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同伦分析法求解非线性多孔收缩表面上黏性磁流体的流动*

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摘要: 研究在非线性多孔收缩表面上黏性磁流体(MHD) 的流动• 先用相似变换简化其控制方程, 然后用同伦分析法(HAM) 求解该简化问题• 用图表的形式对问题的相关参数进行讨论, 发现在有磁流体时, 收缩解存在• 同时得到, 在不同参数下 $f''(0)$ 的解是收敛的•

关 键 词: MHD 流动; 驻点流动; 收缩表面; HAM 解

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引 言

边界上的速度朝向一个固定点的现象称为收缩现象, 如上升并且收缩着的气球等, 经常出现这种现象• 目前对收缩现象的研究有限, 文献[1- 9] 对该问题作了关注• 但因无法将涡流局限在边界层内, 使得在某些情况下, 收缩表面解不存在• 如果将磁场或者驻点流考虑进去, 这些解就可能会存在• 值得注意的是, 在磁场作用下的导电流体通过多孔薄板时的流动问题, 引起了众多学者的注意• 这个问题在许多工程领域中有着广泛的应用, 如磁流体动力学(MHD) 发电机、等离子、核反应堆、石油开发、地热能量的抽取以及空气动力学中边界层的控制等• 文献[10- 17] 已经在 MHD 的研究中, 考虑了磁场对流体动力学流动的影响•

本文讨论二维导电黏性流体, 流经非线性多孔收缩表面时的解析解• 据作者所知, 非线性收缩表面现象尚未有人进行过研究• 先用合适的相似变换对控制方程进行简化, 然后用同伦分析法(HAM) 求解该简化后的非线性边值问题• HAM 的最新文献可参考文献[18- 28]•

本文用图形和表格对速度和表面摩擦, 给出了相关参数的物理特性, 通过 h 曲线, 对解的收敛性进行了讨论•

1 公式的建立

考虑一个二维稳定磁流体流动问题: 不可压缩黏性流体流经 $y = 0$ 处的非线性多孔收缩表面, 导电流体受到垂直于收缩多孔表面的磁场 $B(x)$ 的作用, 感应磁场忽略不计, 流体运动

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的控制方程为

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0^2(x)}{\rho}u, \tag{2}$$

其中 u 和 v 分别为 x 和 y 方向上的速度分量, ν 为动黏度, ρ 为密度, σ 为流体的导电率• 方程(2)中忽略了内部电场和磁极化效应, 根据文献[17]有

$$B(x)=B_0x^{(n-1)/2}• \tag{3}$$

与非线性多孔收缩表面相应的边界条件如下:

$$u(x,0)=-cx^n,\ v(x,0)=-V_0x^{(n-1)/2}, \tag{4}$$

$$u(x,y)\to 0\qquad (y\to\infty), \tag{5}$$

其中 V_0 为薄板多孔性的参数• 引入相似变量和无量纲变量如下:

$$\eta=\sqrt{\frac{c(n+1)}{2\nu}}x^{(n-1)/2}y,\quad u=cx^nf'(\eta), \tag{6}$$

$$v=-\sqrt{\frac{c\nu(n+1)}{2}}x^{(n-1)/2}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right]• \tag{7}$$

由方程(6)和(7)知, 不可压缩条件(1)得到自动满足, 且动量方程(2)成为

$$f'''+ff''-\beta f'^2-Mf'=0• \tag{8}$$

问题的边界条件(4)和(5)成为

$$f(0)=K,\ f'(0)=-1,\ f'(\infty)=0, \tag{9}$$

其中 K 为壁面质量传递参数, M 为磁场参数, β 为无量纲参数, 它们有如下关系:

$$\beta=\frac{2n}{n+1},\ M=\frac{2\sigma B_0^2}{\rho c(1+n)},\ K=\frac{V_0}{\sqrt{c\nu(n+1)/2}}• \tag{10}$$

2 同伦分析法求解

为了用 HAM 求解方程(8), 选取

$$f(\eta)=K-1+\exp(-\eta) \tag{11}$$

作为 f 的初始近似值, 且将

$$\mathcal{L}[\hat f(\eta;p)]=\frac{\partial^3\hat f(\eta;p)}{\partial\eta^3}+\frac{\partial^2\hat f(\eta;p)}{\partial\eta^2} \tag{12}$$

作为辅助线性运算, 并且满足

$$\mathcal{L}[C_1+C_2e^{\eta}+C_3e^{-\eta}]=0, \tag{13}$$

其中 $C_i\,(i=1,2,3)$ 为任意常数• 如果 $p\in[0,1]$ 为一个内嵌参数, h 为非零的辅助参数• 于是, 第零阶变形方程可表示为

$$(1-p)\mathcal{L}\left[\hat f(\eta;p)-f(\eta)\right]=ph\mathcal{N}\left[\hat f(\eta;p)\right], \tag{14}$$

$$\hat f(0;p)=K,\ \hat f'(0;p)=1,\ \hat f'(\infty;p)=0, \tag{15}$$

其中

$$\mathcal{N}\left[\hat f(\eta;p)\right]=\frac{\partial^3\hat f(\eta;p)}{\partial\eta^3}+\hat f(\eta;p)\frac{\partial^2\hat f(\eta;p)}{\partial\eta^2}-\beta\left(\frac{\partial\hat f(\eta;p)}{\partial\eta}\right)^2-M^2\frac{\partial\hat f(\eta;p)}{\partial\eta}, \tag{16}$$

第 m 阶变形方程可表示为

$$\mathscr{L}[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar \mathscr{R}_m(\eta), \tag{17}$$

$$f_m(0) = 0,\ f'_m(0) = 0,\ f'_m(\infty) = 0, \tag{18}$$

$$\mathscr{R}_m(\eta) = f'''_{m-1}(\eta) - M^2 f'_{m-1}(\eta) + \sum_{k=0}^{m-1}[f_{m-1-k}f''_k - \beta f'_{m-1-k}f'_k], \tag{19}$$

其中

$$\chi_m = \begin{cases} 0, & m \leqslant 1, \\ 1, & m > 1. \end{cases} \tag{20}$$

使用符号计算软件 MATHEMATICA，得到方程(17)前面 N 阶的近似值．则 f 的解为

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta) = \lim_{N\to\infty}\left[\sum_{m=0}^{N} a^0_{m,0} + \sum_{n=1}^{2N+1} e^{-n\eta}\left(\sum_{m=n-1}^{2N} a_{m,n}\right)\right], \tag{21}$$

其中 $f_m(\eta)$ 的系数 $a_{m,n}$ 由给定的边界条件以及选取的初始近似方程(11)给出，数值分析见后面的图形．$a_{m,n}$ 的计算过程见附录部分．

3　解的收敛性及讨论

同伦分析法给出了该问题的级数解(21)，式中含有非零辅助参数 $\hbar$，用于调整和控制其解的收敛性．正如 Liao[21] 指出，应画出 $\hbar$ 曲线，确定辅助参数 $\hbar$ 容许取值的范围，以期保证解的收敛性．图 1 ～ 图 3 分别给出了不同参数 M, K 和 β 下，第 20 阶、第 20 阶和第 17 阶的 $\hbar$ 曲线图．从这些图中可以看出，$\hbar$ 的取值范围分别为

$$-1.7 \leqslant \hbar \leqslant -0.8,$$
$$-1.7 \leqslant \hbar \leqslant -0.8,$$
$$-1.7 \leqslant \hbar \leqslant -0.8.$$

表 1～ 表 3 给出了不同参数 M, K 和 β 时，HAM 解的不同阶次的近似值．从这些表中可以看出，随着参数 M, K 和 β 的增加，$f''(0)$ 值也增加．

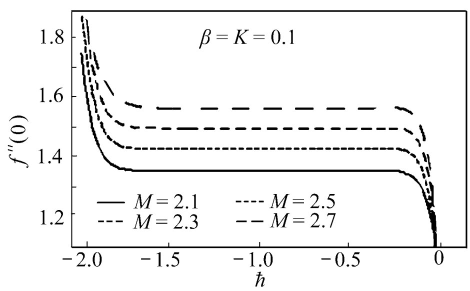

图 1　当 $\beta = 0.1$, $K = 0.1$ 时，不同 M 值时的 $f''(0)$ ～ $\hbar$ 曲线

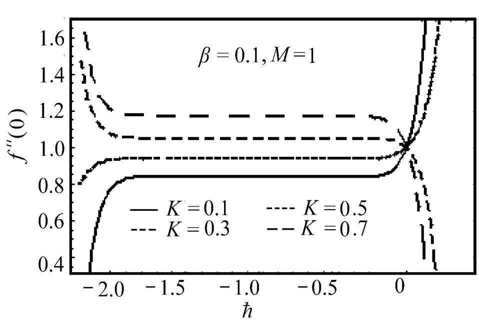

图 2　当 $\beta = 0.1$, $M = 1$ 时，不同 K 值时的 $f''(0)$ ～ $\hbar$ 曲线

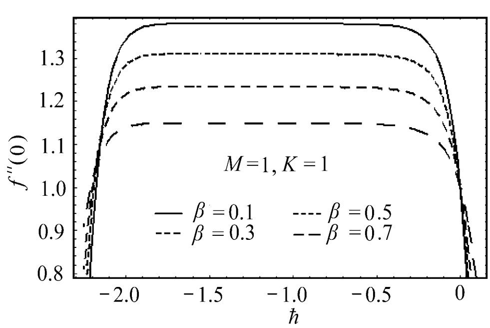

图 3　当 $M = 1$, $K = 1$ 时，不同 β 值时的 $f''(0)$ ～ $\hbar$ 曲线

表 1 当 $\beta=-2, K=1, h=-1$, 使用不同阶 HAM 时, $f''(0)$ 随 M 的变化

M	第 10 阶	第 15 阶	第 20 阶	第 25 阶	第 30 阶	第 35 阶	第 40 阶
2.0	1.618 10	1.618 06	1.618 05	1.618 04	1.618 04	1.618 04	1.618 04
2.1	1.661 97	1.661 92	1.661 91	1.661 90	1.661 90	1.661 90	1.661 90
2.2	1.704 23	1.704 18	1.704 17	1.704 16	1.704 16	1.704 16	1.704 16
2.3	1.745 05	1.745 00	1.745 00	1.744 99	1.744 99	1.744 99	1.744 99
2.4	1.784 56	1.784 53	1.784 53	1.784 53	1.784 52	1.784 52	1.784 52
2.5	1.822 89	1.822 88	1.822 88	1.822 88	1.822 88	1.822 88	1.822 88
2.6	1.860 13	1.860 15	1.860 15	1.860 15	1.860 15	1.860 15	1.860 15
2.7	1.896 39	1.896 43	1.896 42	1.896 42	1.896 42	1.896 42	1.896 42
2.8	1.931 75	1.931 78	1.931 78	1.931 78	1.931 78	1.931 78	1.931 78
2.9	1.966 27	1.966 29	1.966 29	1.966 29	1.966 29	1.966 29	1.966 29
3.0	2.000 01	2.000 00	2.000 00	2.000 00	2.000 00	2.000 00	2.000 00

表 2 当 $M=2, \beta=1, h=-1$, 使用不同阶 HAM 时, $f''(0)$ 随 K 的变化

K	第 10 阶	第 15 阶	第 20 阶	第 25 阶	第 30 阶	第 35 阶	第 40 阶
0.0	1.270 98	1.270 95	1.270 95	1.270 95	1.270 95	1.270 94	1.270 94
0.1	1.318 80	1.318 75	1.318 74	1.318 74	1.318 74	1.318 74	1.318 74
0.2	1.368 64	1.368 58	1.368 56	1.368 56	1.368 55	1.368 55	1.368 55
0.3	1.420 51	1.420 44	1.420 42	1.420 41	1.420 41	1.420 41	1.420 41
0.4	1.474 43	1.474 34	1.474 32	1.474 31	1.474 31	1.474 30	1.474 30
0.5	1.530 36	1.530 28	1.530 25	1.530 24	1.530 24	1.530 24	1.530 24
0.6	1.588 31	1.588 22	1.588 20	1.588 19	1.588 19	1.588 19	1.588 19
0.7	1.648 23	1.648 16	1.648 14	1.648 13	1.648 13	1.648 12	1.648 12
0.8	1.710 10	1.710 04	1.710 02	1.710 01	1.710 01	1.710 01	1.710 01
0.9	1.773 86	1.773 81	1.773 80	1.773 80	1.773 79	1.773 79	1.773 79
1.0	1.839 46	1.839 43	1.839 43	1.839 42	1.839 42	1.839 42	1.839 42

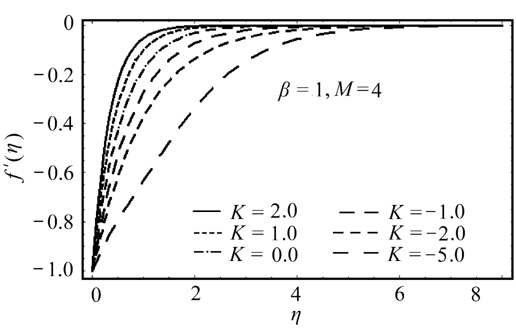

图 4 当 $M=4, \beta=1$ 时, 不同质量吸入参数 K 时的速度分布曲线

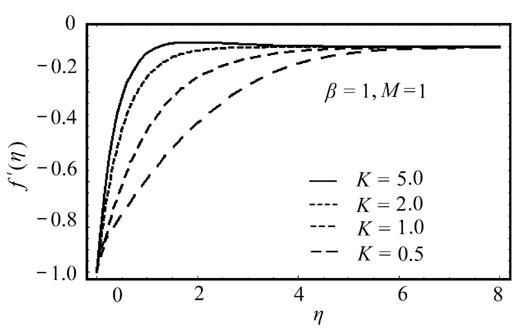

图 5 当 $M=1, \beta=1$ 时, 不同质量吸入参数 K 时的速度分布曲线

图 4~ 图 5 分别给出了 $M=4$, $\beta=1$ 和 $M=1$, $\beta=1$ 时, 质量吸入参数 K 对流速分布的影响• 从图 4 中可以看出, 当 $M=4$ 时, 随着质量的射入, 速度分布偏离壁面且边界层增厚• 从图 5 可以看出, 当 $M=1$, 质量吸入参数最大时, 边界层紧贴壁面• 从而可知在取 $\beta=1$ 时, 本

文的结果与文献[4]是一致的•

表 3　当 $M = 2, K = 1, h = -1$，使用不同阶 HAM 时，$f''(0)$ 随 β 的变化

β	第 10 阶	第 15 阶	第 20 阶	第 25 阶	第 30 阶	第 35 阶	第 40 阶
0.0	1.862 05	1.862 02	1.862 01	1.862 01	1.862 01	1.862 01	1.862 01
0.1	1.839 46	1.839 43	1.839 43	1.839 42	1.839 42	1.839 42	1.839 42
0.2	1.816 53	1.816 50	1.816 49	1.816 48	1.816 48	1.816 48	1.816 48
0.3	1.793 23	1.793 19	1.793 18	1.793 18	1.793 18	1.793 18	1.793 18
0.4	1.769 54	1.769 51	1.769 49	1.769 49	1.769 49	1.769 49	1.769 49
0.5	1.745 45	1.745 41	1.745 40	1.745 40	1.745 39	1.745 39	1.745 39
0.6	1.720 94	1.720 89	1.720 88	1.720 88	1.720 88	1.720 87	1.720 87
0.7	1.695 97	1.695 93	1.695 92	1.695 91	1.695 91	1.695 91	1.695 91
0.8	1.670 53	1.670 49	1.670 47	1.670 47	1.670 46	1.670 46	1.670 46
0.9	1.644 59	1.644 54	1.644 53	1.644 52	1.644 52	1.644 52	1.644 52
1.0	1.618 10	1.618 06	1.618 05	1.618 04	1.618 04	1.618 04	1.618 04

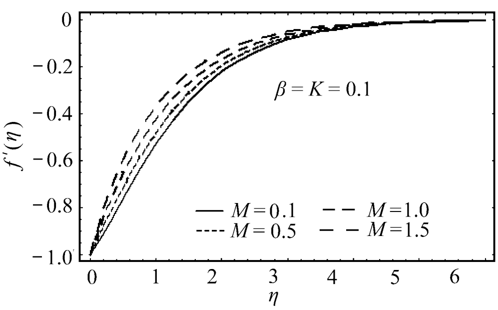

图 6　当 $K = 0.1, \beta = 0.1$ 时，不同 M 值时的解曲线

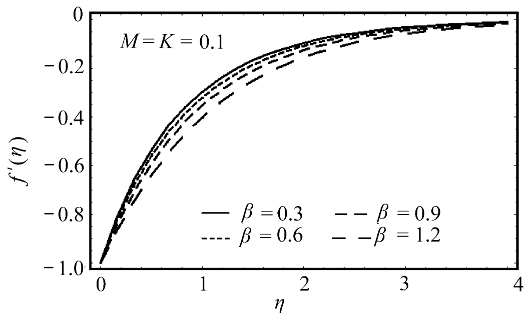

图 7　当 $K = 0.1, M = 0.1$ 时，不同 β 值时的解曲线

从图 6 可以看出，速度分布偏离壁面，并随着磁场参数 M 的增加，边界层变厚•　从图 7 可以看出，速度分布偏离壁面，并随着无量纲参数 β 的增加，其值减小，边界层也逐渐变厚•

附　　录

$$f_m(\eta) = \sum_{n=0}^{2m+1} a_{m,n} e^{-n\eta}, \quad f'_m(\eta) = \sum_{n=0}^{2m+1} (-n) a_{m,n} e^{-n\eta},$$

后面一式改写为

$$f'_m(\eta) = \sum_{n=0}^{2m+1} a_{1\,m,n} e^{-n\eta},$$

其中　$a_{1\,m,n} = (-n) a_{m,n}$•

又　$$f''_m(\eta) = \sum_{n=0}^{2m+1} (-n) a_{1\,m,n} e^{-n\eta},$$

改写为　$$f''_m(\eta) = \sum_{n=0}^{2m+1} a_{2\,m,n} e^{-n\eta},$$

其中　$a_{2\,m,n} = (-n) a_{1\,m,n}$•

又　$$f'''_m(\eta) = \sum_{n=0}^{2m+1} (-n) a_{2\,m,n} e^{-n\eta},$$

改写为 $f_m'''(\eta) = \sum_{n=0}^{2m+1} a_{3m,n} e^{-n\eta}$,

其中 $a_{3m,n} = (-n) a_{2m,n}$•

由 $f_{m-1}'''(\eta) = \sum_{n=0}^{2m-1} a_{3m-1,n} e^{-n\eta}$, $f_{m-1}'(\eta) = \sum_{n=0}^{2m-1} a_{1m-1,n} e^{-n\eta}$,

得到 $f_{m-1-k} f_k'' = \sum_{r=0}^{2m-2k-1} a_{m-1-k,r} e^{-r\eta} \sum_{s=0}^{2k+1} a_{2k,s} e^{-s\eta}$

即 $f_{m-1-k} f_k'' = \sum_{r=0}^{2m-2k-1} e^{-(r+s)\eta} \sum_{s=0}^{2k+1} a_{2k,s} a_{m-1-k,r}$•

令 $r+s=n$, 则

$$f_{m-1-k} f_k'' = \sum_{n=0}^{2m} e^{-n\eta} \sum_{s=\max(0,\,n-2m+2k+1)}^{s=\min(n,\,2k+1)} a_{2k,s} a_{m-1-k,n-s}$$

即 $$\sum_{k=0}^{m-1} f_{m-1-k} f_k'' = \sum_{n=0}^{2m} e^{-n\eta} \left[\sum_{k=0}^{m-1} \sum_{s=\max(0,\,n-2m+2k+1)}^{s=\min(n,\,2k+1)} a_{2k,s} a_{m-1-k,n-s} \right],$$

$$\sum_{k=0}^{m-1} f_{m-1-k} f_k'' = \sum_{n=0}^{2m} e^{-n\eta} \delta_{1m,n},$$

其中 $$\delta_{1m,n} = \left[\sum_{k=0}^{m-1} \sum_{s=\max(0,\,n-2m+2k+1)}^{s=\min(n,\,2k+1)} a_{2k,s} a_{m-1-k,n-s} \right]•$$

类似地

$$\sum_{k=0}^{m-1} f_{m-1-k}' f_k' = \sum_{n=0}^{2m} e^{-n\eta} \delta_{2m,n},$$

其中 $$\delta_{2m,n} = \left[\sum_{k=0}^{m-1} \sum_{s=\max(0,\,n-2m+2k+1)}^{s=\min(n,\,2k+1)} a_{1k,s} a_{m-1-k,n-s} \right]•$$

又

$$h\mathcal{R}_{1m}(\eta) = f_{m-1}'''(\eta) - M^2 f_{m-1}'(\eta) + \sum_{k=0}^{m-1} \left[f_{m-1-k} f_k'' - \beta f_{m-1-k}' f_k' \right] =$$

$$h \sum_{n=0}^{2m-1} e^{-n\eta} \left[a_{3m-1,n} - M^2 a_{1m-1,n} \right] + h \sum_{n=0}^{2m} e^{-n\eta} \left[\delta_{1m,n} - \beta \delta_{2m,n} \right] =$$

$$h \sum_{n=0}^{2m+1} \chi_{2m-n+1} e^{-n\eta} \left[a_{3m-1,n} - M^2 a_{1m-1,n} \right] + h \sum_{n=0}^{2m+1} \chi_{2m-n+1} e^{-n\eta} \left[\delta_{1m,n} - \beta \delta_{2m,n} \right],$$

$$h\mathcal{R}_{1m}(\eta) = \sum_{n=0}^{2m+1} \Delta_{m,n} e^{-n\eta},$$

其中 $\Delta_{m,n} = h\chi_{2m-n+1} \left[a_{3m-1,n} - M^2 a_{1m-1,n} + \delta_{1m,n} - \beta \delta_{2m,n} \right]$•

又

$$\mathcal{L}\left[f_m(\eta) - \chi_m f_{m-1}(\eta) \right] = h\mathcal{R}_{1m}(\eta),$$

$$\mathcal{L}\left[f_m(\eta) - \chi_m f_{m-1}(\eta) \right] = \sum_{n=0}^{2m+1} \Delta_{m,n} e^{-n\eta}•$$

方程两边乘以 $\mathcal{L}^{-1}$ 得

$$f_m(\eta) - \chi_m f_{m-1}(\eta) = \sum_{n=0}^{2m+1} \Delta_{m,n} \mathcal{L}^{-1}(e^{-n\eta}) =$$

$$\sum_{n=0}^{2m+1} \Delta_{m,n} \frac{1}{D^3 + D^2} e^{-n\eta} + c_1^m + c_2^m \eta + c_3^m e^{-\eta} =$$

$$\sum_{n=2}^{2m+1} \Delta_{m,n} \frac{1}{-n^3 + n^2} e^{-n\eta} + c_1^m + c_2^m \eta + c_3^m e^{-\eta} =$$

$$\sum_{n=2}^{2m+1} \Delta_{m,n} \frac{1}{n^2(1-n)} e^{-n\eta} + c_1^m + c_2^m \eta + c_3^m e^{-\eta} =$$

$$-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-(n+2)\eta}+c_1^m+c_2^m\eta+c_3^m e^{-\eta}.$$

由于 $f_m(0)=0$,

有

$$0=-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}+c_1^m+c_3^m,$$

$$f'_m(\eta)-\chi_m f'_{m-1}(\eta)=-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{-(n+2)}{(n+2)^2(n+1)}\left\{e^{-(n+2)\eta}\right\}+c_2^m-c_3^m e^{-\eta}.$$

由于 $f'_m(0)=0$,

有

$$0=\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{n+2}{(n+2)^2(n+1)}+c_2^m-c_3^m,$$

$$0=\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}+c_2^m-c_3^m.$$

由于 $f'_m(\infty)=0$,

有

$$0=c_2^m,$$

$$c_3^m=\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)},$$

$$c_1^m=\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}=$$
$$\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right],$$

$$f_m(\eta)-\chi_m f_{m-1}(\eta)=-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-(n+2)\eta}+$$
$$\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right]+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}e^{-\eta},$$

$$f_m(\eta)=\chi_m f_{m-1}(\eta)-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-(n+2)\eta}+$$
$$\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right]+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}e^{-\eta},$$

$$\sum_{n=0}^{2m+1}a_{m,n}e^{-n\eta}=\chi_m\sum_{n=0}^{2m-1}a_{m-1,n}e^{-n\eta}-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-(n+2)\eta}+$$
$$\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right]+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}e^{-\eta},$$

$$\sum_{n=0}^{2m+1}a_{m,n}e^{-n\eta}=\sum_{n=0}^{2m+1}\chi_m\chi_{2m-n+1}a_{m-1,n}e^{-n\eta}-\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-(n+2)\eta}+$$
$$\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right]+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+2)(n+1)}e^{-\eta},$$

$$a_{m,0}=\chi_m\chi_{2m+1}a_{m-1,0}+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)}\left[\frac{1}{(n+2)}-1\right],$$

$$a_{m,1}=\chi_m\chi_{2m}a_{m-1,1}+\sum_{n=0}^{2m+1}\iddots_{m,n+2}\frac{1}{(n+1)(n+2)},$$

$$a_{m,n}=\chi_m\chi_{2m-n+1}a_{m-1,n}+\iddots_{m,n+2}\frac{1}{(n+2)^2(n+1)}e^{-2\eta}.$$

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MHD Flow of a Viscous Fluid on a Non- Linear Porous Shrinking Sheet by Homotopy Analysis Method

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Abstract: The MHD flow of a viscous fluid towards a non- linear porous shrinking sheet was investigated. The governing equations were simplified by similarity transformations and then the reduced problem was solved by homotopy analysis method (HAM). The pertinent parameters appeared in the problem were discussed graphically and through the tables. It is found that the shrinking solutions in the presence of MHD exit. It is also observed from the tables that solutions for $f''(0)$ with different values of parameters are convergent.

Key words: MHD flow; stagnation flow; shrinking sheet; HAM solutions