文章编号:1000-0887(2011)06-0647-11

内窥镜中 Walter B 流体的蠕动流*

S·纳迪姆[1], N·S·阿克巴[1], T·哈亚特[1,2], A·A·珩迪[2]

(1. 真纳大学 数学系, 45320,伊斯兰堡 44000,巴基斯坦;
2. 沙特国王大学 理学院 物理系,1846 信箱,利雅得 11321, 沙特阿拉伯)

摘要: 研究内窥镜中 Walter B 流体的蠕动流,在圆柱坐标系中建立问题的模型,目的是研究内窥镜对 Walter B 流体蠕动流的影响. 以 delta 为摄动参数,使用正规的摄动法求出解析解. 利用数值积分,求得压力增量和摩擦力的近似解析解. 用图形给出了 Walter B 流体所显现参数的影响.

关 键 词: 蠕动流; Walter B 流体; 内窥镜; 摄动解

中图分类号: O357.2 **文献标志码:** A

DOI: 10.3879/j.issn.1000-0887.2011.06.002

引 言

最近,不同的流动几何条件下,在对 Newton 流体和非 Newton 流体蠕动流问题的研究中,许多研究人员是着眼于对某些实际现象的了解,例如,尿液从肾脏通过输尿管到膀胱的传输,食糜在胃肠道中运动,精子在男性生殖管道中的传输,卵子在女性输卵管中的运动,以及小血管中的血液的循环[1-3]. 此外,在生物力学系统出现了许多实际应用的机械装置,例如机械手(finger)、滚动泵(roller pumps)、心肺机(heart lungs machines)等.

不久前,为了正确了解不同情况下的蠕动流,出现了一些解析、数值和实验的研究. 关于这个专题的最新发展,详见文献[4-7]. 最近,Mekheimer 和 Elmaboud 的文献[8],在一个内窥镜中,调查耦合应力流体的蠕动流. 与此相应,内窥镜对蠕动流的影响,对医生诊断就非常重要,内窥镜还有许多临床的应用,是非常重要的工具,对于人体器官中的许多问题,提供可靠的真实缘由,例如,胃小肠就是通过蠕动泵输送流体的. 内窥镜还像当代医学科学中使用的一根导管. 对该问题的重要研究请参见文献[9-17].

根据以上分析,本文的目的是讨论内窥镜中 Walter B 流体的蠕动流[18]. Walter B 流体是粘弹性流体,粘弹性反映了很多血液参数的累积效应,如像血浆的粘度,红血球的可变形能力、聚合能力,以及血球的比容. 一些反映粘弹性影响的重要的研究可参见文献[19-23]. 用正规的摄动法,解析地求解控制方程,数值计算了压力增量和摩擦力表达式. 最后,画出压力增量、摩擦力和压力梯度图形,讨论不同显示度参数的物理特征.

* **收稿日期:** 2010-09-17; **修订日期:** 2011-03-30
作者简介: S. Nadeem(联系人. Tel: +92-5190642182; Fax: +92-92512275341; E-mail: snqau@hotmail.com).
本文原文为英文,吴承平 译,张禄坤 校.

1 数学模型

对一不可压缩流体,其质量平衡和动量平衡方程如下:

$$\operatorname{div}\boldsymbol{V}=0, \tag{1}$$

$$\rho\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}=\operatorname{div}\boldsymbol{S}+\rho\boldsymbol{f}, \tag{2}$$

其中,ρ 为密度,$\boldsymbol{V}$ 为速度向量,$\boldsymbol{S}$ 为 Cauchy 应力张量,$\boldsymbol{f}$ 为给定的体力,$\mathrm{d}/\mathrm{d}t$ 为物质时间导数. Walter B 流体的本构方程为[18,23]

$$\boldsymbol{S}=-\bar{P}\boldsymbol{I}+\boldsymbol{\tau}, \tag{3}$$

$$\boldsymbol{\tau}=2\eta_0\boldsymbol{e}-2k_0\frac{\delta\boldsymbol{e}}{\delta t}, \tag{4}$$

$$\boldsymbol{e}=\nabla\boldsymbol{V}+(\nabla\boldsymbol{V})^{\mathrm{T}}, \tag{5}$$

$$\frac{\delta\boldsymbol{e}}{\delta t}=\frac{\partial\boldsymbol{e}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{e}-\boldsymbol{e}\,\nabla\boldsymbol{V}-(\nabla\boldsymbol{V})^{\mathrm{T}}\boldsymbol{e}, \tag{6}$$

其中,$-\bar{P}\boldsymbol{I}$ 为球形应力张量,因为流体是不可压缩性的;$\boldsymbol{\tau}$ 为应力偏量张量,η_0 为粘性系数,$\boldsymbol{e}$ 为应变率张量,$\delta/\delta t$ 表示与物质运动有关的、张量分量的迁移微分.

Walter B 流体是粘弹性的非 Newton 流体,具有削弱或增强抗剪性能的功性.因而作为蠕动流模型,Walter B 流体模型是合适的,因为它具有粘弹性特性.粘弹性反映了很多血液参数的累积效应,例如血浆的粘度,红血球的可变形能力、聚合能力,血球比容等.当发生了变形,粘弹性这样一种材料性质,展示出粘性和弹性双重特征.因此粘弹性流体很容易在内部流动,因为弹性材料应变的即时性,当它被伸长后,一旦恢复到原来状态,应力立即消失.因此,Walter B 流体是一个重要的流动模型,非常贴近我们实际的生命体系.

2 数学公式

考虑一不可压缩的 Walter B 流体,在两个同心的水平管道内蠕动传输,流动以正弦波传播,沿管壁的波速为常速度 c.管壁表面的几何关系如图 1.

$$\bar{R}_1=a_1, \tag{7a}$$

$$\bar{R}_2=a_2+b\sin\frac{2\pi}{\lambda}(\bar{Z}-c\bar{t}), \tag{7b}$$

其中,a_1,a_2 为入口处管道的内外半径,b 为波幅,λ 为波长,c 为传播速度,$\bar{t}$ 为时间.考虑建立柱面坐标系$(\bar{R},\bar{Z})$,$\bar{Z}$ 轴沿管道中心线,$\bar{R}$ 轴与中心线垂直.

对一个不可压缩的流动,在固定坐标系中的控制方程为

$$\frac{\partial\bar{U}}{\partial\bar{R}}+\frac{\bar{U}}{\bar{R}}+\frac{\partial\bar{W}}{\partial\bar{Z}}=0, \tag{8}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{R}}+\bar{W}\frac{\partial}{\partial\bar{Z}}\right)\bar{U}=-\frac{\partial\bar{P}}{\partial\bar{R}}+\frac{1}{\bar{R}}\frac{\partial}{\partial\bar{R}}(\bar{R}\bar{\tau}_{\bar{R}\bar{R}})+\frac{\partial}{\partial\bar{Z}}(\bar{\tau}_{\bar{R}\bar{Z}})-\frac{\bar{\tau}_{\bar{\theta}\bar{\theta}}}{\bar{R}}, \tag{9}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}}+\bar{U}\frac{\partial}{\partial\bar{R}}+\bar{W}\frac{\partial}{\partial\bar{Z}}\right)\bar{W}=-\frac{\partial\bar{P}}{\partial\bar{Z}}+\frac{1}{\bar{R}}\frac{\partial}{\partial\bar{R}}(\bar{R}\bar{\tau}_{\bar{R}\bar{Z}})+\frac{\partial}{\partial\bar{Z}}(\bar{\tau}_{\bar{Z}\bar{Z}}). \tag{10}$$

引入波动坐标系 $(\bar{r},\bar{z})$ 如下式，速度 c 远离固定坐标系 $(\bar{R},\bar{Z})$：

$$\bar{z}=\bar{Z}-c\bar{t},\ \bar{r}=\bar{R}, \tag{11}$$

$$\bar{w}=\bar{W}-c,\ \bar{u}=\bar{U}, \tag{12}$$

其中，$\bar{U}$,$\bar{W}$ 和 $\bar{u}$,$\bar{w}$ 分别为固定坐标系和移动坐标系中，径向和轴向的速度分量.

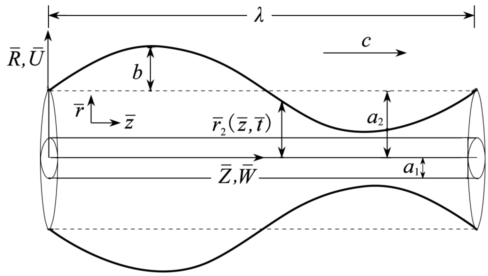

图 1 管道的几何表示

Fig. 1 Geometry of the problem

以中心线为对称，且壁面不出现滑移时的边界条件为

$$\bar{u}=0,\ \bar{w}=-c,\qquad \bar{r}=\bar{r}_1, \tag{12a}$$

$$\bar{u}=-c\frac{\mathrm{d}\bar{r}_2}{\mathrm{d}z},\ \bar{w}=-c,\qquad \bar{r}=\bar{r}_2=a_2+b\sin\frac{2\pi}{\lambda}(\bar{z}). \tag{12b}$$

定义

$$\begin{cases} R=\dfrac{\bar{R}}{a_2},\ r=\dfrac{\bar{r}}{a_2},\ Z=\dfrac{\bar{Z}}{\lambda},\ z=\dfrac{\bar{z}}{\lambda},\ W=\dfrac{\bar{W}}{c},\ w=\dfrac{\bar{w}}{c},\ \alpha=\dfrac{k_0c}{\eta_0a_2}, \\ U=\dfrac{\lambda\bar{U}}{a_2c},\ u=\dfrac{\lambda\bar{u}}{a_2c},\ P=\dfrac{a_2^2\bar{P}}{c\lambda\eta_0},\ t=\dfrac{c\bar{t}}{\lambda},\ \delta=\dfrac{a_2}{\lambda},\ Re=\dfrac{\rho ca_2}{\eta_0}, \\ r_2=\dfrac{\bar{r}_2}{a_2}=1+\phi\sin(2\pi z),\ \bar{\tau}_{ij}=\dfrac{\lambda\bar{\tau}_{ij}}{c\eta_0}\quad(i=j),\ \bar{\tau}_{ij}=\dfrac{a_2\bar{\tau}_{ij}}{c\eta_0}\quad(i\neq j), \\ \phi=\dfrac{b}{a_2},\ r_1=\dfrac{\bar{r}_1}{a_1}=\varepsilon. \end{cases} \tag{13}$$

α 为 Walter B 流体参数，ε 为半径比，ϕ 为振幅比.

利用上述无量纲量，方程(8)～(10)改写为

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{14}$$

$$Re\delta^3\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u=-\frac{\partial P}{\partial r}+\delta^2\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{\partial}{\partial z}(\tau_{rz})-\frac{\tau_{\theta\theta}}{r}\right], \tag{15}$$

$$Re\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)w=-\frac{\partial P}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\delta^2\frac{\partial}{\partial z}(\tau_{zz}), \tag{16}$$

其中

$$\tau_{rr}=2\frac{\partial u}{\partial r}-2\alpha\left[\delta u\frac{\partial^2u}{\partial r^2}+\delta w\frac{\partial^2u}{\partial r\partial z}-2\delta\left(\frac{\partial u}{\partial r}\right)^2-\frac{\partial w}{\partial r}\left(\frac{\partial u}{\partial z}\delta^2-\frac{\partial w}{\partial r}\right)\right],$$

$$\tau_{rz}=\left(\frac{\partial u}{\partial z}\delta^2-\frac{\partial w}{\partial r}\right)-\alpha\left[\delta^3u\frac{\partial^2u}{\partial r^2}+\delta u\frac{\partial^2w}{\partial r^2}+w\delta\frac{\partial^2w}{\partial r\partial z}+w\frac{\partial^2u}{\partial z^2}\delta^3-\right.$$
$$\left.2\frac{\partial u}{\partial r}\frac{\partial u}{\partial z}\delta^3-\delta\left(\frac{\partial u}{\partial z}\delta^2-\frac{\partial w}{\partial r}\right)\left(\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}\right)-\frac{\partial w}{\partial r}\frac{\partial w}{\partial z}\delta\right],$$

$$\tau_{zz}=2\frac{\partial w}{\partial z}-2\alpha\left[\delta u\frac{\partial^2w}{\partial r\partial z}+\delta w\frac{\partial^2w}{\partial z^2}-\frac{\partial u}{\partial z}\delta\left(\frac{\partial u}{\partial z}\delta^2-\frac{\partial w}{\partial r}\right)+2\delta\left(\frac{\partial w}{\partial z}\right)^2\right],$$

$$\tau_{\theta\theta}=2\frac{u}{r}-2\alpha\delta\left[u\frac{\partial u}{\partial z}-\frac{3u^2}{r^2}\right],$$

这里，δ 为波参数，Re 为 Reynolds 数，α 为 Walter B 流体参数. 由式(15)和(16)消去压力梯度，得到

$$\frac{\partial}{\partial r}\left[-Re\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)w+\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\delta^2\frac{\partial\tau_{zz}}{\partial z}\right]=$$

$$\frac{\partial}{\partial z}\left[-Re\delta^3\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u+\delta^2\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr})+\frac{\partial\tau_{rz}}{\partial z}-\frac{\tau_{\theta\theta}}{r}\right]\right]. \tag{17}$$

相应的无量纲形式的边界条件为

$$u=0,\ w=-1,\qquad r=r_1=\varepsilon, \tag{18}$$

$$u=-\frac{\mathrm{d}r_2}{\mathrm{d}z},\ w=-1,\qquad r=r_2=1+\phi\sin(2\pi z). \tag{19}$$

3　问 题 的 解

3.1　摄动解

由于方程(17)的高度非线性，因此不可能求得精确解. 我们转而寻求摄动解，用 δ^2 作为摄动参数，展开 u,w,F,τ,P：

$$w=w_0+\delta^2w_1+O(\delta^4), \tag{20a}$$

$$u=u_0+\delta^2u_1+O(\delta^4), \tag{20b}$$

$$F=F_0+\delta^2F_1+O(\delta^4), \tag{20c}$$

$$\tau=\tau_{ij}^{(0)}+\delta^2\tau_{ij}^{(1)}+O(\delta^4), \tag{20d}$$

$$P=P_0+\delta^2P_1+O(\delta^4). \tag{20e}$$

利用上述方程，满足边界条件(18)和(19)，直到 δ^2 阶的解为

$$w=-1+\left(\frac{r^2}{4}+a_1\ln r+a_2\right)\frac{\partial P}{\partial z}+\delta^2(b_{15}r^5+b_{16}r^4+b_{17}r^3+b_{18}r^2+b_{19}r+$$

$$b_{20}r^2\ln r+b_{21}r^3\ln r+b_{22}\ln r+b_{23}(\ln r)^2+b_{24}r^2(\ln r)^2+b_{25}), \tag{21}$$

$$\frac{\mathrm{d}P}{\mathrm{d}z}=\frac{2F+(r_2^2-r_1^2)}{a_3}+\delta^2\left(\frac{-2a_4}{a_3}\right), \tag{22}$$

其中

$$a_1=\frac{r_1^2-r_2^2}{4(\ln r_2-\ln r_1)},\ a_2=-\frac{r_1^2\ln r_2-r_2^2\ln r_1}{4(\ln r_2-\ln r_1)},$$

$$a_3=\frac{r_2^4-r_1^4}{8}+a_1(r_2^2\ln r_2-r_1^2\ln r_1)-a_1\frac{r_2^2-r_1^2}{2}+a_2(r_2^2-r_1^2),$$

$$a_4=b_{15}\left(\frac{r_2^7-r_1^7}{7}\right)+b_{16}\left(\frac{r_2^6-r_1^6}{6}\right)+b_{17}\left(\frac{r_2^5-r_1^5}{5}\right)+b_{18}\left(\frac{r_2^4-r_1^4}{4}\right)+b_{19}\left(\frac{r_2^3-r_1^3}{3}\right)+$$

$$b_{20}\left(\frac{r_2^4\ln r_2-r_1^4\ln r_1}{4}-\frac{r_2^4-r_1^4}{16}\right)+b_{21}\left(\frac{r_2^5\ln r_2-r_1^5\ln r_1}{5}-\frac{r_2^5-r_1^5}{25}\right)+$$

$$b_{22}\left(\frac{r_2^2\ln r_2-r_1^2\ln r_1}{2}-\frac{r_2^2-r_1^2}{4}\right)+$$

$$b_{23}\left(\frac{r_2^2(\ln r_2)^2-r_1^2(\ln r_1)^2-r_2^2(\ln r_2)+r_1^2(\ln r_1)}{2}+\frac{r_2^2-r_1^2}{4}\right)+$$

$$b_{24}\left(\frac{r_2^4(\ln r_2)^2 - r_1^2(\ln r_1)^2}{2} + \frac{-r_2^2(\ln r_2) + r_1^2(\ln r_1)}{4} - \frac{r_2^2 - r_1^2}{32}\right) +$$

$$b_{27}\left(\frac{r_2^2(\ln r_2) - r_1^2(\ln r_1)^2}{2} - \frac{r_2^2 - r_1^2}{4}\right) - b_{25}\frac{r_2^2 - r_1^2}{2} - b_{27}\ln r_1\left(\frac{r_2^2 - r_1^2}{4}\right),$$

$$b_1 = \left(\frac{\mathrm{d}P_0}{\mathrm{d}x}\right)', \ b_2 = \frac{b_1(r_1^2 - r_2^2)}{4(\ln r_2 - \ln r_1)}, \ b_3 = -\frac{b_1(r_1^2\ln r_2 - r_2^2\ln r_1)}{4(\ln r_2 - \ln r_1)} - 1,$$

$$b_4 = -\frac{1}{16}(b_1' r_1^4 + b_2'(2\ln r_1 - 1)r_1^2 + 8b_3' r_1^2), \ b_5 = -\frac{3b_1'}{8} + \frac{\alpha}{2}b_1'^2 - \frac{3}{4}b_1' + \alpha b_1^2,$$

$$b_6 = \frac{b_2'}{2} - (b_2' + b_3') + 2\alpha a_1 b_1^2 - b_2', \ b_7 = -b_2', \ b_8 = 2\alpha a_1 b_1^2 - 6b_4 - 4\alpha a_1^2 b_1^2,$$

$$b_9 = b_5' + \frac{b_1''}{2}, \ b_{10} = (b_6'' + b_1'' a_1 + a_1'' b_1 + a_1' b_1'), \ b_{11} = -\frac{b_2''}{2} + b_3'', \ b_{12} = \frac{b_1'}{4} + \frac{b_1''}{4},$$

$$b_{13} = 2a_1'' b_1 + 3b_1' a_1' + a_1 b_1'', \ b_{14} = 2a_2'' b_1 + 3b_1' a_2', \ b_{15} = \frac{b_1''}{600}, \ b_{16} = \frac{b_9}{32} - \frac{b_{12}}{16} - \frac{b_1''}{64},$$

$$b_{17} = -\frac{3b_2''}{27}, \ b_{18} = -\frac{b_{10}}{4} + \frac{b_7'}{8} + \frac{b_{11}}{4} + \frac{b_7'}{4} - \frac{b_{14}}{8} - \frac{b_{13}}{4} - \frac{b_3''}{4}, \ b_{19} = 2b_4',$$

$$b_{20} = \frac{b_{10}}{4} - \frac{b_7'}{4} - \frac{b_7'}{8} - \frac{b_{14}}{8} - \frac{b_{13}}{4} - \frac{b_2'}{4}, \ b_{21} = \frac{b_2''}{9}, \ b_{22} = -b_4', \ b_{23} = -\frac{b_8'}{4}, \ b_{24} = \frac{b_7'}{4},$$

$$b_{25} = b_{15}r_1^5 + b_{16}r_1^4 + b_{17}r_1^3 + b_{18}r_1^2 + b_{19}r_1 + b_{20}r_1^2\ln r_1 + b_{21}r_1^3\ln r_1 +$$
$$b_{22}\ln r_1 + b_{23}(\ln r_1)^2 + b_{24}r_1^2(\ln r_1)^2,$$

$$b_{26} = b_{15}r_2^5 + b_{16}r_2^4 + b_{17}r_2^3 + b_{18}r_2^2 + b_{19}r_2 + b_{20}r_2^2\ln r_2 + b_{21}r_2^3\ln r_2 +$$
$$b_{22}\ln r_2 + b_{23}(\ln r_2)^2 + b_{24}r_2^2(\ln r_2)^2,$$

$$b_{27} = \frac{b_{26} - b_{25}}{\ln r_1 - \ln r_2}, \ b_{28} = -\frac{b_{15}'}{7}, \ b_{29} = -\frac{b_{16}'}{6}, \ b_{30} = -\frac{b_{17}'}{5} + \frac{b_{21}'}{4},$$

$$b_{31} = -\frac{1}{16}\frac{\mathrm{d}P_1}{\mathrm{d}z} - \frac{b_{18}'}{4} - \frac{b_{20}'}{16} - \frac{b_{24}'}{32},$$

$$b_{32} = -\frac{b_{19}'}{3}, \ b_{33} = -\frac{a_1'}{4}\frac{\mathrm{d}P_1}{\mathrm{d}z} - a_2'\frac{\mathrm{d}P_1}{\mathrm{d}z} + \frac{b_{22}'}{4} - \frac{b_{23}'}{4} + \frac{b_{27}'}{4} + \frac{b_{25}'}{2} + \frac{b_{27}'}{2}\ln r_1,$$

$$b_{34} = -\frac{a_1'}{2} - \frac{b_{22}'}{2} + \frac{b_{23}'}{2} - \frac{b_{27}'}{2}, \ b_{35} = -\frac{b_{20}'}{4} - \frac{b_{24}'}{8}, \ b_{36} = -\frac{b_{21}'}{5}, \ b_{37} = -\frac{b_{23}'}{2},$$

$$b_{38} = -\frac{b_{24}'}{4},$$

$$b_{39} = -(b_{28}r_1^7 + b_{29}r_1^6 + b_{30}r_1^5 + b_{31}r_1^4 + b_{32}r_1^3 + b_{33}r_1^2 + b_{34}r_1^2\ln r_1 +$$
$$b_{35}r_1^4\ln r_1 + b_{36}r_1^5\ln r_1 + b_{37}r_1^2(\ln r_1)^2 + b_{38}r_1^4(\ln r_1)^2).$$

压力增量 ΔP,内外管道摩擦力分别为 $F^{(\mathrm{i})}$,$F^{(\mathrm{o})}$, 以下面公式给出:

$$\Delta P = \int_0^1 \frac{\mathrm{d}P}{\mathrm{d}z}\mathrm{d}z, \tag{23}$$

$$F^{(\mathrm{o})} = \int_0^1 r_1^2\left(-\frac{\mathrm{d}P}{\mathrm{d}z}\right)\mathrm{d}z, \tag{24}$$

$$F^{(\mathrm{i})} = \int_0^1 r_2^2\left(-\frac{\mathrm{d}P}{\mathrm{d}z}\right)\mathrm{d}z, \tag{25}$$

其中 dP/dz 由式(22)定义.

3.2　数值解

本问题同时考虑采用打靶法数值地求解方程(17)～(19),并将该数值解与摄动解作比较,两种解法的数值比较见图 2.

特殊情况

表 1　$\varepsilon = 0.1, z = 0.5, \phi = 0.2, r_1 = 0.1$ 时速度场的比较

Table 1　Velocity field for $\varepsilon = 0.1, z = 0.5, \phi = 0.2, r_1 = 0.1$

r	results of Newtonian fluid	our results when $\alpha = 0, \delta = 0$	results of ref. [17] when $\lambda_1 = 0$
0.1	−1.000	−1.000	−1.000
0.2	−1.020	−1.020	−1.020
0.3	−1.030	−1.030	−1.030
0.4	−1.033	−1.033	−1.033
0.5	−1.034	−1.034	−1.034
0.6	−1.032	−1.032	−1.032
0.7	−1.030	−1.030	−1.030
0.8	−1.020	−1.020	−1.020
0.9	−1.010	−1.010	−1.010
1.0	−1.000	−1.000	−1.000

4　图形结果和讨论

本节给出速度场、压力增量、摩擦力和轴向压力梯度的图形(见图 2～图 20).

图 2 给出了速度场的数值解和摄动解的比较.

图 3 将我们 Walter B 的结果约简后,和 Newton 流体比较.

图 4 表示了 Walter B 流体与 Newton 流体的差别.压力增量的数值计算采用 Mathematica 软件.

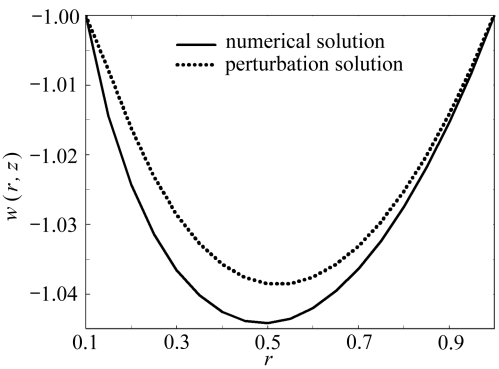

图 2　$\varepsilon = 0.1, z = 0.5, \phi = 0.2, r_1 = 0.1, \alpha_1 = 0.1, \delta = 0.1$ 时的速度场 $w(r,z)$

Fig. 2　Velocity field for $\varepsilon = 0.1, z = 0.5, \phi = 0.2, r_1 = 0.1, \alpha_1 = 0.1, \delta = 0.1$

图 5～图 8 给出了,不同的半径比 ε、波幅比 ϕ、波长 δ 和 Walter B 流体参数 α 时,压力增量 ΔP 与体积流量 Q 的关系.图 5 ～ 图 8 表明,压力增量与体积流量互成反比,同时,当 $\Delta P > 0$ 时出现蠕动泵,$\Delta P = 0$ 时进入无泵区,并进入 $\Delta P < 0$ 的辅助泵区.图 5 ～ 图 8 可以看出,随着 α 和 ε 的增大,压力增量在减小,同时随着 δ 和 ϕ 的增大而减小.还可以看到,图 5 在 $-3 \leqslant Q \leqslant 0.5$ 区域中出现蠕动泵,图 6 ～ 图 8 在 $-1 \leqslant Q \leqslant 0.5$ 区域中出现蠕动泵,还有一些其他方法可以增加蠕动泵的出现.

图 9 ～ 图 16 描述了摩擦力的变化.可以看到,摩擦力与压力增量的变化正好相反.

图 17～图 20 给出了不同参数下压力梯度的变化.可以看到,区间 $z \in [0, 0.5]$ 和 $[1.1, 1.5]$ 中,压力梯度很小,而在区间 $z \in [0.51, 1]$ 中,压力梯度很大.还可以得到,δ,ε 和 ϕ 的增大,引起压力梯度的增大,而 α 的增大,却引起压力梯度的减小.

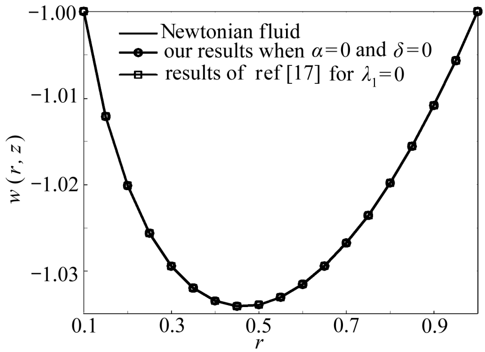

图 3 $\varepsilon = 0.1$, $z = 0.5$, $\phi = 0.2$, $r_1 = 0.1$ 时的速度场 $w(r,z)$

Fig. 3 Velocity field for $\varepsilon = 0.1$, $z = 0.5$, $\phi = 0.2$, $r_1 = 0.1$

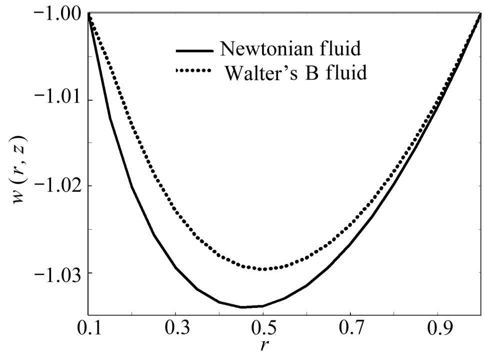

图 4 $\varepsilon = 0.1$, $z = 0.5$, $\phi = 0.2$, $r_1 = 0.1$, $\alpha = 0.1$, $\delta = 0.1$ 时的速度场 $w(r,z)$

Fig. 4 Velocity field for $\varepsilon = 0.1$, $z = 0.5$, $\phi = 0.2$, $r_1 = 0.1$, $\alpha = 0.1$, $\delta = 0.1$

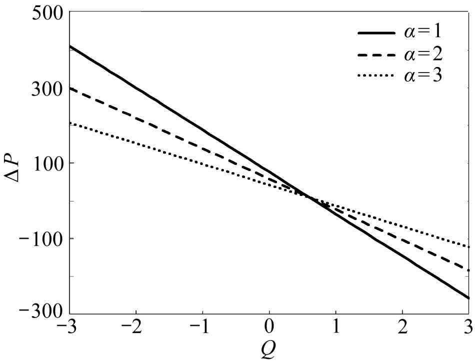

图 5 $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$ 时压力增量 ΔP 与流量 Q 的关系

Fig. 5 Pressure rise versus flow rate for $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$,

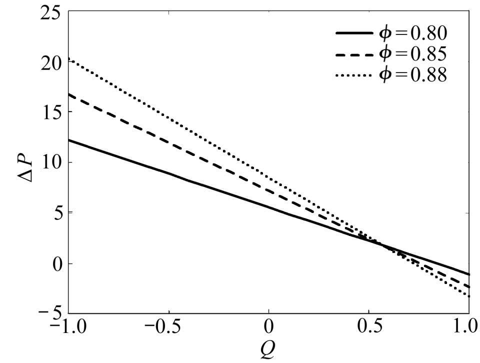

图 6 $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$ 时压力增量 ΔP 与流量 Q 的关系

Fig. 6 Pressure rise versus flow rate for $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$,

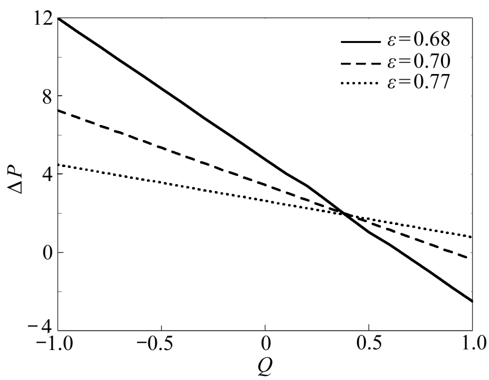

图 7 $n = 4$, $We = 0.001$, $\phi = 0.5$ 时压力增量 ΔP 与流量 Q 的关系

Fig. 7 Pressure rise versus flow rate for $n = 4$, $We = 0.001$, $\phi = 0.5$

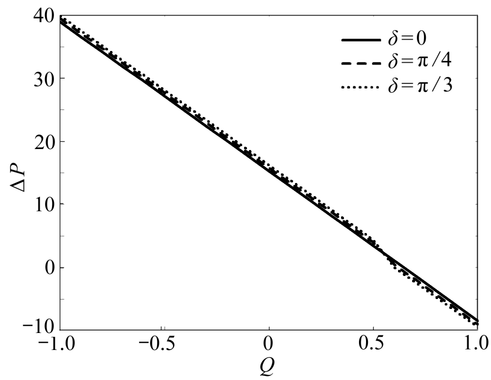

图 8 $\alpha = 0.04$, $\delta = 0.01$, $\phi = 0.6$ 时压力增量 ΔP 与流量 Q 的关系

Fig. 8 Pressure rise versus flow rate for $\alpha = 0.04$, $\delta = 0.01$, $\phi = 0.6$

致谢 本文第二作者作为客座教授感谢沙特国王大学的资助(KSU-VPP-117)。

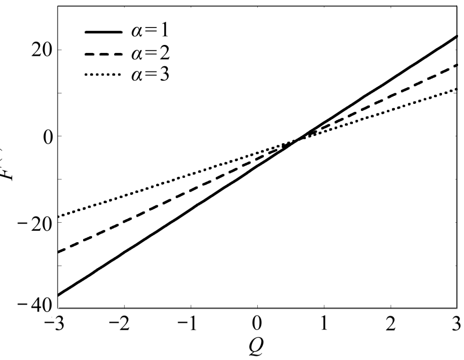

图 9 $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$ 时外管摩擦力 $F^{(o)}$ 随流量 Q 的变化

Fig. 9 Frictional force (on outer tube) versus flow rate for $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$

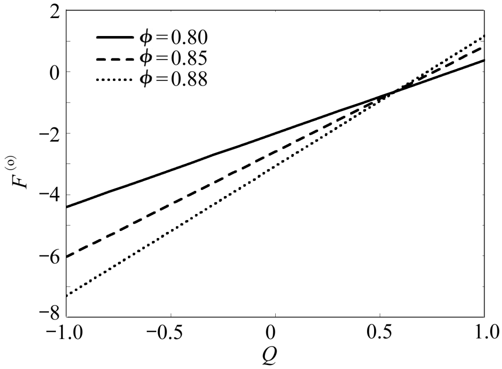

图 10 $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$ 时外管摩擦力 $F^{(o)}$ 随流量 Q 的变化

Fig. 10 Frictional force (on outer tube) versus flow rate for $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$

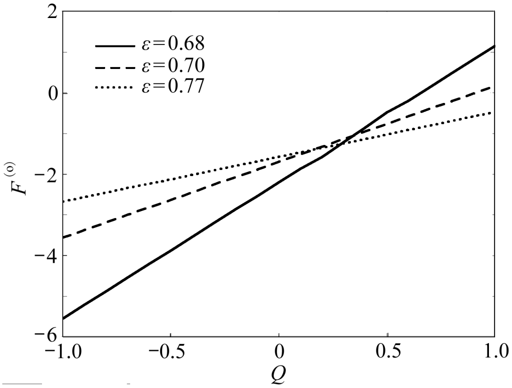

图 11 $n = 4$, $We = 0.001$, $\phi = 0.5$ 时外管内摩擦力 $F^{(o)}$ 随流量 Q 的变化

Fig. 11 Frictional force (on outer tube) versus flow rate for $n = 4$, $We = 0.001$, $\phi = 0.5$

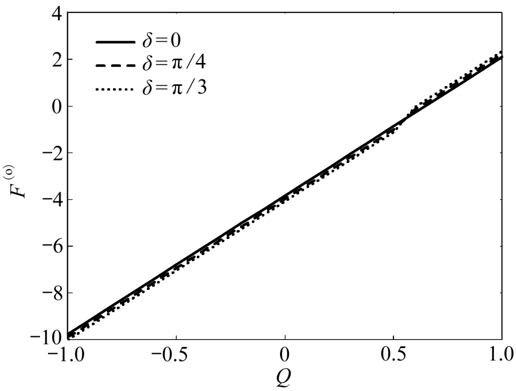

图 12 $\alpha = 0.04$, $\delta = 0.01$, $\phi = 0.6$ 时外管内摩擦力 $F^{(o)}$ 随流量 Q 的变化

Fig. 12 Frictional force (on outer tube) versus flow rate for $\alpha = 0.04$, $\delta = 0.01$, $\phi = 0.6$

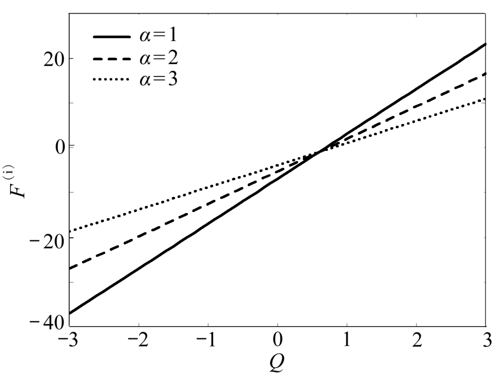

图 13 $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$ 时内管外摩擦力 $F^{(i)}$ 随流量 Q 的变化

Fig. 13 Frictional force (on inner tube) versus flow rate for $\varepsilon = 0.4$, $\delta = 0.01$, $\phi = 0.2$

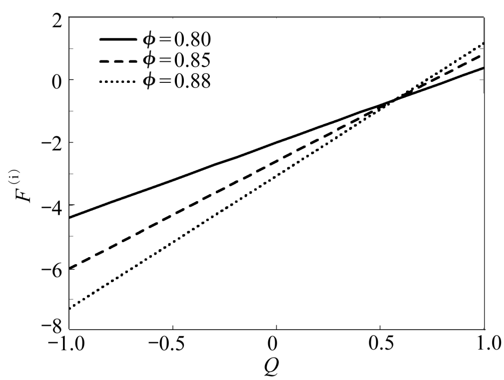

图 14 $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$ 时内管外摩擦力 $F^{(i)}$ 随流量 Q 的变化

Fig. 14 Frictional force (on inner tube) versus flow rate for $\varepsilon = 0.6$, $\delta = 0.01$, $\alpha = 0.04$

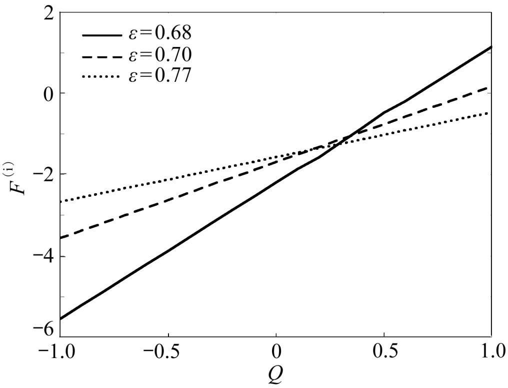

图 15　$n = 4$，$We = 0.001$，$\phi = 0.5$ 时内管摩擦力 $F^{(i)}$ 随流量 Q 的变化

Fig. 15　Frictional force (on inner tube) versus flow rate for $n = 4$, $We = 0.001$, $\phi = 0.5$

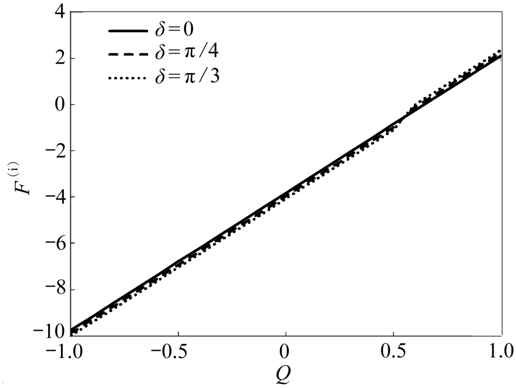

图 16　$\alpha = 0.04$，$\delta = 0.01$，$\phi = 0.6$ 时内管摩擦力 $F^{(i)}$ 随流量 Q 的变化

Fig. 16　Frictional force (on inner tube) versus flow rate for $\alpha = 0.04$, $\delta = 0.01$, $\phi = 0.6$

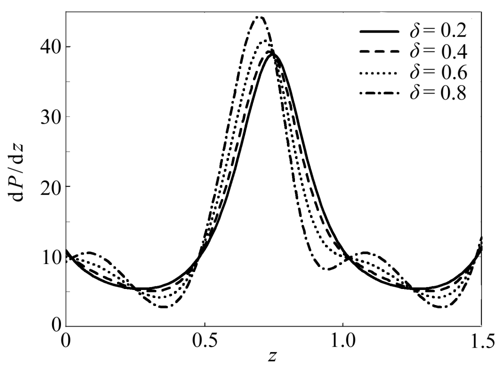

图 17　$\varepsilon = 0.5$，$\phi = 0.3$，$\alpha = 0.4$，$Q = 2$ 时压力梯度 dP/dz 随 z 的变化

Fig. 17　Pressure gradient versus z for $\varepsilon = 0.5$，$\phi = 0.3$，$\alpha = 0.4$，$Q = 2$

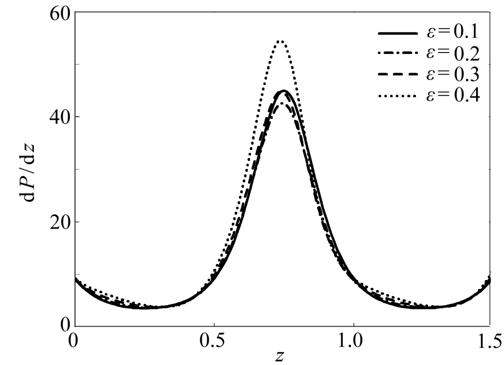

图 18　$\delta = 0.4$，$\phi = 0.4$，$\alpha = 0.4$，$Q = 2$ 时压力梯度 dP/dz 随 z 的变化

Fig. 18　Pressure gradient versus z for $\delta = 0.4$，$\phi = 0.4$，$\alpha = 0.4$，$Q = 2$

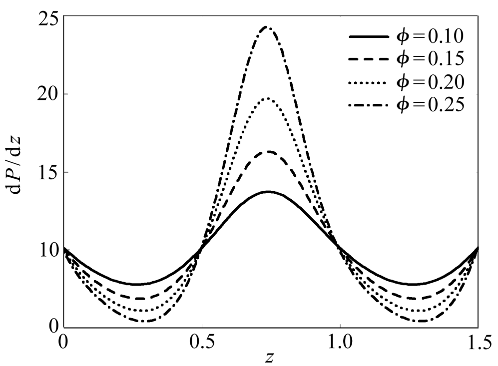

图 19　$\delta = 0.4$，$\varepsilon = 0.4$，$\alpha = 0.4$，$Q = 2$ 时压力梯度 dP/dz 随 z 的变化

Fig. 19　Pressure gradient versus z for $\delta = 0.4$，$\varepsilon = 0.4$，$\alpha = 0.4$，$Q = 2$

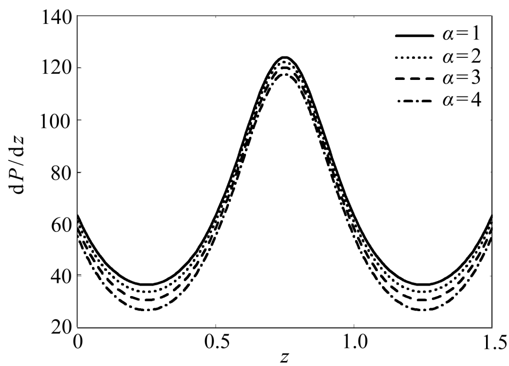

图 20　$\delta = 0.8$，$\varepsilon = 0.4$，$\phi = 0.4$，$Q = 2$ 时压力梯度 dP/dz 随 z 的变化

Fig. 20　Pressure gradient versus z for $\delta = 0.8$，$\varepsilon = 0.4$，$\phi = 0.4$，$Q = 2$

参考文献：

[1] Nadeem S, Akbar N S. Influence of heat transfer on a peristaltic transport of Herschel Bulkley fluid in a non-uniform inclined tube[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2009, **14**(12): 4100-4113.

[2] El Hakeem A, El Naby A, El Misery A E M, El Shamy I I. Separation in the flow through peristaltic motion of a Carreau fluid in uniform tube[J]. *Physica A*, 2004, **343**: 1-14.

[3] Ealshahed M, Haroun M H. Peristaltic transport of Johnson-Segalman fluid under effect of a magnetic field[J]. *Mathematical Problems in Engineering*, 2005, **6**(2005):663-677.

[4] Nadeem S, Akbar N S. Effects of heat transfer on the peristaltic transport of MHD Newtonian fluid with variable viscosity: application of Adomian decomposition method[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2009, **14**(11): 3844-3855.

[5] Nadeem S, Akbar N S. Influence of heat transfer on a peristaltic transport of Herschel Bulkley fluid in a non-uniform inclined tube[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2009, **14**(12): 4100-4113.

[6] El Hakeem A, El Naby A, El Misery A E M, El Shamy I I. Hydromagnetic flow of fluid with variable viscosity in a uniform tube with peristalsis[J]. *J Phys A*, 2003, **36**(31): 8535-8547.

[7] Haroun M H. Effect of Deborah number and phase difference on peristaltic transport of a third order fluid in an asymmetric channel[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2007, **12**(8): 1464-1480.

[8] Mekheimer Kh S, Elmaboud Y Abd. Peristaltic flow of a couple stress fluid in an annulus: application of an endoscope[J]. *Phy A*, 2008, **387**(11): 2403-2415.

[9] El Hakeem A, El Naby A, El Misery A E M, El Shamy I I. Effects of an endoscope and generalized Newtonian fluid on peristaltic motion[J]. *Appl Math Comput*, 2002, **128**(1): 19-35.

[10] Mekheimer Kh S, Elmaboud Y Abd. The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian fluid in a vertical annulus: an application of an endoscope [J]. *Phys Lett A*, 2008, **372**(10):1657-1665.

[11] Hayat T, Momoniat E, Mahmood F M. Peristaltic MHD flow of a third grade fluid with an endoscope and variable viscosity[J]. *J Nonlinear Math Phys*,2008, **15**(1): 91-104.

[12] Hayat T, Ahmed N, Ali N. Effects of an endoscope and the magnetic field on the peristalsis involving Jeffrey fluid[J]. *Communications in Nonlinear Science and Numerical Simulation*, 2008, **13**(8): 1581-1591.

[13] Hakeem A M, El Naby A, El Misery A E M, El Shamy I I. Effects of an endoscope and fluid with variable viscosity on peristaltic motion[J]. *Appl Math Comput*, 2004, **158**(2): 497-511.

[14] El Misery A M, El Hakeem A, El Naby A, El Nagar A H. Effects of a fluid with variable viscosity and an endoscope on peristaltic motion[J]. *J Phys Soc Jpn*, 2001, **72**: 89-93.

[15] Hayat T, Ali N. Effects of an endoscope on peristaltic flow of a micropolar fluid[J]. *Math Comput Model*, 2008, **48**(5/6): 721-733.

[16] Hayat T, Ali N, Asghar S, Siddiqui A M. Exact peristaltic flow in tubes with an endoscope [J]. *Appl Math Comput*, 2006, **182**(1):359-368.

[17] Hayat T, Momoniat E, Mahomed F M. Endoscope effects on MHD peristaltic flow of a power-law fluid[J]. *Mathematical Problems in Engineering*, 2006, **2006**(6): 1-19.

[18] Baris S. Steady three-dimensional flow of a Walter's B' fluid in a vertical channel[J]. *Turkish J Eng Env Sci*, 2002, **26**: 385-394.

[19] Ariel P D. Flow of viscoelastic fluids through a porous channel—Ⅰ[J]. *International Journal for Numerical Methods in Fluids*, 1993, **17**(7): 605-633.

[20] Ariel P D. A new finite-difference algorithm for computing the boundary layer flow of viscoelastic fluids in hydromagnetics[J]. *Comput Methods in Appl Mech Eng*, 1995, **124**(1/2): 1-13.

[21] Ariel P D. On extra boundary condition in the stagnation point flow of a second grade fluid [J]. *Int J Eng Sci*, 2002, **40**(2): 145-162.

[22] Sadeghy K, Sharifi M. Local similarity solution for the flow of a second grade viscoelastic fluid above a moving plate[J]. *Int J Non-Linear Mech*, 2004, **39**(8):1265-1273.

[23] Nadeem S, Akbar N S. Peristaltic flow of Walter's B fluid in a uniform inclined tube[J]. *Journal of Biorheology*, 2010, **24**(1): 22-28.

Peristaltic Flow of Walter's B Fluid in an Endoscope

S. Nadeem[1], Noreen Sher Akbar[1], T. Hayat[1,2], Awatif A. Hendi[2]

(1. *Department of Mathematics*, *Quaid-i-Azam University*, 45320, *Islamabad* 44000, *Pakistan*;

2. *Department of Physics*, *Faculty of Science*, *King Saud University*, *P. O. Box* 1846, *Riyadh* 11321, *Saudi Arabia*)

Abstract: The peristaltic flow of a Walter's B fluid in an endoscope was studied. The problem was model in cylindrical coordinates system. The main theme of the present analysis was to study the endoscopic effects on the peristaltic flow of Walter's B fluid, because to the best of authors' knowledge no investigation had been made up to yet in peristaltic literature to study the Walter's B fluid in an endoscope. The analytical solutions were carried out using regular perturbation method by taking delta as erturbation parameter. The approximate analytical solutions for pressure rise and friction forces were evaluated using numerical integration. The effects of emerging parameters of Walter's B fluid were presented graphically.

Key words: peristaltic flow; Walter's B fluid; endoscope; perturbation solution