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剪力非局部因子对双壁碳纳米管中弯曲波频散特性的影响*

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摘要: 基于应力梯度理论修正的 Timoshenko(铁木辛柯)梁模型,结合管间 Van der Waals(范德华)力,研究了非局部效应对双壁碳纳米管中弹性波各阶模态的相速度、频率、临界频率、内外管振幅比等频散特性的影响.数值模拟结果表明:在应力梯度理论修正模型下,对应同一波数,4 阶模态的相速度、频率均随剪力非局部因子的增大而减小,且随着剪力非局部因子的等幅增加,相速度、频率的减小并没有明显的等幅规律;值得注意的是,3、4 阶模态的内外管振幅比并不是随着剪力非局部因子的增加依次减小;在波数较高阶段,剪力非局部因子对双壁碳纳米管中波的相速度、频率、渐近频率、内外管振幅比等频散特性均有显著影响.

关 键 词: 双壁碳纳米管; 弯曲波传播; 应力梯度理论; 频散关系
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引 言

近年来,碳纳米管作为一种具有潜在用途的特殊材料引起了很多学者的广泛关注.例如,它可用于制备原子力显微镜、场发射器件,作为飞行器的隐形涂料等等.因此,研究碳纳米管中波的传播特性,进而深入了解碳纳米管的力学性能具有重要的意义.Wang 和 Hu[1] 用分子动力学的方法给出了单壁碳纳米管中弯曲波的频散特性曲线,这种方法多适用于由少数原子或分子组成的系统.在不考虑非局部效应的情况下,弹性梁模型[2-3] 在研究碳纳米管中波的频散关系方面已有一些应用.2003 年,Peddieson 等[4] 指出纳尺度结构广泛存在非局部效应,Lu 等[5-6] 也指出非局部效应在与纳米技术应用有关的分析研究中起着重要作用,后续有一些研究[7-8] 在考虑非局部效应的影响下分析了碳纳米管的振动[9-10] 和屈曲[11] 等问题.

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目前,碳纳米管中波传播问题的求解主要有分子动力学方法和连续介质力学方法等[2].作为一种考虑了非局部效应的连续介质模型,本文利用应力梯度修正的 Timoshenko 梁模型,在考虑转动惯量和剪切变形的情况下,结合层间 Van der Waals 力,重点分析了变化的剪力非局部因子对双壁碳纳米管中弯曲波的相速度、频率、振幅比等频散特性的影响.

1　应力梯度修正的 Timoshenko 梁模型的建立

在非局部弹性理论中[12],定义于一点 x 处的应力状态是由这个结构上所有点的应变共同决定的[1].为研究微结构的非局部效应对其动力学特性的影响,在处理一维梁模型时,取应力梯度的本构关系有

$$\left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\sigma = E\varepsilon, \tag{1}$$

$$\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\tau = G\gamma, \tag{2}$$

其中 E,G 是弹性模量;切应变 $\gamma = \partial w/\partial x - \phi$,$\phi$ 是截面转角;l_1,l_2 分别表示非局部弹性材料的弯矩非局部因子和剪力非局部因子.

为建立梁的动力学方程,建立 Oxy 坐标系,使 x 方向与梁的轴向重合.则梁的弯矩 M 和剪力 Q 表示如下:

$$M = \int_A y\sigma \mathrm{d}A, \tag{3}$$

$$Q = \int_A \tau \mathrm{d}A = A\beta G, \tag{4}$$

其中 $\sigma = \sigma_x$,$\tau = \sigma_{xy}$,y 表示相应点离中性轴的距离,A 表示梁的横截面面积,I 是惯性矩,β 是横截面剪切系数.

由方程(1)~(4),有

$$\left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)M = - EI\frac{\partial \phi}{\partial x}, \tag{5}$$

$$\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)Q = GA\beta\left(\frac{\partial w}{\partial x} - \phi\right). \tag{6}$$

梁的平衡方程为[13]

$$\rho A\frac{\partial^2 w}{\partial t^2} = \frac{\partial Q}{\partial x}, \tag{7}$$

$$\rho I\frac{\partial^2 \phi}{\partial t^2} = Q - \frac{\partial M}{\partial x}. \tag{8}$$

由方程(5)~(8)得

$$\left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\left(\rho A\frac{\partial^2 w}{\partial t^2} - \rho I\frac{\partial^3 \phi}{\partial x\partial t^2}\right) + EI\frac{\partial^3 \phi}{\partial x^3} = 0, \tag{9}$$

$$\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\rho A\frac{\partial^2 w}{\partial t^2} - GA\beta\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \phi}{\partial x}\right) = 0. \tag{10}$$

方程(9)和(10)是应力梯度理论修正的 Timoshenko 梁模型的控制方程.

基于上述应力梯度理论修正的 Timoshenko 梁模型,分析了非局部效应对单壁碳纳米管中

弯曲波频散关系的影响[14].并通过将所得结果与分子动力学[1]及应力梯度理论修正的 Euler(欧拉)梁模型结果对比,证实了模型的可行性与优越性.基于此,拟结合管间 Van der Waals 力,将上述应力梯度修正的 Timoshenko 梁模型推广到研究非局部效应对双壁碳纳米管中弯曲波频散特性的影响.

2 双壁碳纳米管中弯曲波的频散关系

为分析弯矩非局部因子和剪力非局部因子不同时,非局部效应对双壁碳纳米管中弯曲波频散特性的影响,结合控制方程(9)、(10),在考虑管间 Van der Waals 力时有

$$\begin{cases} \left(1 - l_1^2 \dfrac{\partial^2}{\partial x^2}\right) GA_1\beta\left(\dfrac{\partial w_1}{\partial x} - \phi_1\right) - \left(1 - l_1^2 \dfrac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right)\rho I_1 \dfrac{\partial^2 \phi_1}{\partial t^2} + \\ \quad \left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right) EI_1 \dfrac{\partial^2 \phi_1}{\partial x^2} = 0, \\ \left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right)\rho A_1 \dfrac{\partial^2 w_1}{\partial t^2} - GA_1\beta\left(\dfrac{\partial^2 w_1}{\partial x^2} - \dfrac{\partial \phi_1}{\partial x}\right) = \left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right) C(w_2 - w_1), \\ \left(1 - l_1^2 \dfrac{\partial^2}{\partial x^2}\right) GA_2\beta\left(\dfrac{\partial w_2}{\partial x} - \phi_2\right) - \left(1 - l_1^2 \dfrac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right)\rho I_2 \dfrac{\partial^2 \phi_2}{\partial t^2} + \\ \quad \left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right) EI_2 \dfrac{\partial^2 \phi_2}{\partial x^2} = 0, \\ \left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right)\rho A_2 \dfrac{\partial^2 w_2}{\partial t^2} - GA_2\beta\left(\dfrac{\partial^2 w_2}{\partial x^2} - \dfrac{\partial \phi_2}{\partial x}\right) = -\left(1 - l_2^2 \dfrac{\partial^2}{\partial x^2}\right) C(w_2 - w_1), \end{cases} \tag{11}$$

其中位移分量中下标 1,2 分别用来区分内外管的变量, $C(w_2 - w_1)$ 表示轴向单位长度上两管间 Van der Waals 力.在小振幅的线性振动中,两相邻管间任一点处的相互作用力可以用其位移差的线性函数来表示[3].管间作用系数 C 表示为[11]: $C = 320(2\bar{r}) \times 10^{-3}\ \mathrm{J/m^2}/0.16a^2 = 9.918\,667(2\bar{r}) \times 10^{19}\ \mathrm{N/m^3}$,内径 $\bar{r} = (d_1 + d_2)/4$,碳原子晶格间距 $a = 0.142 \times 10^{-8}$ cm .

在双壁碳纳米管的控制方程(11)中消去截面转角 ϕ_1,ϕ_2,即得关于 w_1,w_2 的位移方程:

$$\begin{aligned} &\left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\left[-C(w_2 - w_1) + \rho A_1 \frac{\partial^2 w_1}{\partial t^2}\right] - \\ &\quad \left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\rho I_1\left[\frac{C}{GA_1\beta}\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial t^2}\right) - \right. \\ &\quad \left.\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\frac{\rho A_1}{GA_1\beta}\frac{\partial^4 w_1}{\partial t^4} + \frac{\partial^4 w_1}{\partial x^2 \partial t^2}\right] + \\ &\quad \left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right) EI_1\left[\frac{C}{GA_1\beta}\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_2}{\partial x^2} - \frac{\partial^2 w_1}{\partial x^2}\right) - \right. \\ &\quad \left.\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\frac{\rho A_1}{GA_1\beta}\frac{\partial^4 w_1}{\partial x^2 \partial t^2} + \frac{\partial^4 w_1}{\partial x^4}\right] = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &\left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\left[C(w_2 - w_1) + \rho A_2 \frac{\partial^2 w_2}{\partial t^2}\right] - \\ &\quad \left(1 - l_1^2 \frac{\partial^2}{\partial x^2}\right)\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\rho I_2\left[-\frac{C}{GA_2\beta}\left(1 - l_2^2 \frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_2}{\partial t^2} - \frac{\partial^2 w_1}{\partial t^2}\right) - \right. \end{aligned}$$

$$\left(1-l_2^2\frac{\partial^2}{\partial x^2}\right)\frac{\rho A_2}{GA_2\beta}\frac{\partial^4 w_1}{\partial t^4}+\frac{\partial^4 w_2}{\partial x^2\partial t^2}\Bigg]+$$

$$\left(1-l_2^2\frac{\partial^2}{\partial x^2}\right)EI_2\left[-\frac{C}{GA_2\beta}\left(1-l_2^2\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial^2 w_1}{\partial x^2}\right)-\right.$$

$$\left.\left(1-l_2^2\frac{\partial^2}{\partial x^2}\right)\frac{\rho A_2}{GA_2\beta}\frac{\partial^4 w_2}{\partial x^2\partial t^2}+\frac{\partial^4 w_2}{\partial x^4}\right]=0; \tag{13}$$

再代入波函数表达式 $w_1(x,t)=\bar{w}_1\exp[\mathrm{i}k(x-ct)]$, $w_2(x,t)=\bar{w}_2\exp[\mathrm{i}k(x-ct)]$, 整理得

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{Bmatrix}\bar{w}_1\\ \bar{w}_2\end{Bmatrix}=0, \tag{14}$$

其中

$$A_{11}=-\frac{1}{c_S^2}k^4\delta_1\delta_2^2c^4+k^2\delta_2\left(\frac{c_L^2}{c_S^2}k^2\delta_2+\frac{\varepsilon_1}{c_S^2}\delta_1\delta_2+\frac{1}{\eta_1^2}\delta_1+k^2\delta_1\right)c^2-\left(\frac{c_L^2\varepsilon_1}{c_S^2}k^2\delta_2^2+\frac{\varepsilon_1}{\eta_1^2}\delta_1\delta_2+c_L^2k^4\delta_2\right),$$

$$A_{12}=-\frac{\varepsilon_1}{c_S^2}k^2\delta_1\delta_2^2c^2+\left(\frac{\varepsilon_1}{\eta_1^2}\delta_1\delta_2+\frac{c_L^2}{c_S^2}\varepsilon_1k^2\delta_2^2\right),$$

$$A_{21}=-\frac{\varepsilon_2}{c_S^2}k^2\delta_1\delta_2^2c^2+\left(\frac{\varepsilon_2}{\eta_2^2}\delta_1\delta_2+\frac{c_L^2\varepsilon_2}{c_S^2}k^2\delta_2^2\right),$$

$$A_{22}=-\frac{1}{c_S^2}k^4\delta_1\delta_2^2c^4+k^2\delta_2\left(\frac{c_L^2}{c_S^2}k^2\delta_2+\frac{\varepsilon_2}{c_S^2}\delta_1\delta_2+\frac{\delta_1}{\eta_2^2}+k^2\delta_1\right)c^2-\left(\frac{c_L^2\varepsilon_2}{c_S^2}k^2\delta_2^2+\frac{\varepsilon_2}{\eta_2^2}\delta_1\delta_2+c_L^2k^4\delta_2\right).$$

且有

$$c_L=\sqrt{\frac{E}{\rho}},\ c_S=\sqrt{\frac{G\beta}{\rho}},\ \eta_1=\sqrt{\frac{I_1}{A_1}},\ \eta_2=\sqrt{\frac{I_2}{A_2}},$$

$$\varepsilon_1=\frac{C}{\rho A_1},\ \varepsilon_2=\frac{C}{\rho A_2},\ \delta_1=1+l_1^2k^2,\ \delta_2=1+l_2^2k^2.$$

为研究双壁碳纳米管中波的传播特性,求方程(14)的非平凡解,即

$$A_{11}A_{22}-A_{12}A_{21}=0. \tag{15}$$

代入整理有

$$B_8c^8+B_6c^6+B_4c^4+B_2c^2+B_0=0, \tag{16}$$

其中

$$B_8=\frac{1}{c_S^4}k^8\delta_1^2\delta_2^4,$$

$$B_6=-\frac{1}{c_S^2}k^6\delta_1\delta_2^3\left[\delta_1\left(2k^2+\frac{\varepsilon_1+\varepsilon_2}{c_S^2}\delta_2+\frac{1}{\eta_1^2}+\frac{1}{\eta_2^2}\right)+\frac{2c_L^2}{c_S^2}k^2\delta_2\right],$$

$$B_4 = k^4\delta_1^2\delta_2^2\left(k^2 + \frac{1}{\eta_1^2}\right)\left(k^2 + \frac{1}{\eta_2^2}\right) + \frac{k^4\delta_1\delta_2^3}{c_S^2}\left[\left(1 + \frac{1}{\eta_1^2} + \frac{1}{\eta_2^2}\right)\delta_1(\varepsilon_1 + \varepsilon_2) + c_L^2k^2\left(4k^2 + \frac{1}{\eta_1^2} + \frac{1}{\eta_2^2}\right)\right] + \frac{c_L^2k^6\delta_2^4}{c_S^4}[c_L^2k^2 + 2\delta_1(\varepsilon_1 + \varepsilon_2)],$$

$$B_2 = -k^2\delta_1\delta_2^2\left[c_L^2k^4\left(\frac{1}{\eta_1^2} + \frac{1}{\eta_2^2} + 2\right) + \varepsilon_1\delta_1\left(\frac{1}{\eta_1^2\eta_2^2} + \frac{k^2}{\eta_1^2}\right) + \varepsilon_2\delta_1\left(\frac{1}{\eta_1^2\eta_2^2} + \frac{k^2}{\eta_2^2}\right)\right] - \frac{c_L^2}{c_S^2}k^4\delta_2^3\left[(\varepsilon_1 + \varepsilon_2)\left(2k^2 + \frac{1}{\eta_1^2} + \frac{1}{\eta_2^2}\right)\delta_1 + 2c_L^2k^4\right] - \frac{c_L^4}{c_S^4}k^6\delta_2^4(\varepsilon_1 + \varepsilon_2),$$

$$B_0 = c_L^2k^4\delta_2^2\left[\left(\frac{\varepsilon_1}{\eta_1^2} + \frac{\varepsilon_2}{\eta_2^2}\right)\delta_1 + c_L^2k^4\right] + \frac{c_L^4}{c_S^2}k^6\delta_2^3(\varepsilon_1 + \varepsilon_2).$$

从方程(16)知,双壁碳纳米管有4阶模态的相速度和频率值,代入 $k=\omega/c$,且令 $k\to 0$,可得双壁碳纳米管的截止频率为

$$\omega_{c1}^2 = \frac{c_S^2}{\eta_1^2},\ \omega_{c2}^2 = \frac{c_S^2}{\eta_2^2},\ \omega_{c3}^2 = \varepsilon_1 + \varepsilon_2. \tag{17}$$

从方程(17)可以明显看出,截止频率与碳纳米管的非局部因子无关,但与 Van der Waals 系数有关.同理,令 $k\to\infty$,即得在应力梯度理论修正的 Timoshenko 梁模型下的另一组临界频率,又称渐近频率:

$$\omega_{a1}^2 = \frac{c_L^2}{l_1^2},\ \omega_{a2}^2 = \frac{c_S^2}{l_2^2},\ \omega_{a3}^2 = \frac{c_S^2}{l_2^2} + \frac{l_1^2}{l_2^2}\varepsilon_1 + \varepsilon_2. \tag{18}$$

从上式明确看出,双壁碳纳米管的渐近频率受非局部因子和 Van der Waals 系数的共同影响.

从方程(14)有,双壁碳纳米管外管与内管的振幅比为

$$\left(\frac{\bar{w}_2}{\bar{w}_1}\right)_j = \left[-\frac{1}{c_S^2}k^4\delta_1\delta_2^2c_j^4 + k^2\delta_2\left(\frac{c_L^2}{c_S^2}k^2\delta_2 + \frac{\varepsilon_1}{c_S^2}\delta_1\delta_2 + \frac{1}{\eta_1^2}\delta_1 + k^2\delta_1\right)c_j^2 - \left(\frac{c_L^2\varepsilon_1}{c_S^2}k^2\delta_2^2 + \frac{\varepsilon_1}{\eta_1^2}\delta_1\delta_2 + c_L^2k^4\delta_2\right)\right]\Big/\left[-\frac{\varepsilon_1}{c_S^2}k^2\delta_1\delta_2^2c_j^2 + \left(\frac{\varepsilon_1}{\eta_1^2}\delta_1\delta_2 + \frac{c_L^2}{c_S^2}\varepsilon_1k^2\delta_2^2\right)\right],$$

$$j = 1,2,3,4. \tag{19}$$

3 数 值 算 例

下面用具体数值算例,分析双壁碳纳米管中弯曲波的频散特性.

碳纳米管的材料和几何参数如表1所示,剪切模量可通过 $G=E/[2(1+\nu)]$ 导出,壁厚 $t=0.34$ nm,$d_1=7$ nm.

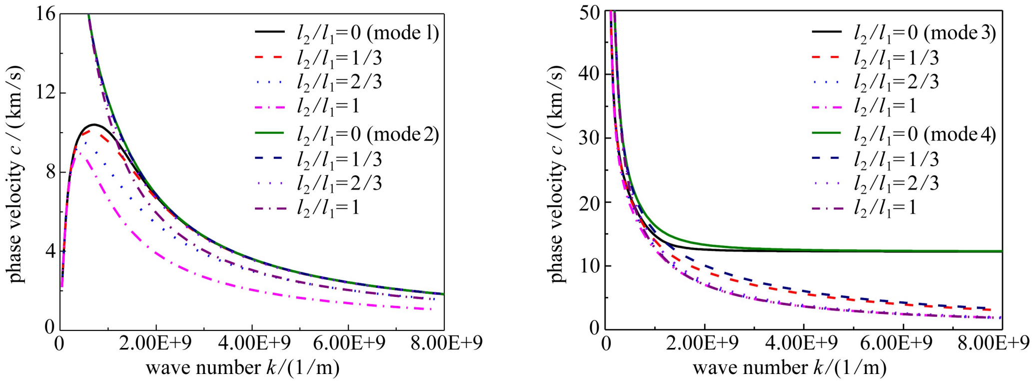
图1是应力梯度修正的 Timoshenko 梁模型下,双壁碳纳米管中弯曲波各阶模态的相速度 c 随波数 k 的变化曲线.取弯矩非局部因子 $l_1=0.4\bar{r}$,剪力非局部因子根据 l_2/l_1 在0到1之间取一组值.综合各阶模态的相速度曲线,发现在波数 $k<2.8\times10^8\ \mathrm{m}^{-1}$ 时,剪力非局部因子对其相速度影响相对较小,而在波数稍大阶段,剪力非局部因子的变化对各阶模态的相速度均有明显的影响.在相同波数下,各阶模态的相速度均随剪力非局部因子的增大而减小,当剪力非局部因子增大到等于弯矩非局部因子时,相速度达到最小值,此时即退化到两个非局部因子相

等[15-16]的情况.

表 1　双壁碳纳米管的材料参数[15]

Table 1　Parameters of double-walled carbon nanotubes[15]

	$E/(\mathrm{N/m^2})$	ν	$\rho/(\mathrm{kg/m^3})$	β
nanotube	1.06×10^{12}	0.25	2 270	0.8



（a）1,2 阶模态相速度 c 随波数 k 的变化曲线

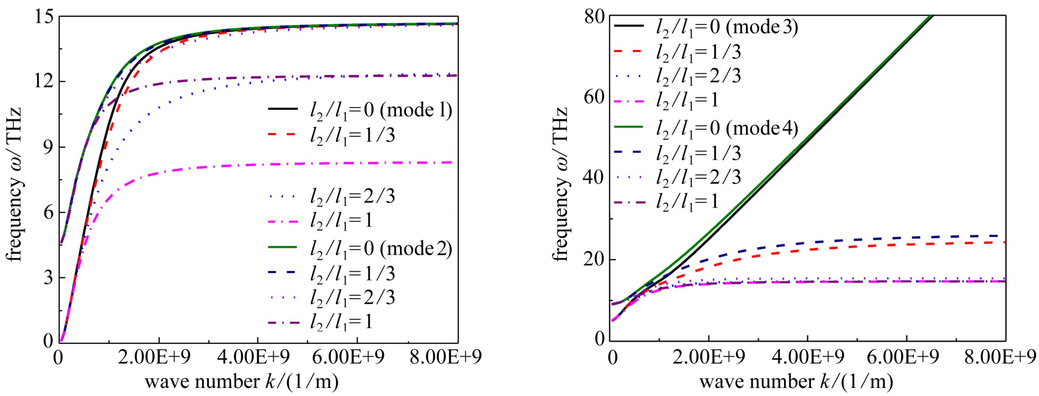
（a）The phase velocity c vs. wave number k of mode 1 and mode 2

（b）3,4 阶模态相速度 c 随波数 k 的变化曲线

（b）The phase velocity c vs. wave number k of mode 3 and mode 4

图 1　修正的 Timoshenko 梁模型下双壁碳纳米管中弯曲波在不同剪力非局部因子取值时相速度 c 随波数 k 的变化曲线

Fig.1　Effects of nonlocal shear factor on the phase velocity c vs. wave number k for different modes of double-walled carbon nanotubes, based on the modified Timoshenko beam model



（a）1,2 阶模态频率 ω 随波数 k 的变化曲线

（a）The frequency ω vs. wave number k of mode 1 and mode 2

（b）3,4 阶模态频率 ω 随波数 k 的变化曲线

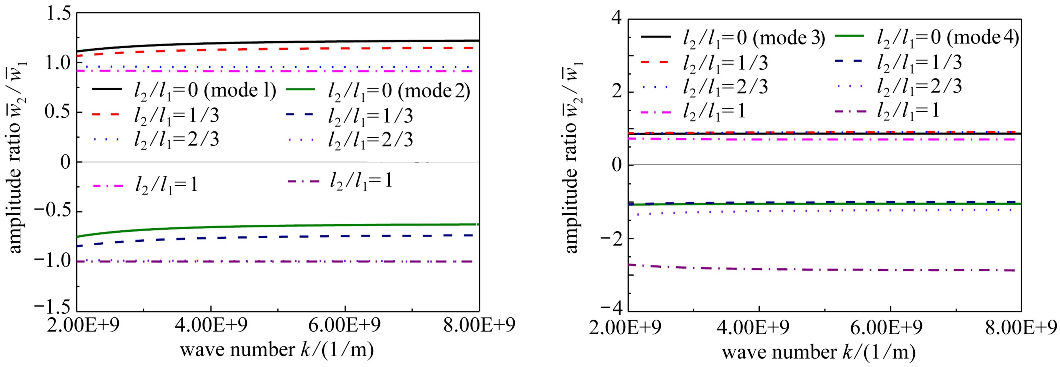
（b）The frequency ω vs. wave number k of mode 3 and mode 4

图 2　不同剪力非局部因子下双壁碳纳米管中弯曲波的频率 ω 随波数 k 的变化曲线

Fig.2　Effects of nonlocal shear factor on the frequency ω vs. wave number k for different modes of double-walled carbon nanotubes, based on the modified Timoshenko beam model

图 2 用应力梯度修正的 Timoshenko 梁模型给出了双壁碳纳米管中弯曲波的频率 ω 随波数 k 的变化曲线.分别取 $l_2/l_1=0,1/3,2/3,1$ 这 4 组值来研究剪力非局部因子对弯曲波各阶模态下频率的影响,l_1 取值同图 1.从图中可以看出,随波数 k 的增大,剪力非局部因子对频率的

影响也越来越明显.在同一波数下,各阶模态的频率均随剪力非局部因子的增大而减小,且当剪力非局部因子增大到等于弯矩非局部因子时,频率值达到最小.同时发现在 1,2 阶模态中,当剪力非局部因子以同等幅度增加时,其取值本身已经较大时频率的减幅比其本身取值小时更明显;而在第 3,4 阶模态中,剪力非局部因子以同等幅度增加时,则得到与前面相反的结果.



（a）1,2 阶模态外管与内管振幅比随波数 k 的变化曲线

（a）The amplitude ratio vs. wave number k of mode 1 and mode 2

（b）3,4 阶模态外管与内管振幅比随波数 k 的变化曲线

（b）The amplitude ratio vs. wave number k of mode 3 and mode 4

图 3 不同剪力非局部因子下双壁碳纳米管外管与内管振幅比随波数 k 的变化曲线

Fig.3 Effects of nonlocal shear factor on the amplitude ratio vs. wave number k of double-walled carbon nanotubes, based on the modified Timoshenko beam model

图 3 给出了双壁碳纳米管外管与内管的振幅比随波数 k 的变化曲线.在剪力非局部因子分别取 $l_2/l_1 = 0, 1/3, 2/3, 1$ 时分析其对各阶模态振幅比的影响.从图 3(a)所示第 1,2 阶模态振幅比看出,随着剪力非局部因子的增大,外管与内管的振幅比在 1 周围出现小的下降过程,但随着剪力非局部因子的等幅增加,振幅比的减小没有明显的等幅规律.值得注意的是,图 3(b)中 3,4 阶模态的振幅比并不是随着剪力非局部因子的增加依次减小.同时,可以从图中 4 阶模态得到同样的规律,即随着波数的增大,双壁碳纳米管的振幅比趋于平缓.

4 结 论

本文用应力梯度修正的 Timoshenko 梁模型,结合管间 Van der Waals 力分析了剪力非局部因子对双壁碳纳米管中弯曲波频散关系的影响.通过分析一系列剪力非局部因子取值下的频散曲线,可以得到如下结论:剪力非局部因子对双壁碳纳米管各阶模态下的相速度曲线均有明显的影响,在相同波数下,相速度随剪力非局部因子的增大而减小,当剪力非局部因子增大到等于弯矩非局部因子时相速度达到最小值;由频率波数曲线知,当剪力非局部因子以同等幅度增加时,不同阶模态下频率的变化幅度存在明显差异;另外,由双壁碳纳米管外管与内管的振幅比曲线可知,随着波数的增大,各阶模态的振幅比均趋于平缓.

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Effects of Nonlocal Shear Factor on Flexural Wave Dispersion in Double-Walled Carbon Nanotubes

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Abstract: The effects of nonlocal factors on the wave dispersion in the double-walled carbon nanotube (DWCNT) were analyzed with the modified Timoshenko beam model modified based on the stress gradient theory. Coupling with Van der Waals force, the dispersion characteristics, such as phase velocity, frequency, critical frequency and amplitude ratio of outer tube to inner tube, were studied. The results show that: for a given wave number, both the first 4 modes' phase velocities and frequencies of the DWCNT decrease with rise of the nonlocal shear factor, in which no obvious linear law is found. It is notable that for the 3rd and 4th modes the amplitude ratios do not decrease with the rise of the nonlocal shear factor. Meanwhile, the nonlocal shear factor has prominent effects on the wave dispersion characteristics of the DWCNT especially at relatively higher wave numbers.

Key words: double-walled carbon nanotubes; wave propagation; stress gradient theory; dispersion relation

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