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正交异性双材料的 II 型界面裂纹尖端场*

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摘要:　通过引入含 16 个待定实系数和两个实应力奇异指数的应力函数, 再借助边界条件, 得到了两个八元非齐次线性方程组• 求解该方程组, 在双材料工程参数满足适当条件下, 确定了两个实应力奇异指数• 根据极限唯一性定理, 求出了全部系数, 得到了应力函数的表示式• 代入相应的力学公式, 推出了当特征方程组两个判别式都小于 0 时, 每种材料的裂纹尖端应力强度因子、应力场和位移场的理论解• 裂纹尖端附近的应力和位移有混合型断裂特征, 但没有振荡奇异性和裂纹面相互嵌入现象• 作为特例, 当两种正交异性材料相同时, 可以推出正交异性单材料 II 型断裂的应力奇异指数、应力强度因子公式、应力场、位移场表示式•

关　键　词:　II 型界面裂纹;　应力强度因子;　双材料;　正交异性
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引　　言

双材料界面附近容易存在缺陷(裂纹、夹杂等), 引发应力集中或裂纹扩展, 导致结合强度的低下• 复合材料及其连接处的缺陷是引起其强度降低的主要原因, 因此, 许多研究者[1- 5]对于界面裂纹的问题进行了大量的研究工作• 但对于界面裂纹问题, 裂纹尖端场应力和位移存在振荡奇异性及裂纹面相互嵌入现象[4- 7], 这在物理上是不合理的, 到现在为止这一问题还没有完全得到解决•

本文将研究单材料平面断裂问题的复变函数法、待定系数法[8]推广到双材料界面裂纹问题, 通过定义含待定实系数的应力函数, 并假设在应力函数的表示式中包含两个应力奇异指数[9- 13] $\lambda_m(m = 1, 2)$, 利用边界条件, 得到两个八元非齐次线性方程组, 由此推出当特征方程组的判别式 $\Delta_1 < 0$, $\Delta_2 < 0$ 时, 每种材料 $j = 1$ 或 $j = 2$ 的裂纹尖端应力强度因子、应力场和位移场, 它们都受到两个应力奇异指数 λ_1, λ_2 的共同影响• 裂纹尖端附近应力和位移没有振荡奇

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异性和裂纹面相互嵌入现象. 作为特例, 当两种正交异性材料相同时, 可以推出正交异性单材料II型断裂的已有结果[8,14].

1 基本方程

设 $y>0$ 部分为第1种正交异性材料 $(j=1)$; $y<0$ 部分为第2种正交异性材料 $(j=2)$; $y=0, |x|<a$ 部分为裂纹面; $y=0, |x|>a$ 部分为双材料粘接界面. x 和 y 为点 M 的直角坐标, r 和 θ 为从裂纹右端点起度量的点 M 的极坐标, 如图1所示. $E_{j1}, E_{j2}, \nu_{j1}, \nu_{j2}, \mu_j (j=1,2)$ 为材料弹性常数. 常数 $E_{j1}, E_{j2}, \nu_{j1}, \nu_{j2}$ 满足 Maxwell 关系:

$$\frac{\nu_{j1}}{E_{j1}}=\frac{\nu_{j2}}{E_{j2}} \quad (j=1,2). \tag{1}$$

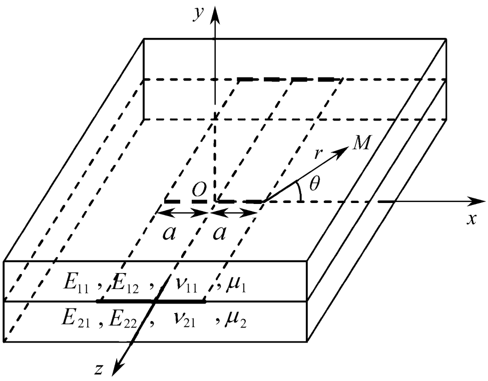

图1 正交异性双材料界面裂纹模型

每种材料的柔度系数 $(b_{11})_j, (b_{12})_j, (b_{22})_j, (b_{66})_j$ 与它的弹性常数 $E_{j1}, E_{j2}, \nu_{j1}, \nu_{j2}, \mu_j$ 之间, 满足

$$\begin{cases} (b_{11})_j=\dfrac{1}{E_{j1}}, \\ (b_{12})_j=(b_{21})_j=-\dfrac{\nu_{j1}}{E_{j1}}, \\ (b_{22})_j=\dfrac{1}{E_{j2}}, \ (b_{66})_j=\dfrac{1}{\mu_j} \end{cases} \quad (j=1,2). \tag{2}$$

设 $U_j (j=1,2)$ 为应力函数, 应力与应力函数之间的关系为[11,15]

$$(\sigma_x)_j=\frac{\partial^2 U_j}{\partial y^2}, \ (\sigma_y)_j=\frac{\partial^2 U_j}{\partial x^2}, \ (\tau_{xy})_j=-\frac{\partial^2 U_j}{\partial x \partial y} \quad (j=1,2). \tag{3}$$

本文讨论平面应力情况下, 正交异性双材料II型界面裂纹问题, 有一位于正交异性双材料之间的II型界面裂纹长度为 $2a$. 在不考虑体力情况下, 正交各向异性双材料的控制方程可表示为[11,14]

$$(b_{22})_j\frac{\partial^4 U_j}{\partial x^4}+[2(b_{12})_j+(b_{66})_j]\frac{\partial^4 U_j}{\partial x^2\partial y^2}+(b_{11})_j\frac{\partial^4 U_j}{\partial y^4}=0 \quad (j=1,2), \tag{4}$$

在边界上满足条件[5,11]:

$$y=0, |x|<a: \quad (\sigma_y)_1=(\sigma_y)_2=0, \ (\tau_{xy})_1=(\tau_{xy})_2=-\tau, \tag{5}$$

$$y=0, |x|>a: \quad (\sigma_y)_1=(\sigma_y)_2, \ (\tau_{xy})_1=(\tau_{xy})_2, \ (u)_1=(u)_2, \ (v)_1=(v)_2, \tag{6}$$

$$\sqrt{x^2+y^2}\to+\infty: \quad (\sigma_y)_1=(\sigma_y)_2=0, \ (\tau_{xy})_1=(\tau_{xy})_2=0, \tag{7}$$

其中, $(u)_j, (v)_j$ 是 x, y 方向上的位移.

2 待定系数法

令 $U_j=U(x+s_j y)$, 可得控制方程(4)的特征方程[8,14]:

$$(b_{11})_j s_j^4+[2(b_{12})_j+(b_{66})_j]s_j^2+(b_{22})_j=0 \quad (j=1,2), \tag{8}$$

这是双二次方程, 其判别式:

$$\Delta_j=\left[\frac{2(b_{12})_j+(b_{66})_j}{(b_{11})_j}\right]^2-4\frac{(b_{22})_j}{(b_{11})_j} \quad (j=1,2). \tag{9}$$

本文仅讨论 $\Delta_1 < 0$ 和 $\Delta_2 < 0$ 的情形. 特征方程(8)的根为

$$s_{jk} = (-1)^{k-1}\alpha_j + \mathrm{i}\beta_j,\ s_{j(k+2)} = (-1)^{k-1}\alpha_j - \mathrm{i}\beta_j \qquad (j, k = 1, 2), \tag{10}$$

其中 $\beta_j > \alpha_j > 0$, 且

$$\alpha_j^2 - \beta_j^2 = -\frac{2(b_{12})_j + (b_{66})_j}{2(b_{11})_j},\ \alpha_j^2 + \beta_j^2 = \sqrt{\frac{(b_{22})_j}{(b_{11})_j}} \qquad (j = 1, 2). \tag{11}$$

设

$$z_{jk} = x + s_{jk}y = x_{jk} + \mathrm{i}y_{jk} \qquad (j, k = 1, 2). \tag{12}$$

利用复变函数、微积分知识及式(12), 可证明

$$\frac{\partial}{\partial x_{jk}} = \frac{\partial}{\partial x},\ \frac{\partial}{\partial y_{jk}} = -\frac{(-1)^{k-1}\alpha_j}{\beta_j}\frac{\partial}{\partial x} + \frac{1}{\beta_j}\frac{\partial}{\partial y}. \tag{13}$$

利用上式及式(12), 将控制方程(4)化为广义重调和方程:

$$\left(\frac{\partial^2}{\partial x_{j1}^2} + \frac{\partial^2}{\partial y_{j1}^2}\right)\left(\frac{\partial^2}{\partial x_{j2}^2} + \frac{\partial^2}{\partial y_{j2}^2}\right) U_j = 0 \qquad (j = 1, 2), \tag{14a}$$

$$\left(\frac{\partial^2}{\partial x_{j2}^2} + \frac{\partial^2}{\partial y_{j2}^2}\right)\left(\frac{\partial^2}{\partial x_{j1}^2} + \frac{\partial^2}{\partial y_{j1}^2}\right) U_j = 0 \qquad (j = 1, 2) \tag{14b}$$

即
$$\nabla_{j1}^2\nabla_{j2}^2 U_j = \nabla_{j2}^2\nabla_{j1}^2 U_j = 0 \qquad (j = 1, 2). \tag{15}$$

由方程(15)和复数表示式(12)易知, 复变量 $z_{jk}(j, k = 1, 2)$ 的解析函数的实部或虚部是控制方程(4)的解[15-16]. 考虑到 $k, m = 1, 2$, 可选取应力函数为含两个实应力奇异指数 λ_1, λ_2 的下列级数:

$$U_j(x, y) = \sum_{m=1}^{2}\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\lambda_m} - \mathrm{i}B_{jk,\lambda_m})U_{jk,\lambda_m}(z_{jk})], \tag{16a}$$

$$U_{jk,\lambda_m}(z_{jk}) = \frac{\tau}{(\lambda_m + 2)(\lambda_m + 1)}(z_{jk} - a)^{\lambda_m + 2} \qquad (z_{jk} \neq a;\ j, k, m = 1, 2), \tag{16b}$$

其中, A_{jk,λ_m}, B_{jk,λ_m} 为待定实系数.

将式(16)代入式(3), 利用式(12), 得到应力表达式如下:

$$(\sigma_x)_j = \sum_{m=1}^{2}\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\lambda_m} - \mathrm{i}B_{jk,\lambda_m})s_{jk}^2 U_{jk,\lambda_m}(z_{jk})], \tag{17a}$$

$$(\sigma_y)_j = \sum_{m=1}^{2}\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\lambda_m} - \mathrm{i}B_{jk,\lambda_m})U_{jk,\lambda_m}(z_{jk})], \tag{17b}$$

$$(\tau_{xy})_j = -\sum_{m=1}^{2}\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\lambda_m} - \mathrm{i}B_{jk,\lambda_m})s_{jk}U_{jk,\lambda_m}(z_{jk})], \tag{17c}$$

$$U_{jk,\lambda_m}(z_{jk}) = \tau(z_{jk} - a)^{\lambda_m} \qquad (j, k, m = 1, 2). \tag{17d}$$

有了应力, 借助应力-应变、应变-位移公式可以推出位移的应力函数表达式.

将应力、位移公式代入边界条件(5)、(6), 注意到当 $\lambda_1 \neq \lambda_2$ 时, r^{λ_1} 与 r^{λ_2} 线性无关, 得到关于系数 $B_{11,\lambda_m}, B_{12,\lambda_m}, A_{11,\lambda_m}, A_{12,\lambda_m}, B_{21,\lambda_m}, B_{22,\lambda_m}, A_{21,\lambda_m}, A_{22,\lambda_m}(m = 1, 2)$ 的两个八元非齐次线性方程组:

$$r^{\lambda_m}[(\sin\lambda_m\pi)B_{11,\lambda_m} + (\sin\lambda_m\pi)B_{12,\lambda_m} + (\cos\lambda_m\pi)A_{11,\lambda_m} + (\cos\lambda_m\pi)A_{12,\lambda_m}] = 0, \tag{18a}$$

$$r^{\lambda_m}[(\sin\lambda_m\pi)B_{21,\lambda_m} + (\sin\lambda_m\pi)B_{22,\lambda_m} - (\cos\lambda_m\pi)A_{21,\lambda_m} - (\cos\lambda_m\pi)A_{22,\lambda_m}] = 0, \tag{18b}$$

$$r^{\lambda_m}[(\beta_1\cos\lambda_m\pi+\alpha_1\sin\lambda_m\pi)B_{11,\lambda_m}+(\beta_1\cos\lambda_m\pi-\alpha_1\sin\lambda_m\pi)B_{12,\lambda_m}+(\alpha_1\cos\lambda_m\pi-\beta_1\sin\lambda_m\pi)A_{11,\lambda_m}-(\alpha_1\cos\lambda_m\pi+\beta_1\sin\lambda_m\pi)A_{12,\lambda_m}]=\frac{1}{2}, \tag{18c}$$

$$r^{\lambda_m}[(\beta_2\cos\lambda_m\pi-\alpha_2\sin\lambda_m\pi)B_{21,\lambda_m}+(\beta_2\cos\lambda_m\pi+\alpha_2\sin\lambda_m\pi)B_{22,\lambda_m}+(\alpha_2\cos\lambda_m\pi+\beta_2\sin\lambda_m\pi)A_{21,\lambda_m}-(\alpha_2\cos\lambda_m\pi-\beta_2\sin\lambda_m\pi)A_{22,\lambda_m}]=\frac{1}{2}, \tag{18d}$$

$$r^{\lambda_m}[A_{11,\lambda_m}+A_{12,\lambda_m}-A_{21,\lambda_m}-A_{22,\lambda_m}]=0, \tag{18e}$$

$$r^{\lambda_m}[\beta_1B_{11,\lambda_m}+\beta_1B_{12,\lambda_m}+\alpha_1A_{11,\lambda_m}-\alpha_1A_{12,\lambda_m}-\beta_2B_{21,\lambda_m}-\beta_2B_{22,\lambda_m}-\alpha_2A_{21,\lambda_m}+\alpha_2A_{22,\lambda_m}]=0, \tag{18f}$$

$$r^{\lambda_m}\left[\frac{2\alpha_1\beta_1}{E_{11}}B_{11,\lambda_m}-\frac{2\alpha_1\beta_1}{E_{11}}B_{12,\lambda_m}+\frac{\alpha_1^2-\beta_1^2-\nu_{11}}{E_{11}}A_{11,\lambda_m}+\frac{\alpha_1^2-\beta_1^2-\nu_{11}}{E_{11}}A_{12,\lambda_m}-\frac{2\alpha_2\beta_2}{E_{21}}B_{21,\lambda_m}+\frac{2\alpha_2\beta_2}{E_{21}}B_{22,\lambda_m}-\frac{\alpha_2^2-\beta_2^2-\nu_{21}}{E_{21}}A_{21,\lambda_m}-\frac{\alpha_2^2-\beta_2^2-\nu_{21}}{E_{21}}A_{22,\lambda_m}\right]=0, \tag{18g}$$

$$\begin{aligned}r^{\lambda_m}\Bigg\{&\beta_1\left[\left(\frac{\beta_1^2-3\alpha_1^2+\nu_{11}}{E_{11}}-\frac{1}{\mu_1}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{11}+1)}{E_{11}}-\frac{1}{\mu_1}\right)\right]B_{11,\lambda_m}+\\&\beta_1\left[\left(\frac{\beta_1^2-3\alpha_1^2+\nu_{11}}{E_{11}}-\frac{1}{\mu_1}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{11}+1)}{E_{11}}-\frac{1}{\mu_1}\right)\right]B_{12,\lambda_m}+\\&\alpha_1\left[\left(\frac{3\beta_1^2-\alpha_1^2+\nu_{11}}{E_{11}}-\frac{1}{\mu_1}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{11}+1)}{E_{11}}-\frac{1}{\mu_1}\right)\right]A_{11,\lambda_m}-\\&\alpha_1\left[\left(\frac{3\beta_1^2-\alpha_1^2+\nu_{11}}{E_{11}}-\frac{1}{\mu_1}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{11}+1)}{E_{11}}-\frac{1}{\mu_1}\right)\right]A_{12,\lambda_m}-\\&\beta_2\left[\left(\frac{\beta_2^2-3\alpha_2^2+\nu_{21}}{E_{21}}-\frac{1}{\mu_2}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{21}+1)}{E_{21}}-\frac{1}{\mu_2}\right)\right]B_{21,\lambda_m}-\\&\beta_2\left[\left(\frac{\beta_2^2-3\alpha_2^2+\nu_{21}}{E_{21}}-\frac{1}{\mu_2}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{21}+1)}{E_{21}}-\frac{1}{\mu_2}\right)\right]B_{22,\lambda_m}-\\&\alpha_2\left[\left(\frac{3\beta_2^2-\alpha_2^2+\nu_{21}}{E_{21}}-\frac{1}{\mu_2}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{21}+1)}{E_{21}}-\frac{1}{\mu_2}\right)\right]A_{21,\lambda_m}+\\&\alpha_2\left[\left(\frac{3\beta_2^2-\alpha_2^2+\nu_{21}}{E_{21}}-\frac{1}{\mu_2}\right)+\frac{1}{\lambda_m}\left(\frac{2(\nu_{21}+1)}{E_{21}}-\frac{1}{\mu_2}\right)\right]A_{22,\lambda_m}\Bigg\}=0,\end{aligned} \tag{18h}$$

其中 $m=1,2$.

利用行列式性质,可求出系数矩阵的行列式为

$$|A_{\lambda_m}|=16r^{8\lambda_m}\alpha_1^2\alpha_2^2\left[\left(\frac{1}{\lambda_m}e_{12}+f_{12}\right)f_{12}+g_{12}h_{12}\cot^2\lambda_m\pi\right]\sin^4\lambda_m\pi, \tag{19}$$

其中, e_{12}, f_{12}, g_{12}, h_{12} 为双材料工程参数, 且

$$e_{12}=\left(\frac{2(\nu_{11}+1)}{E_{11}}-\frac{1}{\mu_1}\right)-\left(\frac{2(\nu_{21}+1)}{E_{21}}-\frac{1}{\mu_2}\right), \tag{20a}$$

$$f_{12}=\frac{\alpha_1^2+\beta_1^2-\nu_{11}}{E_{11}}-\frac{\alpha_2^2+\beta_2^2-\nu_{21}}{E_{21}}, \tag{20b}$$

$$g_{12}=\frac{2\beta_1}{E_{11}}+\frac{2\beta_2}{E_{21}}, \tag{20c}$$

$$h_{12}=(\alpha_1^2+\beta_1^2)\frac{2\beta_1}{E_{11}}+(\alpha_2^2+\beta_2^2)\frac{2\beta_2}{E_{21}}. \tag{20d}$$

为使非齐次线性方程组(18)有解,应满足系数矩阵和增广矩阵的秩相等• 利用初等变换可证明,若非齐次线性方程组(18)的系数矩阵和增广矩阵的秩相等,则其秩必等于7,因而每个非齐次线性方程组有解,且有无穷多解(含一个自由未知量)• 由于 $\operatorname{rank}(A_{\lambda_m}) = 7$, 必有

$$|A_{\lambda_m}| = 16r^{8\lambda_m}\alpha_1^2\alpha_2^2\left[\left(\frac{1}{\lambda_m}e_{12}+f_{12}\right)f_{12}+g_{12}h_{12}\cot^2\lambda_m\pi\right]\sin^4\lambda_m\pi = 0$$

$$(m = 1, 2)\bullet \qquad (21)$$

若 $\sin\lambda_m\pi = 0$, 则 $\lambda_m = n\ (m = 1, 2;\ n = 0, \pm 1, \pm 2, \dots)$, 与双材料工程参数 $e_{12}, f_{12}, g_{12}, h_{12}$ 无关,应舍去•

若

$$\left(\frac{1}{\lambda_m}e_{12}+f_{12}\right)f_{12}+g_{12}h_{12}\cot^2\lambda_m\pi = 0 \qquad (m = 1, 2), \qquad (22)$$

其中含因子 $\cot\lambda_m\pi$, 可选取应力奇异指数为[9-13]

$$\lambda_m = -n-\frac{1}{2}+\varepsilon_m \qquad (m = 1, 2;\ n = 0, 1, 2, \dots), \qquad (23)$$

其中 ε_m 是实双材料弹性常数,而 n 所取的值由边界条件(7)确定•

将式(23)代入式(16),注意到 k, m, n 所取的值,可确定应力函数为下列级数:

$$U_j(x, y) = \sum_{n=0}^{+\infty}\sum_{m=1}^{2}\sum_{k=1}^{2}\operatorname{Re}[(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})U_{jk,\varepsilon_m}(z_{jk})] \qquad (j = 1, 2), \qquad (24a)$$

$$U_{jk,\varepsilon_m}(z_{jk}) = \frac{\tau}{(n-3/2-\varepsilon_m)(n-1/2-\varepsilon_m)(z_{jk}-a)^{n-3/2-\varepsilon_m}} \qquad (j = 1, 2), \qquad (24b)$$

在裂纹尖端附近 $(z_{jk}\to a;\ r\to 0)$, 式(23)中 $n = 0$, 有

$$\lambda_m = -\frac{1}{2}+\varepsilon_m \qquad (m = 1, 2)\bullet \qquad (25)$$

将式(25)代入式(22),得到

$$\left(-\frac{2}{1-2\varepsilon_m}e_{12}+f_{12}\right)f_{12}+g_{12}h_{12}\tan^2\varepsilon_m\pi = 0 \qquad (m = 1, 2)\bullet \qquad (26)$$

已知函数 $1/(1-2\varepsilon_m)$ 和 $\tan\varepsilon_m\pi$ 的幂级数展开式是

$$\frac{1}{1-2\varepsilon_m} = 1+2\varepsilon_m+4\varepsilon_m^2+\cdots \qquad \left(|\varepsilon_m| < \frac{1}{2}\right), \qquad (27a)$$

$$\tan\varepsilon_m\pi = \pi\varepsilon_m+\frac{1}{3}\pi^3\varepsilon_m^3+\cdots \qquad \left(|\varepsilon_m| < \frac{1}{2}\right)\bullet \qquad (27b)$$

将式(27)代入式(26),略去 ε_m 的三阶及三阶以上的充分小量,得到

$$(g_{12}h_{12}\pi^2-8e_{12}f_{12})\varepsilon_m^2-4e_{12}f_{12}\varepsilon_m-(2e_{12}-f_{12})f_{12} = 0 \qquad (m = 1, 2)\bullet \qquad (28)$$

解此一元二次方程,当判别式

$$\Delta = 4f_{12}[(2e_{12}-f_{12})g_{12}h_{12}\pi^2-4(3e_{12}-2f_{12})e_{12}f_{12}] > 0 \qquad (29)$$

时,得到两个实根:

$$\varepsilon_m = \frac{2e_{12}f_{12}+(-1)^{m-1}\sqrt{f_{12}[(2e_{12}-f_{12})g_{12}h_{12}\pi^2-4(3e_{12}-4f_{12})e_{12}f_{12}]}}{g_{12}h_{12}\pi^2-8e_{12}f_{12}}$$

$$(m = 1, 2)\bullet \qquad (30)$$

对于正交异性双材料II型界面裂纹问题,只有当双材料工程参数满足条件(29)时,可由式(30)求出两个实双材料弹性常数 $\varepsilon_1, \varepsilon_2$• 将 $\varepsilon_1, \varepsilon_2$ 代入式(25),得到两个实应力奇异指数 λ_1, λ_2•

采用顺序消元法求解非齐次线性方程组(18),考虑到系数由边界条件(5)、(6)确定,求出系数后 r^{λ_m} 作为因子并入应力函数里面,所以得到16个系数的求解公式如下:

$$B_{11,\varepsilon_m}=\frac{\alpha_2}{\alpha_1}[f_{12}-g_{12}(\beta_1+\alpha_1\tan\varepsilon_m\pi)](\tan\varepsilon_m\pi)a_{22,\varepsilon_m},\tag{31a}$$

$$B_{12,\varepsilon_m}=-\frac{\alpha_2}{\alpha_1}[f_{12}-g_{12}(\beta_1-\alpha_1\tan\varepsilon_m\pi)](\tan\varepsilon_m\pi)a_{22,\varepsilon_m},\tag{31b}$$

$$A_{11,\varepsilon_m}=\frac{\alpha_2}{\alpha_1}[f_{12}-g_{12}(\alpha_1-\beta_1\tan\varepsilon_m\pi)\tan\varepsilon_m\pi]a_{22,\varepsilon_m}+\frac{1}{4\alpha_1\sin\varepsilon_m\pi},\tag{31c}$$

$$A_{12,\varepsilon_m}=-\frac{\alpha_2}{\alpha_1}[f_{12}+g_{12}(\alpha_1+\beta_1\tan\varepsilon_m\pi)\tan\varepsilon_m\pi]a_{22,\varepsilon_m}-\frac{1}{4\alpha_1\sin\varepsilon_m\pi},\tag{31d}$$

$$B_{21,\varepsilon_m}=-[f_{12}+g_{12}(\beta_2-\alpha_2\tan\varepsilon_m\pi)](\tan\varepsilon_m\pi)a_{22,\varepsilon_m},\tag{31e}$$

$$B_{22,\varepsilon_m}=[f_{12}+g_{12}(\beta_2+\alpha_2\tan\varepsilon_m\pi)](\tan\varepsilon_m\pi)a_{22,\varepsilon_m},\tag{31f}$$

$$A_{21,\varepsilon_m}=[f_{12}-g_{12}(\alpha_2+\beta_2\tan\varepsilon_m\pi)\tan\varepsilon_m\pi]a_{22,\varepsilon_m}+\frac{1}{4\alpha_2\sin\varepsilon_m\pi},\tag{31g}$$

$$A_{22,\varepsilon_m}=-[f_{12}+g_{12}(\alpha_2-\beta_2\tan\varepsilon_m\pi)\tan\varepsilon_m\pi]a_{22,\varepsilon_m}-\frac{1}{4\alpha_2\sin\varepsilon_m\pi},\tag{31h}$$

其中 a_{22,ε_m} 是自由未知量.

3 应力强度因子

将式(25)代入式(17)中,若记

$$(\sigma_x)_{j,\varepsilon_m}=\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})s_{jk}^2U_{jk,\varepsilon_m}(z_{jk})],\tag{32a}$$

$$(\sigma_y)_{j,\varepsilon_m}=\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})U_{jk,\varepsilon_m}(z_{jk})],\tag{32b}$$

$$(\tau_{xy})_{j,\varepsilon_m}=-\sum_{k=1}^{2}\mathrm{Re}[(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})s_{jk}U_{jk,\varepsilon_m}(z_{jk})],\tag{32c}$$

$$U_{jk,\varepsilon_m}(z_{jk})=\frac{\tau}{(z_{jk}-a)^{1/2-\varepsilon_m}}\quad(j,m=1,2),\tag{32d}$$

则式(17)可简记为

$$(\sigma_x)_j=\sum_{m=1}^{2}(\sigma_x)_{j,\varepsilon_m},\quad(\sigma_y)_j=\sum_{m=1}^{2}(\sigma_y)_{j,\varepsilon_m},\quad(\tau_{xy})_j=\sum_{m=1}^{2}(\tau_{xy})_{j,\varepsilon_m}.\tag{33}$$

通过观察式(32)、(33),引入应力强度因子如下:

$$(K_{\mathrm{II}})_j=-\sum_{m=1}^{2}\sum_{k=1}^{2}\lim_{z_{jk}\to a}\mathrm{Re}[(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})s_{jk}(-1)^jU_{jk,\varepsilon_m}(z_{jk})],\tag{34a}$$

$$(K_{\mathrm{I}})_j=\sum_{m=1}^{2}\sum_{k=1}^{2}\lim_{z_{jk}\to a}\mathrm{Re}[(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})(-1)^jU_{jk,\varepsilon_m}(z_{jk})],\tag{34b}$$

$$(K)_j=(K_{\mathrm{I}})_j-\mathrm{i}(K_{\mathrm{II}})_j\quad(j=1,2).\tag{34c}$$

考虑到两个裂纹尖端 $z_{jk}=a$ 和 $z_{jk}=-a$ 处都存在应力奇异性,在式(34)中选取

$$U_{jk,\varepsilon_m}(z_{jk})=\frac{\tau z_{jk}^{1-2\varepsilon_m}}{(z_{jk}-a)^{1/2-\varepsilon_m}(z_{jk}+a)^{1/2-\varepsilon_m}}\quad(j,k,m=1,2).\tag{35}$$

在裂纹尖端 $z_{jk}=a$ 附近,有

$$U_{jk,\varepsilon_m}(z_{jk}) = \left(\frac{a}{2}\right)^{1/2-\varepsilon_m}\frac{\tau}{(z_{jk}-a)^{1/2-\varepsilon_m}} \quad (z_{jk}\to a;\ j,k,m=1,2)\bullet \tag{36}$$

将式(32d)与式(36)对照,式(36)多一个常数因子• 考虑到系数公式(31)中有一个自由未知量 a_{22,ε_m}, 所以在应力表示式(33),(32)中,仍可选取系数与式(31)相同•

充分考虑到边界条件,取

$$(-1)^{1/2} = \mathrm{i},\ (-1)^{\varepsilon_m} = [\mathrm{e}^{\mathrm{i}(\pm\pi)}]^{\varepsilon_m} = \cos\varepsilon_m\pi + \mathrm{i}(-1)^{j-1}\sin\varepsilon_m\pi\bullet \tag{37}$$

由式(35)、(37)、(12)推出

$$\lim_{z_{jk}\to a^-}[(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}U_{jk,\varepsilon_m}(z_{jk})] = \tau(\pi a)^{1/2-\varepsilon_m}[\sin\varepsilon_m\pi + \mathrm{i}(-1)^j\cos\varepsilon_m\pi], \tag{38a}$$

$$\lim_{z_{jk}\to a^+}[(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}U_{jk,\varepsilon_m}(z_{jk})] = \tau(\pi a)^{1/2-\varepsilon_m}\bullet \tag{38b}$$

根据极限的唯一性定理,当 $z_{jk}\to a^-$ 和 $z_{jk}\to a^+$ 时,取得相同的极限 $(K_{\mathrm{II}})_j$, 由式(34a)有

$$\sum_{m=1}^{2}\sum_{k=1}^{2}\lim_{z_{jk}\to a^-}\mathrm{Re}\left\{(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}[-(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})s_{jk}U_{jk,\varepsilon_m}(z_{jk})]\right\} =$$
$$\sum_{m=1}^{2}\sum_{k=1}^{2}\lim_{z_{jk}\to a^+}\mathrm{Re}\left\{(2\pi|z_{jk}-a|)^{1/2-\varepsilon_m}[-(A_{jk,\varepsilon_m}-\mathrm{i}B_{jk,\varepsilon_m})s_{jk}U_{jk,\varepsilon_m}(z_{jk})]\right\}\bullet \tag{39}$$

将式(38)、(10)代入式(39),得到

$$[1+(-1)^j\sin\varepsilon_m\pi][\alpha_j(A_{j1,\varepsilon_m}-A_{j2,\varepsilon_m})+\beta_j(B_{j1,\varepsilon_m}+B_{j2,\varepsilon_m})] -$$
$$(\cos\varepsilon_m\pi)[\beta_j(A_{j1,\varepsilon_m}+A_{j2,\varepsilon_m})-\alpha_j(B_{j1,\varepsilon_m}-B_{j2,\varepsilon_m})] = 0$$
$$(j,m=1,2)\bullet \tag{40}$$

在上式中令 $j=1$, 将式(31a)~(31d)代入式(40),求出

$$a^{(1)}_{22,\varepsilon_m} = -\frac{1-\sin\varepsilon_m\pi}{4\alpha_2 f_{12}\sin\varepsilon_m\pi}\bullet \tag{41}$$

将式(41)代入式(31a)~(31d),推出8个系数公式如下:

$$B_{11,\varepsilon_m} = -\frac{[f_{12}-g_{12}(\beta_1+\alpha_1\tan\varepsilon_m\pi)](1-\sin\varepsilon_m\pi)}{4\alpha_1 f_{12}\cos\varepsilon_m\pi}, \tag{42a}$$

$$B_{12,\varepsilon_m} = \frac{[f_{12}-g_{12}(\beta_1-\alpha_1\tan\varepsilon_m\pi)](1-\sin\varepsilon_m\pi)}{4\alpha_1 f_{12}\cos\varepsilon_m\pi}, \tag{42b}$$

$$A_{11,\varepsilon_m} = \frac{f_{12}\cos\varepsilon_m\pi+g_{12}(\alpha_1-\beta_1\tan\varepsilon_m\pi)(1-\sin\varepsilon_m\pi)}{4\alpha_1 f_{12}\cos\varepsilon_m\pi}, \tag{42c}$$

$$A_{12,\varepsilon_m} = -\frac{f_{12}\cos\varepsilon_m\pi-g_{12}(\alpha_1+\beta_1\tan\varepsilon_m\pi)(1-\sin\varepsilon_m\pi)}{4\alpha_1 f_{12}\cos\varepsilon_m\pi} \quad (m=1,2)\bullet \tag{42d}$$

将式(38b)、(10)代入式(34a)、(34b),得到

$$(K_{\mathrm{II}})_j = \sum_{m=1}^{2}\tau(\pi a)^{1/2-\varepsilon_m}[\alpha_j(A_{j1,\varepsilon_m}-A_{j2,\varepsilon_m})+\beta_j(B_{j1,\varepsilon_m}+B_{j2,\varepsilon_m})], \tag{43a}$$

$$(K_{\mathrm{I}})_j = (-1)^j\sum_{m=1}^{2}\tau(\pi a)^{1/2-\varepsilon_m}(A_{j1,\varepsilon_m}+A_{j2,\varepsilon_m})\bullet \tag{43b}$$

将式(42)代入式(43),得到材料 $j=1$ 的应力强度因子公式如下:

$$(K_{\mathrm{II}})_1 = \sum_{m=1}^{2}\frac{\tau(\pi a)^{1/2-\varepsilon_m}}{2} = \sum_{m=1}^{2}(K_{\mathrm{II}})_{1,\varepsilon_m}, \tag{44a}$$

$$(K_{\mathrm{I}})_1 = -\sum_{m=1}^{2}\frac{\tau(\pi a)^{1/2-\varepsilon_m}g_{12}(1-\sin\varepsilon_m\pi)}{2f_{12}\cos\varepsilon_m\pi} = \sum_{m=1}^{2}(K_{\mathrm{I}})_{1,\varepsilon_m}\bullet \tag{44b}$$

同理可求出材料 $j=2$ 的系数和应力强度因子公式如下:

$$B_{21,\varepsilon_m}=\frac{[f_{12}+g_{12}(\beta_2-\alpha_2\tan\varepsilon_m\pi)](1+\sin\varepsilon_m\pi)}{4\alpha_2 f_{12}\cos\varepsilon_m\pi}, \tag{45a}$$

$$B_{22,\varepsilon_m}=-\frac{[f_{12}+g_{12}(\beta_2+\alpha_2\tan\varepsilon_m\pi)](1+\sin\varepsilon_m\pi)}{4\alpha_2 f_{12}\cos\varepsilon_m\pi}, \tag{45b}$$

$$A_{21,\varepsilon_m}=-\frac{f_{12}\cos\varepsilon_m\pi-g_{12}(\alpha_2+\beta_2\tan\varepsilon_m\pi)(1+\sin\varepsilon_m\pi)}{4\alpha_2 f_{12}\cos\varepsilon_m\pi}, \tag{45c}$$

$$A_{22,\varepsilon_m}=\frac{f_{12}\cos\varepsilon_m\pi+g_{12}(\alpha_2-\beta_2\tan\varepsilon_m\pi)(1+\sin\varepsilon_m\pi)}{4\alpha_2 f_{12}\cos\varepsilon_m\pi}\qquad(m=1,2), \tag{45d}$$

$$(K_{\mathrm{II}})_2=\sum_{m=1}^{2}\frac{\tau(\pi a)^{1/2-\varepsilon_m}}{2}=\sum_{m=1}^{2}(K_{\mathrm{II}})_{2,\varepsilon_m}, \tag{46a}$$

$$(K_{\mathrm{I}})_2=\sum_{m=1}^{2}\frac{\tau(\pi a)^{1/2-\varepsilon_m}g_{12}(1+\sin\varepsilon_m\pi)}{2f_{12}\cos\varepsilon_m\pi}=\sum_{m=1}^{2}(K_{\mathrm{I}})_{2,\varepsilon_m}. \tag{46b}$$

4 裂纹尖端场

由式(35) 易知, 在裂纹尖端 $z_{jk}=a$ 附近, 有

$$\lim_{z_{jk}\to a}\left\{[2\pi(z_{jk}-a)]^{1/2-\varepsilon_m}U_{jk,\varepsilon_m}(z_{jk})\right\}=\tau(\pi a)^{1/2-\varepsilon_m}\qquad(m=1,2), \tag{47}$$

由此看到

$$U_{jk,\varepsilon_m}(z_{jk})=\frac{\tau(\pi a)^{1/2-\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(z_{jk}-a)^{1/2-\varepsilon_m}}, \tag{48a}$$

$$\frac{1}{(z_{jk}-a)^{1/2-\varepsilon_m}}=\mathrm{Re}\frac{1}{(z_{jk}-a)^{1/2-\varepsilon_m}}+\mathrm{i}\,\mathrm{Im}\frac{1}{(z_{jk}-a)^{1/2-\varepsilon_m}}$$
$$(z_{jk}\to a;\ j,k,m=1,2). \tag{48b}$$

令 $j=1$, 将式(48)、(42)、(44)、(40) 代入式(32)、(33), 推出第 1 种正交异性材料 $j=1$ 的 II型界面裂纹尖端附近 $(z_{1k}\to a)$ 的应力表示式为

$$\begin{aligned}(\sigma_x)_1=&\sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_1}\left\{-(\alpha_1^2-\beta_1^2)\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\right.\\&2\alpha_1\beta_1\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\\&(\alpha_1^2-\beta_1^2)\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\\&\left.2\alpha_1\beta_1\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\right\}+\\&\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{\alpha_1^2+\beta_1^2}{2\alpha_1}\left\{-\beta_1(\tan\varepsilon_m\pi)\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\right.\\&\alpha_1\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]-\\&\beta_1\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\\&\left.\alpha_1(\tan\varepsilon_m\pi)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\right\},\end{aligned} \tag{49a}$$

$$(\sigma_y)_1 = \sum_{m=1}^{2}\frac{(K_{\rm II})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_1}\Bigg\{-\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+ \left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}+ \sum_{m=1}^{2}\frac{(K_{\rm I})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_1}\Bigg\{-\beta_1(\tan\varepsilon_m\pi)\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+ \alpha_1\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+ \beta_1\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+ \alpha_1(\tan\varepsilon_m\pi)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}, \tag{49b}$$

$$(\tau_{xy})_1 = \sum_{m=1}^{2}\frac{(K_{\rm II})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_1}\Bigg\{-\beta_1\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+\alpha_1\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]- \beta_1\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]- \alpha_1\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}- \sum_{m=1}^{2}\frac{(K_{\rm I})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{\alpha_1^2+\beta_1^2}{2\alpha_1}\Bigg\{\mathrm{Re}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]+ (\tan\varepsilon_m\pi)\mathrm{Im}\left[\frac{1}{(z_{11}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{12}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}. \tag{49c}$$

借助有关公式[11,14-15]，可以得到材料 $j=1$ 的II型界面裂纹尖端附近的位移表示式为

$$(u)_1 = \sum_{m=1}^{2}\frac{(K_{\rm II})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_1}\times \Bigg\{-\frac{\alpha_1^2-\beta_1^2-\nu_{11}}{E_{11}}\mathrm{Re}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]+ \frac{2\alpha_1\beta_1}{E_{11}}\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Re}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]+ \frac{\alpha_1^2-\beta_1^2-\nu_{11}}{E_{11}}\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\mathrm{Im}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]+ \frac{2\alpha_1\beta_1}{E_{11}}\mathrm{Im}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]\Bigg\}+ \sum_{m=1}^{2}\frac{(K_{\rm I})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{\alpha_1^2+\beta_1^2+\nu_{11}}{(1+2\varepsilon_m)\alpha_1E_{11}}\Bigg\{\beta_1(\tan\varepsilon_m\pi)\mathrm{Re}[(z_{11}-a)^{1/2+\varepsilon_m}- (z_{12}-a)^{1/2+\varepsilon_m}]+\alpha_1\mathrm{Re}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]- \beta_1\mathrm{Im}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]+ \alpha_1(\tan\varepsilon_m\pi)\mathrm{Im}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]\Bigg\}, \tag{50a}$$

$$(v)_1 = \sum_{m=1}^{2}\frac{(K_{\rm II})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_1(\alpha_1^2+\beta_1^2)}\Bigg\{-\alpha_1\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)+\frac{1}{E_{12}}\right]\times$$

$$\operatorname{Re}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]+\beta_1\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)-\frac{1}{E_{12}}\right]\times$$
$$\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\operatorname{Re}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]+$$
$$\alpha_1\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)+\frac{1}{E_{12}}\right]\left(\frac{1}{\cos\varepsilon_m\pi}-\tan\varepsilon_m\pi\right)\times$$
$$\operatorname{Im}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]+$$
$$\beta_1\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)-\frac{1}{E_{12}}\right]\operatorname{Im}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]\Bigg\}+$$
$$\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{1,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_1(\alpha_1^2+\beta_1^2)}\times$$
$$\Bigg\{-\frac{2\alpha_1\beta_1}{E_{11}}(\tan\varepsilon_m\pi)\operatorname{Re}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]+$$
$$\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)^2+\frac{1}{E_{12}}(\alpha_1^2-\beta_1^2)\right]\operatorname{Re}[(z_{11}-a)^{1/2+\varepsilon_m}-(z_{12}-a)^{1/2+\varepsilon_m}]+$$
$$\frac{2\alpha_1\beta_1}{E_{11}}\operatorname{Im}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]+\left[-\frac{\nu_{11}}{E_{11}}(\alpha_1^2+\beta_1^2)^2+\right.$$
$$\left.\frac{1}{E_{12}}(\alpha_1^2-\beta_1^2)\right](\tan\varepsilon_m\pi)\operatorname{Im}[(z_{11}-a)^{1/2+\varepsilon_m}+(z_{12}-a)^{1/2+\varepsilon_m}]\Bigg\}\cdot \tag{50b}$$

同理,得到第2种正交异性材料$j=2$的II型界面裂纹尖端附近$(z_{2k}\to a)$的应力和位移表示式如下:

$$(\sigma_x)_2=\sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_2}\Bigg\{-(\alpha_2^2-\beta_2^2)\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$
$$2\alpha_2\beta_2\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$
$$(\alpha_2^2-\beta_2^2)\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\operatorname{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$
$$2\alpha_2\beta_2\operatorname{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}+\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{(\alpha_2^2+\beta_2^2)}{2\alpha_2}\times$$
$$\Bigg\{\beta_2(\tan\varepsilon_m\pi)\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$
$$\alpha_2\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]-$$
$$\beta_2\operatorname{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]-$$
$$\alpha_2(\tan\varepsilon_m\pi)\operatorname{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}, \tag{51a}$$

$$(\sigma_y)_2=\sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_2}\Bigg\{-\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$
$$\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\operatorname{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]\Bigg\}+$$
$$\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_2}\Bigg\{\beta_2(\tan\varepsilon_m\pi)\operatorname{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+$$

$$
\begin{aligned}
&\alpha_2\mathrm{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+\\
&\beta_2\mathrm{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]-\\
&\alpha_2(\tan\varepsilon_m\pi)\mathrm{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]\Bigg\},
\end{aligned}
\tag{51b}
$$

$$
\begin{aligned}
(\tau_{xy})_2 =& \sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{2\alpha_2}\Bigg\{-\beta_2\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\mathrm{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\right.\\
&\left.\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+\alpha_2\mathrm{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]-\\
&\beta_2\mathrm{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]-\\
&\alpha_2\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\mathrm{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}+\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+\\
&\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{\alpha_2^2+\beta_2^2}{2\alpha_2}\Bigg\{-\mathrm{Re}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]+\\
&(\tan\varepsilon_m\pi)\mathrm{Im}\left[\frac{1}{(z_{21}-a)^{1/2-\varepsilon_m}}-\frac{1}{(z_{22}-a)^{1/2-\varepsilon_m}}\right]\Bigg\},
\end{aligned}
\tag{51c}
$$

$$
\begin{aligned}
(u)_2 =& \sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_2}\times\\
&\Bigg\{-\frac{\alpha_2^2-\beta_2^2-\nu_{21}}{E_{21}}\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\frac{2\alpha_2\beta_2}{E_{21}}\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\frac{\alpha_2^2-\beta_2^2-\nu_{21}}{E_{21}}\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\frac{2\alpha_2\beta_2}{E_{21}}\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]\Bigg\}+\\
&\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_2}\Bigg\{-\frac{\beta_2(\alpha_2^2+\beta_2^2+\nu_{21})}{E_{21}}(\tan\varepsilon_m\pi)\times\\
&\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\frac{\alpha_2(\alpha_2^2+\beta_2^2-\nu_{21})}{E_{21}}\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]-\\
&\frac{\beta_2(\alpha_2^2+\beta_2^2+\nu_{21})}{E_{21}}\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]-\\
&\frac{\alpha_2(\alpha_2^2+\beta_2^2-\nu_{21})}{E_{21}}(\tan\varepsilon_m\pi)\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]\Bigg\},
\end{aligned}
\tag{52a}
$$

$$
\begin{aligned}
(v)_2 =& \sum_{m=1}^{2}\frac{(K_{\mathrm{II}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_2(\alpha_2^2+\beta_2^2)}\Bigg\{-\alpha_2\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)+\frac{1}{E_{22}}\right]\times\\
&\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\beta_2\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)-\frac{1}{E_{22}}\right]\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\times
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]+\alpha_2\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)+\frac{1}{E_{22}}\right]\times\\
&\left(\frac{1}{\cos\varepsilon_m\pi}+\tan\varepsilon_m\pi\right)\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\beta_2\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)-\frac{1}{E_{22}}\right]\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]\Big\}+\\
&\sum_{m=1}^{2}\frac{(K_{\mathrm{I}})_{2,\varepsilon_m}}{(2\pi)^{1/2-\varepsilon_m}}\frac{1}{(1+2\varepsilon_m)\alpha_2(\alpha_2^2+\beta_2^2)}\times\\
&\Big\{\frac{2\alpha_2\beta_2}{E_{22}}(\tan\varepsilon_m\pi)\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)^2+\frac{1}{E_{22}}(\alpha_2^2-\beta_2^2)\right]\mathrm{Re}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]+\\
&\frac{2\alpha_2\beta_2}{E_{22}}\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}+(z_{22}-a)^{1/2+\varepsilon_m}]-\\
&\left[-\frac{\nu_{21}}{E_{21}}(\alpha_2^2+\beta_2^2)^2+\frac{1}{E_{22}}(\alpha_2^2-\beta_2^2)\right]\times\\
&\mathrm{Im}[(z_{21}-a)^{1/2+\varepsilon_m}-(z_{22}-a)^{1/2+\varepsilon_m}]\Big\}\cdot
\end{aligned}\tag{52b}
$$

5 特 例

若两种正交异性材料 $j=1, j=2$ 相同时, 有

$$E_{jk}=E_k,\ \nu_{jk}=\nu_k,\ \mu_j=\mu\qquad(j,k=1,2),\tag{53a}$$

$$(b_{11})_j=\frac{1}{E_1}=b_{11},\ (b_{12})_j=-\frac{\nu_1}{E_1}=b_{12},\ (b_{22})_j=\frac{1}{E_2}=b_{22},\tag{53b}$$

$$\alpha_j=\alpha,\ \beta_j=\beta\qquad(j=1,2),\tag{54}$$

$$
\begin{aligned}
z_{jk}-a&=\mathrm{Re}(z_{jk}-a)+\mathrm{i}\,\mathrm{Im}(z_{jk}-a)=\\
&\mathrm{Re}(z_k-a)+\mathrm{i}(-1)^{j-1}\mathrm{Im}(z_k-a),
\end{aligned}\tag{55a}
$$

$$\frac{1}{(z_{jk}-a)^{1/2-\varepsilon_m}}=\mathrm{Re}\frac{1}{(z_k-a)^{1/2-\varepsilon_m}}+\mathrm{i}(-1)^{j-1}\mathrm{Im}\frac{1}{(z_k-a)^{1/2-\varepsilon_m}},\tag{55b}$$

$$(z_{jk}-a)^{1/2+\varepsilon_m}=\mathrm{Re}(z_k-a)^{1/2+\varepsilon_m}+\mathrm{i}(-1)^{j}\mathrm{Im}(z_k-a)^{1/2+\varepsilon_m}\cdot\tag{55c}$$

将式(53)、(54)先代入式(20), 再代入式(21)、(22)、(30)、(23)和(25), 得到

$$|A_{\lambda_m}|=256r^{8\lambda_m}\alpha^4\beta^2\frac{\alpha^2+\beta^2}{E_1^2}\cot^2\lambda_m\pi\sin^4\lambda_m\pi=0\qquad(m=1,2),\tag{56}$$

$$16r^{8\lambda_m}\beta^2\frac{\alpha^2+\beta^2}{E_1^2}\cot^2\lambda_m\pi=0\qquad(m=1,2),\tag{57}$$

$$\varepsilon_1=\varepsilon_2=0,\tag{58}$$

$$\lambda_m=-n-\frac{1}{2}\qquad(m=1,2;\ n=0,1,2,\cdots),\tag{59}$$

$$\lambda_1=\lambda_2=-\frac{1}{2},\tag{60}$$

其中式(60)与正交异性单材料II型断裂的应力奇异指数 $\lambda=-1/2$ 是吻合的•

考虑到两个应力奇异指数和每种材料 $j=1$ 或 $j=2$ 的断裂性态对单材料的协同影响, 正交异性单材料的应力强度因子确定如下:

$$K_{\mathrm{II}} = \frac{[(K_{\mathrm{II}})_1 + (K_{\mathrm{II}})_2]_{\varepsilon_m = 0}}{2} = \sum_{m=1}^{2} \lim_{\varepsilon_m \to 0} [(K_{\mathrm{II}})_{1,\varepsilon_m} + (K_{\mathrm{II}})_{2,\varepsilon_m}]/2 = \tau(\pi a)^{1/2}, \tag{61a}$$

$$K_{\mathrm{I}} = \frac{[(K_{\mathrm{I}})_1 + (K_{\mathrm{I}})_2]_{\varepsilon_m = 0}}{2} = \sum_{m=1}^{2} \lim_{\varepsilon_m \to 0} [(K_{\mathrm{I}})_{1,\varepsilon_m} + (K_{\mathrm{I}})_{2,\varepsilon_m}]/2 = 0, \tag{61b}$$

这与正交异性单材料II型裂纹的应力强度因子相同•

鉴于每种材料 $j = 1$ 或 $j = 2$ 的应力 $(\sigma_x)_j, (\sigma_y)_j, (\tau_{xy})_j$; 位移 $(u)_j, (v)_j$ 和两个应力奇异指数 λ_1, λ_2 对单材料的应力和位移的协同影响, 利用式(49) ~ (52), 注意到式(44)、(46)、(54)、(55), 推出与文献[8, 14]相同的正交异性单材料II型裂纹尖端附近 $(z_k \to a)$ 的应力、位移表示式如下:

$$\sigma_x = \frac{[(\sigma_x)_1 + (\sigma_x)_2]_{\varepsilon_m = 0}}{2} = \sum_{m=1}^{2} \lim_{\varepsilon_m \to 0} [(\sigma_x)_{1,\varepsilon_m} + (\sigma_x)_{2,\varepsilon_m}]/2 = \frac{K_{\mathrm{II}}}{\sqrt{2}} \frac{1}{2\alpha} \left\{ (\alpha^2 - \beta^2) \mathrm{Re}\left[\frac{1}{(z_2 - a)^{1/2}} - \frac{1}{(z_1 - a)^{1/2}} \right] + 2\alpha\beta \mathrm{Im}\left[\frac{1}{(z_2 - a)^{1/2}} + \frac{1}{(z_1 - a)^{1/2}} \right] \right\}, \tag{62a}$$

$$\sigma_y = \frac{[(\sigma_y)_1 + (\sigma_y)_2]_{\varepsilon_m = 0}}{2} = \sum_{m=1}^{2} \lim_{\varepsilon_m \to 0} [(\sigma_y)_{1,\varepsilon_m} + (\sigma_y)_{2,\varepsilon_m}]/2 = \frac{K_{\mathrm{II}}}{\sqrt{2}} \frac{1}{2\alpha} \mathrm{Re}\left[\frac{1}{(z_2 - a)^{1/2}} - \frac{1}{(z_1 - a)^{1/2}} \right], \tag{62b}$$

$$\tau_{xy} = \frac{[(\tau_{xy})_1 + (\tau_{xy})_2]_{\varepsilon_m = 0}}{2} = \sum_{m=1}^{2} \lim_{\varepsilon_m \to 0} [(\tau_{xy})_{1,\varepsilon_m} + (\tau_{xy})_{2,\varepsilon_m}]/2 = \frac{K_{\mathrm{II}}}{\sqrt{2}} \frac{1}{2\alpha} \left\{ \alpha \mathrm{Re}\left[\frac{1}{(z_1 - a)^{1/2}} + \frac{1}{(z_2 - a)^{1/2}} \right] + \beta \mathrm{Im}\left[\frac{1}{(z_2 - a)^{1/2}} - \frac{1}{(z_1 - a)^{1/2}} \right] \right\}, \tag{62c}$$

$$u = \frac{[(u)_1 + (u)_2]_{\varepsilon_m = 0}}{2} = \frac{K_{\mathrm{II}}}{\sqrt{2}} \frac{1}{\alpha} \left\{ [b_{11}(\alpha^2 - \beta^2) + b_{12}] \times \mathrm{Re}[(z_2 - a)^{1/2} - (z_1 - a)^{1/2}] + 2b_{11}\alpha\beta \mathrm{Im}[(z_1 - a)^{1/2} + (z_2 - a)^{1/2}] \right\}, \tag{63a}$$

$$v = \frac{[(v)_1 + (v)_2]_{\varepsilon_m = 0}}{2} = \frac{K_{\mathrm{II}}}{\sqrt{2}} \frac{1}{\alpha(\alpha^2 + \beta^2)} \times \left\{ -\alpha[b_{12}(\alpha^2 + \beta^2) + b_{22}] \mathrm{Re}[(z_1 - a)^{1/2} + (z_2 - a)^{1/2}] + \beta[b_{12}(\alpha^2 + \beta^2) - b_{22}] \mathrm{Im}[(z_1 - a)^{1/2} - (z_2 - a)^{1/2}] \right\}. \tag{63b}$$

6 应 力 曲 线

测定 3 组正交异性双材料, 得到如表 1 所示的材料性能数据•

表 1　　3组双材料中每种材料的弹性常数

双材料		E_1 / GPa	E_2 / GPa	ν_{12}	μ_{12} / GPa
A	材料 $j = 1$	50	9	0.33	11
	材料 $j = 2$	22.1	16	0.167 5	9.21
B	材料 $j = 1$	40	10.5	0.33	9.2
	材料 $j = 2$	18	14	0.167 5	8.4
C	材料 $j = 1$	46	11.4	0.33	9.935
	材料 $j = 2$	20.1	18	0.167 5	9.21

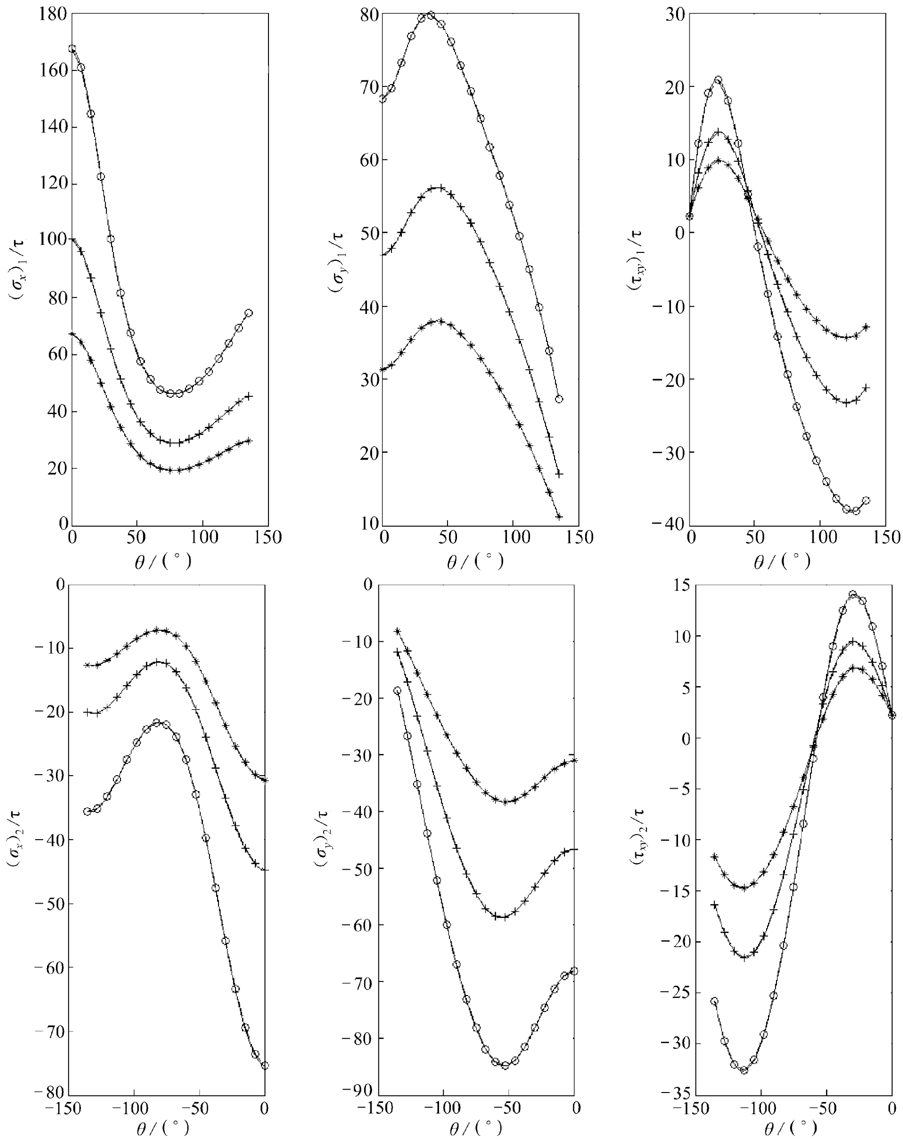

图 2　双材料应力曲线

将每种材料的弹性常数代入式(2)、(9)、(10)、(20)、(30)及式(25)，得到如表 2 所示的判别式、特征值、双材料弹性常数及应力奇异指数•

表 2 双材料性能数据

双材料		判别式 Δ_j	特征值 $\alpha_j+i(-1)^{k-1}\beta_j$		双材料弹性常数 ε_m	应力奇异指数 λ_m
A	材料 $j=1$	-7.1255	α_1 β_1	0.4551 1.4662	$\varepsilon_1=0.0303$	$\lambda_1=-0.4697$
	材料 $j=2$	-1.2626	α_2 β_2	0.2674 1.0506	$\varepsilon_2=-0.0285$	$\lambda_2=-0.5285$
B	材料 $j=1$	-1.6380	α_1 β_1	0.2323 1.3776	$\varepsilon_1=0.0544$	$\lambda_1=-0.4456$
	材料 $j=2$	-1.8745	α_2 β_2	0.3391 1.0094	$\varepsilon_2=-0.0484$	$\lambda_2=-0.5484$
C	材料 $j=1$	-0.3787	α_1 β_1	0.1089 1.4131	$\varepsilon_1=0.0471$	$\lambda_1=-0.4529$
	材料 $j=2$	-1.0537	α_2 β_2	0.2579 0.9951	$\varepsilon_2=-0.0427$	$\lambda_2=-0.5427$

在图 2 中, 给出了 3 组双材料当 r/a 为常数时, 应力 σ_x, σ_y, τ_{xy} 随极角 θ 的变化关系• 曲线上带"°"、"*"、"+"的点分别是双材料 A、双材料 B 和双材料 C 当 $r/a=0.1$, θ 分别取 0°、7.5°、15°、22.5°、30° …时, 由计算公式所求出的 $(\sigma_x)_j/\tau$, $(\sigma_y)_j/\tau$, $(\tau_{xy})_j/\tau(j=1,2)$ 的值画出的相应点, 利用样条函数拟合成的应力曲线•

7 结 论

(i) 本文采用与文献[13] 不同的力学模型, 通过引入新的应力函数, 利用复变函数、待定系数方法推出了 II 型界面裂纹尖端附近的应力强度因子、应力场和位移场;

(ii) 在双材料工程参数满足适当条件下, 求出了实双材料弹性常数 $\varepsilon_m(m=1,2)$ 和实应力奇异指数 $\lambda_m=-1/2+\varepsilon_m(m=1,2)$;

(iii) II 型界面裂纹尖端附近的应力、位移有混合型断裂特征, 但因为应力奇异指数是实数, 所以应力、位移没有振荡奇异性和裂纹面相互嵌入现象;

(iv) 作为特例, 取两种正交异性复合材料相同, 所得到的应力奇异指数、应力强度因子、应力场、位移场与单材料裂纹尖端场的经典结果完全相同, 从而相互验证了正确性•

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Crack- Tip Field on Mode II Interface Crack of Double Dissimilar Orthotropic Composite Materials

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Abstract: Two systems of non- homogeneous linear equations in 8 unknowns were obtained by introducing two stress functions containing 16 undetermined coefficients and two real stress singularity exponents with the help of boundary conditions. By solving the above systems of non- homogeneous linear equations, the two real stress singularity exponents can be determined when the double material engineering parameters meet certain conditions. The expression of the stress function and all the coefficients were got by the unique theorem of limit. By substituting them into corresponding mechanics equations, theoretical solutions to the stress intensity factor, the stress field and the displacement field near the crack tip of each material can be obtained when the discriminants of the characteristic equations are less than zero. Stress and displacement near crack tip show mixed crack characteristics but no stress oscillation or crack surfaces overlap. As an example, when the two orthotropic materials are the same, the stress singularity exponent, the stress intensity factor, the stress field and the expression for the displacement field of the orthotropic single material can be deduced.

Key words: mode II interface crack; stress intensity factors; double materials; orthotropic