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离散时间型复值神经网络的全局指数周期性*

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摘要: 复值神经网络是神经网络的一个分支,也是最近几年快速发展的一个领域,在图像处理、模式识别、联想记忆等方面有广泛的应用.目前,对于复值神经网络动力学方面的研究主要集中在稳定性上,对于离散时间型复值神经网络周期性的研究还几乎没有.首先将连续时间型复值神经网络模型离散化得到离散时间型复值神经网络模型,然后利用 M- 矩阵理论、不等式技巧和 Lyapunov 方法,获得了全局指数周期性的一个充分条件,最后给出的具有仿真的数值例子验证了获得结果的有效性.

关 键 词: 离散; 复值神经网络; 时滞; 全局指数周期性
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引 言

近些年来,由于时滞神经网络在信号处理、模式识别、工程优化以及联想处理等方面有着广泛的应用,人们对于时滞神经网络的动力学特征展开了广泛的研究[1-9].在很多应用,如光电子、语音合成、信息流中,我们必须考虑复信号,从而需要研究复值神经网络,因此一些复值神经网络的模型纷纷被提出并加以了研究[10-19],而复值神经网络也可以解决一些实值神经网络无法解决的问题.

在复值神经网络中,状态变量、连接权及激活函数均为复函数.一般来说,复值神经网络由于结构上的差异,性质要比实值神经网络更复杂.在实值神经网络中,激活函数 $f(\cdot)$ 通常选择光滑(连续可微)且有界的函数,例如 sigmoidal 函数.然而,在复数域内,根据 Liouville 定理[17],有界整函数必为常数,即若 $f(z)$ 在复数域 $\mathbf{C}$ 内有界且连续,则 $f(z)$ 是常数.从而如果我们在复值神经网络中选择有界解析的函数 $f(z)$ 作为激活函数,则 $f(z)$ 在整个复数域 $\mathbf{C}$ 内恒为常数,显然这不符合我们的要求.这就意味着复值神经网络的激活函数不能同时满足有界和解析两个条件,可见,激活函数是复值神经网络与实值神经网络的最大区别之一,也是研究复值神经

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网络的关键,故而很有必要从激活函数入手,对复值神经网络的动力学性质加以研究.在文献[18]中,作者利用 Lyapunov 泛函和线性矩阵不等式研究了一类离散时间型复值神经网络的有界性和稳定性.在文献[13]中,作者对连续时间型复值神经网络的激活函数进行了细统的分类和讨论,给出了平衡点的存在唯一性,以及相应的全局稳定性的充分条件.众所周知,平衡点可以看作是周期解的一种特殊情况(周期为任意常数),从这个意义上说,对周期解的研究是对平衡点研究的推广.目前对于复值神经网络的动力学方面的研究还主要集中在稳定性上,对于周期性,特别是离散时间型复值神经网络的周期性研究较少.首先利用离散化的方法得到连续时间型复值神经网络的离散化模拟,进而利用 M- 矩阵和不等式分析等方法,给出全局指数周期性的充分条件.

1 预备知识

首先,给出复值神经网络的激活函数需要满足的条件,以及 M- 矩阵的定义.

设 $f_j(\cdot)(j=1,2,\cdots,n)$ 为复值函数.设 $z=x+\mathrm{i}y$,其中 i 表示虚数单位,即,$\mathrm{i}=\sqrt{-1}$.$f_j(z)$ 可以分离实部和虚部如下:$f_j(z)=f_j^{\mathrm{R}}(x,y)+\mathrm{i}f_j^{\mathrm{I}}(x,y)$,其中,$f_j^{\mathrm{R}}(\cdot,\cdot):R^2\to\mathbf{R}$, $f_j^{\mathrm{I}}(\cdot,\cdot):R^2\to\mathbf{R}$.

设 $f_j^{\mathrm{R}}(\cdot,\cdot)$, $f_j^{\mathrm{I}}(\cdot,\cdot)(j=1,2,\cdots,n)$ 满足二元函数的 Lipschtiz 条件,即,对于 $x,x',y,y'\in\mathbf{R}$,存在正常数 λ_j^{RR}, λ_j^{RI}, λ_j^{IR}, λ_j^{II} 使得

$$\begin{cases}|f_j^{\mathrm{R}}(x,y)-f_j^{\mathrm{R}}(x',y')|\le\lambda_j^{\mathrm{RR}}|x-x'|+\lambda_j^{\mathrm{RI}}|y-y'|,\\ |f_j^{\mathrm{I}}(x,y)-f_j^{\mathrm{I}}(x',y')|\le\lambda_j^{\mathrm{IR}}|x-x'|+\lambda_j^{\mathrm{II}}|y-y'|.\end{cases}\tag{1}$$

定义 1[20] 设矩阵 $\boldsymbol{A}=(a_{ij})_{n\times n}$ 的次对角线元非正.若下述条件之一成立,则 $\boldsymbol{A}$ 为非奇异的 M- 矩阵:

1) $\boldsymbol{A}$ 的所有主子式均为正;

2) $\boldsymbol{A}$ 的所有对角线元均为正,且存在一个正定的对角矩阵 $\boldsymbol{\Lambda}=\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_n)$ 使得 $\boldsymbol{A\Lambda}$ 严格对角占优,即

$$a_{ii}\lambda_i>\sum_{j\ne i}|a_{ij}|\lambda_j,\qquad i=1,2,\cdots,n;$$

3) $\boldsymbol{A}$ 的所有顺序主子式均为正.

下面讨论复值神经网络的离散化.离散时间型系统实际上是连续时间型系统的数值离散化,在将连续时间型系统离散化的过程中,其动力学特征需要被完整地保留在离散时间型系统中.采用文献[8]中的方法,对下述连续时间型复值神经网络进行离散化:

$$\dot{\boldsymbol{z}}(t)=-\boldsymbol{D}\boldsymbol{z}+\boldsymbol{A}\boldsymbol{f}(\boldsymbol{z}(t))+\boldsymbol{B}\boldsymbol{g}(\boldsymbol{z}(t-\boldsymbol{\tau}))+\boldsymbol{s}(t),\tag{2}$$

其中,$\boldsymbol{z}=(z_1,z_2,\cdots,z_n)^{\mathrm{T}}\in C^n$ 是状态向量,$\boldsymbol{D}=\mathrm{diag}(d_1,d_2,\cdots,d_n)\in R^{n\times n}$,$d_j>0(j=1,2,\cdots,n)$ 是自反馈连接权矩阵,$\boldsymbol{A}=(a_{jk})_{n\times n}\in C^{n\times n}$ 和 $\boldsymbol{B}=(b_{jk})_{n\times n}\in C^{n\times n}$ 分别是不含时滞和具有时滞的连接权矩阵,$\boldsymbol{f}(\boldsymbol{z}(t))=(f_1(z_1(t)),f_2(z_2(t)),\cdots,f_n(z_n(t)))^{\mathrm{T}}:C^n\to C^n$ 和 $\boldsymbol{g}(\boldsymbol{z}(t-\boldsymbol{\tau}))=(g_1(z_1(t-\tau_1)),g_2(z_2(t-\tau_2)),\cdots,g_n(z_n(t-\tau_n)))^{\mathrm{T}}:C^n\to C^n$ 分别是不含时滞和具有时滞的向量值激活函数,其每一个分量是复值非线性函数,$\tau_k(k=1,2,\cdots,n)$ 是常数时滞,$\boldsymbol{s}(t)=(s_1(t),s_2(t),\cdots,s_n(t))^{\mathrm{T}}\in C^n$ 是外部输入的向量值函数.

将上述连续复值神经网络的状态向量、连接权矩阵、激活函数以及外部输入函数分离成实部和虚部如下:$\boldsymbol{x}=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}$ 和 $\boldsymbol{y}=(y_1,y_2,\cdots,y_n)^{\mathrm{T}}$ 分别是 $\boldsymbol{z}$ 的实部和虚部,$\boldsymbol{A}^{\mathrm{R}}=(a_{jk}^{\mathrm{R}})_{n\times n}$ 和 $\boldsymbol{A}^{\mathrm{I}}=(a_{jk}^{\mathrm{I}})_{n\times n}$ 分别是 $\boldsymbol{A}$ 的实部和虚部,$\boldsymbol{B}^{\mathrm{R}}=(b_{jk}^{\mathrm{R}})_{n\times n}$ 和 $\boldsymbol{B}^{\mathrm{I}}=(b_{jk}^{\mathrm{I}})_{n\times n}$ 分别是 $\boldsymbol{B}$ 的实

部和虚部，$\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x},\boldsymbol{y})=(f_1^{\mathrm{R}}(x_1,y_1),f_2^{\mathrm{R}}(x_2,y_2),\cdots,f_n^{\mathrm{R}}(x_n,y_n))^{\mathrm{T}}$ 和 $\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{x},\boldsymbol{y})=(f_1^{\mathrm{I}}(x_1,y_1),f_2^{\mathrm{I}}(x_2,y_2),\cdots,f_n^{\mathrm{I}}(x_n,y_n))^{\mathrm{T}}$ 分别是 $\boldsymbol{f}(z)$ 的实部和虚部，$\boldsymbol{g}^{\mathrm{R}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})=(g_1^{\mathrm{R}}(x_1(t-\tau_1),y_1(t-\tau_1)),g_2^{\mathrm{R}}(x_2(t-\tau_2),y_2(t-\tau_2)),\cdots,g_n^{\mathrm{R}}(x_n(t-\tau_n),y_n(t-\tau_n)))^{\mathrm{T}}$ 和 $\boldsymbol{g}^{\mathrm{I}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})=(g_1^{\mathrm{I}}(x_1(t-\tau_1),y_1(t-\tau_1)),g_2^{\mathrm{I}}(x_2(t-\tau_2),y_2(t-\tau_2)),\cdots,g_n^{\mathrm{I}}(x_n(t-\tau_n),y_n(t-\tau_n)))^{\mathrm{T}}$ 分别是 $\boldsymbol{g}(\boldsymbol{z}(t-\boldsymbol{\tau}))$ 的实部和虚部，$\boldsymbol{s}^{\mathrm{R}}(t)=(s_1^{\mathrm{R}}(t),s_2^{\mathrm{R}}(t),\cdots,s_n^{\mathrm{R}}(t))^{\mathrm{T}}$ 和 $\boldsymbol{s}^{\mathrm{I}}(t)=(s_1^{\mathrm{I}}(t),s_2^{\mathrm{I}}(t),\cdots,s_n^{\mathrm{I}}(t))^{\mathrm{T}}$ 分别是 $\boldsymbol{s}(t)$ 的实部和虚部.从而复值神经网络式(2)可以被分离为如下形式：

$$\begin{cases}\dot{\boldsymbol{x}}=-\boldsymbol{D}\boldsymbol{x}+\boldsymbol{A}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x},\boldsymbol{y})-\boldsymbol{A}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{x},\boldsymbol{y})+\boldsymbol{B}^{\mathrm{R}}\boldsymbol{g}^{\mathrm{R}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})-\boldsymbol{B}^{\mathrm{I}}\boldsymbol{g}^{\mathrm{I}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})+\boldsymbol{s}_j^{\mathrm{R}}(t),\\ \dot{\boldsymbol{y}}=-\boldsymbol{D}\boldsymbol{y}+\boldsymbol{A}^{\mathrm{I}}\boldsymbol{f}^{\mathrm{R}}(\boldsymbol{x},\boldsymbol{y})+\boldsymbol{A}^{\mathrm{R}}\boldsymbol{f}^{\mathrm{I}}(\boldsymbol{x},\boldsymbol{y})+\boldsymbol{B}^{\mathrm{I}}\boldsymbol{g}^{\mathrm{R}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})+\boldsymbol{B}^{\mathrm{R}}\boldsymbol{g}^{\mathrm{I}}(\boldsymbol{x}^{\tau},\boldsymbol{y}^{\tau})+\boldsymbol{s}_j^{\mathrm{I}}(t).\end{cases} \tag{3}$$

写成分量形式为

$$\begin{aligned}\dot{x}_j(t)=&-d_jx_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}(x_k(t),y_k(t))-\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}(x_k(t),y_k(t))+\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}(x_k(t-\tau_k),y_k(t-\tau_k))-\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}(x_k(t-\tau_k),y_k(t-\tau_k))+s_j^{\mathrm{R}}(t),\end{aligned} \tag{4a}$$

$$\begin{aligned}\dot{y}_j(t)=&-d_jy_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}(x_k(t),y_k(t))+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}(x_k(t),y_k(t))+\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}(x_k(t-\tau_k),y_k(t-\tau_k))+\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}(x_k(t-\tau_k),y_k(t-\tau_k))+s_j^{\mathrm{I}}(t).\end{aligned} \tag{4b}$$

对上式进行逼近，可以得到：对于 $i=1,2,\cdots,n$，$t\in([t/h]h,[t/h]h+h)$，有

$$\begin{aligned}\dot{x}_j(t)=&-d_jx_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}\left(x_k\left(\left[\frac{t}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h\right)\right)-\\&\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}\left(x_k\left(\left[\frac{t}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h\right)\right)+\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}\left(x_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right)\right)-\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}\left(x_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right)\right)+s_j^{\mathrm{R}}\left(\left[\frac{t}{h}\right]h\right),\end{aligned} \tag{5a}$$

$$\begin{aligned}\dot{y}_j(t)=&-d_jy_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}\left(x_k\left(\left[\frac{t}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h\right)\right)-\\&\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}\left(x_k\left(\left[\frac{t}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h\right)\right)+\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}\left(x_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right)\right)-\\&\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}\left(x_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right),y_k\left(\left[\frac{t}{h}\right]h-\left[\frac{\tau_k}{h}\right]h\right)\right)+s_j^{\mathrm{I}}\left(\left[\frac{t}{h}\right]h\right),\end{aligned} \tag{5b}$$

其中，h 为正数，表示一致的离散化步长，$[t/h]$ 表示 t/h 的整数部分，$[\tau_k/h]$ 表示 τ_k/h 的整数部分.我们记

$$\left[\frac{t}{h}\right]=m,\ \left[\frac{\tau_k}{h}\right]=\sigma_k,\qquad m,\sigma_k=0,1,2,\cdots;\ x_j(mh)=x_j(m).$$

从而式(5a)和式(5b)可改为

$$\dot{x}_j(t)=-d_jx_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}(x_k(m),y_k(m))-\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))-\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{R}}(m),\tag{6a}$$

$$\dot{y}_j(t)=-d_jy_j(t)+\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}(x_k(m),y_k(m))+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{I}}(m).\tag{6b}$$

上式又可改为

$$\frac{\mathrm{d}}{\mathrm{d}t}(x_j(t)\mathrm{e}^{d_jt})=\mathrm{e}^{d_jt}\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}(x_k(m),y_k(m))-\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))-\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{R}}(m)\Big],\tag{7a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(y_j(t)\mathrm{e}^{d_jt})=-\mathrm{e}^{d_jt}\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}(x_k(m),y_k(m))+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{I}}(m)\Big].\tag{7b}$$

上式在区间 $[mh,t)$ 上积分,其中 $t<(m+1)h$, 可得

$$x_j(t)\mathrm{e}^{d_jt}-x_j(m)\mathrm{e}^{d_jmh}=\left(\frac{\mathrm{e}^{d_jt}-\mathrm{e}^{d_jmh}}{d_j}\right)\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}(x_k(m),y_k(m))-\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))-\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{R}}(m)\Big],\tag{8a}$$

$$y_j(t)\mathrm{e}^{d_jt}-y_j(m)\mathrm{e}^{d_jmh}=\left(\frac{\mathrm{e}^{d_jt}-\mathrm{e}^{d_jmh}}{d_j}\right)\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}(x_k(m),y_k(m))+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{I}}(m)\Big].\tag{8b}$$

上式中,令 $t\to(m+1)h$,整理可得:对于 $i=1,2,\cdots,n$, m 为非负整数,有

$$x_j(m+1)=x_j(m)\mathrm{e}^{-d_j h}+\theta_j(h)\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{R}}(x_k(m),y_k(m))-\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))-\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{R}}(m)\Big],\tag{9a}$$

$$y_j(m+1)=y_j(m)\mathrm{e}^{-d_j h}+\theta_j(h)\Big[\sum_{k=1}^{n}a_{jk}^{\mathrm{I}}f_k^{\mathrm{R}}(x_k(m),y_k(m))+\sum_{k=1}^{n}a_{jk}^{\mathrm{R}}f_k^{\mathrm{I}}(x_k(m),y_k(m))+\sum_{k=1}^{n}b_{jk}^{\mathrm{I}}g_k^{\mathrm{R}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+\sum_{k=1}^{n}b_{jk}^{\mathrm{R}}g_k^{\mathrm{I}}(x_k(m-\sigma_k),y_k(m-\sigma_k))+s_j^{\mathrm{I}}(m)\Big],\tag{9b}$$

其中

$$\theta_j(h)=\frac{1-\mathrm{e}^{-d_j h}}{d_j},\qquad j=1,2,\cdots,n.$$

由 $d_j>0,h>0$ 可知 $\theta_j(h)>0$，从而式(9a)和(9b)即为连续神经网络式(2)的离散化.易知，当 $h\to 0+$ 时，离散式(9a)和(9b)收敛于连续式(3).

将式(9)重新写成复值函数的形式，可得式(2)的离散化形式：

$$z_j(m+1)=z_j(m)\mathrm{e}^{-d_j h}+\theta_j(h)\Big[\sum_{k=1}^{n}a_{jk}f_k(z_k(m))+\sum_{k=1}^{n}b_{jk}g_k(z_k(m-\sigma_k))+s(m)\Big].\tag{10}$$

外部输入函数 $s_j^{\mathrm{R}}(\cdot)$ 和 $s_j^{\mathrm{I}}(\cdot)$ 为周期函数，即存在正整数 ω 使得

$$s_j^{\mathrm{R}}(m+\omega)=s_j^{\mathrm{R}}(m),\ s_j^{\mathrm{I}}(m+\omega)=s_j^{\mathrm{I}}(m).$$

2 主要结果

定理 1 设复值神经网络式(10)的激活函数满足假设 H_1.若 $\bar{\boldsymbol{D}}-\bar{\boldsymbol{A}}\bar{\boldsymbol{K}}-\bar{\boldsymbol{B}}\bar{\boldsymbol{L}}$ 为非奇异的 M-矩阵，其中

$$\bar{\boldsymbol{D}}=\begin{pmatrix}\boldsymbol{D}&\boldsymbol{0}\\\boldsymbol{0}&\boldsymbol{D}\end{pmatrix},\ \bar{\boldsymbol{A}}=\begin{pmatrix}|\boldsymbol{A}^{\mathrm{R}}|&|\boldsymbol{A}^{\mathrm{I}}|\\|\boldsymbol{A}^{\mathrm{I}}|&|\boldsymbol{A}^{\mathrm{R}}|\end{pmatrix},\ \bar{\boldsymbol{B}}=\begin{pmatrix}|\boldsymbol{B}^{\mathrm{R}}|&|\boldsymbol{B}^{\mathrm{I}}|\\|\boldsymbol{B}^{\mathrm{I}}|&|\boldsymbol{B}^{\mathrm{R}}|\end{pmatrix},\ \bar{\boldsymbol{K}}=\begin{pmatrix}\boldsymbol{K}^{\mathrm{RR}}&\boldsymbol{K}^{\mathrm{RI}}\\\boldsymbol{K}^{\mathrm{IR}}&\boldsymbol{K}^{\mathrm{II}}\end{pmatrix},$$

$$\bar{\boldsymbol{L}}=\begin{pmatrix}\boldsymbol{L}^{\mathrm{RR}}&\boldsymbol{L}^{\mathrm{RI}}\\\boldsymbol{L}^{\mathrm{IR}}&\boldsymbol{L}^{\mathrm{II}}\end{pmatrix},\ |\boldsymbol{A}^{\mathrm{R}}|=(|a_{jk}^{\mathrm{R}}|)_{n\times n},\ |\boldsymbol{A}^{\mathrm{I}}|=(|a_{jk}^{\mathrm{I}}|)_{n\times n},\ |\boldsymbol{B}^{\mathrm{R}}|=(|b_{jk}^{\mathrm{R}}|)_{n\times n},$$

$$|\boldsymbol{B}^{\mathrm{I}}|=(|b_{jk}^{\mathrm{I}}|)_{n\times n},\ \boldsymbol{K}^{\mathrm{RR}}=\mathrm{diag}(\lambda_1^{\mathrm{RR}},\lambda_2^{\mathrm{RR}},\cdots,\lambda_n^{\mathrm{RR}}),\ \boldsymbol{K}^{\mathrm{RI}}=\mathrm{diag}(\lambda_1^{\mathrm{RI}},\lambda_2^{\mathrm{RI}},\cdots,\lambda_n^{\mathrm{RI}}),$$

$$\boldsymbol{K}^{\mathrm{IR}}=\mathrm{diag}(\lambda_1^{\mathrm{IR}},\lambda_2^{\mathrm{IR}},\cdots,\lambda_n^{\mathrm{IR}}),\ \boldsymbol{K}^{\mathrm{II}}=\mathrm{diag}(\lambda_1^{\mathrm{II}},\lambda_2^{\mathrm{II}},\cdots,\lambda_n^{\mathrm{II}}),$$

$$\boldsymbol{L}^{\mathrm{RR}}=\mathrm{diag}(\mu_1^{\mathrm{RR}},\mu_2^{\mathrm{RR}},\cdots,\mu_n^{\mathrm{RR}}),\ \boldsymbol{L}^{\mathrm{RI}}=\mathrm{diag}(\mu_1^{\mathrm{RI}},\mu_2^{\mathrm{RI}},\cdots,\mu_n^{\mathrm{RI}}),$$

$$\boldsymbol{L}^{\mathrm{IR}}=\mathrm{diag}(\mu_1^{\mathrm{IR}},\mu_2^{\mathrm{IR}},\cdots,\mu_n^{\mathrm{IR}}),\ \boldsymbol{L}^{\mathrm{II}}=\mathrm{diag}(\mu_1^{\mathrm{II}},\mu_2^{\mathrm{II}},\cdots,\mu_n^{\mathrm{II}}),$$

则式(10)存在全局指数周期解.

证明 令 $\boldsymbol{z}(m,\phi)$ 和 $\boldsymbol{z}(m,\psi)$ 分别表示经过 $z_0=\phi$ 和 $z_0=\psi$ 的解，其实部分别为 $\boldsymbol{x}(m,\phi^{\mathrm{R}})$ 和 $\boldsymbol{x}(m,\psi^{\mathrm{R}})$，虚部分别为 $\boldsymbol{y}(m,\phi^{\mathrm{I}})$ 和 $\boldsymbol{y}(m,\psi^{\mathrm{I}})$，并有

$$\boldsymbol{x}(m,\phi^{\mathrm{R}})=(x_1(m,\phi_1^{\mathrm{R}}),\ x_2(m,\phi_2^{\mathrm{R}}),\cdots,\ x_n(m,\phi_n^{\mathrm{R}}))^{\mathrm{T}},$$

$$\boldsymbol{x}(m,\psi^{\mathrm{R}})=(x_1(m,\psi_1^{\mathrm{R}}),\ x_2(m,\psi_2^{\mathrm{R}}),\cdots,x_n(m,\psi_n^{\mathrm{R}}))^{\mathrm{T}},$$

$$\boldsymbol{y}(m,\phi^{\mathrm{R}})=(y_1(m,\phi_1^{\mathrm{R}}),\ y_2(m,\phi_2^{\mathrm{R}}),\cdots,y_n(m,\phi_n^{\mathrm{R}}))^{\mathrm{T}},$$
$$\boldsymbol{y}(m,\psi^{\mathrm{R}})=(y_1(m,\psi_1^{\mathrm{R}}),\ y_2(m,\psi_2^{\mathrm{R}}),\cdots,y_n(m,\psi_n^{\mathrm{R}}))^{\mathrm{T}}.$$

则由式(1)可得

$$\begin{aligned}
&|x_j(m+1,\phi_j^{\mathrm{R}})-x_j(m+1,\psi_j^{\mathrm{R}})|\leqslant|x_j(m,\phi_j^{\mathrm{R}})-x_j(m,\psi_j^{\mathrm{R}})|\mathrm{e}^{-d_jh}+\\
&\quad\theta_j(h)\Big\{\sum_{k=1}^{n}|a_{jk}^{\mathrm{R}}|[\mu_k^{\mathrm{RR}}|x_k(m,\phi_k^{\mathrm{R}})-x_k(m,\psi_k^{\mathrm{R}})|+\mu_k^{\mathrm{RI}}|y_k(m,\phi_k^{\mathrm{I}})-\\
&\quad y_k(m,\psi_k^{\mathrm{I}})|]\times\sum_{k=1}^{n}|a_{jk}^{\mathrm{I}}|[\mu_k^{\mathrm{IR}}|x_k(m,\phi_k^{\mathrm{R}})-x_k(m,\psi_k^{\mathrm{R}})|+\\
&\quad\mu_k^{\mathrm{II}}|y_k(m,\phi_k^{\mathrm{I}})-y_k(m,\psi_k^{\mathrm{I}})|]+\\
&\quad\sum_{k=1}^{n}|b_{jk}^{\mathrm{R}}|[\lambda_k^{\mathrm{RR}}|x_k(m-\sigma_k,\phi_k^{\mathrm{R}})-x_k(m-\sigma_k,\psi_k^{\mathrm{R}})|+\\
&\quad\lambda_k^{\mathrm{RI}}|y_k(m-\sigma_k,\phi_k^{\mathrm{I}})-y_k(m-\sigma_k,\psi_k^{\mathrm{I}})|]+\\
&\quad\sum_{k=1}^{n}|b_{jk}^{\mathrm{I}}|[\lambda_k^{\mathrm{IR}}|x_k(m-\sigma_k,\phi_k^{\mathrm{R}})-x_k(m-\sigma_k,\psi_k^{\mathrm{R}})|+\\
&\quad\lambda_k^{\mathrm{II}}|y_k(m-\sigma_k,\phi_k^{\mathrm{I}})-y_k(m-\sigma_k,\psi_k^{\mathrm{I}})|]\Big\},
\end{aligned}\tag{11a}$$

$$\begin{aligned}
&|y_j(m+1,\phi_j^{\mathrm{I}})-y_j(m+1,\psi_j^{\mathrm{I}})|\leqslant|y_j(m,\phi_j^{\mathrm{I}})-y_j(m,\psi_j^{\mathrm{I}})|\mathrm{e}^{-d_jh}+\\
&\quad\theta_j(h)\Big\{\sum_{k=1}^{n}|a_{jk}^{\mathrm{I}}|[\mu_k^{\mathrm{RR}}|x_k(m,\phi_k^{\mathrm{R}})-x_k(m,\psi_k^{\mathrm{R}})|+\mu_k^{\mathrm{RI}}|y_k(m,\phi_k^{\mathrm{I}})-\\
&\quad y_k(m,\psi_k^{\mathrm{I}})|]+\sum_{k=1}^{n}|a_{jk}^{\mathrm{R}}|[\mu_k^{\mathrm{IR}}|x_k(m,\phi_k^{\mathrm{R}})-x_k(m,\psi_k^{\mathrm{R}})|+\\
&\quad\mu_k^{\mathrm{II}}|y_k(m,\phi_k^{\mathrm{I}})-y_k(m,\psi_k^{\mathrm{I}})|]+\\
&\quad\sum_{k=1}^{n}|b_{jk}^{\mathrm{I}}|[\lambda_k^{\mathrm{RR}}|x_k(m-\sigma_k,\phi_k^{\mathrm{R}})-x_k(m-\sigma_k,\psi_k^{\mathrm{R}})|+\\
&\quad\lambda_k^{\mathrm{RI}}|y_k(m-\sigma_k,\phi_k^{\mathrm{I}})-y_k(m-\sigma_k,\psi_k^{\mathrm{I}})|]\times\sum_{k=1}^{n}|b_{jk}^{\mathrm{R}}|[\lambda_k^{\mathrm{IR}}|x_k(m-\sigma_k,\phi_k^{\mathrm{R}})-\\
&\quad x_k(m-\sigma_k,\psi_k^{\mathrm{R}})|+\lambda_k^{\mathrm{II}}|y_k(m-\sigma_k,\phi_k^{\mathrm{I}})-y_k(m-\sigma_k,\psi_k^{\mathrm{I}})|]\Big\}.
\end{aligned}\tag{11b}$$

令

$$\begin{cases}u_j(m)=\mathrm{e}^{\varepsilon m}|x_j(m,\phi_j^{\mathrm{R}})-x_j(m,\psi_j^{\mathrm{R}})|,\\ v_j(m)=\mathrm{e}^{\varepsilon m}|y_j(m,\phi_j^{\mathrm{I}})-y_j(m,\psi_j^{\mathrm{I}})|.\end{cases}\tag{12}$$

由式(11a)和式(11b)及式(12)可得

$$\begin{aligned}
&u_j(m+1)=\mathrm{e}^{\varepsilon(m+1)}|x_j(m+1,\phi_j^{\mathrm{R}})-x_j(m+1,\psi_j^{\mathrm{R}})|\leqslant\\
&\quad\mathrm{e}^{\varepsilon-d_jh}u_j(m)+\theta_j(h)\mathrm{e}^{\varepsilon}\Big[\sum_{k=1}^{n}(|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{RR}}+|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{IR}})u_k(m)+\\
&\quad\sum_{k=1}^{n}(|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{RI}}+|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{II}})v_k(m)+\\
&\quad\sum_{k=1}^{n}(|b_{jk}^{\mathrm{R}}|\lambda_k^{\mathrm{RR}}+|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{IR}})u_k(m-\sigma_k)+\\
&\quad\sum_{k=1}^{n}(|b_{jk}^{\mathrm{R}}|\lambda_k^{\mathrm{RI}}+|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{II}})v_k(m-\sigma_k)\Big],
\end{aligned}\tag{13a}$$

$$v_j(m+1)=\mathrm{e}^{\varepsilon(m+1)}|y_j(m+1,\phi_j^{\mathrm{I}})-y_j(m+1,\psi_j^{\mathrm{I}})|\leqslant$$

$$e^{\varepsilon-d_jh}v_j(m)+\theta_j(h)e^{\varepsilon}\Big[\sum_{k=1}^{n}(|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{RR}}+|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{IR}})u_k(m)+$$
$$\sum_{k=1}^{n}(|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{RI}}+|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{II}})v_k(m)+$$
$$\sum_{k=1}^{n}(|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{RR}}+|b_{jk}^{\mathrm{R}}|\lambda_k^{\mathrm{IR}})u_k(m-\sigma_k)+$$
$$\sum_{k=1}^{n}(|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{RI}}+|b_{jk}^{\mathrm{R}}|\lambda_k^{\mathrm{II}})v_k(m-\sigma_k)\Big].\tag{13b}$$

由于 $\bar{\boldsymbol{D}}-\bar{\boldsymbol{A}}\bar{\boldsymbol{K}}-\bar{\boldsymbol{B}}\bar{\boldsymbol{L}}$ 为非奇异的 M- 矩阵,则由定义 1,可得

$$\alpha_j d_j-\sum_{k=1}^{n}\alpha_k(|a_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{RR}}+|a_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{IR}})-\sum_{k=1}^{n}\beta_k(|a_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{RR}}+|a_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{IR}})-$$
$$\sum_{k=1}^{n}\alpha_k(|b_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{RR}}+|b_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{IR}})-\sum_{k=1}^{n}\beta_k(|b_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{RR}}+|a_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{IR}})>0,\tag{14a}$$

$$\beta_j d_j-\sum_{k=1}^{n}\alpha_k(|a_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{RI}}+|a_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{II}})-\sum_{k=1}^{n}\beta_k(|a_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{RI}}+|a_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{II}})-$$
$$\sum_{k=1}^{n}\alpha_k(|b_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{RI}}+|b_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{II}})-\sum_{k=1}^{n}\beta_k(|b_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{RI}}+|b_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{II}})>0.\tag{14b}$$

考虑如下两个函数:

$$F_j^{\mathrm{R}}(\varepsilon_j)=\sum_{j=1}^{n}\Big[\frac{\alpha_j}{\theta_j(h)}(e^{\varepsilon_j}-1)-e^{\varepsilon_j}\alpha_j d_j+e^{\varepsilon_j}\sum_{k=1}^{n}\alpha_k(a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{RR}}+a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{IR}})+$$
$$e^{\varepsilon_j}\sum_{k=1}^{n}\alpha_k(b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{RR}}+b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{IR}})+e^{\varepsilon_j}\sum_{k=1}^{n}\beta_k(a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{RR}}+a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{IR}})+$$
$$e^{\varepsilon_j}\sum_{k=1}^{n}\beta_k(b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{RR}}+b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{IR}})\Big],\tag{15a}$$

$$F_j^{\mathrm{I}}(\varepsilon_j)=\sum_{j=1}^{n}\Big[\frac{\beta_j}{\theta_j(h)}(e^{\varepsilon_j}-1)-e^{\varepsilon_j}\beta_j d_j+e^{\varepsilon_j}\sum_{k=1}^{n}\alpha_k(a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{RI}}+a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{II}})+$$
$$e^{\varepsilon_j}\sum_{k=1}^{n}\alpha_k(b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{RI}}+b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{II}})+e^{\varepsilon_j}\sum_{k=1}^{n}\beta_k(a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{RI}}+a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{II}})+$$
$$e^{\varepsilon_j}\sum_{k=1}^{n}\beta_k(b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{RI}}+b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{II}})\Big].\tag{15b}$$

由式(14a)和式(14b)及式(15a)和式(15b)可知, $F_j^{\mathrm{R}}(0)=F_j^{\mathrm{I}}(0)=1$,从而由 $F_j^{\mathrm{R}}(\cdot)$ 和 $F_j^{\mathrm{I}}(\cdot)$ 在 $[0,\infty)$ 上的连续性可知,存在 $\varepsilon>0$ 使得

$$F_j^{\mathrm{R}}(\varepsilon)=\sum_{j=1}^{n}\Big[\frac{\alpha_j}{\theta_j(h)}(e^{\varepsilon}-1)-e^{\varepsilon}\alpha_j d_j+e^{\varepsilon}\sum_{k=1}^{n}\alpha_k(a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{RR}}+a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{IR}})+$$
$$e^{\varepsilon}\sum_{k=1}^{n}\alpha_k(b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{RR}}+b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{IR}})+e^{\varepsilon}\sum_{k=1}^{n}\beta_k(a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{RR}}+a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{IR}})+$$
$$e^{\varepsilon}\sum_{k=1}^{n}\beta_k(b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{RR}}+b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{IR}})\Big]>0,\tag{16a}$$

$$F_j^{\mathrm{I}}(\varepsilon)=\sum_{j=1}^{n}\Big[\frac{\beta_j}{\theta_j(h)}(e^{\varepsilon}-1)-e^{\varepsilon}\beta_j d_j+e^{\varepsilon}\sum_{k=1}^{n}\alpha_k(a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{RI}}a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{II}})+$$
$$e^{\varepsilon}\sum_{k=1}^{n}\alpha_k(b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{RI}}+b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{II}})+e^{\varepsilon}\sum_{k=1}^{n}\beta_k(a_{kj}^{\mathrm{I}}\mu_j^{\mathrm{RI}}+a_{kj}^{\mathrm{R}}\mu_j^{\mathrm{II}})+$$

$$\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\beta_k(b_{kj}^{\mathrm{I}}\lambda_j^{\mathrm{RI}}+b_{kj}^{\mathrm{R}}\lambda_j^{\mathrm{II}})\Big]>0. \tag{16b}$$

本文选择如下的 Lyapunov 函数：

$$V(m)=\sum_{j=1}^{n}\frac{\alpha_j}{\theta_j(h)}u_j(m)+\sum_{j=1}^{n}\frac{\beta_j}{\theta_j(h)}v_j(m)+\sum_{j=1}^{n}\xi_j\sum_{k=m-\sigma_j}^{m-1}\tilde{x}_j(k)+\sum_{j=1}^{n}\eta_j\sum_{k=m-\sigma_j}^{m-1}\tilde{y}_j(k),$$

其中

$$\xi_j=\mathrm{e}^{\varepsilon}[\alpha_j(b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{RR}}+b_{jk}^{\mathrm{I}}\lambda_k^{\mathrm{IR}})+\beta_j(b_{jk}^{\mathrm{I}}\lambda_k^{\mathrm{RR}}+b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{IR}})],$$

$$\eta_j=\mathrm{e}^{\varepsilon}[\alpha_j(b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{RI}}+b_{jk}^{\mathrm{I}}\lambda_k^{\mathrm{II}})+\beta_j(b_{jk}^{\mathrm{I}}\lambda_k^{\mathrm{RI}}+b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{II}})].$$

沿上式计算差分 $\Delta V(m)=V(m+1)-V(m)$，可得

$$\begin{aligned}\Delta V(m)\leqslant&\sum_{j=1}^{n}\frac{\alpha_j}{\theta_j(h)}(\mathrm{e}^{\varepsilon-d_jh}-1)u_j(m)+\mathrm{e}^{\varepsilon}\sum_{j=1}^{n}\alpha_j\Big[\sum_{k=1}^{n}(|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{RR}}+|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{IR}})u_k(m)+\\&\sum_{k=1}^{n}(|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{RI}}+|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{II}})v_k(m)+\sum_{k=1}^{n}(b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{RR}}+|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{IR}})u_k(m-\sigma_k)+\\&\sum_{k=1}^{n}(b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{RI}}+|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{II}})v_k(m-\sigma_k)\Big]+\sum_{j=1}^{n}\frac{\beta_j}{\theta_j(h)}(\mathrm{e}^{\varepsilon-d_jh}-1)v_j(m)+\\&\mathrm{e}^{\varepsilon}\sum_{j=1}^{n}\beta_j\Big[\sum_{k=1}^{n}(|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{RR}}+|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{IR}})u_k(m)+\sum_{k=1}^{n}(|a_{jk}^{\mathrm{I}}|\mu_k^{\mathrm{RI}}+|a_{jk}^{\mathrm{R}}|\mu_k^{\mathrm{II}})v_k(m)+\\&\sum_{k=1}^{n}(|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{RR}}+b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{IR}})u_k(m-\sigma_k)+\sum_{k=1}^{n}(|b_{jk}^{\mathrm{I}}|\lambda_k^{\mathrm{RI}}+b_{jk}^{\mathrm{R}}\lambda_k^{\mathrm{II}})v_k(m-\sigma_k)\Big]+\\&\sum_{j=1}^{n}\xi_j[u_k(m)-u_k(m-\sigma_k)]+\sum_{j=1}^{n}\eta_j[v_k(m)-v_k(m-\sigma_k)]=\\&\sum_{j=1}^{n}\Big[\frac{\alpha_j}{\theta_j(h)}(\mathrm{e}^{\varepsilon}-1)-\mathrm{e}^{\varepsilon}\alpha_jd_j+\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\alpha_k(|a_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{RR}}+|a_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{IR}})+\\&\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\alpha_k(|b_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{RR}}+|b_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{IR}})+\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\beta_k(|a_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{RR}}+|a_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{IR}})+\\&\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\beta_k(|b_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{RR}}+|b_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{IR}})\Big]u_j(m)+\sum_{j=1}^{n}\Big[\frac{\beta_j}{\theta_j(h)}(\mathrm{e}^{\varepsilon}-1)-\\&\mathrm{e}^{\varepsilon}\beta_jd_j+\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\alpha_k(|a_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{RI}}+|a_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{II}})+\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\alpha_k(|b_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{RI}}+|b_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{II}})+\\&\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\beta_k(|a_{kj}^{\mathrm{I}}|\mu_j^{\mathrm{RI}}+|a_{kj}^{\mathrm{R}}|\mu_j^{\mathrm{II}})+\mathrm{e}^{\varepsilon}\sum_{k=1}^{n}\beta_k(|b_{kj}^{\mathrm{I}}|\lambda_j^{\mathrm{RI}}+|b_{kj}^{\mathrm{R}}|\lambda_j^{\mathrm{II}})\Big]v_j(m).\end{aligned} \tag{17}$$

由式(16a)和式(16b)可知 $\Delta V(m)<0$，即 $V(m)<V(0)$. 由于

$$\begin{aligned}V(m)\geqslant&\sum_{j=1}^{n}\frac{\alpha_j}{\theta_j(h)}u_j(m)+\sum_{j=1}^{n}\frac{\beta_j}{\theta_j(h)}v_j(m)\geqslant\\&\mathrm{e}^{\varepsilon m}\min_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}\Big[\sum_{j=1}^{n}|x_j(m,\phi_j^{\mathrm{R}})-x_j(m,\psi_j^{\mathrm{R}})|+\\&\sum_{j=1}^{n}|y_j(m,\phi_j^{\mathrm{I}})-x_j(m,\psi_j^{\mathrm{I}})|\Big]=\\&\mathrm{e}^{\varepsilon m}\min_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}\sum_{j=1}^{2n}|w_j(m,\phi_j)-w_j(m,\psi_j)|,\end{aligned}$$

其中

$$w_j(m,\phi_j)=\begin{cases}x_j(m,\phi_j^{\mathrm{R}}), & j=1,2,\cdots,n,\\ y_{j-n}(m,\phi_j^{\mathrm{I}}), & j=n+1,n+2,\cdots,2n,\end{cases}$$

$$w_j(m,\psi_j)=\begin{cases}x_j(m,\psi_j^{\mathrm{R}}), & j=1,2,\cdots,n,\\ y_{j-n}(m,\psi_j^{\mathrm{I}}), & j=n+1,n+2,\cdots,2n.\end{cases}$$

以及

$$\begin{aligned}V(0)=&\sum_{j=1}^{n}\frac{\alpha_j}{\theta_j(h)}u_j(0)+\sum_{j=1}^{n}\frac{\beta_j}{\theta_j(h)}v_j(0)+\\&\sum_{j=1}^{n}\sum_{k=1}^{n}\xi_j\sum_{l=-\sigma_k}^{-1}u_k(l)+\sum_{j=1}^{n}\sum_{k=1}^{n}\eta_j\sum_{l=-\sigma_k}^{-1}v_k(l)\leqslant\\&\max_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}\left[\sum_{j=1}^{n}|\phi_j^{\mathrm{R}}-\psi_j^{\mathrm{R}}|+\sum_{j=1}^{n}|\phi_j^{\mathrm{I}}-\psi_j^{\mathrm{I}}|\right]+\\&\max_j\{(\xi_j+\eta_j)\sigma_j\}\left[\sum_{j=1}^{n}|\phi_j^{\mathrm{R}}-\psi_j^{\mathrm{R}}|+\sum_{j=1}^{n}|\phi_j^{\mathrm{I}}-\psi_j^{\mathrm{I}}|\right]=\\&\left\{\max_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}+(\xi_j+\eta_j)\sigma_j\right\}\sum_{j=1}^{2n}|w_j(0,\phi_j)-w_j(0,\psi_j)|.\right.\end{aligned}$$

从而有

$$\begin{aligned}&\mathrm{e}^{\varepsilon m}\min_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}\sum_{j=1}^{2n}|w_j(m,\phi_j)-w_j(m,\psi_j)|\leqslant\\&\quad\left\{\max_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}+\max_j\{(\xi_j+\eta_j)\sigma_j\}\right\}\sum_{j=1}^{2n}|w_j(0,\phi_j)-w_j(0,\psi_j)|.\end{aligned}$$

故有

$$\sum_{j=1}^{2n}|w_j(m,\phi_j)-w_j(m,\psi_j)|\leqslant\mathrm{e}^{-\varepsilon m}\gamma\sum_{j=1}^{2n}|w_j(0,\phi_j)-w_j(0,\psi_j)|,$$

其中

$$\gamma=\frac{\max_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}+(\xi_j+\eta_j)\sigma_j\right\}}{\min_j\left\{\frac{\alpha_j+\beta_j}{\theta_j(h)}\right\}},$$

可以取一个正整数 M, 使得

$$\gamma\,\mathrm{e}^{-M\omega}\leqslant\frac{1}{2}.$$

定义 Poincaré 映射 $H:\mathbf{C}\rightarrow\mathbf{C}$ 为 $H(\phi)=w_\omega(\phi)$, 则由上式可得

$$\|H^M\phi-H^M\varphi\|\leqslant\frac{1}{2}\|\phi-\varphi\|,$$

即, H^M 为压缩映射, 从而存在唯一的不动点 $\phi^*\in\mathbf{C}$ 使得 $H^M(\phi^*)=\phi^*$. 故 $H^M(H\phi^*)=H(H^M\phi^*)=H\phi^*$,从而可得 $H\phi^*\in\mathbf{C}$ 是 H^M 的不动点,从而有 $H\phi^*=\phi^*$,即 $w_\omega\phi^*=\phi^*$.设 $\boldsymbol{w}(m,\phi^*)=(\boldsymbol{x}(m,\phi^*)^{\mathrm{T}},\boldsymbol{y}(m,\phi^*)^{\mathrm{T}})^{\mathrm{T}}$ 为式(9a)和式(9b)经过 $(0,\phi^*)$ 的解,由 $\boldsymbol{s}^{\mathrm{R}}(m+\omega)=\boldsymbol{s}^{\mathrm{R}}(m)$, $\boldsymbol{s}^{\mathrm{I}}(m+\omega)=\boldsymbol{s}^{\mathrm{I}}(m)$,可知 $\boldsymbol{w}(m+\omega,\phi^*)$ 也是式(9a)和式(9b)的解.注意到 $w_{m+\omega}(\phi^*)=w_m(w_\omega(\phi^*))=w_m(\phi^*)$,故有 $\boldsymbol{w}(m+\omega,\phi^*)=\boldsymbol{w}(m,\phi^*)$,从而 $\boldsymbol{w}(m,\phi^*)$ 是式(9a)和式(9b)

的周期为 ω 的周期解. □

3　仿真结果

本节给出一个实例来验证上述的结果.

例 1　考虑下述由两个神经元构成的离散时间型复值神经网络:

$$\dot{z} = -Dz + Af(z) + Bg(z(t-\tau)) + s(t), \tag{18}$$

其中

$$D = \begin{pmatrix} 9 & 0 \\ 0 & 8 \end{pmatrix},\ A = \begin{pmatrix} 3+\mathrm{i} & -2-4\mathrm{i} \\ 1+2.5\mathrm{i} & 3+2\mathrm{i} \end{pmatrix},\ B = \begin{pmatrix} -2-5\mathrm{i} & 3+\mathrm{i} \\ -3-2\mathrm{i} & 4+3\mathrm{i} \end{pmatrix},$$

$$f_j(z_j) = \frac{1-\exp(-x_j)}{1+\exp(-x_j)} + \mathrm{i}\frac{1}{1+\exp(-y_j)} \qquad (j=1,2),$$

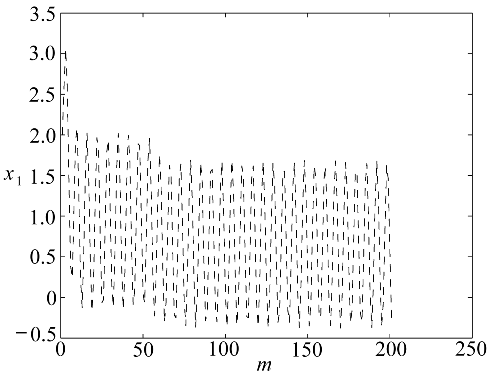
$$g_j(z_j) = \frac{1-\exp(-y_j)}{1+\exp(-y_j)} + \mathrm{i}\frac{1}{1+\exp(-x_j)} \qquad (j=1,2),$$

$$s(t) = (\sin t - 2\mathrm{i}\cos t,\ 3\cos(t+1) + \mathrm{i}\sin(t-1))^{\mathrm{T}}.$$

容易计算得

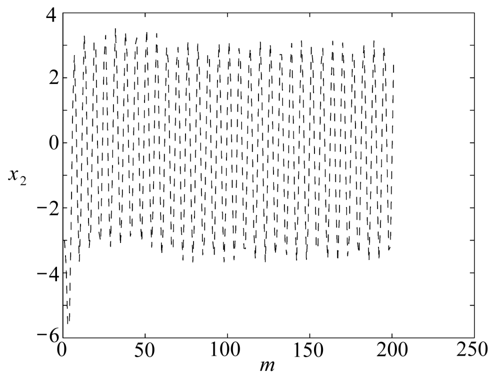
$$\bar{D} = \begin{pmatrix} 8 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix},\ \bar{A} = \begin{pmatrix} 2 & 3 & 3 & 1 \\ 4 & 1 & 2 & 2 \\ 3 & 1 & 2 & 3 \\ 2 & 2 & 4 & 1 \end{pmatrix},\ \bar{B} = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 3 & 3 & 4 & 2 \\ 2 & 1 & 1 & 2 \\ 4 & 2 & 3 & 3 \end{pmatrix},$$

$$\bar{K} = \begin{pmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix},\ \bar{L} = \begin{pmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.25 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \end{pmatrix}.$$



（a）例 1 中 x_1 关于时间的瞬态响应

（a）Transient state of x_1 in example 1



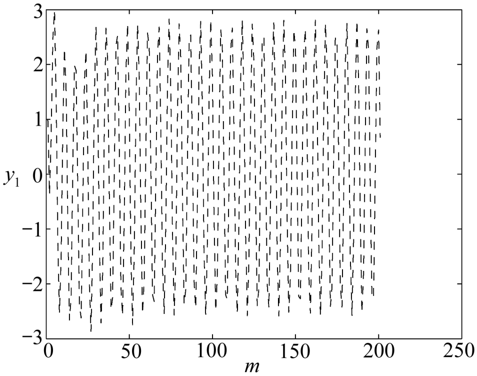
（b）例 1 中 x_2 关于时间的瞬态响应

（b）Transient state of x_2 in example 1

图 1　例 1 中实部关于时间的瞬态响应

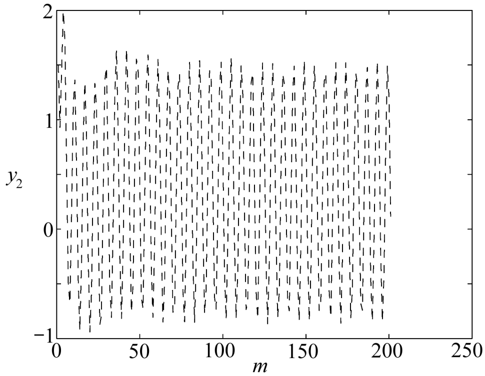
Fig.1　Transient states of the real part of the neural network in example 1

从而有



(a) 例 1 中 y_1 关于时间的瞬态响应

(a) Transient state of y_1 in example 1



(b) 例 1 中 y_2 关于时间的瞬态响应

(b) Transient state of y_2 in example 1

图 2 例 1 中虚部关于时间的瞬态响应

Fig.2 Transient states of the imaginary part of the neural network in example 1

$$\bar{\boldsymbol{D}} - \bar{\boldsymbol{A}}\bar{\boldsymbol{K}} - \bar{\boldsymbol{B}}\bar{\boldsymbol{L}} = \begin{pmatrix} 7 & -3/4 & -3/2 & -7/2 \\ -3/4 & 13/2 & -11/4 & -3 \\ -3/4 & -7/4 & 7 & -3/2 \\ -11/4 & -3/2 & -3/2 & 5 \end{pmatrix}.$$

容易验证 $\bar{\boldsymbol{D}} - \bar{\boldsymbol{A}}\bar{\boldsymbol{K}} - \bar{\boldsymbol{B}}\bar{\boldsymbol{L}}$ 是非奇异的 M- 矩阵，故由定理 1 可知式(18)存在全局指数周期解.图 1 和图 2 给出了网络的实部和虚部关于时间的瞬态响应图.

4 结 论

周期性是神经网络动力学的重要组成部分,但是最近对于离散复值神经网络的周期性研究较少.本文采用离散化的方法,对连续复值神经网络作离散化,得到相应的离散模拟,并采用 M- 矩阵等方法,得到了全局指数周期性的充分条件.下一步还将研究,当激活函数满足其他条件时,相应的周期性条件,以及多周期性的充分条件.

参考文献(References):

[1] Cao J. Periodic oscillation and exponential stability of delayed CNNs[J]. *Phyics Letter A*, 2000, **270**(3/4): 157-163.

[2] Cao J, Wang J. Global exponential stability and periodicity of recurrent neural networks with time delays[J]. *IEEE Transactions on Circuits and Systems-I: Regular Papers*, 2005, **52**(5): 920-931.

[3] Chen B, Wang J. Global exponential periodicity of a class of recurrent neural networks with oscillating parameters and time-varying delays[J]. *IEEE Transactions on Neural Networks*, 2005, **16**(6): 1440-1448.

[4] Chen T. Global exponential stability of delayed Hopfield neural networks[J]. *Neural Networks*, 2001, **14**(8): 977-980.

[5] Ji Y. Global attractivity of almost periodic sequence solutions of delayed discrete-time neural networks[J]. *Arabian Journal for Science and Engineering*, 2011, **36**(7): 1447-1459.

[6] Li C, Liao X, Yu J. Complex valued recurrent neural network with IIR neural model: training and applications[J]. *Circuits Systems Signal Processing*, 2002, **21**(5): 461-471.

[7] Liao X, Wang L, Yu P. *Stability of Dynamical Systems*[M]. Vol **5**. Monograph Series on Nonlinear Science and Complexity. Amsterdam, The Netherlands: Elsevier, 2007.

[8] Mohamad S, Gopalsamy K. Dynamics of a class of discrete-time neural networks and their continuous-time counterparts[J]. *Mathematics and Computers in Simulation*, 2000, **53**(1/2): 1-39.

[9] Sun C, Feng C B. Exponential periodicity of continuous-time and discrete-time neural networks with delays[J]. *Neural Processing Letters*, 2004, **19**(2): 131-146.

[10] Bohner M, Rao V S H, Sanyal S. Global stability of complex-valued neural networks on time scales[J]. *Differential Equations and Dynamical Systems*, 2011, **19**(1/2): 3-11.

[11] Hirose A. *Complex-Valued Neural Networks: Theories and Applications*[M]. Singapore: World Scientific, 2003.

[12] Hirose A. *Complex-Valued Neural Networks*[M]. Heidelberg, German: Springer, 2006.

[13] Hu J, Wang J. Global stability of complex-valued recurrent neural networks with time-delays [J]. *IEEE Transcations on Neural Networks and Learning Systems*, 2012, **23**(6): 853-865.

[14] Jankowski S, Lozowski A, Zurada J. Complex-valued multistate neural associative memory [J]. *IEEE Transactions on Neural Networks*, 1996, **7**(6): 1491-1496.

[15] Lee D L. Improving the capacity of complex-valued neural networks with a modified gradient descent learning rule[J]. *IEEE Transactions on Neural Networks*, 2001, **12**(2): 439-443.

[16] Lee D L. Relaxation of the stability condition of the complex-valued neural networks[J]. *IEEE Transactions on Neural Networks*, 2001, **12**(5): 1260-1262.

[17] Rudin R. *Real and Complex Analysis*[M]. New York: McGraw-Hill, 1987.

[18] DUAN Cheng-jun, SONG Qian-kun. Boundedness and stability for discrete-time delayed neural network with complex-valued linear threshold neurons[J]. *Discrete Dynamics in Nature and Society*, 2010, **2010**: 1-19.

[19] Kobayashi M. Exceptional reducibility of complex-valued neural networks[J]. *IEEE Transactions on Neural Networks*, 2010, **21**(7): 1060-1072.

[20] Berman A, Plemmons R J. *Nonnegative Matrices in the Mathematical Sciences*[M]. New York: Academic Press, 1979.

Global Exponential Periodicity of Discrete-Time Complex-Valued Neural Networks With Time-Delays

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Abstract: Since the last decade, complex-valued neural networks have been rapidly developed and applied in various research areas, but few research has been done on the periodicity on discrete-time complex-valued neural networks. The periodicity of discrete-time complex-valued neural networks with time-delays was investigated. With the discretization technique, the discrete-time analogue of the continuous-time system with periodic input was formulated, and a sufficient condition for checking the global exponential periodicity of the considered neural networks was obtained. Numeric simulation verifieds validity of the analysis.

Key words: discrete-time; complex-valued neural networks; time-delay; global exponential periodicity

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