24+0.00i\end{bmatrix},$$

$$W_1=\begin{bmatrix}2.32-0.01i & 2.92+2.98i\\ 2.91-2.98i & 7.66+0.01i\end{bmatrix},\ W_2=\begin{bmatrix}-10.67-0.40i & 1.43+1.04i\\ 0.97-1.32i & -7.43+0.38i\end{bmatrix},$$

$$W_3=\begin{bmatrix}6.75+0.02i & 0.11+2.35i\\ 0.12-2.34i & 9.29-0.02i\end{bmatrix},\ W_4=\begin{bmatrix}0.64+0.02i & 0.22+1.44i\\ 0.24-1.42i & 2.32-0.03i\end{bmatrix},$$

$$Q=\begin{bmatrix}4.71+0.06i & 1.74+2.80i\\ 2.32-2.85i & 8.77+0.37i\end{bmatrix},\ U=\begin{bmatrix}6.64-17.49i & -14.11+17.88i\\ -14.36+12.11i & -1.75-2.66i\end{bmatrix},$$

$$M_1=\mathrm{diag}(166.37,\ 125.08),\ M_2=\mathrm{diag}(30.90,\ 29.11),\ M_3=\mathrm{diag}(16.05,\ 15.41),$$

$$M_4=\mathrm{diag}(91.42,\ 79.38),\ G=\mathrm{diag}(30.78,\ 34.29).$$

根据 $K=Q^{-1}U$, 得

$$K=\begin{bmatrix}4.10-4.60i & -4.45+5.62i\\ -1.06+3.97i & -0.98-3.19i\end{bmatrix}.$$

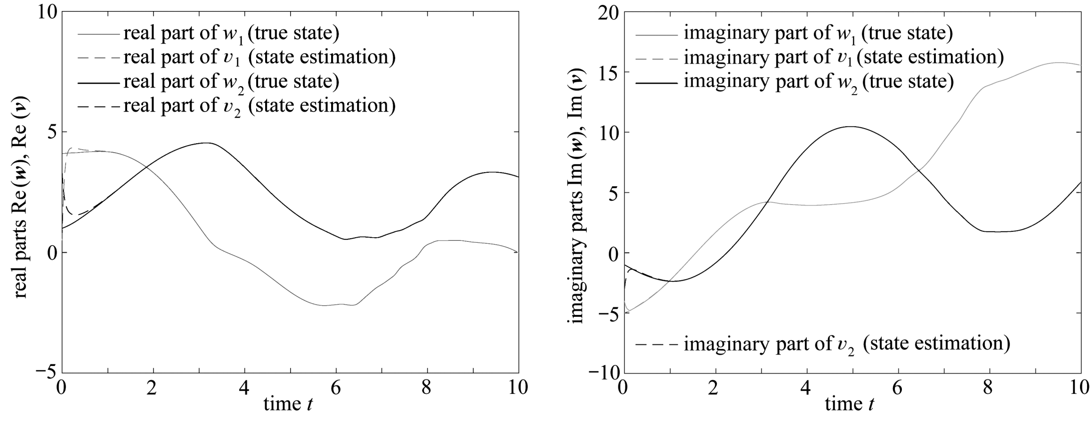

图 1　神经元状态 $w(t)$ 和估计状态 $v(t)$ 的实部、虚部状态轨迹

Fig. 1　State trajectories of real and imaginary parts for neuronal state $w(t)$ and estimation state $v(t)$

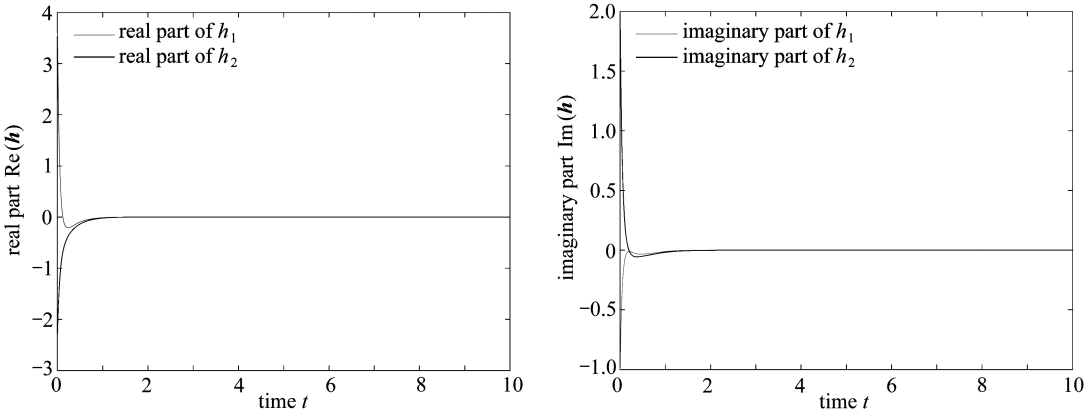

图 2 误差状态模型(5)的实部、虚部状态轨迹

Fig. 2 State trajectories of real and imaginary parts of neuronal state $\boldsymbol{h}(t)$ for system(5)

根据定理 1，CVNNs(1)的误差状态模型(5)是全局渐近稳定的. CVNNs(1)的初值为 $\boldsymbol{w}(t)=[2.4-3\mathrm{i} \quad 4-2\mathrm{i}]^{\mathrm{T}}$，CVNNs(3)的初值为 $\boldsymbol{v}(t)=[0.5-4\mathrm{i} \quad 1.3-3\mathrm{i}]^{\mathrm{T}}$. 仿真结果图 1、2 验证了用于设计延迟 CVNNs 状态估计器的方法的有效性.

4 结 论

本文研究了具有泄漏时滞、加性离散时变时滞、加性分布时变时滞复数神经网络的状态估计问题. 在不要求复数神经网络分解为两个实数神经网络的条件下，给出了保证误差神经网络状态模型的全局渐近稳定性的充分性判据. 同时，本文所考虑的时变时滞可以是不可微分的. 最后，通过一个数值仿真算例验证了理论分析的有效性. 笔者后续将研究具有加性时变时滞的分数阶复数神经网络以及四元数神经网络的相关动力学.

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State Estimation of Complex-Valued Neural Networks With Leakage Delay and Mixed Additive Time-Varying Delays

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Abstract: The state estimation of complex-valued neural networks with leakage delay and both discrete and distributed additive time-varying delays was studied. In the case where the activation function of the network was not required to be separated, through construction of the appropriate Lyapunov-Krasovskii functionals, and with the free weight matrix, the matrix inequality and the reciprocal convex combination method, the state of the neuron was estimated by means of observable output measurements. In addition, complex-valued linear matrix inequalities related to time delays were given to ensure the global asymptotic stability of the error-state model. Finally, numerical simulation examples verify the validity of the theoretical analysis.

Key words: leakage delay; additive time-varying delay; complex-valued neural network; linear matrix inequality; state estimation

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