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低压-发电机转子系统弯扭耦合情况下的组合共振研究*

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(我刊编委陈予恕来稿)

摘要: 考虑转子系统弯扭耦合作用,建立汽轮发电机组低压缸转子和发电机转子在次同步谐振作用下的非线性模型. 应用平均法研究在次同步谐振的情况下发生组合共振的解析解. 并得到分岔方程. 应用奇异性理论,得到系统参数和其动态行为的关系. 运用数值方法对所得结果进行验证,对发生组合共振和不发生组合共振的情况进行了数值比较. 该结果对工程实际应具有一定参考价值.

关 键 词: 转子弯扭耦合振动; 次同步谐振; 非线性动力学; 组合共振

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引 言

次同步谐振是机械电力系统的主要问题之一. 理解这一现象对实际的工程技术人员尤为重要. 大多数的电力旋转机械由驱动机械和被驱动机械组成, 通常这将引起系统的弯扭耦合振动.

以往研究轴系次同步问题时,往往仅研究扭转问题,实际上,弯曲振动和扭转振动是密不可分的. 单纯研究弯曲振动或扭转振动是不全面的. 对于弯扭耦合不对中问题,李明等[1-2]、AL-Hussain和Redmond[3-4]进行了研究. 对于弯扭耦合裂纹问题, 何成兵等[5]、李小彭等[6]、Papadopoulos等[7-8]做了大量工作. 在弯扭耦合碰摩的研究中, 李永强等[9]、孙政策等[10]对碰摩中的运动特性进行了详细的分析.

本文建立了汽轮发电机组低压缸转子和发电机转子的双转子模型,并在次同步谐振的条件下,研究了系统的弯扭耦合问题. 得出分岔方程. 应用奇异性理论,在转迁集和分岔图的基础上,得到了系统参数和各类振动的响应关系. 同时应用数值方法进行验证,对大电机组轴系的次同步谐振的机理有了深入的了解.

1 模型的建立及求解

图1给出了发电机组低压缸转子和发电机转子的简化模型,其中将低压缸和发电机简化

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为两个圆盘. 假设系统的坐标向量为 $\boldsymbol{q}=[x_1,y_1,\theta_1,x_2,y_2,\theta_2]^{\mathrm{T}}$,其中 x_1,y_1,x_2,y_2 分别是圆盘1和圆盘2在 x 方向和 y 方向的弹性位移. e_1 和 e_2 是两个圆盘的偏心量. $\dot{\phi}_1$ 和 $\dot{\phi}_2$ 分别是两圆盘的扭转角速度.

系统的动能为

$$T=\frac{1}{2}m_1[(\dot{x}_1-e_1\sin(\phi_1)\dot{\phi}_1)^2+(\dot{y}_1+e_1\cos(\phi_1)\dot{\phi}_1)^2]+\frac{1}{2}m_2[(\dot{x}_2-e_2\sin(\phi_2)\dot{\phi}_2)^2+(\dot{y}_2+e_2\cos(\phi_2)\dot{\phi}_2)^2]+\frac{1}{2}J_1\dot{\phi}_1^2+\frac{1}{2}J_2\dot{\phi}_2^2, \tag{1}$$

其中, $\phi_i=\omega t+\theta_i(i=1,2)$.

系统的势能为

$$V=\frac{1}{2}k_{x_1}x_1^2+\frac{1}{2}k_{y_1}y_1^2+\frac{1}{2}k_{x_2}x_2^2+\frac{1}{2}k_{y_2}y_2^2+\frac{1}{2}k_{x_3}(x_1-x_2)^2+\frac{1}{2}k_{y_3}(y_1-y_2)^2+\frac{1}{2}k_{t_1}\theta_1^2+\frac{1}{2}k_{t_2}\theta_2^2+\frac{1}{2}k_{t_3}(\theta_1-\theta_2)^2+m_1gy_{1c}+m_2gy_{2c}. \tag{2}$$

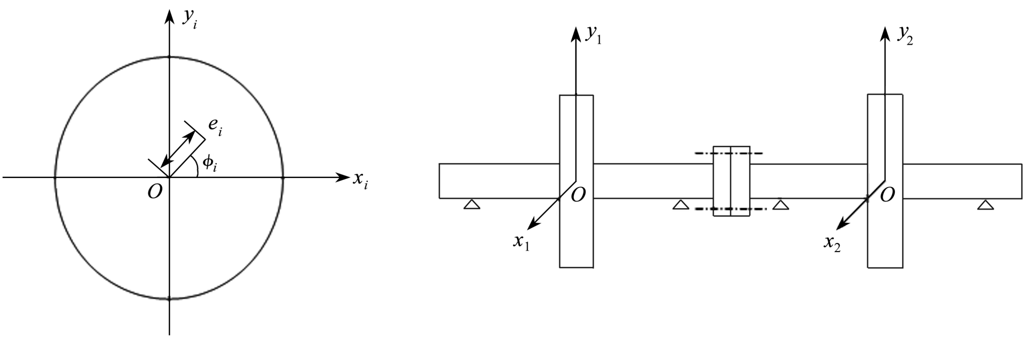


图1　两盘转子模型

Fig. 1　Model of rotor system

根据 Lagrange 函数和 Lagrange 方程:

$$L=T-V, \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=Q_i, \tag{4}$$

可求得系统的运动方程为

$$m_1\ddot{x}_1+c_1\dot{x}_1+k_{x_1}x_1+c_3(\dot{x}_1-\dot{x}_2)+k_{x_3}(x_1-x_2)=m_1e_1(\dot{\phi}_1^2\cos\phi_1+\ddot{\phi}_1\sin\phi_1)+F_{x_1}, \tag{5a}$$

$$m_1\ddot{y}_1+c_1\dot{y}_1+k_{y_1}y_1+c_3(\dot{y}_1-\dot{y}_2)+k_{y_3}(y_1-y_2)=m_1e_1(\dot{\phi}_1^2\sin\phi_1-\ddot{\phi}_1\cos\phi_1)+F_{y_1}-m_1g, \tag{5b}$$

$$(J_1+m_1e_1^2)\ddot{\phi}_1+c_{t_1}\dot{\phi}_1+k_{t_1}\theta_1+c_{t_3}(\dot{\phi}_1-\dot{\phi}_2)+k_{t_3}(\theta_1-\theta_2)=m_1e_1(\ddot{x}_1\sin\phi_1-(\ddot{y}_1+g)\cos\phi_1)+M_{t_1}, \tag{5c}$$

$$m_2\ddot{x}_2+c_2\dot{x}_2+k_{x_2}x_2+c_3(\dot{x}_2-\dot{x}_1)+k_{x_3}(x_2-x_1)=m_2e_2(\dot{\phi}_2^2\cos\phi_2+\ddot{\phi}_2\sin\phi_2)+F_{x_2}, \tag{5d}$$

$$m_2\ddot{y}_2+c_2\dot{y}_2+k_{y_2}y_2+c_3(\dot{y}_2-\dot{y}_1)+k_{y_3}(y_2-y_1)=m_2e_2(\dot{\phi}_2^2\sin\phi_2-\ddot{\phi}_2\cos\phi_2)+F_{y_2}-m_2g, \tag{5e}$$

$$(J_2+m_2e_2^2)\ddot{\phi}_2+c_{t_2}\dot{\phi}_2+k_{t_2}\theta_2+c_{t_3}(\dot{\phi}_2-\dot{\phi}_1)+k_{t_3}(\theta_2-\theta_1)=$$
$$m_2e_2(\ddot{x}_2\sin\phi_2-(\ddot{y}_2+g)\cos\phi_2)+M_{t_2}, \tag{5f}$$

其中, $m_i, J_i, F_{x_i}, F_{y_i}$ 和 M_{t_i} 分别为第 i 个盘的等效圆盘质量、转动惯量、在 x 方向和 y 方向的外力和外转矩($i=1,2$). $k_{x_j}, k_{y_j}, c_j, c_{t_j}, k_{t_j}$ 分别为 x 方向和 y 方向的弯曲刚度、阻尼系数、扭转刚度系数($j=1,2,3$).

引入无量纲化参数变换:

$$X_1=x_1/c,\ Y_1=y_1/c,\ X_2=x_2/c,\ Y_2=y_2/c,\ \tau=\omega t,$$

其中 ω 为激励频率. 则运动方程(5)变为

$$\begin{cases}
X_1''+c_{11}X_1'+k_{x_{11}}X_1+c_{12}(X_1'-X_2')+k_{x_{12}}(X_1-X_2)= \\
\qquad E_1((1+\theta_1')^2\cos\phi_1+\theta_1''\sin\phi_1)+\dfrac{F_{x_1}}{m_1\omega^2c}, \\
Y_1''+c_{11}Y_1'+k_{y_{11}}Y_1+c_{12}(Y_1'-Y_2')+k_{y_{12}}(Y_1-Y_2)= \\
\qquad E_1((1+\theta')^2\sin\phi_1-\theta_1''\cos\phi_1)+\dfrac{F_{y_1}}{m_1\omega^2c}-G_1, \\
\theta_1''+c_{t_{11}}\theta_1'+k_{t_{11}}\theta_1+c_{t_{12}}(\theta_1'-\theta_2')+k_{t_{12}}(\theta_1-\theta_2)= \\
\qquad E_2(X_1''\sin\phi_1-Y_1''\cos\phi_1)+\dfrac{M_{t_1}}{(J_1+m_1e_1^2)\omega^2}-G_2\cos\phi_1-c_{t_{11}}, \\
X_2''+c_{21}X_2'+k_{x_{21}}X_2+c_{22}(X_2'-X_1')+k_{x_{22}}(X_2-X_1)= \\
\qquad E_3((1+\theta_2')^2\cos\phi_2+\theta_2''\sin\phi_2)+\dfrac{F_{x_2}}{m_2\omega^2c}, \\
Y_2''+c_{21}Y_2'+k_{y_{21}}Y_2+c_{12}(Y_2'-Y_1')+k_{y_{22}}(Y_2-Y_1)= \\
\qquad E_3((1+\theta_2')^2\sin\phi_2-\theta_2''\cos\phi_2)+\dfrac{F_{y_2}}{m_2\omega^2c}-G_1, \\
\theta_2''+c_{t_{21}}\theta_1'+k_{t_{21}}\theta_1+c_{t_{22}}(\theta_2'-\theta_1')+k_{t_{22}}(\theta_2-\theta_1)= \\
\qquad E_4(X_2''\sin\phi_2-Y_2''\cos\phi_2)+\dfrac{M_{t_2}}{(J_2+m_2e_2^2)\omega^2}-G_4\cos\phi_2-c_{t_{21}},
\end{cases} \tag{6}$$

其中用到如下无量纲参数:

$$k_{x_{11}}=\frac{k_{x_1}}{m_1\omega^2},\ k_{x_{12}}=\frac{k_{x_3}}{m_1\omega^2},\ c_{11}=\frac{c_1}{m_1\omega},\ c_{12}=\frac{c'}{m_1\omega},\ E_1=\frac{e_1}{c},\ E_2=\frac{m_1e_1c}{J_1+m_1e_1^2},$$

$$G_1=\frac{g}{\omega^2c},\ G_2=\frac{m_1e_1g}{(J_1+m_1e_1^2)\omega^2},\ k_{t_{11}}=\frac{k_{t_1}}{(J_1+m_1e_1^2)\omega^2},\ k_{t_{12}}=\frac{k_{t_3}}{(J_1+m_1e_1^2)\omega^2},$$

$$c_{t_{11}}=\frac{c_{t_1}}{(J_1+m_1e_1^2)\omega},\ c_{t_{12}}=\frac{c_{t_3}}{(J_1+m_1e_1^2)\omega},\ k_{x_{21}}=\frac{k_{x_2}}{m_2\omega^2},\ k_{x_{22}}=\frac{k_{x_3}}{m_2\omega^2},$$

$$c_{21}=\frac{c_2}{m_2\omega},\ c_{22}=\frac{c_3}{m_2\omega},\ E_3=\frac{e_2}{c},\ E_4=\frac{m_2e_2c}{J_2+m_2e_2^2},\ G_4=\frac{m_2e_2g}{(J_2+m_2e_2^2)\omega^2},$$

$$k_{t_{21}}=\frac{k_{t_2}}{(J_2+m_2e_2^2)\omega^2},\ k_{t_{22}}=\frac{k_{t_3}}{(J_2+m_2e_2^2)\omega^2},\ c_{t_{21}}=\frac{c_{t_2}}{(J_2+m_2e_2^2)\omega},$$

$$c_{t_{22}}=\frac{c_{t_3}}{(J_2+m_2e_2^2)\omega},\ k_{y_{11}}=\frac{k_{y_1}}{m_1\omega^2},\ k_{y_{12}}=\frac{k_{y_3}}{m_1\omega^2},\ k_{y_{21}}=\frac{k_{y_2}}{m_2\omega^2},\ k_{y_{22}}=\frac{k_{y_3}}{m_2\omega^2},$$
$$M_{t_1}=M_{t_{11}}+M_{t_{12}}\sin\left(\frac{\upsilon\tau}{\omega}\right).\tag{7}$$

方程(6)是转子系统的非线性方程，可以采用平均法分析系统的动态行为[11].引入下列变换：

$$\begin{cases}x_1=z_1,\ \dot{x}_1=z_2,\\ y_1=z_3,\ \dot{y}_1=z_4,\\ \theta_1=z_5,\ \dot{\theta}_1=z_6,\\ x_2=z_7,\ \dot{x}_2=z_8,\\ y_2=z_9,\ \dot{y}_2=z_{10},\\ \theta_2=z_{11},\ \dot{\theta}_2=z_{12}.\end{cases}\tag{8}$$

系统变为典则形式：

$$\begin{Bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\\ \dot z_5\\ \dot z_6\\ \dot z_7\\ \dot z_8\\ \dot z_9\\ \dot z_{10}\\ \dot z_{11}\\ \dot z_{12}\end{Bmatrix}=\begin{Bmatrix}z_2\\ \frac{-(-J_1-m_1e_1^2)}{J_1}(-(k_{x_{11}}+k_{x_{12}})z_1+k_{x_{12}}z_7)-\frac{(-J_1-m_1e_1^2)e_1\cos\phi_1}{J_1c}+\varepsilon f_2\\ z_4\\ -G_1-(k_{y_{11}}+k_{y_{12}})z_3+k_{y_{12}}z_9+\frac{e_1\sin\phi_1}{c}+\varepsilon f_4\\ z_6\\ \frac{(J_1+m_1e_1^2)}{J_1}\left(\frac{M_{t_{11}}}{(J_1+m_1e_1^2)\omega^2}-(k_{t_{11}}+k_{t_{12}})z_5+k_{t_{12}}z_{11}\right)+\varepsilon f_6\\ z_8\\ \frac{-(-J_2-m_2e_2^2)}{J_2}(k_{x_{22}}z_1-(k_{x_{21}}+k_{x_{22}})z_7)-\frac{(-J_2-m_2e_2^2)e_2\cos\phi_2}{J_2c}+\varepsilon f_8\\ z_{10}\\ -G_3+k_{y_{22}}z_3-(k_{y_{21}}+k_{y_{22}})z_9+\frac{e_2}{c}\sin\phi_2+\varepsilon f_{10}\\ z_{12}\\ \frac{(J_2+m_2e_2^2)}{J_2}\left(\frac{M_{t_2}}{(J_2+m_2e_2^2)\omega^2}+k_{t_{22}}z_5-(k_{t_{21}}+k_{t_{22}})z_{11}\right)+\varepsilon f_{12}\end{Bmatrix},\tag{9}$$

其中非线性力详见附录.

2 解析解计算

方程(9)的线性方程为

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \\ \dot{z}_{10} \\ \dot{z}_{11} \\ \dot{z}_{12} \end{Bmatrix} = \begin{Bmatrix} z_2 \\ \dfrac{-(-J_1 - m_1 e_1^2)}{J_1}(-(k_{x_{11}} + k_{x_{12}})z_1 + k_{x_{12}} z_7) - \dfrac{(-J_1 - m_1 e_1^2) e_1 \cos\phi_1}{J_1 c} \\ z_4 \\ -(k_{y_{11}} + k_{y_{12}}) z_3 + k_{y_{12}} z_9 + \dfrac{e_1 \sin\phi_1}{c} \\ z_6 \\ \dfrac{(J_1 + m_1 e_1^2)}{J_1}(-(k_{t_{11}} + k_{t_{12}}) z_5 + k_{t_{12}} z_{11}) \\ z_8 \\ \dfrac{-(-J_2 - m_2 e_2^2)}{J_2}(k_{x_{22}} z_1 - (k_{x_{21}} + k_{x_{22}}) z_7) - \dfrac{(-J_2 - m_2 e_2^2) e_2 \cos\phi_2}{J_2 c} \\ z_{10} \\ k_{y_{22}} z_3 - (k_{y_{21}} + k_{y_{22}}) z_9 + \dfrac{e_2}{c}\sin\phi_2 \\ z_{12} \\ \dfrac{(J_2 + m_2 e_2^2)}{J_2}(k_{t_{22}} z_5 - (k_{t_{21}} + k_{t_{22}}) z_{11}) \end{Bmatrix}, \tag{10}$$

方程(10)可以简化为

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \boldsymbol{A}z + f(\gamma t). \tag{11}$$

其中 $\boldsymbol{A}$ 只含有质量和刚度而不含有阻尼和非线性力.

故其齐次部分的特征方程只有简单的纯虚根：$\pm \mathrm{i}\omega_1$，$\pm \mathrm{i}\omega_2$，$\pm \mathrm{i}\omega_3$，$\pm \mathrm{i}\omega_4$，$\pm \mathrm{i}\omega_5$，$\pm \mathrm{i}\omega_6$，方程(11)齐次部分的特解为

$$\begin{cases} \mathrm{Re}_{mi}(\omega_i t) = P_{mi}\sin(\omega_i t) - Q_{mi}\cos(\omega_i t), \\ \mathrm{Im}_{mi}(\omega_i t) = P_{mi}\sin(\omega_i t) + Q_{mi}\cos(\omega_i t), \end{cases} \tag{12}$$

其中 P_{mi} 和 Q_{mi} 为实数.

方程(11)齐次部分的共轭方程组为

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\boldsymbol{A}^{\mathrm{T}} y, \tag{13}$$

其中 $\boldsymbol{A}^{\mathrm{T}}$ 为 $\boldsymbol{A}$ 的转置方程.

方程(13)的特解为

$$\begin{cases} \mathrm{Re}'_{mn}(\omega_n t) = C_{mn}\cos(\omega_n t) - D_{mn}\sin(\omega_n t), \\ \mathrm{Im}'_{mn}(\omega_n t) = C_{mn}\sin(\omega_n t) + D_{mn}\cos(\omega_n t). \end{cases} \tag{14}$$

引入下列变换：

$$z_s = A_1\ \mathrm{Re}_{s_1}(\theta_1) + z_s^* \qquad (s = 3,4,9,10), \tag{15}$$

$$z_k = A_2\ \mathrm{Re}_{k_2}(\theta_2) + z_k^* \qquad (k = 3,4,9,10), \tag{16}$$

$$z_j = A_3\ \mathrm{Re}_{s_3}(\theta_3) + z_j^* \qquad (j = 5,6,11,12), \tag{17}$$

其中，$\mathrm{Re}_{mi} = P_{mi}\cos\theta_i - Q_{mi}\sin\theta_i$（$m = s$，$i = 1$；$m = k$，$i = 2$；$m = j$，$i = 3$），并且 z_s^*，z_k^*，z_j^* 为

方程(10)的特解. 当 $A_i=\text{const}$, $\mathrm{d}\theta_i/\mathrm{d}t=\omega_i(i=1,2,3)$, 方程(15)、(16)和(17)为方程(10)的通解. 故

$$\frac{\mathrm{dRe}_{mi}(\theta_i)}{\mathrm{d}\theta_i}=-\mathrm{Im}_{mi}(\theta_i),\quad \sum_{i=1}^{q}a_i\,\mathrm{Im}_{mi}(\theta_i)=-\sum_{\beta=1}^{n}a_{m\beta}\sum_{i=1}^{q}a_i\,\mathrm{Re}_{mi}(\theta_i)$$

$$(m=s,\ i=1;\ m=k,\ i=2;\ m=j,\ i=3).$$

如果 $A_i\neq\text{const},\mathrm{d}\theta_i/\mathrm{d}t\neq\omega_i(i=1,2,3)$, 将方程(15)、(16)和(17)代入方程(10)可以得到:

$$\frac{\mathrm{d}A_1}{\mathrm{d}t}\mathrm{Re}_{s_1}(\theta_1)-\mathrm{Im}_{s_1}(\theta_1)\left(\frac{\mathrm{d}\theta_1}{\mathrm{d}t}-\lambda_1\right)=\varepsilon f_s,\tag{18}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}t}\mathrm{Re}_{k_2}(\theta_2)-\mathrm{Im}_{k_2}(\theta_2)\left(\frac{\mathrm{d}\theta_2}{\mathrm{d}t}-\lambda_2\right)=\varepsilon f_k,\tag{19}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}t}\mathrm{Re}_{j_3}(\theta_3)-\mathrm{Im}_{j_3}(\theta_3)\left(\frac{\mathrm{d}\theta_3}{\mathrm{d}t}-\lambda_3\right)=\varepsilon f_j.\tag{20}$$

方程(12)和(14)存在正交性:

$$\begin{cases}\sum\limits_m \mathrm{Re}_{mi}(\theta_i)\mathrm{Re}'_{mn}(\theta_n)=\sum\limits_m \mathrm{Im}_{mi}(\theta_i)\mathrm{Im}'_{mn}(\theta_n)=0,\\ \sum\limits_m \mathrm{Re}_{mi}(\theta_i)\mathrm{Im}'_{mn}(\theta_n)=\delta_{in},\quad \sum\limits_m \mathrm{Im}_{mn}(\theta_i)\mathrm{Re}'_{mn}(\theta_n)=-\delta_{in},\end{cases}\tag{21}$$

其中

$$\delta_{in}=\begin{cases}\Delta i & (i=n),\\ 0 & (i\neq n).\end{cases}$$

以 $\mathrm{Im}'_{mn}(\theta_n)$ 和 $\mathrm{Re}'_{mn}(\theta_n)$ 分别乘方程(18)、(19)和(20),再将 m 从 1 ~ 4 求和,则可得方程的标准形式

$$\begin{cases}\dfrac{\mathrm{d}A_1}{\mathrm{d}t}=\varepsilon\dfrac{1}{\Delta_1}\sum\limits_s f_s\mathrm{Re}'_{s_1}(\theta_1)=\varepsilon\psi_1(A,\theta),\\ \dfrac{\mathrm{d}\theta_1}{\mathrm{d}t}=\lambda_1-\varepsilon\dfrac{1}{\Delta_1A_1}\sum\limits_s f_s\mathrm{Im}'_{s_1}(\theta_1)=\lambda_1-\varepsilon\psi_1^*(A,\theta),\end{cases}\tag{22}$$

$$\begin{cases}\dfrac{\mathrm{d}A_2}{\mathrm{d}t}=\varepsilon\dfrac{1}{\Delta_1}\sum\limits_k f_k\mathrm{Re}'_{k_2}(\theta_2)=\varepsilon\psi_2(A,\theta),\\ \dfrac{\mathrm{d}\theta_2}{\mathrm{d}t}=\lambda_2-\varepsilon\dfrac{1}{\Delta_2A_2}\sum\limits_k f_{k_2}\mathrm{Im}'_{k_2}(\theta_2)=\lambda_2-\varepsilon\psi_2^*(A,\theta),\end{cases}\tag{23}$$

$$\begin{cases}\dfrac{\mathrm{d}A_3}{\mathrm{d}t}=\varepsilon\dfrac{1}{\Delta_3}\sum\limits_j f_j\mathrm{Re}'_{j_3}(\theta_3)=\varepsilon\psi_3(A,\theta),\\ \dfrac{\mathrm{d}\theta_3}{\mathrm{d}t}=\lambda_3-\varepsilon\dfrac{1}{\Delta_3A_3}\sum\limits_j f_j\mathrm{Im}'_{j_3}(\theta_3)=\lambda_3-\varepsilon\psi_3^*(A,\theta),\end{cases}\tag{24}$$

其中, $\Delta_1=\sum\limits_s \mathrm{Re}_{s_1}\mathrm{Re}'_{s_1}$, $\Delta_2=\sum\limits_k \mathrm{Re}_{k_2}\mathrm{Re}'_{k_2}$, $\Delta_3=\sum\limits_k \mathrm{Re}_{k_3}\mathrm{Re}'_{k_3}$, λ_1 和 λ_2 是系统的前两阶弯曲固有频率, λ_3 是第一阶扭转固有频率.

方程(10)的齐次部分的特解为

$$\mathrm{Re}_{1k}=\cos(\lambda_k t),$$

$$\mathrm{Re}_{7k}=\frac{(-\lambda_1^2J_1+J_1k_{x_{11}}+J_1k_{x_{12}}+m_1e_1^2k_{x_{11}}+m_1e_1^2k_{x_{12}})\cos(\lambda_k t)}{k_{x_{12}}(J_1+m_1e_1^2)},$$

$$\mathrm{Re}_{2k} = -\lambda_1 \sin(\lambda_k t),$$

$$\mathrm{Re}_{8k} = \frac{-\lambda_1(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{x_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\sin(\lambda_k t)}{k_{x_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Re}_{3k} = \cos(\lambda_k t),\ \mathrm{Re}_{9k} = \frac{(-\lambda_2^2 + k_{y_{11}} + k_{y_{12}})\cos(\lambda_k t)}{k_{y_{12}}},$$

$$\mathrm{Re}_{4k} = -\lambda_2 \sin(\lambda_k t),\ \mathrm{Re}_{10k} = \frac{-\lambda_2(-\lambda_2^2 + k_{y_{11}} + k_{y_{12}})\sin(\lambda_k t)}{k_{y_{12}}},$$

$$\mathrm{Re}_{53} = \cos(\lambda_3 t),$$

$$\mathrm{Re}_{113} = \frac{(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\cos(\lambda_3 t)}{k_{t_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Re}_{63} = -\lambda_3 \sin(\lambda_3 t),$$

$$\mathrm{Re}_{123} = \frac{-\lambda_3(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\sin(\lambda_3 t)}{k_{t_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Im}_{1k} = \sin(\lambda_k t),$$

$$\mathrm{Im}_{7k} = \frac{(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{x_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\sin(\lambda_k t)}{k_{x_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Im}_{2k} = \lambda_1 \cos(\lambda_k t),$$

$$\mathrm{Im}_{8k} = \frac{\lambda_1(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{z_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\cos(\lambda_k t)}{k_{x_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Im}_{3k} = \sin(\lambda_k t),\ \mathrm{Im}_{9k} = \frac{(-\lambda_2^2 + k_{y_{11}} + k_{y_{12}})\sin(\lambda_k t)}{k_{y_{12}}},$$

$$\mathrm{Im}_{4k} = \lambda_1 \cos(\lambda_k t),\ \mathrm{Im}_{10k} = \frac{\lambda_2(-\lambda_2^2 + k_{y_{11}} + k_{y_{12}})\cos(\lambda_k t)}{k_{y_{12}}},$$

$$\mathrm{Im}_{53} = \sin(\lambda_3 t),$$

$$\mathrm{Im}_{113} = \frac{(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\sin(\lambda_3 t)}{k_{t_{12}}(J_1 + m_1 e_1^2)},$$

$$\mathrm{Im}_{63} = \lambda_3 \cos(\lambda_3 t),$$

$$\mathrm{Im}_{123} = \frac{\lambda_3(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\cos(\lambda_3 t)}{k_{t_{12}}(J_1 + m_1 e_1^2)},$$

其中, Re 是解的实部, Im 是解的虚部.

同样,方程(10)共轭方程齐次部分的特解为

$$\mathrm{Re}'_{1k} = \cos(\lambda_k t),$$

$$\mathrm{Re}'_{7k} = \frac{J_2(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{x_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\cos(\lambda_k t)}{k_{x_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$\mathrm{Re}'_{2k} = -\frac{\sin(\lambda_k t)}{\lambda_1},$$

$$\mathrm{Re}'_{8k} = \frac{(\lambda_1^2 J_1 - J_1 k_{x_{11}} - J_1 k_{x_{12}} - m_1 e_1^2 k_{x_{11}} - m_1 e_1^2 k_{x_{12}}) J_2 \sin(\lambda_k t)}{\lambda_1 k_{x_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Re'_{3k} = \cos(\lambda_k t),\ Re'_{9k} = \frac{-(\lambda_2^2 - k_{y_{11}} - k_{y_{12}})\cos(\lambda_k t)}{k_{y_{22}}},$$

$$Re'_{4k} = -\frac{\sin(\lambda_k t)}{\lambda_2},\ Re'_{10k} = \frac{(\lambda_2^2 - k_{y_{11}} - k_{y_{12}})\sin(\lambda_k t)}{\lambda_2 k_{y_{22}}},$$

$$Re'_{53} = \cos(\lambda_3 t),$$

$$Re'_{113} = \frac{J_2(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\cos(\lambda_3 t)}{k_{t_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Re'_{63} = -\frac{\sin(\lambda_3 t)}{\lambda_3},$$

$$Re'_{123} = \frac{(\lambda_3^2 J_1 - J_1 k_{t_{11}} - J_1 k_{t_{12}} - m_1 e_1^2 k_{t_{11}} - m_1 e_1^2 k_{t_{12}}) J_2 \sin(\lambda_3 t)}{\lambda_3 k_{t_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Im'_{1k} = \sin(\lambda_k t),$$

$$Im'_{7k} = \frac{J_2(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{x_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\sin(\lambda_1 t)}{k_{x_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Im'_{2k} = \frac{\cos(\lambda_k t)}{\lambda_1},$$

$$Im'_{8k} = \frac{J_2(-\lambda_1^2 J_1 + J_1 k_{x_{11}} + J_1 k_{x_{12}} + m_1 e_1^2 k_{x_{11}} + m_1 e_1^2 k_{x_{12}})\cos(\lambda_1 t)}{\lambda_1 k_{x_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Im'_{3k} = \sin(\lambda_k t),\ Im'_{9k} = \frac{-(\lambda_2^2 - k_{y_{11}} - k_{y_{12}})\sin(\lambda_2 t)}{k_{y_{22}}},$$

$$Im'_{4k} = \frac{\cos(\lambda_k t)}{\lambda_2},\ Im'_{10k} = \frac{-(\lambda_2^2 - k_{y_{11}} - k_{y_{12}})\cos(\lambda_2 t)}{\lambda_2 k_{y_{22}}},$$

$$Im'_{53} = \sin(\lambda_3 t),$$

$$Im'_{113} = \frac{J_2(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\sin(\lambda_3 t)}{k_{t_{22}} J_1 (J_2 + m_2 e_2^2)},$$

$$Im'_{63} = \frac{\cos(\lambda_3 t)}{\lambda_3},$$

$$Im'_{123} = \frac{J_2(-\lambda_3^2 J_1 + J_1 k_{t_{11}} + J_1 k_{t_{12}} + m_1 e_1^2 k_{t_{11}} + m_1 e_1^2 k_{t_{12}})\cos(\lambda_3 t)}{\lambda_3 k_{t_{22}} J_1 (J_2 + m_2 e_2^2)},$$

其中, Re' 是解的实部, Im' 是解的虚部.

为解方程(22)、(23)和(24)应用平均法,引入 K-B 变换

$$\begin{cases} A_k = a_k + \varepsilon U_k(t, y, \vartheta), \\ \theta_k = \lambda_k + \vartheta_k + \varepsilon V_k(t, y, \vartheta), \end{cases} \tag{25}$$

$$\begin{cases} \dfrac{da_k}{dt} = \varepsilon Y_k(y, \vartheta) + \varepsilon^2 Y_k^*(t, y, \vartheta, \varepsilon), \\ \dfrac{d\vartheta_k}{dt} = \lambda_k - \omega + \varepsilon Z_k(y, \vartheta) + \varepsilon^2 Z_k^*(t, y, \vartheta, \varepsilon). \end{cases} \tag{26}$$

在次同步和组合共振情况下, λ_1, ω 和 λ_3 的关系为

$$\lambda_3 - v/\omega = \varepsilon\sigma,\ \lambda_1 + \lambda_3 \approx 1.$$

应用平均法,系统的第一阶近似解为

$$\begin{cases}\dfrac{\mathrm{d}a_1}{\mathrm{d}\tau}=f_{11}\sin(\vartheta_1)+f_{12}\sin(\vartheta_1+\vartheta_3)+f_{13}\cos(\vartheta_1+\vartheta_3)+f_{14},\\ a_1\dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}\tau}=f_{101}\cos(\vartheta_1)+f_{102}\cos(\vartheta_1+\vartheta_3)+f_{103}\sin(\vartheta_1+\vartheta_3)+f_{104},\\ \dfrac{\mathrm{d}a_3}{\mathrm{d}\tau}=f_{31}\cos(\vartheta_3)+f_{32}\sin(\vartheta_1+\vartheta_3)+f_{33}\cos(\vartheta_1+\vartheta_3)+f_{34},\\ a_3\dfrac{\mathrm{d}\vartheta_3}{\mathrm{d}\tau}=f_{301}\sin(\vartheta_3)+f_{302}\cos(\vartheta_1+\vartheta_3)+f_{303}\sin(\vartheta_1+\vartheta_3)+f_{304},\end{cases} \tag{27}$$

其中

$$\begin{cases}f_{101}=f_{11},f_{102}=f_{12},\\ f_{103}=-f_{13},f_{301}=-f_{31},\\ f_{302}=f_{32},f_{303}=-f_{33}.\end{cases}$$

令

$$\begin{cases}F_1=(f_{12}^2+f_{13}^2)^{1/2},\\ \cos\varphi_1=\dfrac{f_{12}}{F_1},\ \sin\varphi_1=\dfrac{f_{13}}{F_1},\\ F_3=(f_{32}^2+f_{33}^2)^{1/2},\\ \cos\varphi_2=\dfrac{f_{33}}{F_3},\ \sin\varphi_2=\dfrac{f_{32}}{F_3},\end{cases} \tag{28}$$

方程(27)可变换为

$$\begin{cases}\dfrac{\mathrm{d}a_1}{\mathrm{d}\tau}=f_{11}\sin(\vartheta_1)+F_1\sin(\vartheta_1+\vartheta_3+\varphi_1)+f_{14},\\ a_1\dfrac{\mathrm{d}\vartheta_1}{\mathrm{d}\tau}=f_{11}\cos(\vartheta_1)+F_1\cos(\vartheta_1+\vartheta_3+\varphi_1)+f_{104},\\ \dfrac{\mathrm{d}a_3}{\mathrm{d}\tau}=f_{31}\cos(\vartheta_3)+F_3\cos(\vartheta_1+\vartheta_3-\varphi_2)+f_{34},\\ a_3\dfrac{\mathrm{d}\vartheta_3}{\mathrm{d}\tau}=-f_{31}\sin(\vartheta_3)-F_3\sin(\vartheta_1+\vartheta_3-\varphi_2)+f_{304}.\end{cases} \tag{29}$$

方程(29)的定常解满足下列方程:

$$\begin{cases}f_{11}\sin(\vartheta_1)+F_1\sin(\vartheta_1+\vartheta_3+\varphi_1)=-f_{14},\\ f_{11}\cos(\vartheta_1)+F_1\cos(\vartheta_1+\vartheta_3+\varphi_1)=-f_{104},\\ f_{31}\cos(\vartheta_3)+F_3\cos(\vartheta_1+\vartheta_3-\varphi_2)=-f_{34},\\ f_{31}\sin(\vartheta_3)+F_3\sin(\vartheta_1+\vartheta_3-\varphi_2)=f_{304}.\end{cases} \tag{30}$$

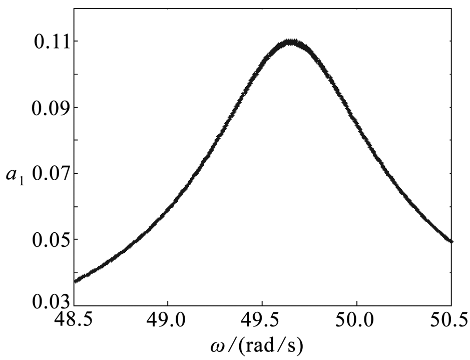
从方程(30)可以得到分岔方程

$$\begin{cases}G_1=\cos^2(\vartheta_1)+\sin^2(\vartheta_1)-1=0,\\ G_3=\cos^2(\vartheta_3)+\sin^2(\vartheta_3)-1=0.\end{cases} \tag{31}$$

结构参数如下:

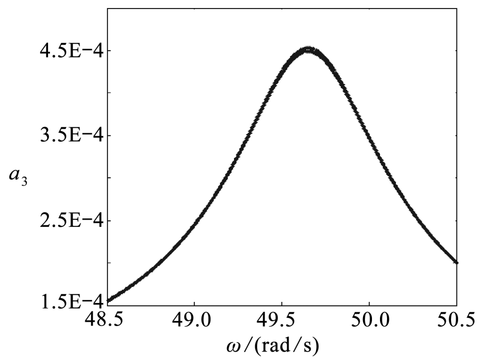
$$\begin{cases} e_1 = 5 \times 10^{-5}\ \text{m},\ e_2 = 5 \times 10^{-5}\ \text{m},\ J_2 = 10\ 000\ \text{kg}\cdot\text{m}^2, \\ J_1 = (1/8) \times 10\ 000\ \text{kg}\cdot\text{m}^2,\ m_2 = 30\ 000\ \text{kg},\ m_1 = 9\ 400\ \text{kg}, \\ c = 7 \times 10^{-4}\ \text{m},\ g = 9.8\ \text{m/s}^2,\ \omega = 314\ \text{rad/s}, \\ k_{x_3} = 1.55 \times 10^9\ \text{N/m},\ k_{x_2} = 1.51 \times 10^9\ \text{N/m}, \\ k_{x_1} = 1.6 \times 10^9\ \text{N/m},\ k_{y_3} = 1.6 \times 10^9\ \text{N/m}, \\ k_{y_2} = 1.7 \times 10^9\ \text{N/m},\ k_{y_1} = 1.8 \times 10^9\ \text{N/m}, \\ k_{t_1} = 1.7 \times 10^7\ \text{N/m},\ k_{t_2} = 1.7 \times 10^7\ \text{N/m},\ k_{t_3} = 1.7 \times 10^7 \text{N/m}, \\ C_1 = 10\ 000\ \text{N}\cdot\text{s/m},\ C_2 = 10\ 000\ \text{N}\cdot\text{s/m},\ C_3 = 10\ 000\ \text{N}\cdot\text{s/m}, \\ C_{t_1} = 3\ 000\ \text{N}\cdot\text{s/m},\ C_{t_2} = 3\ 000\ \text{N}\cdot\text{s/m},\ C_{t_3} = 3\ 000\ \text{N}\cdot\text{s/m}, \\ f_{x_1} = 0, f_{y_1} = 0, f_{x_2} = 0, f_{y_2} = 0,\ M_{t_{11}} = 0.9 \times 10^5\ \text{N}\cdot\text{m}, \\ M_{t_{12}} = 10^4\ \text{N}\cdot\text{m},\ M_{t_2} = -10^5\ \text{N}\cdot\text{m},\ M_{t_1} = M_{t_{11}} + M_{t_{12}} \times \sin(\upsilon t/\omega), \end{cases} \tag{32}$$

系统的响应曲线如图 2 所示.



(a) 弯曲响应曲线

(a) Response curve of bending direction



(b) 扭转响应曲线

(b) Response curve of torsional direction

图 2 发电机幅频响应曲线

Fig. 2 Response curves of generator

以往的文献指出,在系统发生次同步谐振的情况下,转子轴系的扭转振动幅值会大幅增加,因此需要研究扭转振动. 我们的结果如图 2 可知:在次同步谐振和组合共振的双重作用下,系统的弯曲振动幅值和扭转振动幅值都有大幅增加,在此种情况下必须要考虑轴系的弯扭耦合振动.

3 两个状态变量的奇异性分析

引入下列变换

$$\lambda_3 - \upsilon/\omega = \lambda,$$

方程(31)可变换为

$$\begin{aligned} G_1 = {} & \beta_1 a_1^4 + (\beta_2 \lambda^2 a_3^2 + \beta_3 a_3^2 + \beta_0) a_1^2 + \\ & a_3^4(\beta_4 \lambda^4 + \beta_5 \lambda^2 + \beta_6) + a_3^2(\beta_7 \lambda^2 + \beta_8) + \beta_9, \end{aligned} \tag{33}$$

$$\begin{aligned} G_3 = {} & (\alpha_1 \lambda^2 + \alpha_2 \lambda + \alpha_3) a_1^4 + ((\alpha_4 \lambda + \alpha_5 \lambda^3 + \alpha_6 \lambda^2 + \alpha_7) a_3^2 + \alpha_8 \lambda^2 + \\ & \alpha_9 \lambda + \alpha_{10}) a_1^2 + (\alpha_{11} \lambda + \alpha_{12} \lambda^3 + \alpha_{13} \lambda^2 + \alpha_{14} + \alpha_{15} \lambda^4) a_3^4 + \end{aligned}$$

$$(\alpha_{16}\lambda+\alpha_{17}\lambda^3+\alpha_{18}\lambda^2+\alpha_{19})a_3^2+\alpha_{20}\lambda^2+\alpha_{21}\lambda+\alpha_{22}, \tag{34}$$

其中，a_1 和 a_3 是状态变量，λ 是分岔参数，β_9，α_{22} 是开折参数。

两个状态变量的转迁集为[12-13]：

$$\begin{cases} B=\{\alpha\in R^k:\exists(z,\lambda), \\ \quad \text{使得 } G(z,\lambda,\alpha)=0, \\ \quad \operatorname{rank}(DG)_{z,\lambda,\alpha}\leqslant 1\}, \\ H=\{\alpha\in R^k:\exists(z,\lambda), \\ \quad \text{使得 } G(z,\lambda,\alpha)=0, \\ \quad v\neq 0,\ (dG)_{z,\lambda,\alpha}\cdot v=0, \\ \quad (d^2G)_{z,\lambda,\alpha}(v,v)\in \\ \quad \operatorname{range}(dG)_{z,\lambda,\alpha}\}, \\ D=\{\alpha\in R^k:\exists(z_1,z_2,\lambda), \\ \quad \text{使得 } z_1\neq z_2 G(z_i,\lambda,\alpha)=0, \\ \quad \det(dG)_{z_i,\lambda,\alpha}=0,\ i=1,2\}. \end{cases} \tag{35}$$

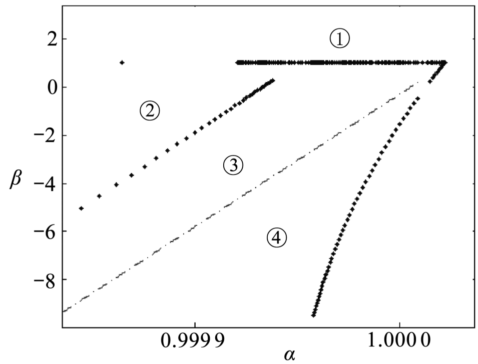
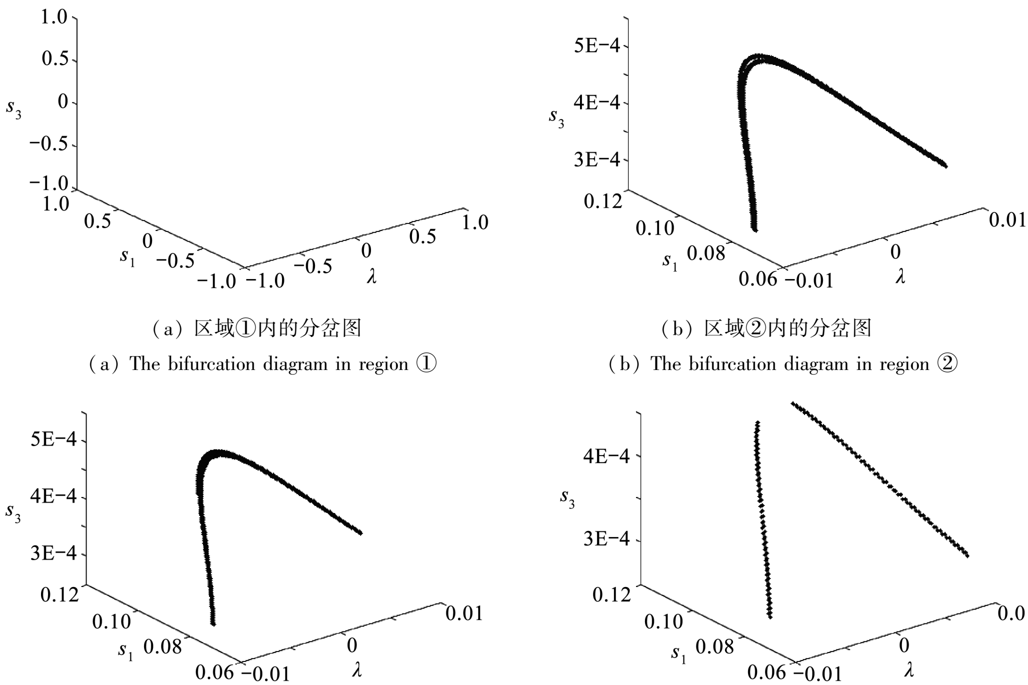


图 3 转迁集

Fig. 3 The transition sets



（a）区域①内的分岔图

（a）The bifurcation diagram in region ①

（b）区域②内的分岔图

（b）The bifurcation diagram in region ②

（c）区域③内的分岔图

（c）The bifurcation diagram in region ③

（d）区域④内的分岔图

（d）The bifurcation diagram in region ④

图 4 不同保持域内的分岔图

Fig. 4 The bifurcation diagrams in different persistent regions

由方程(33)～方程(35)，转迁集可以描述为如图 3，并且在不同保持域上的分岔图在图 4 中给出。

从图 3 可以推出：转迁集将参数平面分为 4 个不同的保持域，分岔图在不同的保持域上不

同. 从图 4 中可以得出：在参数平面内系统有 4 种不同的分岔模式，这为系统的动力学分析和设计提供了理论指导.

4　数值仿真

4.1　非组合共振情况

在次同步和非组合共振情况下，系统参数满足如下情况：

$$\lambda_3 - \upsilon/\omega = \varepsilon\sigma,\ \lambda_1 + \lambda_3 - 1 \neq 0,$$

系统的结构参数选取如下：

$$\begin{cases} e_1 = 5\times10^{-5}\ \mathrm{m},\ e_2 = 5\times10^{-5}\ \mathrm{m},\ J_2 = 10\,000\ \mathrm{kg\cdot m^2}, \\ J_1 = (1/8)\times10\,000\ \mathrm{kg\cdot m^2},\ m_2 = 30\,000\ \mathrm{kg},\ m_1 = 9\,400\ \mathrm{kg}, \\ c = 7\times10^{-4}\ \mathrm{m},\ g = 9.8\ \mathrm{m/s^2},\ \omega = 314\ \mathrm{rad/s}, \\ k_{x_3} = 1.55\times10^9\ \mathrm{N/m},\ k_{x_2} = 1.51\times10^9\ \mathrm{N/m},\ k_{y_3} = 1.6\times10^9\ \mathrm{N/m}, \\ k_{y_2} = 1.7\times10^9\ \mathrm{N/m},\ k_{y_1} = 1.8\times10^9\ \mathrm{N/m},\ k_{t_1} = 1.7\times10^7\ \mathrm{N/m}, \\ k_{t_2} = 1.7\times10^7\ \mathrm{N/m},\ k_{t_3} = 1.7\times10^7\ \mathrm{N/m}, \\ C_1 = 10\,000\ \mathrm{N\cdot s/m},\ C_2 = 10\,000\ \mathrm{N\cdot s/m},\ C_3 = 10\,000\ \mathrm{N\cdot s/m}, \\ C_{t_1} = 3\,000\ \mathrm{N\cdot s/m},\ C_{t_2} = 3\,000\ \mathrm{N\cdot s/m},\ C_{t_3} = 3\,000\ \mathrm{N\cdot s/m}, \\ f_{x_1} = 0,\ f_{y_1} = 0,\ f_{x_2} = 0,\ f_{y_2} = 0,\ M_{t_{11}} = 0.9\times10^5\ \mathrm{N\cdot m}, \\ M_{t_{12}} = 10^4\ \mathrm{N\cdot m},\ M_{t_2} = -10^5\ \mathrm{N\cdot m},\ M_{t_1} = M_{t_{11}} + M_{t_{12}}\times\sin\left(\dfrac{\upsilon t}{\omega}\right), \end{cases} \tag{36}$$

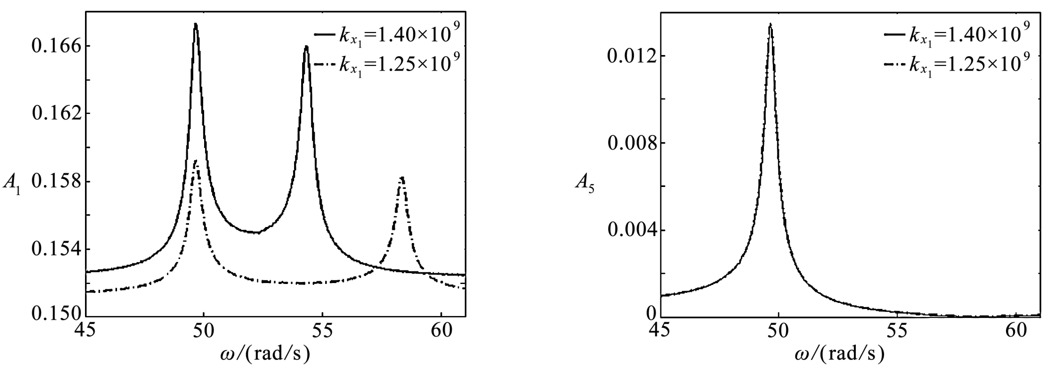
数值计算结果如图 5 所示.

从图 5 可以得出：在次同步和非组合共振情况下，在扭转固有频率附近扭转振动的振幅大幅增加，并且弯曲振幅有 2 个峰值，其中一个是由扭转的固有频率引起的弯曲振幅的变化，另一个是由非共振因素 $1-\lambda_1$ 引起的. 在 $\lambda_1+\lambda_3-1$ 趋近于0 的过程中，弯曲振动的振幅将越来越大. 次同步组合共振会发生在 $\lambda_1+\lambda_3\approx1$ 的情况下.

4.2　组合共振情况

在次同步组合共振的情况下，系统满足

$$\lambda_3 - \upsilon/\omega = \varepsilon\sigma,\ \lambda_1 + \lambda_3 \approx 1,$$

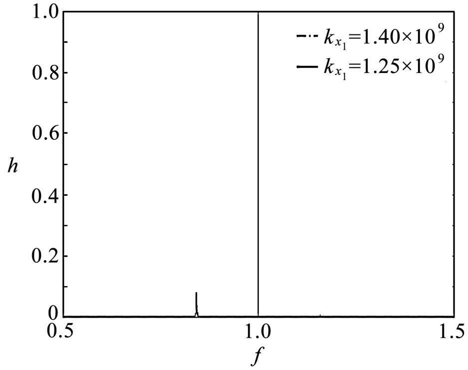


（a）弯曲响应曲线

（a）Response curve of the bending direction

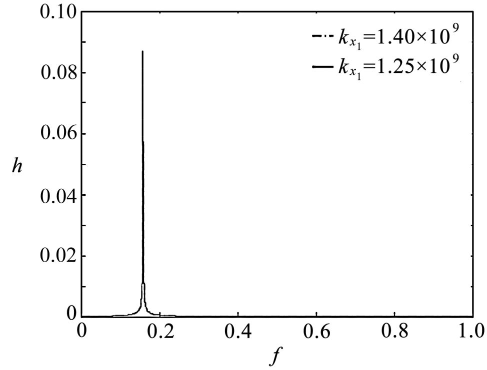
（b）扭转响应曲线

（b）Response curve of the torsional direction



（c）弯曲频谱

（c）Spectrum curve of the bending direction

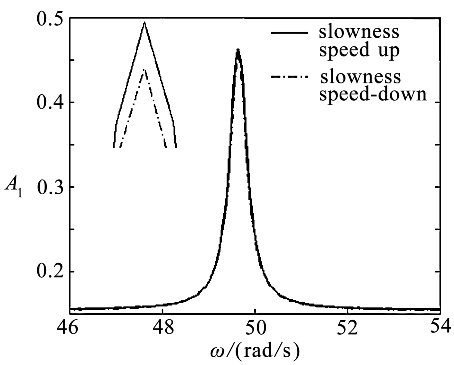


（d）扭转频谱

（d）Spectrum curve of the torsional direction

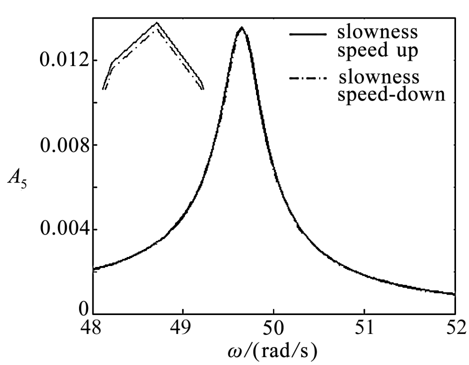
图5 次同步非组合共振发电机的幅频特性曲线和频谱曲线

Fig. 5 Response curve and spectrum curve of generator for the case of sub-synchronous resonance and uncombined resonance



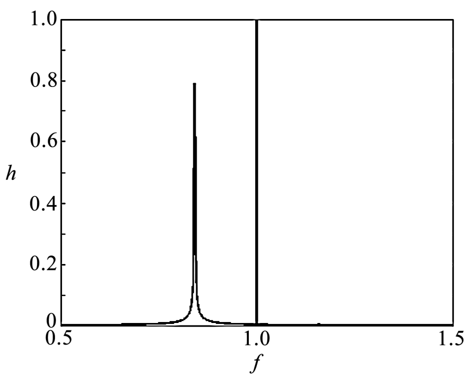
（a）弯曲方向响应曲线

（a）Response curve of the bending direction



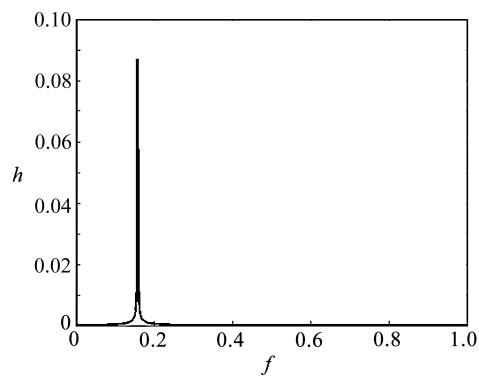
（b）扭转方向响应曲线

（b）Response curve of the torsional direction



（c）弯曲频谱

（c）Spectrum curve of the bending direction



（d）扭转频谱

（d）Spectrum curve of the torsional direction

图6 次同步组合共振发电机的幅频特性曲线和频谱曲线

Fig. 6 Response curve and spectrum curve of generator for the case of sub-synchronous resonance and combined resonance

系统结构参数选取如方程(32)，次同步组合共振发电机的幅频特性曲线和频谱曲线，如图5

所示.

由图 6 可以得出:当发生次同步组合共振和次同步非组合共振时,扭转振动的变化是很大的,在次同步非组合共振的情况下,弯曲振动会出现 2 个小峰值,第 1 个峰值是由于弯扭耦合作用中次同步振动发生引起的扭转振动增加而引起的弯曲振动. 第 2 个峰值:由频谱分析可得,在弯曲振动的频率成分中会有激励频率加减扭固有转振动的频率成分. 也是由弯扭耦合引起的. 在次同步组合共振的情况下,两个峰值重合,并且重合的峰值比非组合共振时两个峰值的和还要大很多,在次同步组合共振情况下,将可能引发严重的故障,应当引起高度的重视. 在加速和减速两种情况下,系统的弯曲振动响应曲线会略有不同,扭转振动共振曲线基本相同.

5 结 论

本文建立了低发转子系统弯扭的非线性模型. 经过分析得到结论如下.

1) 偏心是引起弯扭耦合的原因之一,因此应尽量减小不平衡量以降低弯扭耦合的影响.

2) 在次同步组合共振的情况下,不仅引起扭转振动的变化,而且也引起弯曲振动的变化,仅考虑在次同步情况下对轴系扭转的影响是不全面的.

3) 在次同步非组合共振时,弯曲振动会出现两个峰值,其中一个是由扭转的固有频率引起的弯曲振幅的变化,另一个是由非共振因素 $1-\lambda_1$ 引起的(见图 5(a)),且比非次同步情况略大,此结果为故障诊断提供了一种识别方法.

4) 在组合共振情况下,弯曲振动会剧烈增加,快速 Fourier 变换显示,弯曲振动固有频率成分大幅增加. 弯曲振动与扭转振动能量交换通道打开. 应该尽量避免设计上出现此种情况.

5) 在转迁集的不同参数区间内,系统的动态行为不同. 为动力学分析和设计提供了指导.

附 录

$$
\begin{aligned}
f_2 = {} & \frac{-m_1e_1^3(1+z_6)^2\cos(t+\theta_1)^3}{J_1c} - \frac{m_1e_1^2}{J_1}\Bigg(-(c_{11}+c_{12})z_2 + \frac{f_{x_1}}{m_1\omega^2c} - (k_{x_{11}}+k_{x_{12}})z_1 + \\
& k_{x_{12}}z_7 + c_{12}z_8\Bigg)\cos(t+\theta_1)^2 + \Bigg(\frac{-m_1e_1^3(1+z_6)^2\sin(t+\theta_1)^2}{J_1c} + \\
& \Bigg(\frac{-m_1e_1^2}{J_1}\Bigg(k_{y_{12}}z_9 + \frac{f_{y_1}}{m_1\omega^2c} - G_1 - (k_{y_{11}}+k_{y_{12}})z_3 - (c_{11}+c_{12})z_4 + c_{12}z_{10}\Bigg) - \\
& \frac{e_1(J_1+m_1e_1^2)G_2}{cJ_1}\Bigg)\sin(t+\theta_1) - \frac{(-J_1-m_1e_1^2)_1e_1(1+z_6)^2}{J_1c}\Bigg)\cos(t+\theta_1) + \\
& \frac{e_1(J_1+m_1e_1^2)}{cJ_1}\Bigg(-(c_{t_{11}}+c_{t_{12}})(1+z_6) + \frac{M_{t_1}}{(J_1+m_1e_1^2)\omega^2} - (k_{t_{11}}+k_{t_{12}})z_5 + \\
& k_{t_{12}}z_{11} + c_{t_{12}}(1+z_{12})\Bigg)\sin(t+\theta_1) - \frac{(-J_1-m_1e_1^2)}{J_1}\Bigg(-(c_{11}+c_{12})z_2 + \frac{f_{x_1}}{m_1\omega^2c} + c_{12}z_8\Bigg) + \\
& \frac{(-J_1-m_1e_1^2)e_1}{J_1c}\cos(t+\theta_1),
\end{aligned}
$$

$$
\begin{aligned}
f_4 = {} & \Bigg(\frac{m_1e_1^2}{J_1}\Bigg(k_{y_{12}}z_9 + \frac{f_{y_1}}{m_1\omega^2c} - G_1 - (k_{y_{11}}+k_{y_{12}})z_3 - (c_{11}+c_{12})z_4 + c_{12}z_{10}\Bigg) + \\
& \frac{e_1(J_1+m_1e_1^2)G_2}{cJ_1}\Bigg)\cos(t+\theta_1)^2 + \Bigg(\frac{-m_1e_1^2}{J_1}\Bigg(-(c_{11}+c_{12})z_2 + \frac{f_{x_1}}{m_1\omega^2c} - \\
& (k_{x_{11}}+k_{x_{12}})z_1 + k_{x_{12}}z_7 + c_{12}z_8\Bigg)\sin(t+\theta_1) - \frac{e_1(J_1+m_1e_1^2)}{cJ_1}\times\Bigg(-(c_{t_{11}}+c_{t_{12}})(1+z_6) +
\end{aligned}
$$

$$\frac{M_{t_1}}{(J_1+m_1e_1^2)\omega^2}-(k_{t_{11}}+k_{t_{12}})z_5+k_{t_{12}}z_{11}+c_{t_{12}}(1+z_{12})\Big)\Big)\cos(t+\theta_1)+$$

$$\frac{e_1}{c}(1+z_6)^2\sin(t+\theta_1)+\frac{f_{y_1}}{m_1\omega^2c}-G_1-(c_{11}+c_{12})z_4+c_{12}z_{10}-\frac{e_1}{c}\sin(t+\theta_1),$$

$$f_6=\Big(\frac{-m_1e_1c}{J_1}\Big(k_{y_{12}}z_9+\frac{f_{y_1}}{m_1\omega^2c}-G_1-(k_{x_{11}}+k_{x_{12}})z_3-(c_{11}+c_{12})z_4+c_{12}z_{10}\Big)-$$

$$\frac{(J_1+m_1e_1^2)/(J_1G_2)}{J_1}\Big)\cos(t+\theta_1)+\frac{m_1e_1c}{J_1}\Big(-(c_{11}+c_{12})z_2+\frac{f_{x_1}}{m_1\omega^2c}-(k_{x_{11}}+k_{x_{12}})z_1+$$

$$k_{x_{12}}z_7+c_{12}z_8\Big)\sin(t+\theta_1)+\frac{(J_1+m_1e_1^2)}{J_1}\Big(-(c_{t_{11}}+c_{t_{12}})(1+z_6)+$$

$$\frac{M_{t_1}}{(J_1+m_1e_1^2)\omega^2}+c_{t_{12}}(1+z_{12})\Big),$$

$$f_8=\frac{-m_2e_2^3}{J_2c}(1+z_6)^2\cos(t+\theta_2)^3-\frac{m_2e_2^2}{J_2}\Big(c_{22}z_2+\frac{f_{x_2}}{m_2\omega^2c}+$$

$$k_{22}z_1-(k_{x_{21}}+k_{x_{22}})z_7-(c_{21}+c_{22})z_8\Big)\cos(t+\theta_2)^2+\Big(\frac{-e_2^3m_2}{J_2c}(1+z_6)^2\sin(t+\theta_2)^2+$$

$$\Big(\frac{-e_2^2m_2}{J_2}\Big(-(k_{y_{21}}+k_{y_{22}})z_9+\frac{f_{y_2}}{m_2\omega^2c}-G_3+k_{x_{22}}z_3+c_{22}z_4-(c_{21}+c_{22})z_{10}\Big)-$$

$$\frac{e_2}{cJ_2}(J_2+m_2e_2^2)G_4\Big)\sin(t+\theta_2)-\frac{(-J_2-m_2e_2^2)e_2\,(1+z_6)^2}{cJ_2}\Big)\cos(t+\theta_2)+$$

$$\frac{e_2}{J_2c}(J_2+m_2e_2^2)\Big(c_{t_{22}}(1+z_6)+\frac{M_{t_2}}{(J_2+m_2e_2^2)\omega^2}+k_{t_{22}}z_5-(k_{t_{21}}+k_{t_{22}})z_{11}-$$

$$(c_{t_{21}}+c_{t_{22}})(1+z_{12})\Big)\sin(t+\theta_2)-\frac{(-J_2-m_2e_2^2)}{J_2}\Big(c_{22}z_2+\frac{f_{x_2}}{m_2\omega^2c}-(c_{21}+c_{22})z_8\Big)-$$

$$\Big(\frac{-(-J_2-m_2e_2^2)e_2}{J_2c}\Big)\cos(t+\theta_2),$$

$$f_{10}=\Big(\frac{e_2^2m_2}{J_2}\Big(-(k_{y_{21}}+k_{y_{22}})z_9+\frac{f_{y_2}}{m_2\omega^2c}-G_3+k_{x_{22}}z_3+c_{22}z_4-(c_{21}+c_{22})z_{10}\Big)+$$

$$\frac{e_2}{J_2c}(J_2+m_2e_2^2)G_4\Big)\cos(t+\theta_2)^2+\Big(\frac{-m_2e_2^2}{J_2}\Big(c_{22}z_2+\frac{f_{x_2}}{m_2\omega^2c}+k_{x_{22}}z_1-$$

$$(k_{x_{21}}+k_{x_{22}})z_7-(c_{21}+c_{22})z_8\Big)\sin(t+\theta_2)-\frac{e_2}{cJ_2}(J_2+m_2e_2^2)\Big(c_{t_{22}}(1+z_6)+$$

$$\frac{M_{t_2}}{(J_2+m_2e_2^2)\omega^2}+k_{t_{22}}z_5-(k_{t_{21}}+k_{t_{22}})z_{11}-(c_{t_{21}}+c_{t_{22}})(1+z_{12})\Big)\Big)\cos(t+\theta_2)+$$

$$\frac{e_2}{c}(1+z_6)^2\sin(t+\theta_2)+\frac{f_{y_2}}{m_2\omega^2c}-G_3+c_{22}z_4-(c_{21}+c_{22})z_{10}-\frac{e_2}{c}\sin(t+\theta_2),$$

$$f_{12}=\Big(\frac{-m_2e_2c}{J_2}\Big(-(k_{y_{21}}+k_{y_{22}})z_9+\frac{f_{y_2}}{m_2\omega^2c}-G_3+k_{x_{22}}z_3+c_{22}z_4-(c_{21}+c_{22})z_{10}\Big)-$$

$$\frac{(J_2+m_2e_2^2)G_4}{J_2}\Big)\cos(t+\theta_2)+\frac{m_2e_2c}{J_2}\Big(c_{22}z_2+\frac{f_{x_2}}{m_2\omega^2c}+k_{x_{22}}z_1-(k_{x_{21}}+k_{x_{22}})z_7-$$

$$(c_{21}+c_{22})z_8\Big)\sin(t+\theta_2)+\frac{(J_2+m_2e_2^2)}{J_2}\Big(c_{t_{22}}(1+z_6)+\frac{M_{t_2}}{(J_2+m_2e_2^2)\omega^2}-$$

$$(c_{t_{21}}+c_{t_{22}})(1+z_{12})\Big).$$

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Study on Combined Resonance of Low Pressure Cylinder-Generator Rotor System With Bending-Torsion Coupling

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Abstract: The nonlinear model of the low pressure cylinder-generator rotor system was presented for the study of sub-synchronous resonance and combined resonance. Analytical results were obtained by the averaging method. The transition sets and bifurcation diagrams were obtained by the singularity theory for two-state variable system. The bifurcation characteristics were analyzed, which can provide a basis for the optimal design and fault diagnosis of the rotor system. Finally, the theoretical results are verified by the numerical results.

Key words: bending-torsion coupling vibration of rotor systems; sub-synchronous resonance; nonlinear dynamics of rotor; combined resonance of bending-torsion coupling vibration