文章编号:1000-0887(2013)04-0364-09

基于厚板拉伸振动精确化方程求解动应力集中问题*

胡 超[1,2], 周传平[1], 佟广清[2], 刘殿魁[3]

(1. 同济大学 航空航天与力学学院,上海 200092;
2. 扬州大学 建筑科学与工程学院,江苏 扬州 225127;
3. 哈尔滨工程大学 航天与建筑学院,哈尔滨 150001)

摘要: 过去,对拉伸平板考虑应力集中的工程设计多借鉴弹性力学平面问题分析求解结果,例如弹性力学 Kirsch 问题的解或弹性动力学平面问题的解. 基于厚板拉伸振动精确化方程,对含圆孔平板中弹性波散射与动应力集中问题进行了研究. 研究结果表明:1)两种模型得到的开孔附近的应力是不同的;2)当入射波波数变大或者说入射波频率变高时,动应力集中系数最大值趋于单位 1. 含孔平板拉伸振动的动应力集中系数最大值达到 3.30,以及基于弹性动力学平面问题模型得到的结果为 2.77. 对数值计算结果做了分析讨论, 可以看到,当孔径厚度比是 $a/h = 0.10$ 时,基于平板拉伸振动精确化方程得到的动应力集中系数可以达到最大值,超出基于弹性动力学平面问题所得到结果的 19%. 分析方法和数值计算结果可望能在工程平板结构的动力学分析和强度设计中得到应用.

关 键 词: 平板拉伸振动精确化方程; 弹性波散射与动应力集中; 厚壁结构动力学; 剪应力一阶矩

中图分类号: O347.4; TU311.3　　**文献标志码:** A

DOI: 10.3879/j.issn.1000-0887.2013.04.005

引 言

平板是航空航天、土木建筑和机械工程中广泛采用的典型结构. 为满足工程设计需要不可避免地要在平板上开孔. 开孔将降低结构的承载能力和减少结构的使用寿命. 因此,国内外许多专家和学者对平板开孔静/动应力集中问题进行了研究[1-8].

过去,对拉伸平板考虑应力集中的工程设计多借鉴弹性力学平面问题分析求解结果,例如弹性力学 Kirsch 问题的解,或弹性动力学平面问题即二维区域弹性波散射与动应力集中结果. 显然,利用弹性动力学二维模型求解应力集中问题与平板实际结构特点有很大的差异. 有必要建立更为精化的平板拉伸振动模型,并基于此模型对结构内的动应力集中问题进行研究. 随着现代科学技术的发展,工程结构设计趋于轻型化,而实现轻型化的途径是采用先进材料和完善结构设计理论.

* **收稿日期:** 2013-01-07; **修订日期:** 2013-03-08
基金项目: 国家自然科学基金资助项目(10572045;51276129); 航天创新科学研究基金资助项目
作者简介: 胡超(1961—), 男,哈尔滨人,特聘教授,所长(通讯作者. E-mail:chaohu@tongji.edu.cn).

直到目前,基于平板拉伸振动支配方程分析求解开孔动应力集中的工作尚未见文献报道.本文将基于文献[9]给出的厚板拉伸振动精确化方程,对含圆孔平板中弹性波散射与动应力集中问题进行研究,并给出可供工程应用的分析方法和数值结果.

1 平板拉伸振动方程及其求解

根据文献[9],平板拉伸自由振动的精确化方程为

$$\nabla^2\nabla^2 E - 12\left(\frac{1}{h^2} + \frac{2-\kappa^2}{24(1-\kappa)}T_2^2\right)\nabla^2 E + \frac{3}{1-\kappa}\left(\frac{1}{h^2} + \frac{1+3\kappa}{24}T_2^2\right)T_2^2 E = 0, \tag{1a}$$

$$\left[(3-2\kappa)\nabla^2 - T_2^2 - \frac{24}{h^2}\right](\nabla^2 - T_1^2)F = -\left[(3-4\kappa)\nabla^2 - (1-2\kappa^2)T_2^2 - \frac{24}{h^2}(1-2\kappa)\right]W, \tag{1b}$$

$$\nabla^2 f - T_2^2 f = 0, \tag{1c}$$

式中, E, F, f 是平板拉伸振动的 3 个广义位移函数;

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

为 Laplace 算子;

$$T_j^2 = \frac{1}{c_j^2}\frac{\partial^2}{\partial t^2} \qquad (j=1,2);$$

c_1, c_2 分别为弹性纵波和横波速度:

$$c_1^2 = \frac{\lambda+2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho};\ \kappa = \frac{1-2\nu}{2(1-\nu)};$$

λ, μ 是材料的 Lamé 常数; ν, ρ 分别表示平板材料的 Poisson 比和密度; h 表示平板的厚度.

不失一般性,研究问题的谐和振动解,设

$$E = \tilde{E}e^{-i\omega t},\ F = \tilde{F}e^{-i\omega t},\ f = \tilde{f}e^{-i\omega t}, \tag{2}$$

其中, ω 是平板拉伸振动的圆频率; i 是虚数单位.

在以下分析中略去时间因子和广义位移函数上的符号 ~. 将式(2)代入到式(1)可得方程

$$\prod_{j=1}^{2}(\nabla^2 + \alpha_j^2)E = 0, \tag{3a}$$

$$\nabla^2 f + k_2^2 f = 0, \tag{3b}$$

式中, $\alpha_j(j=1,2)$ 是弹性波波数,满足方程

$$\alpha^4 + 12\left(\frac{1}{h^2} - \frac{2-\kappa^2}{24(1-\kappa)}k_2^2\right)\alpha^2 - \frac{3}{1-\kappa}\left(\frac{1}{h^2} - \frac{1+3\kappa}{24}k_2^2\right)k_2^2 = 0,$$

$$k_j^2 = \omega^2/c_j^2 \qquad (j=1,2).$$

方程(3)描述的散射波一般解可描述为

$$E = \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} A_{mn}H_n^{(1)}(\alpha_m r)e^{in\theta}, \tag{4a}$$

$$F = \sum_{m=1}^{2}\sum_{n=-\infty}^{\infty} A_{mn}\delta_m H_n^{(1)}(\alpha_m r)e^{in\theta}, \tag{4b}$$

$$f = \sum_{n=-\infty}^{\infty} B_n K_n(k_2 r)e^{in\theta}, \tag{4c}$$

式中，$\delta_j(j=1,2)$ 是散射波函数的比例系数，

$$\delta_j = \frac{(3-4\kappa)\alpha_j^2h^2 - (1-2\kappa^2)k_2^2h^2 + 24(1-2\kappa)}{[(3-2\kappa)\alpha_j^2h^2 - k_2^2h^2 + 24](\alpha_j^2 - k_1^2)},$$

$\mathrm{H}_n^{(1)}(\cdot)$ 是 Hankel 函数；$\mathrm{K}_n(\cdot)$ 是虚宗量 Bessel 函数；$A_{mn}(m=1,2)$ 和 B_n 是散射波模式系数，可由开孔边界条件确定.

基于平板拉伸振动精确化理论，平板结构中广义内力的表达式为[9]

$$N_r = \int_{-h/2}^{h/2}\sigma_r\mathrm{d}z = 2\lambda\frac{\sin\left(\frac{h}{2}\square_1\right)}{\square_1}(\nabla^2F+E) + 4\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial\theta}\frac{\sin\left(\frac{h}{2}\square_2\right)}{\square_2}f\right] + \frac{\partial^2}{\partial r^2}\left[\sum_{j=1}^{2}(-1)^{j-1}\frac{\sin\left(\frac{h}{2}\square_j\right)}{\square_j}T_1^{-2}(\square_1^2F+E) + \frac{\sin\left(\frac{h}{2}\square_1\right)}{\square_1}F\right]\right\}, \tag{5a}$$

$$N_\theta = \int_{-h/2}^{h/2}\sigma_\theta\mathrm{d}z = 2\lambda\frac{\sin\left(\frac{h}{2}\square_1\right)}{\square_1}(\nabla^2F+E) + 4\mu\left\{\frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{\sin\left(\frac{h}{2}\square_2\right)}{\square_2}f + \frac{1}{r}\frac{\partial}{\partial r}\left[\sum_{j=1}^{2}(-1)^{j-1}\frac{\sin\left(\frac{h}{2}\square_j\right)}{\square_j}T_1^{-2}(\square_1^2F+E) + \frac{\sin\left(\frac{h}{2}\square_1\right)}{\square_1}F\right]\right\}, \tag{5b}$$

$$N_{r\theta} = \int_{-h/2}^{h/2}\tau_{r\theta}\mathrm{d}z = \mu\left\{2\left(\nabla^2 - 2\frac{\partial^2}{\partial r^2}\right)\frac{\sin\left(\frac{h}{2}\square_2\right)}{\square_2}f + 4\left(\frac{1}{r}\frac{\partial^2}{\partial\theta\partial r} - \frac{1}{r^2}\frac{\partial}{\partial\theta}\right)\times\left[\sum_{j=1}^{2}(-1)^{j-1}\frac{\sin\left(\frac{h}{2}\square_j\right)}{\square_j}T_1^{-2}(\square_1^2F+E) + \frac{\sin\left(\frac{h}{2}\square_1\right)}{\square_1}F\right]\right\}, \tag{5c}$$

$$Q_r = \int_{-h/2}^{h/2}\tau_{zr}\mathrm{d}z = 0,\ Q_\theta = \int_{-h/2}^{h/2}\tau_{z\theta}\mathrm{d}z = 0, \tag{5d}$$

其中

$$\square_j^2 = \nabla^2 - T_j^2,\ \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}.$$

2 入射波的激发与总波场

设在平板结构端部有一拉压弹性波沿 x 轴正方向入射，其表达式为

$$E^{(\mathrm{I})} = E_0\mathrm{e}^{\mathrm{i}\alpha_1x} = E_0\sum_{n=-\infty}^{\infty}(\mathrm{i})^n\mathrm{J}_n(\alpha_1r)\mathrm{e}^{\mathrm{i}n\theta}, \tag{6a}$$

$$F^{(\mathrm{I})} = \delta_1E^{(\mathrm{I})}, \tag{6b}$$

$$f^{(\mathrm{I})} = 0, \tag{6c}$$

式中，α_1 是入射波波数，E_0 是入射波的幅值.

平板中拉压弹性波总波场可描述为

$$E = E^{(I)} + E^{(S)} = E_0 e^{i\alpha_1 x} + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} A_{mn} H_n^{(1)}(\alpha_m r) e^{in\theta}, \tag{7a}$$

$$F = F^{(I)} + F^{(S)} = \delta_1 E_0 e^{i\alpha_1 x} + \sum_{m=1}^{2} \sum_{n=-\infty}^{\infty} A_{mn} \delta_m H_n^{(1)}(\alpha_m r) e^{in\theta}, \tag{7b}$$

$$f = f^{(I)} + f^{(S)} = \sum_{n=-\infty}^{\infty} B_n K_n(k_2 r) e^{in\theta}. \tag{7c}$$

3 满足开孔边界条件确定模式系数

设平板结构拉压振动时开孔为自由边界条件,厚板理论可以满足 3 个边界条件

$$N_r|_{r=a} = 0,\ N_{r\theta}|_{r=a} = 0,\ Q_r|_{r=a} = 0, \tag{8}$$

其中, a 为平板开孔的半径.

由于剪应力(零阶矩)边界条件自动满足,因此需要补充一个考虑剪应力一阶消失矩的方程,其表达式为

$$M_{Q_r} = \int_{-h/2}^{h/2} z\tau_{zr} dz = -\frac{1}{4\kappa(1-\kappa)} D\left[(1-2\kappa)\frac{\partial}{\partial r}(\Box_1^2 F + E) + 2\kappa\frac{\partial}{\partial r}\Box_1^2 F\right], \tag{9}$$

式中, D 是平板结构的抗弯刚度,

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

将式(7)和(5)代入到式(8)和(9)中,可得到确定模式系数 A_{1n}, A_{2n}, B_n 的无穷代数方程组

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} A_{1n} \\ A_{2n} \\ B_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \tag{10}$$

其中

$$a_{1j} = (1-2\kappa)(1-\alpha_j^2\delta_j)\left[1 + \frac{1}{24}h^2(\alpha_j^2 - k_1^2)\right] H_n^{(1)}(\alpha_j a) + \alpha_j^2\gamma_j\left\{\frac{1}{2}\kappa\delta_j - \frac{1}{48}h^2[(1-\kappa) - (\alpha_j^2 - k_1^2)\delta_j]\right\} \times [H_{n-2}^{(1)}(\alpha_j a) - 2H_n^{(1)}(\alpha_j a) + H_{n+2}^{(1)}(\alpha_j a)] \quad (j = 1,2),$$

$$a_{13} = 2in\kappa\left\{-\frac{1}{a^2}K_n(k_2 a) - \frac{1}{2}\left(\frac{1}{a}k_2\right)[K_{n-1}(k_2 a) + K_{n+1}(k_2 a)]\right\},$$

$$b_1 = -E_0(1-2\kappa)(1-\alpha_1^2\delta_1)\left[1 + \frac{1}{24}h^2(\alpha_1^2 - k_1^2)\right] i^n J_n(\alpha_1 a) - E_0\alpha_1^2 i^n\left\{\frac{1}{2}\kappa\delta_1 - \frac{1}{48}h^2[(1-\kappa) - (\alpha_1^2 - k_1^2)\delta_1]\right\} \times [J_{n-2}(\alpha_1 a) - 2J_n(\alpha_1 a) + J_{n+2}(\alpha_1 a)],$$

$$a_{2j} = in\left\{\kappa\delta_j - \frac{h^2}{24}[(1-\kappa) - \delta_j(\alpha_j^2 - k_1^2)]\right\} \times \left\{\frac{1}{a}\alpha_j[H_{n-1}^{(1)}(\alpha_j a) - H_{n+1}^{(1)}(\alpha_j a)] - \frac{2}{a^2}H_n^{(1)}(\alpha_j a)\right\} \quad (j = 1,2),$$

$$a_{23}=-\frac{1}{2}\kappa k_2^2[\mathrm{K}_{n-2}(k_2a)+4\mathrm{K}_n(k_2a)+\mathrm{K}_{n+2}(k_2a)],$$

$$b_2=-E_0(\mathrm{i})^{n+1}n\left\{\kappa\delta_1-\frac{h^2}{24}[(1-\kappa)-\delta_1(\alpha_1^2-k_1^2)]\right\}\times$$
$$\left\{\frac{1}{a}\alpha_1[\mathrm{J}_{n-1}(\alpha_1a)-\mathrm{J}_{n+1}(\alpha_1a)]-\frac{2}{a^2}\mathrm{J}_n(\alpha_1a)\right\},$$

$$a_{3j}=\alpha_j[(1-2\kappa)-\delta_j(\alpha_j^2-k_1^2)][\mathrm{H}_{n-1}^{(1)}(\alpha_ja)-\mathrm{H}_{n+1}^{(1)}(\alpha_ja)]\qquad(j=1,2),$$

$$b_3=-\mathrm{i}^n\alpha_1[(1-2\kappa)-\delta_1(\alpha_1^2-k_1^2)][\mathrm{J}_{n-1}(\alpha_1a)-\mathrm{J}_{n+1}(\alpha_1a)].$$

开孔边界处动应力集中系数可描述为

$$\sigma_\theta^*=\frac{\sigma_\theta}{\sigma_0}=\frac{\sigma_\theta^{(\mathrm{I})}+\sigma_\theta^{(\mathrm{S})}}{\sigma_0},\tag{11}$$

其中,开孔边界处入射波对应的环向应力为

$$\sigma_\theta^{(\mathrm{I})}=[\lambda_{\mathrm{M}}(1-\delta_1\alpha_1^2)-2\mu_{\mathrm{M}}\delta_1\alpha_1^2\sin^2\theta]E_0\mathrm{e}^{\mathrm{i}\alpha_1x};\tag{12}$$

开孔边界处散射波对应的环向应力表达式为

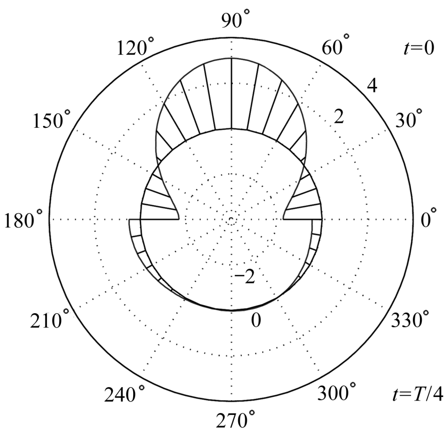
$$\sigma_\theta^{(\mathrm{S})}=\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}[\lambda_{\mathrm{M}}(1-\delta_m\alpha_m^2)-2\mu_{\mathrm{M}}\delta_m\alpha_m^2]A_{mn}\mathrm{H}_n^{(1)}(\alpha_mr)\mathrm{e}^{\mathrm{i}n\theta}+$$
$$\frac{1}{2}\mu_{\mathrm{M}}\sum_{m=1}^{2}\sum_{n=-\infty}^{\infty}\delta_m\alpha_m^2A_{mn}[\mathrm{H}_{n-2}^{(1)}(\alpha_mr)-2\mathrm{H}_n^{(1)}(\alpha_mr)+\mathrm{H}_{n+2}^{(1)}(\alpha_mr)]\mathrm{e}^{\mathrm{i}n\theta}+$$
$$\mathrm{i}n\mu_{\mathrm{M}}\left[\frac{2}{r^2}\sum_{n=-\infty}^{\infty}B_n\mathrm{K}_n(k_2r)+\frac{1}{r}k_2\sum_{n=-\infty}^{\infty}B_n(\mathrm{K}_{n-1}(k_2r)+\mathrm{K}_{n+1}(k_2r))\right]\mathrm{e}^{\mathrm{i}n\theta};\tag{13}$$

而 σ_0 是弹性波入射波沿 x 方向正应力的幅值,

$$\sigma_0=[\lambda_{\mathrm{M}}(1-\delta_1\alpha_1^2)-2\mu_{\mathrm{M}}\delta_1\alpha_1^2]E_0.$$

4 数值算例

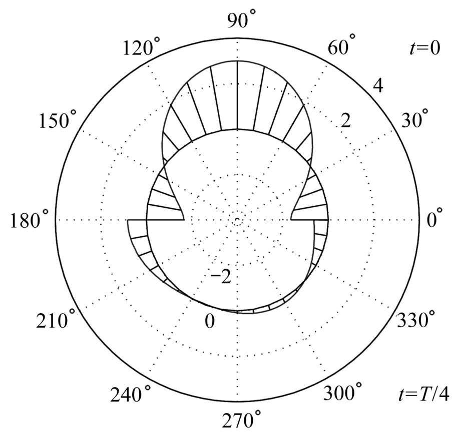
基于本文提出的平板结构拉伸振动精确化理论及其相应的分析计算公式,编制了计算程序,并绘出了动应力分布曲线. 在作数值计算时,Poisson 比取为 $\nu=0.30$,无量纲波数为 $\alpha_1a=0.1\sim5.0$.



$(\alpha_1a=0.1,\ a/h=1.0)$

图 1 拉伸平板孔边动应力分布

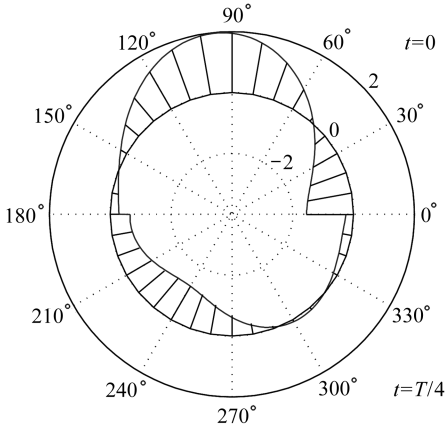
Fig. 1 Dynamic stress in stretching plates



$(\alpha_1a=0.2,\ a/h=0.1)$

图 2 拉伸平板孔边动应力分布

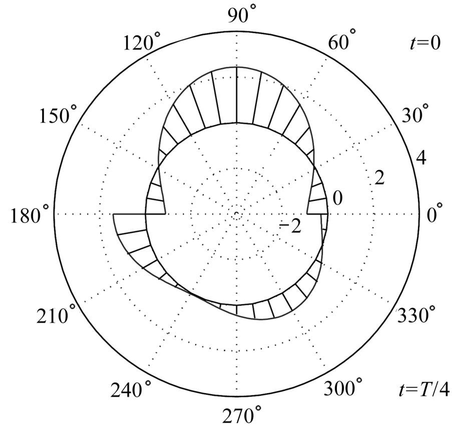
Fig. 2 Dynamic stress in stretching plates



(α₁a = 0.5, a/h = 0.1)

图3　拉伸平板孔边动应力分布

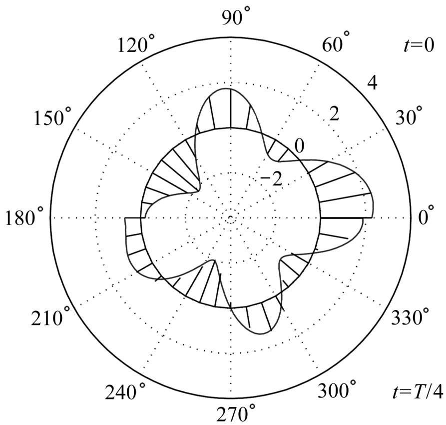
Fig. 3　Dynamic stress in stretching plates



(α₁a = 1.0, a/h = 1.0)

图4　拉伸平板孔边动应力分布

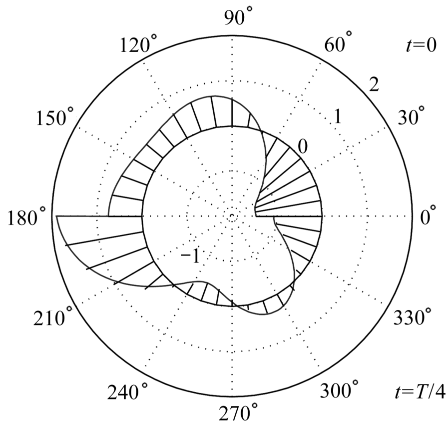
Fig. 4　Dynamic stress in stretching plates



(α₁a = 2.0, a/h = 0.1)

图5　拉伸平板孔边动应力分布

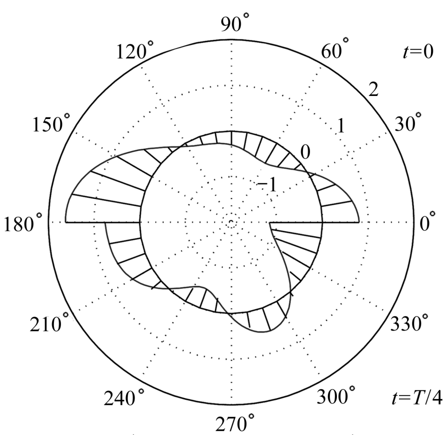
Fig. 5　Dynamic stress in stretching plates



(α₁a = 2.0, a/h = 1.0)

图6　拉伸平板孔边动应力分布

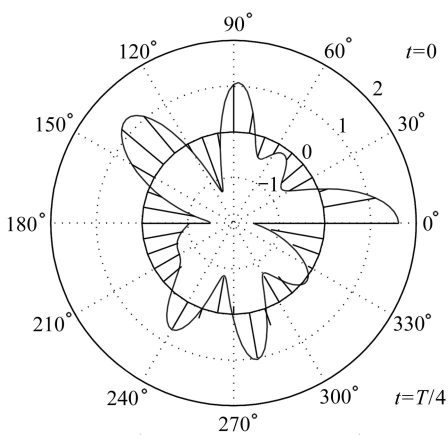
Fig. 6　Dynamic stress in stretching plates



(α₁a = 2.0, a/h = 5.0)

图7　拉伸平板孔边动应力分布

Fig. 7　Dynamic stress in stretching plates

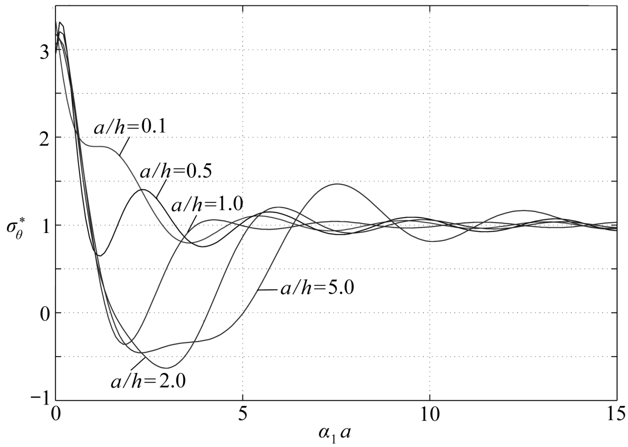


(α₁a = 5.0, a/h = 1.0)

图8　拉伸平板孔边动应力分布

Fig. 8　Dynamic stress in stretching plates

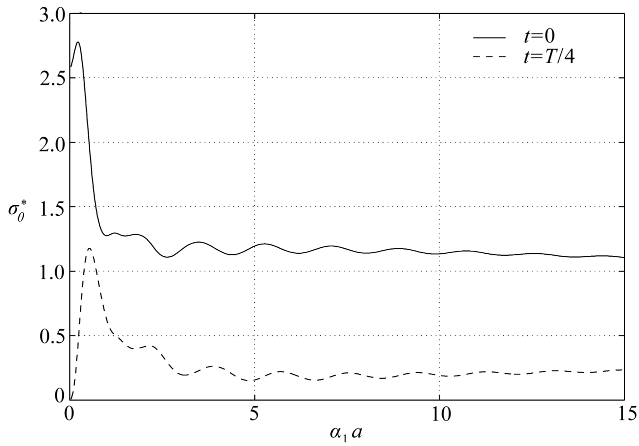
根据平板拉伸振动的精确化方程计算得到的动应力集中系数沿圆孔的分布如图 1 ~8 所示. 对于每张图来说,图的上半部分为 $t=0$ 时动应力的分布,而下半部分为 $t=T/4$ 时动应力的分布. 图 9 是基于平板拉伸的精确化理论分析计算得到的动应力集中系数随无量纲波数 $\alpha_1 a$ 的变化曲线. 图 10 是基于弹性动力学平面问题得到的开孔孔边动应力集中系数随无量纲波数 $\alpha_1 a$ 的变化曲线.



($\nu = 0.3$, $\theta = \pi/2$)

图 9 含圆孔厚板拉伸动应力集中系数随波数变化规律

Fig. 9 Dynamic stress in plate stretching vs dimensionless wave numbers



($\nu = 0.3$, $\theta = \pi/2$)

图 10 平面问题孔边动应力集中系数随波数变化规律

Fig. 10 Dynamic stress in plane vs dimensionless wave numbers

5 结　　论

通过分析计算可以看到:1) 基于平板拉伸振动精确化方程计算得到的动应力结果与基于弹性动力学平面问题的动应力结果其差别还是比较大的;2) 含孔平板拉压波动时的动应力集中系数最大值为 3.30,而基于弹性动力学平面问题[10] 得到的解为 2.77,前者超出后者 19% 之多;3) 当入射波波数变大或者说入射波频率变高时,基于精确化理论得到的动应力集

中系数最大值趋于单位 1,这充分反映了高频动态应力集中的特点.

根据文献[9]的分析与推导可知,平板拉伸振动精确化方程是在没有基于任何假设下推导得到的,其分析计算结果更精确,可用于求解厚壁结构振动和精确化确定结构振动高阶模态. 本文理论与数值计算结果可望能在工程厚壁结构的动力学分析与强度设计、结构精确化设计与结构轻量化中得到应用.

参考文献(References):

[1] 萨文 Γ H. 孔附近的应力集中[M]. 卢鼎霍 译. 北京:科学出版社, 1965. (Savin E H. *Stress Concentrations at the Edge of a Hole*[M]. LU Ding-huo trans. Beijing: Science Press, 1965. (Chinese version))

[2] Liu D K, Hu C. Scattering of flexural wave and dynamic stress concentration in Mindlin thick plates[J]. *Acta Mech Sin*, 1996, **12**(2): 169-185.

[3] Reissner E. The effect of transverse shear deformation on the bending of elastic plates[J]. *ASME*, *J Appl Mech*, 1945, **12**(2):69-77.

[4] Mindlin R D. Influence of rotatory inertia and shear on flexural motions of isotropic elastic plates[J]. *ASME*, *J Appl Mech*, 1951, **18**(2): 31-36.

[5] Pao Y H, Mao C C. Diffractions of flexural waves by a cavity in an elastic plate[J]. *AIAA J*, 1964, **2**(11): 2004-2010.

[6] Vemula C, Norris A N. Flexural wave propagation and scattering on thin plates using Mindlin theory[J]. *Wave Motion*,1997, **26**(1): 1-12.

[7] 胡超, 周传平, Fai Ma, 刘殿魁. 基于平板弯曲振动精确化方程求解动应力集中问题[J]. 力学学报, 2012, **44**(5): 938-942. (HU Chao, ZHOU Chuan-ping, Fai Ma, LIU Dian-kui. Dynamic stress concentrations by using refined equations of plate bending[J]. *Chinese Journal of Theoretical and Applied Mechanics*, 2012, **44**(5): 938-942. (in Chinese))

[8] Hwu C B. Boundary element formulation for the coupled stretching-bending analysis of thin laminated plates[J]. *Engineering Analysis With Boundary Elements*, 2012, **36**(6): 1027-1039.

[9] 胡超, Fai Ma, 马兴瑞, 黄文虎. 厚板弯曲与拉伸振动精化理论及其求解新途径[J]. 中国科学, G 辑, 物理、力学、天文学, 2012, **42**(5): 522-530. (HU Chao, Fai Ma, MA Xing-rui, HUANG Wen-hu. Refined dynamic theory of thick plates in extension-bending and its new formulism[J]. *Sci China*, *ser G*, *Phys*, *Mech*, *Astron*, 2012, **42**(5):522-530. (in Chinese))

[10] 鲍亦兴, 毛昭宙. 弹性波的衍射与动应力集中[M]. 刘殿魁, 苏先樾 译. 北京: 科学出版社, 1993: 230-231. (Pao Y H, Mao C C. *Diffraction of Elastic Waves and Dynamic Stress Concentrations*[M]. LIU Dian-kui, SU Xian-yue trans. Beijing: Science Press, 1993: 230-231. (Chinese version))

Dynamic Stress Concentrations in Stretching Plates by Using the Refined Dynamic Theory

HU Chao[1,2], ZHOU Chuan-ping[1], TONG Guang-qing[2], LIU Dian-kui[2]

(1. *School of Aerospace Engineering and Applied Mechanics*, *Tongji University*, *Shanghai* 200092, *P. R. China*;

2. *College of Civil Science and Engineering*, *Yangzhou University*, *Yangzhou*, *Jiangsu* 225127, *P. R. China*;

3. *College of Aerospace and Civil Engineering*, *Harbin Engineering University*, *Harbin* 150001, *P. R. China*)

Abstract: In the past, the solution of elasticity plane problems is often used to investigate stress concentrations for the engineering design instead of solution of stretching plates. For example, Kirsch's solution and the solution of elastodynamics plane problems. Based on the refined dynamic equation of stretching plates, elastic wave scattering and dynamic stress concentrations in stretching plates with a circular cutout were studied. Numerical results demonstrated that dynamic stress concentration factors in stretching plates were different from the one obtained by elasticity plane problems and dynamic stress concentration factors trended to unit 1 at the high frequency of incident waves. The dynamic stress concentration factor of stretching plates with cutouts is up to a maximum of 3.30, and the one got by using plane problem of elastic dynamics is 2.77. The comparison of the numerical results was made and discussed. It is showed that as the cutout radius ratio to the thickness is $a/h = 0.10$, using the refined equation the dynamic moment factor may approach the maximum value, which is 19% more than the result from the solution of plane problems of elastic dynamics. The results are more accurate because the refined equation of plate stretching is derivative without using any engineering hypotheses. The numerical results and the method can be used to analyze the dynamics and strength of plate-like structures.

Key words: refined vibration equation of plate stretching; elastic wave scattering and dynamic-stress concentrations; dynamics of thick-walled structures; first moment of shear stresses