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基于多切换传输的复变量混沌系统的有限时组合同步控制*

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摘要: 针对一类复变量混沌系统, 研究了基于多切换传输的有限时同步控制问题.首先,针对网络信号在传输过程中的同步模式,分析了多个混沌系统之间的多切换同步行为.其次,基于预设的切换传输规则,给出了有限时组合同步的定义.进而,依据有限时稳定性理论,设计了一类实现快速同步的控制器,并给出了有限时组合同步的充分条件.最后,通过数值仿真和分析,验证了所设计控制方案的有效性.

关 键 词: 复变量混沌系统; 多切换传输; 有限时控制; 组合同步

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引 言

随着信息技术的飞速发展,混沌系统同步控制理论及应用的研究[1-3]已经成为智能控制与通信领域的一个研究热点,人们对混沌同步控制已做了深入的研究并取得了大量的成果[4-9].1982 年,Fowler 等对复数域上的 Lorenz 系统进行了详细的分析[10].自此,复数域上的复变量混沌系统的同步问题受到了越来越多的关注并取得了很多成果[11-14].Mahmoud 等研究了耦合线性复变量混沌系统的投影同步问题[11].Liu 等分别针对确定和不确定复混沌系统的复修正函数投影同步进行了研究[12].Sun 等对带有未知参数的实变量混沌系统和复混沌系统之间的自适应反同步控制方案的设计进行了探索[13].Chen 等研究了多个复变量系统的有限时复函数同步方案的设计问题[14].尽管复混沌系统同步控制已取得了一些研究成果, 但如何给出保密信号及传输过程更安全有效的同步模式和控制方案仍是当今研究者们关注的焦点.

为了提高保密信号传输过程中的安全性,众多复杂的同步机制被提出,诸如,混合同步[7]、传输同步[9]、组合同步[15]、多切换同步[16]等.2008 年,Ucar 等[16]提出了混沌同步的多切换机制,该机制可以较好地提高保密信号传输过程中的安全性.自此,基于多切换模式的混沌

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同步问题的研究受到了关注.Wang 等探讨了不确定混沌系统的多切换同步问题[17].基于组合同步模式[15],实变量混沌系统的多切换组合同步被细致地研究[18-19].Chen 等也分别对实变量混沌系统和复混沌系统的多切换同步控制方案进行了设计[20].然而,上述结果尚未对复变量混沌系统的多切换组合同步进行研究.

在实际应用中,通信系统的同步速度将直接影响系统的通信质量,而同步速度主要取决于误差系统的稳定时间,所以,设计在有限时间内实现快速稳定的混沌同步控制方案具有重要的理论和应用价值.Sun 等分别研究了两个复混沌系统和三个复混沌系统的有限时自适应同步和组合同步控制问题[21-22].Chen 等研究了多个复混沌系统的有限时多切换同步控制方案的设计问题[23].但大部分研究结果主要集中在讨论两个或三个混沌系统之间的有限时同步控制,并未对多个复变量混沌系统的有限时组合同步控制进行深入的研究.Zhang 等给出了一类可更有效实现快速稳定的混沌系统同步误差系统有限时稳定的设计方法[24].基于此,如何给出基于多切换模式多个复混沌系统的有限时组合同步控制方案成为了研究重点.

本文针对复数域上的多个复变量混沌系统,研究切换模式下混沌系统的有限时组合同步控制问题,分析信号传输过程中状态变量之间的多切换同步行为,给出了一类新的,能较好实现快速同步的有限时控制方案,并给出了组合同步误差系统有限时稳定的充分条件,所设计控制方案通过数值仿真进行了有效性验证.

1 系统模型与问题描述

考虑如下描述的多个复变量混沌系统:

$$\begin{cases} \dot{x}_{i1}^{\mathrm{r}}+\mathrm{j}\dot{x}_{i1}^{\mathrm{c}}=f_{i1}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{i1}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i+ \\ \qquad \mathrm{j}[f_{i1}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{i1}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i], \\ \dot{x}_{i2}^{\mathrm{r}}+\mathrm{j}\dot{x}_{i2}^{\mathrm{c}}=f_{i2}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{i2}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i+ \\ \qquad \mathrm{j}[f_{i2}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{i2}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i], \\ \vdots \\ \dot{x}_{in}^{\mathrm{r}}+\mathrm{j}\dot{x}_{in}^{\mathrm{c}}=f_{in}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{in}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i+ \\ \qquad \mathrm{j}[f_{in}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{in}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i], \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_{N1}^{\mathrm{r}}+\mathrm{j}\dot{x}_{N1}^{\mathrm{c}}=f_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{N1}^{\mathrm{r}}+ \\ \qquad \mathrm{j}[f_{N1}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})+\boldsymbol{F}_{N1}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})\boldsymbol{A}_N+v_{N1}^{\mathrm{c}}], \\ \dot{x}_{N2}^{\mathrm{r}}+\mathrm{j}\dot{x}_{N2}^{\mathrm{c}}=f_{N2}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{N2}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{N2}^{\mathrm{r}}+ \\ \qquad \mathrm{j}[f_{N2}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})+\boldsymbol{F}_{N2}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})\boldsymbol{A}_N+v_{N2}^{\mathrm{c}}], \\ \vdots \\ \dot{x}_{Nn}^{\mathrm{r}}+\mathrm{j}\dot{x}_{Nn}^{\mathrm{c}}=f_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{Nn}^{\mathrm{r}}+ \\ \qquad \mathrm{j}[f_{Nn}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})+\boldsymbol{F}_{Nn}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})\boldsymbol{A}_N+v_{Nn}^{\mathrm{c}}], \end{cases} \tag{2}$$

其中, $i=1,2,\cdots,N-1$, $\mathrm{j}=\sqrt{-1}$; $x_{i1}=x_{i1}^{\mathrm{r}}+\mathrm{j}x_{i1}^{\mathrm{c}}$, $x_{i2}=x_{i2}^{\mathrm{r}}+\mathrm{j}x_{i2}^{\mathrm{c}}$, $\cdots$, $x_{in}=x_{in}^{\mathrm{r}}+\mathrm{j}x_{in}^{\mathrm{c}}$, $x_{N1}=x_{N1}^{\mathrm{r}}+\mathrm{j}x_{N1}^{\mathrm{c}}$, $x_{N2}=x_{N2}^{\mathrm{r}}+\mathrm{j}x_{N2}^{\mathrm{c}}$, $\cdots$, $x_{Nn}=x_{Nn}^{\mathrm{r}}+\mathrm{j}x_{Nn}^{\mathrm{c}}$ 分别是系统(1)和(2)的复状态变量且 $\boldsymbol{x}_i=[x_{i1},x_{i2},\cdots,x_{in}]^{\mathrm{T}}$ 和 $\boldsymbol{x}_N=[x_{N1},x_{N2},\cdots,x_{Nn}]^{\mathrm{T}}$; $f_{il}=f_{il}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\mathrm{j}f_{il}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})$ $(l=1,2,\cdots,$

n）和$f_{Nl}=f_{Nl}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\mathrm{j}f_{Nl}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})$是非线性函数；函数矩阵为$\boldsymbol{F}_{il}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})$，$\boldsymbol{F}_{il}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})$，$\boldsymbol{F}_{Nl}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})$和$\boldsymbol{F}_{Nl}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})$；$\boldsymbol{A}_i$和$\boldsymbol{A}_N$是$n\times 1$的实矩阵；r 和 c 分别表示所有状态变量的实部和虚部；$v_{N1}=v_{N1}^{\mathrm{r}}+\mathrm{j}v_{N1}^{\mathrm{c}}$，$v_{N2}=v_{N2}^{\mathrm{r}}+\mathrm{j}v_{N2}^{\mathrm{c}},\cdots$，$v_{Nn}=v_{Nn}^{\mathrm{r}}+\mathrm{j}v_{Nn}^{\mathrm{c}}$是控制输入.

考虑前$N-1$个系统之间的状态变量组合与第N个系统变量间的同步关系，同时假设实现切换传输的变量为$s_{ikl}(k,l=1,2,\cdots,n)$，则可得误差变量：

$$\begin{cases} e_1=x_{N1}-\sum\limits_{i=1}^{N-1}\sum\limits_{l=1}^{n}s_{i1l}x_{il}, \\ e_2=x_{N2}-\sum\limits_{i=1}^{N-1}\sum\limits_{l=1}^{n}s_{i2l}x_{il}, \\ \vdots \\ e_n=x_{Nn}-\sum\limits_{i=1}^{N-1}\sum\limits_{l=1}^{n}s_{inl}x_{il}. \end{cases} \tag{3}$$

为了探索多切换组合同步行为，所对应的切换变量满足如下条件：

$$s_{ikl}\neq 0,\ s_{ik'l}=0,\ s_{ikl'}=0,\quad k,k',l,l'\in\{1,2,\cdots,n\},\ k\neq k',\ l\neq l',s_{ikl}\in\{0,1\}. \tag{4}$$

同时，针对每一个误差变量，在每一个子系统中只选取一个变量进行组合，由此，式(3)对应的误差系统转化为

$$\begin{cases} \dot e_1=\dot x_{N1}-\sum\limits_{i=1}^{N-1}s_{i1l_{i1}}\dot x_{il_{i1}}=\underbrace{\dot x_{N1}^{\mathrm{r}}-\sum\limits_{i=1}^{N-1}s_{i1l_{i1}}\dot x_{il_{i1}}^{\mathrm{r}}}_{e_1^{\mathrm{r}}}+\underbrace{\dot x_{N1}^{\mathrm{c}}-\sum\limits_{i=1}^{N-1}s_{i1l_{i1}}\dot x_{il_{i1}}^{\mathrm{c}}}_{e_1^{\mathrm{c}}}, \\ \dot e_2=\dot x_{N2}-\sum\limits_{i=1}^{N-1}s_{i2l_{i2}}\dot x_{il_{i2}}=\underbrace{\dot x_{N2}^{\mathrm{r}}-\sum\limits_{i=1}^{N-1}s_{i2l_{i2}}\dot x_{il_{i2}}^{\mathrm{r}}}_{e_2^{\mathrm{r}}}+\underbrace{\dot x_{N2}^{\mathrm{c}}-\sum\limits_{i=1}^{N-1}s_{i2l_{i2}}\dot x_{il_{i2}}^{\mathrm{c}}}_{e_1^{\mathrm{c}}}, \\ \vdots \\ \dot e_n=\dot x_{Nn}-\sum\limits_{i=1}^{N-1}s_{inl_{in}}\dot x_{il_{in}}=\underbrace{\dot x_{Nn}^{\mathrm{r}}-\sum\limits_{i=1}^{N-1}s_{inl_{in}}\dot x_{il_{in}}^{\mathrm{r}}}_{e_n^{\mathrm{r}}}+\underbrace{\dot x_{Nn}^{\mathrm{c}}-\sum\limits_{i=1}^{N-1}s_{inl_{in}}\dot x_{il_{in}}^{\mathrm{c}}}_{e_n^{\mathrm{c}}}, \end{cases} \tag{5}$$

其中，$s_{i1l_{i1}},s_{i2l_{i2}},s_{inl_{in}}(\forall l_{i1},l_{i2},l_{in}\in\{1,2,\cdots,n\})$满足切换规则(4).由此,给出如下基于多切换传输的有限时组合同步的定义.

定义 1 针对组合同步误差系统(5),在满足预设的切换规则(4)的基础上，如果存在有限时间T，使得

$$\begin{cases} \lim\limits_{t\to T}\|e_g\|=\lim\limits_{t\to T}\|e_g^{\mathrm{r}}\|+\lim\limits_{t\to T}\|e_g^{\mathrm{c}}\|=0, \\ \|e_g\|=\|e_g^{\mathrm{r}}+\mathrm{j}e_g^{\mathrm{c}}\|\equiv 0, \end{cases}\quad t\geqslant T,\ g=1,2,\cdots,n.$$

则系统(5)有限时渐近稳定，即系统(1)和(2)之间实现了有限时组合同步.

根据定义 1，本文将基于文献[14,24]所给出的有限时控制方法，设计控制器$v_{N1}=v_{N1}^{\mathrm{r}}+\mathrm{j}v_{N1}^{\mathrm{c}},v_{N2}=v_{N2}^{\mathrm{r}}+\mathrm{j}v_{N2}^{\mathrm{c}},\cdots$，$v_{Nn}=v_{Nn}^{\mathrm{r}}+\mathrm{j}v_{Nn}^{\mathrm{c}}$，从而使得误差系统(5)有限时渐近稳定，进而实现多复变量混沌系统的有限时组合同步.

2 主要结果

首先，将系统(1)和(2)代入系统(5)，且将 $\dot{e}_g$ 的实部和虚部分离，可得

$$\begin{cases}\dot{e}_1^{\mathrm{r}}=f_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{N1}^{\mathrm{r}}-\\ \quad\sum_{i=1}^{N-1}s_{i1l_{i1}}(f_{il_{i1}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{il_{i1}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i),\\ \dot{e}_1^{\mathrm{c}}=f_{N1}(x_{N1}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})+\boldsymbol{F}_{N1}(x_{11}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})\boldsymbol{A}_N+v_{N1}^{\mathrm{c}}-\\ \quad\sum_{i=1}^{N-1}s_{i1l_{i1}}(f_{il_{i1}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{il_{i1}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i),\\ \vdots\\ \dot{e}_n^{\mathrm{r}}=f_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{Nn}^{\mathrm{r}}-\\ \quad\sum_{i=1}^{N-1}s_{inl_{in}}(f_{il_{in}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{il_{in}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i),\\ \dot{e}_n^{\mathrm{c}}=f_{Nn}(x_{N1}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})+\boldsymbol{F}_{Nn}(x_{11}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})\boldsymbol{A}_N+v_{Nn}^{\mathrm{c}}-\\ \quad\sum_{i=1}^{N-1}s_{inl_{in}}(f_{il_{in}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{il_{in}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i).\end{cases}\tag{6}$$

由此，复变量混沌系统(1)和(2)之间的有限时组合同步问题就转化为上述误差系统(6)的有限时渐近稳定问题.基于此，设计同步控制方案 $v_{Ng}=v_{Ng}^{\mathrm{r}}+\mathrm{j}v_{Ng}^{\mathrm{c}},g=1,2,\cdots,n$，其中

$$\begin{cases}v_{N1}^{\mathrm{r}}=-f_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})-\boldsymbol{F}_{N1}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+\\ \quad\sum_{i=1}^{N-1}s_{i1l_{i1}}(f_{il_{i1}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{il_{i1}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i)-\\ \quad\boldsymbol{\omega}_1\mathrm{sgn}(e_1^{\mathrm{r}})\,|\,e_1^{\mathrm{r}}\,|^{\eta_1}-p_1e_1^{\mathrm{r}},\\ v_{N1}^{\mathrm{c}}=-f_{N1}(x_{N1}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})-\boldsymbol{F}_{N1}(x_{11}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})\boldsymbol{A}_N+\\ \quad\sum_{i=1}^{N-1}s_{i1l_{i1}}(f_{il_{i1}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{il_{i1}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i)-\\ \quad\boldsymbol{\omega}_1\mathrm{sgn}(e_1^{\mathrm{c}})\,|\,e_1^{\mathrm{c}}\,|^{\eta_1}-p_1e_1^{\mathrm{c}},\\ \vdots\\ v_{Nn}^{\mathrm{r}}=-f_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})-\boldsymbol{F}_{Nn}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+\\ \quad\sum_{i=1}^{N-1}s_{inl_{in}}(f_{il_{in}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{il_{in}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i)-\\ \quad\boldsymbol{\omega}_1\mathrm{sgn}(e_n^{\mathrm{r}})\,|\,e_n^{\mathrm{r}}\,|^{\eta_1}-p_1e_n^{\mathrm{r}},\\ v_{Nn}^{\mathrm{c}}=-f_{Nn}(x_{N1}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})-\boldsymbol{F}_{Nn}(x_{11}^{\mathrm{c}},x_{12}^{\mathrm{c}},\cdots,x_{1n}^{\mathrm{c}})\boldsymbol{A}_N+\\ \quad\sum_{i=1}^{N-1}s_{inl_{in}}(f_{il_{in}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{il_{in}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i)-\\ \quad\boldsymbol{\omega}_1\mathrm{sgn}(e_1^{\mathrm{c}})\,|\,e_1^{\mathrm{c}}\,|^{\eta_1}-p_1e_n^{\mathrm{c}},\end{cases}\tag{7}$$

其中，$\omega_1>0$，$0<\eta_1<1$，$p_1>0$.由此，给出如下误差系统有限时渐近稳定的充分条件.

定理1 针对组合同步误差系统(6)，基于切换规则(4)，如果设计的控制方案满足式

(7)，则系统(6)的误差状态变量可在有限时间 T_1 收敛到 0，其中

$$T_1=\frac{\ln(1+2^{(1-\eta_1)/2}(p_1/\omega_1)(V_1(0))^{(1-\eta_1)/2})}{(1-\eta_1)p_1},$$

则误差系统(6)有限时渐近稳定，即前 $N-1$ 个系统(1) 之间的状态变量组合与第 N 个系统(2)的状态变量之间的有限时同步被实现.

证明 针对误差系统 (6)，选择如下的 Lyapunov 函数：

$$V_1=\frac{1}{2}\sum_{g=1}^{n}(e_g^{\mathrm{r}})^2+\frac{1}{2}\sum_{g=1}^{n}(e_g^{\mathrm{c}})^2,$$

对其求导可得

$$\dot V_1=\sum_{g=1}^{n}(e_g^{\mathrm{r}}\dot e_g^{\mathrm{r}})+\sum_{l=1}^{n}(e_g^{\mathrm{c}}\dot e_g^{\mathrm{c}}).$$

将系统(6)代入上式，可得

$$\begin{aligned}\dot V_1=&\sum_{g=1}^{n}\Bigg(e_g^{\mathrm{r}}\Bigg(f_{Ng}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})+\boldsymbol{F}_{Ng}(x_{N1}^{\mathrm{r}},x_{N2}^{\mathrm{r}},\cdots,x_{Nn}^{\mathrm{r}})\boldsymbol{A}_N+v_{Ng}^{\mathrm{r}}-\\&\sum_{i=1}^{N-1}s_{igl_{ig}}(f_{il_{i1}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})+\boldsymbol{F}_{il_{ig}}(x_{i1}^{\mathrm{r}},x_{i2}^{\mathrm{r}},\cdots,x_{in}^{\mathrm{r}})\boldsymbol{A}_i)\Bigg)\Bigg)+\\&\sum_{l=1}^{n}\Bigg(e_g^{\mathrm{c}}\Bigg(f_{Ng}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})+\boldsymbol{F}_{Ng}(x_{N1}^{\mathrm{c}},x_{N2}^{\mathrm{c}},\cdots,x_{Nn}^{\mathrm{c}})\boldsymbol{A}_N+v_{Ng}^{\mathrm{c}}-\\&\sum_{i=1}^{N-1}s_{igl_{ig}}(f_{il_{i1}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})+\boldsymbol{F}_{il_{ig}}(x_{i1}^{\mathrm{c}},x_{i2}^{\mathrm{c}},\cdots,x_{in}^{\mathrm{c}})\boldsymbol{A}_i)\Bigg)\Bigg),\end{aligned}\tag{8}$$

进而，将式(7)代入式(8)，可得

$$\begin{aligned}\dot V_1=&\sum_{g=1}^{n}(e_g^{\mathrm{r}}(-\omega_1\mathrm{sgn}(e_g^{\mathrm{r}})\,|\,e_g^{\mathrm{r}}\,|^{\eta_1}-p_1e_g^{\mathrm{r}}))+\\&\sum_{g=1}^{n}(e_g^{\mathrm{c}}(-\omega_1\mathrm{sgn}(e_g^{\mathrm{c}})\,|\,e_g^{\mathrm{c}}\,|^{\eta_1}-p_1e_{1l}^{\mathrm{c}})).\end{aligned}$$

根据 sgn(·)的性质，可得

$$\begin{aligned}\dot V_1=&\sum_{g=1}^{n}(-\omega_1\,|\,e_g^{\mathrm{r}}\,|^{\eta_1+1}-p_1(e_g^{\mathrm{r}})^2)+\sum_{l=1}^{n}(-\omega_1\,|\,e_g^{\mathrm{c}}\,|^{\eta_1+1}-p_1(e_g^{\mathrm{c}})^2)=\\&-\omega_1\left(\sum_{g=1}^{n}(|\,e_g^{\mathrm{r}}\,|^{\eta_1+1}+|\,e_{1l}^{\mathrm{c}}\,|^{\eta_1+1})\right)-p_1\left(\sum_{l=1}^{n}((e_g^{\mathrm{r}})^2+(e_g^{\mathrm{c}})^2)\right)=\\&-\omega_1\left(\sum_{g=1}^{n}([(e_g^{\mathrm{r}})^2]^{(\eta_1+1)/2}+[(e_g^{\mathrm{c}})^2]^{(\eta_1+1)/2})\right)-p_1\left(\sum_{g=1}^{n}((e_g^{\mathrm{r}})^2+(e_g^{\mathrm{c}})^2)\right).\end{aligned}$$

基于文献[14,24]，可知

$$\begin{aligned}\dot V_1=&-\omega_12^{(\eta_1+1)/2}\left(\sum_{g=1}^{n}([(e_g^{\mathrm{r}})^2/2]^{(\eta_1+1)/2}+[(e_g^{\mathrm{c}})^2/2]^{(\eta_1+1)/2})\right)-\\&2p_1\left(\sum_{g=1}^{n}((e_g^{\mathrm{r}})^2/2+(e_g^{\mathrm{c}})^2/2)\right)\leqslant\\&-\omega_12^{(\eta_1+1)/2}\left(\sum_{g=1}^{n}((e_g^{\mathrm{r}})^2/2+(e_g^{\mathrm{c}})^2/2)\right)^{(\eta_1+1)/2}-\\&2p_1\left(\sum_{g=1}^{n}((e_g^{\mathrm{r}})^2/2+(e_g^{\mathrm{c}})^2/2)\right)=\end{aligned}$$

$$-\omega_1 2^{(\eta_1+1)/2} V_1^{(\eta_1+1)/2} - 2p_1 V_1 .$$

由此，误差系统 (6) 有限时渐近稳定，且

$$T_1 = \frac{\ln(1 + 2^{(1-\eta_1)/2}(p_1/\omega_1)(V_1(0))^{(1-\eta_1)/2})}{(1-\eta_1)p_1},$$

即前 $N-1$ 个系统(1) 之间的状态变量组合与第 N 个系统(2) 的状态变量之间的有限时同步被实现.

注 1 事实上，在控制方案的设计过程中，由于 $-p_1 e_1^{\mathrm{r}}$，$-p_1 e_1^{\mathrm{c}}$，…，$-p_1 e_n^{\mathrm{r}}$，$-p_1 e_n^{\mathrm{c}}$ 的引入，使得所构造的误差状态变量的衰减速度增加.因此，本文给出的控制方案具有更好的收敛性.

3 数值仿真与分析

考虑如下 3 个复变量混沌系统：

$$\begin{cases} \dot{x}_{11}^{\mathrm{r}} + \mathrm{j}\dot{x}_{11}^{\mathrm{c}} = x_{12}^{\mathrm{r}} x_{13}^{\mathrm{r}} + \mathrm{j} x_{12}^{\mathrm{c}} x_{13}^{\mathrm{r}}, \\ \dot{x}_{12}^{\mathrm{r}} + \mathrm{j}\dot{x}_{12}^{\mathrm{c}} = x_{11}^{\mathrm{r}} - x_{12}^{\mathrm{r}} + \mathrm{j}(x_{11}^{\mathrm{c}} - x_{12}^{\mathrm{c}}), \\ \dot{x}_{13}^{\mathrm{r}} = 1 - x_{11}^{\mathrm{r}} x_{12}^{\mathrm{r}} - x_{11}^{\mathrm{c}} x_{12}^{\mathrm{c}}; \end{cases} \tag{9}$$

$$\begin{cases} \dot{x}_{21}^{\mathrm{r}} + \mathrm{j}\dot{x}_{21}^{\mathrm{c}} = x_{22}^{\mathrm{r}} x_{23}^{\mathrm{r}} + \mathrm{j}(x_{22}^{\mathrm{c}} x_{23}^{\mathrm{r}}), \\ \dot{x}_{22}^{\mathrm{r}} + \mathrm{j}\dot{x}_{22}^{\mathrm{c}} = x_{21}^{\mathrm{r}} - x_{22}^{\mathrm{r}} + \mathrm{j}(x_{21}^{\mathrm{c}} - x_{12}^{\mathrm{c}}), \\ \dot{x}_{23}^{\mathrm{r}} = 1 - x_{21}^{\mathrm{r}} x_{22}^{\mathrm{r}} - x_{11}^{\mathrm{c}} x_{22}^{\mathrm{c}}; \end{cases} \tag{10}$$

$$\begin{cases} \dot{x}_{31}^{\mathrm{r}} + \mathrm{j}\dot{x}_{31}^{\mathrm{c}} = x_{32}^{\mathrm{r}} x_{33}^{\mathrm{r}} + v_{31}^{\mathrm{r}} + \mathrm{j}(x_{32}^{\mathrm{c}} x_{33}^{\mathrm{r}} + v_{31}^{\mathrm{c}}), \\ \dot{x}_{32}^{\mathrm{r}} + \mathrm{j}\dot{x}_{32}^{\mathrm{c}} = x_{31}^{\mathrm{r}} - x_{32}^{\mathrm{r}} + v_{32}^{\mathrm{r}} + \mathrm{j}(x_{31}^{\mathrm{c}} - x_{32}^{\mathrm{c}} + v_{32}^{\mathrm{c}}), \\ \dot{x}_{33}^{\mathrm{r}} = 1 - x_{31}^{\mathrm{r}} x_{32}^{\mathrm{r}} + v_{33}^{\mathrm{r}} - x_{31}^{\mathrm{c}} x_{32}^{\mathrm{c}}, \end{cases} \tag{11}$$

其中，$x_{1l} = x_{1l}^{\mathrm{r}} + \mathrm{j}x_{1l}^{\mathrm{c}}$ 和 $x_{2l} = x_{2l}^{\mathrm{r}} + \mathrm{j}x_{2l}^{\mathrm{c}}$ 是复状态变量，$x_{3l} = x_{3l}^{\mathrm{r}}$ 是实状态变量，$l = 1,2,3$.预设如下的切换规则：

$$\begin{cases} s_{111} = 1,\ s_{112} = s_{113} = 0,\ s_{212} = 1,\ s_{211} = s_{213} = 0, \\ s_{122} = 1,\ s_{121} = s_{123} = 0,\ s_{221} = 1,\ s_{222} = s_{223} = 0, \\ s_{133} = 1,\ s_{131} = s_{132} = 0,\ s_{233} = 1,\ s_{231} = s_{232} = 0. \end{cases} \tag{12}$$

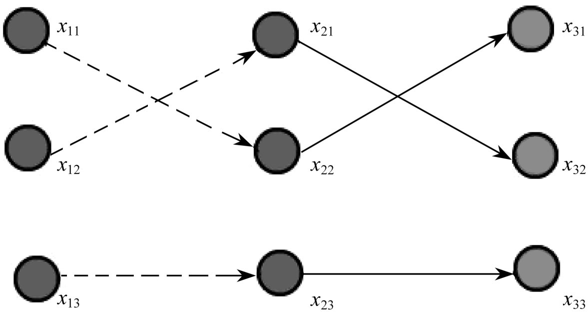

图 1 基于切换规则(12)的多切换传输同步模式

Fig. 1 The diagram for the multi-switching transmission synchronization mode based on rule (12)

图 1 给出了 3 个复混沌系统之间在信号传输过程中的切换规则和组合同步模式，即 $(x_{11}, x_{22}) \rightarrow x_{31}$，$(x_{12}, x_{21}) \rightarrow x_{32}$ 和 $(x_{13}, x_{23}) \rightarrow x_{33}$.从图 1 可以看出，预设的切换规则和建立的

组合同步模式较好地提高了信号传输过程的复杂性.由此，基于切换规则(12)的组合同步误差系统为

$$\begin{cases}\dot{e}_1 = \dot{x}_{31} - \dot{x}_{22} - \dot{x}_{11}, \\ \dot{e}_2 = \dot{x}_{32} - \dot{x}_{21} - \dot{x}_{12}, \\ \dot{e}_3 = \dot{x}_{33} - \dot{x}_{23} - \dot{x}_{13}.\end{cases} \tag{13}$$

针对误差系统(13)，状态初始值为 $\boldsymbol{x}_1(0) = [0.2 + 0.2\mathrm{j}, 0.1 + 0.1\mathrm{j}, 0.1]^{\mathrm{T}}$, $\boldsymbol{x}_2(0) = [0.1 + 0.1\mathrm{j}, 0.3 + 0.2\mathrm{j}, 0.2]^{\mathrm{T}}$ 和 $\boldsymbol{x}_3(0) = [0.5 + 0.6\mathrm{j}, 0.3 + 0.4\mathrm{j}, 0.7]^{\mathrm{T}}$. 由此可得 $\boldsymbol{e}(0) = [0.2 + 0.3\mathrm{j}, -0.1 + 0.1\mathrm{j}, 0.4]^{\mathrm{T}}$.同时，基于所设计的控制方案(7)，相应的参数设定为 $\omega_1 = 1$，$p_1 = 2$，$\eta_1 = 5/7$.图 2 给出了误差系统(13)的状态变化曲线.由图 2 可知，误差系统(13)的状态变量可在有限时间内收敛到 0，实现了有限时渐近稳定，进而可知，3 个复混沌系统(9)~(11)之间实现了多切换传输模式下的有限时组合同步.图 3 给出了基于文献[20-21,25]有限时同步方案设计方法的误差系统(13)的状态变化曲线.通过比较分析，基于本文所给出的同步控制方案，使得所构造误差状态变量的衰减速度增加得更快，给出的控制方案具有更好的收敛性.因此，数值仿真较好地验证了所设计控制方案的可行性和有效性.

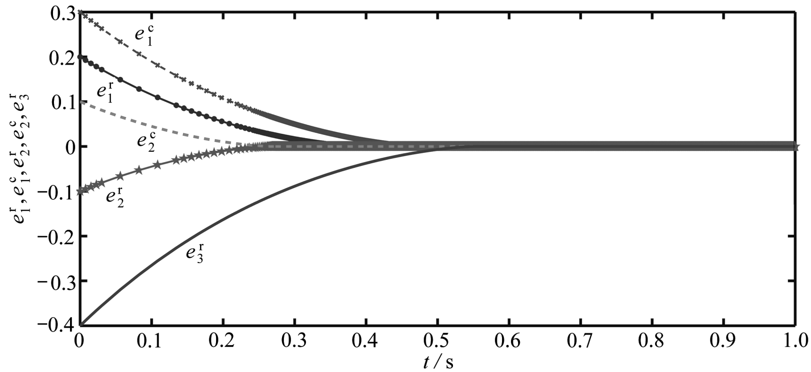

图 2 基于切换规则(12)的误差系统(13)的状态演化

Fig. 2 The state trajectories of error system (13) based on switching rule (12)

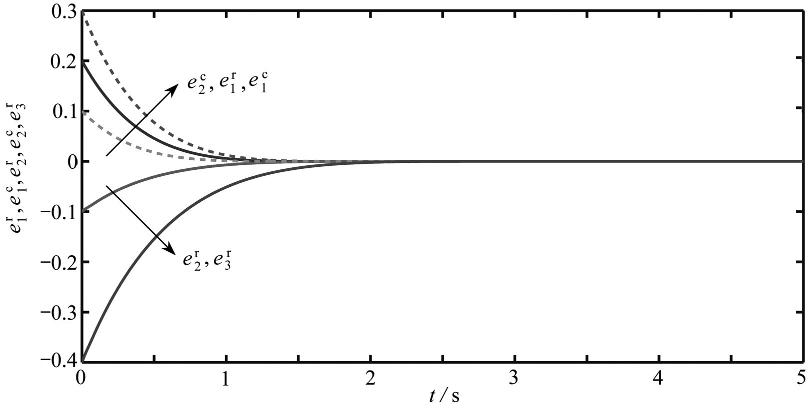

图 3 基于文献[20-21,25]设计方法的误差系统(13)的状态演化

Fig. 3 The state trajectories of error system (13) via the designed method of ref. [20-21,25]

4 结　　论

本文对基于多切换传输的复变量混沌系统的有限时同步控制问题进行了研究，分析了网络信号传输过程中复数域上的混沌系统多切换同步行为，探索了更复杂的同步机制，设计了一类有限时组合同步控制方案，得到了误差系统有限时渐近稳定的充分条件，实现了组合同步误差系统的快速收敛.数值算例较好地展示了所设计控制策略的可行性和有效性.

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Finite-Time Combination Synchronization Control of Complex-Variable Chaotic Systems With Multi-Switching Transmission

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Abstract: The problem of finite-time combination synchronization for a class of complex-variable chaotic systems was investigated. Firstly, for the synchronization mode in signal transmission, the multi-switching synchronization behavior among multiple chaotic systems was analyzed. Secondly, based on the preset switching rules, the definition of finite-time combination synchronization was given. Then, according to the theory of finite-time stability, a kind of controller was designed to realize fast synchronization, and the sufficient conditions were given. Finally, results of numerical simulation and analysis verify the effectiveness of the proposed control scheme.

Key words: complex-variable chaotic system; multi-switching transmission; finite-time control; combination synchronization

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