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解抛物型方程的一族高精度隐式差分格式*

詹涌强[1], 张传林[2]

(1. 华南理工大学广州学院 计算机工程学院, 广州 510800;
2. 暨南大学 数学系, 广州 510632)

摘要: 构造了求解一维抛物型方程的一族高精度隐式差分格式.首先,推导了抛物型方程解的一阶偏导数在特殊节点处的一个差分近似式,利用该差分近似式和二阶中心差商近似式用待定系数法构造了一族隐式差分格式,通过选取适当的参数使格式具有高阶截断误差;然后,利用 Fourier 分析法证明了当 r 大于 1/6 时,差分格式是稳定的.最后,通过数值试验将差分格式的解与具有同样精度的其它差分格式的解和精确解进行了比较,并比较了差分格式与经典差分格式的计算效率.结果说明了差分格式的有效性.

关 键 词: 一维抛物型方程; 隐式差分格式; 截断误差
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引 言

在渗流、扩散、热传导等领域中经常会遇到求解抛物型方程的问题.在一维的情形,其模型为初边值问题:

$$\begin{cases}\dfrac{\partial u}{\partial t}=a\dfrac{\partial^2 u}{\partial x^2}, & 0<x<L,\ t>0,\ a>0,\\ u(x,0)=\varphi(x), & 0\leqslant x\leqslant L,\\ u(0,t)=\mu_1(t),\ u(L,t)=\mu_2(t), & t\geqslant 0.\end{cases}\tag{1}$$

对问题(1)的求解,有限差分法是解决此类问题的常用方法.常见的差分格式[1-2],诸如古典隐格式,Crank-Nicolson 格式和 Dufort-Frankel 格式等,虽都是绝对稳定的,但它们的精度不高.前两者分别是 $O(\tau+h^2)$, $O(\tau^2+h^2)$;后者为 $O(\tau^2+h^2+(\tau/h)^2)$,当 $\tau=h$ 时还失去了相容性.因此,寻找稳定性好且精度高的差分格式就成了当前许多学者所研究的问题.对上述问题,国内外很多学者都进行了研究,有许多好的成果[3-11].在这些研究成果中,有一些高精度的差分格式,如文献[11]构造了一个截断误差达到 $O(\tau^3+h^4)$ 的高精度隐格式,稳定性条件为 $0<r<1/2$,其中 $r=a\tau/h^2$ 为网格步长比,格式精度虽然很高,但稳定性范围较小.本文则利用待定参数法构造了一族稳定性范围大且高精度的隐式格式,稳定性条件为 $r>1/6$,截断误差达到了 $O(\tau^3+h^4)$,并且在数值实验与计算效率中通过与其它差分格式的比较,说明本文格式

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作者简介: 詹涌强(1978—),男,广东潮州人,讲师,硕士(通讯作者. E-mail: zhanyongq@126.com).

是一种好的差分格式.

1 差分近似

设时间步长为 τ,空间步长为 h,对区域$[0,L]\times[0,T]$作矩形剖分,$L=Jh$,$T=N\tau$,J,N 均为正整数,$x_j=jh$,$t_n=n\tau$,并令 $u_j^n=u(x_j,t_n)$ $(j=0,1,\cdots,J;\ n=0,1,\cdots,N)$.

假定问题(1)的解 $u(x,t)$ 是充分光滑的,先建立 $u(x,t)$ 在(x_j,t_{n+1})处对 t 的一阶偏导数的一个差分近似式,将 $u(jh,n\tau)$ 在节点$(jh,(n+1)\tau)$处作 Taylor 展开,有

$$u(jh,n\tau)=$$
$$u(jh,(n+1)\tau)-\frac{\tau}{1!}\frac{\partial u(jh,(n+1)\tau)}{\partial t}+$$
$$\frac{\tau^2}{2!}\frac{\partial^2 u(jh,(n+1)\tau)}{\partial t^2}-\frac{\tau^3}{3!}\frac{\partial^3 u(jh,(n+1)\tau)}{\partial t^3}+\cdots,$$

简记为

$$u_j^n=u_j^{n+1}-\frac{\tau}{1!}\left(\frac{\partial u}{\partial t}\right)_j^{n+1}+\frac{\tau^2}{2!}\left(\frac{\partial^2 u}{\partial t^2}\right)_j^{n+1}-\frac{\tau^3}{3!}\left(\frac{\partial^3 u}{\partial t^3}\right)_j^{n+1}+\cdots. \tag{2}$$

将 $u(jh,(n-1)\tau)$ 在节点$(jh,(n+1)\tau)$处作 Taylor 展开,有

$$u(jh,(n-1)\tau)=$$
$$u(jh,(n+1)\tau)-\frac{2\tau}{1!}\frac{\partial u(jh,(n+1)\tau)}{\partial t}+$$
$$\frac{(2\tau)^2}{2!}\frac{\partial^2 u(jh,(n+1)\tau)}{\partial t^2}-\frac{(2\tau)^3}{3!}\frac{\partial^3 u(jh,(n+1)\tau)}{\partial t^3}+\cdots,$$

简记为

$$u_j^{n-1}=u_j^{n+1}-\frac{(2\tau)}{1!}\left(\frac{\partial u}{\partial t}\right)_j^{n+1}+\frac{(2\tau)^2}{2!}\left(\frac{\partial^2 u}{\partial t^2}\right)_j^{n+1}-\frac{(2\tau)^3}{3!}\left(\frac{\partial^3 u}{\partial t^3}\right)_j^{n+1}+\cdots. \tag{3}$$

将式(2)×2−式(3)×1/2 并化简可得

$$\left(\frac{\partial u}{\partial t}\right)_j^{n+1}=\frac{\frac{3}{2}u_j^{n+1}-2u_j^n+\frac{1}{2}u_j^{n-1}}{\tau}+\frac{1}{3}\tau^2\left(\frac{\partial^3 u}{\partial t^3}\right)_j^{n+1}+\cdots=$$
$$\frac{\frac{3}{2}u_j^{n+1}-2u_j^n+\frac{1}{2}u_j^{n-1}}{\tau}+O(\tau^2).$$

为方便起见,记差商

$$\eta_t u_j^{n+1}=\frac{\frac{3}{2}u_j^{n+1}-2u_j^n+\frac{1}{2}u_j^{n-1}}{\tau},$$

它是 $(\partial u/\partial t)_j^{n+1}$ 的一个差分近似.

2 差分格式的建立

利用差分近似式

$$\delta_x^2 u_j^n = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{h^2},\ \eta_t u_j^{n+1} = \frac{\frac{3}{2}u_j^{n+1} - 2u_j^n + \frac{1}{2}u_j^{n-1}}{\tau},$$

构造如下含参数的 3 层 9 点差分方程：

$$c_1\eta_t u_{j+1}^{n+1} + c_2\eta_t u_j^{n+1} + c_3\eta_t u_{j-1}^{n+1} = c_4 a\delta_x^2 u_j^n + c_5 a\delta_x^2 u_j^{n-1} + (1 - c_4 - c_5)a\delta_x^2 u_j^{n+1}. \tag{4}$$

为了使上式在节点 $(jh, n\tau)$ 处逼近微分方程(1)，考虑到方程(1)的形式，式(4)的待定参数 c_i 应满足 $c_1 + c_2 + c_3 = 1$ 与 $c_4 + c_5 + (1 - c_4 - c_5) = 1$，将式(4)在节点$(jh, n\tau)$处作 Taylor 展开，整理可得

$$\begin{aligned}&(c_1 + c_2 + c_3)a\frac{\partial^2 u}{\partial x^2} + (c_1 - c_3)ah\frac{\partial^3 u}{\partial x^3} + \\ &(c_1 + c_2 + c_3)a^2\tau\frac{\partial^4 u}{\partial x^4} + \frac{1}{2}(c_1 + c_3)ah^2\frac{\partial^4 u}{\partial x^4} + \\ &(c_1 - c_3)a^2h\tau\frac{\partial^5 u}{\partial x^5} + \frac{1}{6}(c_1 + c_2 + c_3)a^3\tau^2\frac{\partial^6 u}{\partial x^6} + \frac{1}{6}(c_1 - c_3)ah^3\frac{\partial^5 u}{\partial x^5} + \\ &\frac{1}{2}(c_1 + c_3)a^2h^2\tau\frac{\partial^6 u}{\partial x^6} + \frac{1}{6}(c_1 - c_3)a^3h\tau^2\frac{\partial^7 u}{\partial x^7} = \\ &a\frac{\partial^2 u}{\partial x^2} + \frac{1}{12}ah^2\frac{\partial^4 u}{\partial x^4} + (1 - c_4 - 2c_5)a^2\tau\frac{\partial^4 u}{\partial x^4} + \frac{1}{2}(1 - c_4)a^3\tau^2\frac{\partial^6 u}{\partial x^6} + \\ &\frac{1}{12}(1 - c_4 - 2c_5)a^2h^2\tau\frac{\partial^6 u}{\partial x^6} + O(\tau^3 + h^4).\end{aligned}$$

为了使格式(4)的截断误差达到 $O(\tau^3 + h^4)$，须满足下面方程组：

$$\begin{cases}c_1 + c_2 + c_3 = 1, \\ c_1 - c_3 = 0, \\ a^2\tau + \frac{1}{2}(c_1 + c_3)ah^2 = \frac{1}{12}ah^2 + (1 - c_4 - 2c_5)a^2\tau, \\ \frac{1}{6}a^3\tau^2 + \frac{1}{2}(c_1 + c_3)a^2h^2\tau = \frac{1}{2}(1 - c_4)a^3\tau^2 + \frac{1}{12}(1 - c_4 - 2c_5)a^2h^2\tau.\end{cases} \tag{5}$$

在方程组(5)中，令 $c_5 = \theta, r = a\tau/h^2$，可解得

$$c_1 = c_3 = \frac{48r^2 + 6r - 1 + 144r^2\theta}{12(6r - 1)},$$

$$c_2 = \frac{-48r^2 + 30r - 5 - 144r^2\theta}{6(6r - 1)},$$

$$c_4 = \frac{-4r - 24r\theta + 2\theta}{6r - 1}.$$

将所得各值代入式(4)，可得截断误差为 $O(\tau^3 + h^4)$ 的一族隐式格式：

$$\left(-48r^2 + 21r - \frac{3}{2} + 12r\theta\right)(u_{j+1}^{n+1} + u_{j-1}^{n+1}) + (96r^2 + 66r - 15 - 24r\theta)u_j^{n+1} =$$

$$(48r^2 + 12r - 2 + 24r\theta)(u_{j+1}^n + u_{j-1}^n) + (-96r^2 + 120r - 20 - 48r\theta)u_j^n +$$

$$\left(-24r^2-3r+\frac{1}{2}-12r\theta\right)(u_{j+1}^{n-1}+u_{j-1}^{n-1})+(48r^2-30r+5+24r\theta)u_j^{n-1}. \quad (6)$$

3　稳定性和收敛性

利用 Fourier 分析法,可算出格式(6)的传播矩阵为

$$\boldsymbol{G}(s)=\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix},$$

其中

$$g_{11}=\frac{72r-12-(96r^2+24r-4+48r\theta)s}{54r-9+(96r^2-42r+3-24r\theta)s},$$

$$g_{12}=\frac{-18r+3+(48r^2+6r-1+24r\theta)s}{54r-9+(96r^2-42r+3-24r\theta)s},$$

$$g_{21}=1,\ g_{22}=0,\ s=\sin^2\frac{kh}{2}\in[0,1].$$

传播矩阵 $\boldsymbol{G}(s)$ 的特征方程为

$$\lambda^2-g_{11}\lambda-g_{12}=0. \quad (7)$$

引理 1[12]　特征方程(7)的根满足 $|\lambda_{1,2}|\leqslant 1$ 的充要条件是

$$|g_{11}|\leqslant 1-g_{12}\leqslant 2. \quad (8)$$

引理 2[12]　差分格式(6)稳定,即矩阵族 $\boldsymbol{G}^n(s)$ $(s\in[0,1],n=1,2,3,\cdots)$ 一致有界的充要条件是

1) $|\lambda_{1,2}|\leqslant 1$($\lambda_{1,2}$ 是方程(7)的两个根);

2) $N_0^1((1-|g_{11}+g_{22}|^2/4)^2)\cap N_0^1(|(g_{11}-g_{22})^2+4g_{12}g_{21}|)\subseteq$

$$N_0^1((g_{11}-g_{22})^2)\cap N_0^1(g_{12}^2)\cap N_0^1(g_{21}^2);$$

其中 $N_0^1(f(s))$ 表示多项式 $f(s)$ 在[0,1]区间内所有实根的集合(重根要重复计).

首先考虑条件 2),由于 $g_{21}=1$,所以 $N_0^1(g_{21}^2)$ 是空集,故条件 2) 成立的充要条件是使 $1-g_{11}^2/4=g_{11}^2+4g_{12}=0$ 成立的 s 或者不存在,或者不属于区间[0,1].由于 $1-g_{11}^2/4=0$ 解得 $g_{11}^2=4$,将其代入 $g_{11}^2+4g_{12}=0$ 得 $g_{12}=-1$,故 $g_{12}\neq-1$ 时,使该等式成立的 s 不存在,条件 2)成立.再由条件 1)和式(8)知,格式(5)稳定的条件为

$$-1+g_{12}\leqslant g_{11}\leqslant 1-g_{12}<2.$$

由 $g_{11}\leqslant 1-g_{12}$ 得

$$\frac{54r-9-(48r^2+18r-3+24r\theta)s}{54r-9+(96r^2-42r+3-24r\theta)s}\leqslant 1. \quad (9)$$

为确定起见,不妨假定

$$54r-9+(96r^2-42r+3-24r\theta)s>0.$$

该式成立的一个充分条件是

$$\begin{cases} 54r-9>0, \\ 96r^2-42r+3-24r\theta>0. \end{cases}$$

由此解得

$$\begin{cases} r > \dfrac{1}{6}, & (10) \\ \theta < \dfrac{32r^2 - 14r + 1}{8r}. & (11) \end{cases}$$

当式(10)、(11)成立时,可验证式(9)也成立.而当式(10)、(11)成立时,由 $1 - g_{12} < 2$ 可得

$$36r - 6 + (144r^2 - 36r + 2)s > 0.$$

该式成立的一个充分条件是

$$\begin{cases} 36r - 6 > 0, & (12) \\ 144r^2 - 36r + 2 > 0. & (13) \end{cases}$$

当式(10)成立时,可验证式(12)、(13)均成立.

再由 $-1 + g_{12} \leqslant g_{11}$ 和式(10)、(11)得

$$144r - 24 + (-48r^2 - 72r + 8 - 96r\theta)s \geqslant 0.$$

上式成立的一个充分条件是

$$\begin{cases} 144r - 24 \geqslant 0, \\ -48r^2 - 72r + 8 - 96r\theta \geqslant 0. \end{cases}$$

由此解得

$$\begin{cases} r \geqslant \dfrac{1}{6}, & (14) \\ \theta \leqslant \dfrac{-6r^2 - 9r + 1}{12r}. & (15) \end{cases}$$

注意到当 $r > 1/6$ 时,条件(15)强于条件(11),综上所述,并根据 Lax 的稳定性与收敛性等价定理可得:

定理 当条件(10)、(15)满足时,差分格式(6)稳定且收敛.

特别地,当 $\theta = (-6r^2 - 9r + 1)/(12r)$ 时,差分格式(6)成为

$$\left(-54r^2 + 12r - \frac{1}{2}\right)(u_{j+1}^{n+1} + u_{j-1}^{n+1}) + (108r^2 + 84r - 17)u_j^{n+1} = (36r^2 - 6r)(u_{j+1}^{n} + u_{j-1}^{n}) + (-72r^2 + 156r - 24)u_j^n + \left(-18r^2 + 6r - \frac{1}{2}\right)(u_{j+1}^{n-1} + u_{j-1}^{n-1}) + (36r^2 - 48r + 7)u_j^{n-1}. \quad (16)$$

4 数 值 例 子

考虑扩散方程

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}, & 0 < x < 1,\ t > 0, \\ u(x,0) = \sin x, & 0 \leqslant x \leqslant 1, \\ u(0,t) = 0,\ u(1,t) = e^{-t}\sin 1, & t \geqslant 0. \end{cases} \quad (17)$$

利用格式(16)求数值解,并与精确解进行比较.

取 $h = 1/20$,$\tau = rh^2 = r/400$,$r = 1/5, 1/2, 1, 2, 4$ 分别计算.为方便起见,用式(17)的精确解 $u(x,t) = e^{-t}\sin x$ 计算第一层的值 u_j^1.将本文格式(16)与文献[11]格式和精确解进行比较,计

算到 $n = 300$ 时的结果如表 1.

表 1 格式(16)和文献[11]格式数值解与精确解的比较

Table 1 Comparison of the results between difference scheme (16), the scheme of reference[11] and the exact solution

r	result	$x = 0.1$	$x = 0.3$	$x = 0.5$	$x = 0.7$	$x = 0.9$
0.2	exact solution	0.085 927 418	0.254 356 599	0.412 645 385	0.554 483 301	0.674 215 719
	scheme (16)	0.085 927 416	0.254 356 596	0.412 645 381	0.554 483 298	0.674 215 717
	reference[11]	0.085 927 415	0.254 356 594	0.412 645 380	0.554 483 296	0.674 215 716
0.5	exact solution	0.068 614 436	0.203 107 869	0.329 504 032	0.442 763 909	0.538 372 186
	scheme (16)	0.068 614 432	0.203 107 858	0.329 504 016	0.442 763 894	0.538 372 179
	reference[11]	0.068 615 924	0.203 112 004	0.329 509 799	0.442 769 523	0.538 374 953
1	exact solution	0.047 157 966	0.139 593 861	0.226 464 588	0.304 306 888	0.370 017 431
	scheme (16)	0.047 157 958	0.139 593 838	0.226 464 557	0.304 306 858	0.370 017 418
	reference[11]			overflow		
2	exact solution	0.022 275 846	0.065 939 471	0.106 974 297	0.143 744 395	0.174 783 858
	scheme (16)	0.022 275 833	0.065 939 437	0.106 974 251	0.143 744 353	0.174 783 839
	reference[11]			overflow		
4	exact solution	0.004 970 413	0.014 713 084	0.023 869 192	0.032 073 710	0.038 999 550
	scheme (16)	0.004 970 403	0.014 713 059	0.023 869 158	0.032 073 678	0.038 999 535
	reference[11]			overflow		

由表 1 看出,对所取的 r,差分解与精确解均有很好的吻合,这与理论分析完全一致.本文格式与文献[11]格式相比,在 $r = 0.2$ 时精确度基本相同,但文献[11]格式稳定性条件为 $r < 1/2$,在 $r = 1/2$ 时文献[11]格式与精确解相比只能精确到小数点后 5 位,在 $r = 1,2,4$ 时文献[11]格式已经完全不收敛,计算结果溢出.而本文格式随着网格比 r 的增大,差分解与精确解的误差并没有逐渐增大,依然吻合得非常好.

5 计算效率

经典的差分格式,如古典隐格式和 Crank-Nicolson 格式等,精度都不高,古典隐格式截断误差为 $O(\tau + h^2)$, Crank-Nicolson 格式的截断误差为 $O(\tau^2 + h^2)$,而本文格式的截断误差为 $O(\tau^3 + h^4)$, 比古典隐格式和 Crank-Nicolson 格式的精度提高了很多,能精确到小数点后第 8 位.利用这 3 种格式求解上面的数值例子,3 种格式的计算效率对比如表 2,其中 J 为空间坐标等份数,N 为时间坐标等份数,测试结果单位为 s .

表 2 3 种差分格式计算效率的比较(单位:s)

Table 2 Comparison of the calculation efficiency between the 3 kinds of difference schemes(unit: s)

difference scheme	$J = 100, N = 500$	$J = 500, N = 500$	$J = 500, N = 1\,000$	$J = 500, N = 2\,000$
classical implicit scheme	0.306 356	11.689 076	23.468 806	46.541 309
C-N scheme	0.320 861	12.057 869	23.975 085	47.584 914
scheme (16)	0.336 311	12.581 965	24.196 551	48.911 162

由表 2 看出,计算所用时间越长则表明计算效率越低,本文格式(16)的计算效率虽然比古典隐格式和 Crank-Nicolson 格式稍低,但是依然有非常高的计算效率.综合考虑本文格式的精度和计算效率看出,本文格式是一种好的差分格式.

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A Family of High Accuracy Implicit Difference Schemes for Solving Parabolic Equations

ZHAN Yong-qiang[1], ZHANG Chuan-lin[2]

(1. *School of Computer Engineering*, *Guangzhou College of South China University of Technology*, *Guangzhou* 510800, *P.R.China*;
2. *Department of Mathematics*, *Jinan University*, *Guangzhou* 510632, *P.R.China*)

Abstract: A family of implicit difference schemes with high accuracy for solving 1-dimensional parabolic equations were given. First, a difference approximation expression of the first order partial derivative of the solution to the parabolic equation was deduced at the special nodes; then this difference approximation expression and the second order central difference quotient approximation were used to construct a family of implicit difference schemes by the method of undetermined coefficients, and appropriate parameters were chosen to endow the schemes with high order truncation errors. In turn, the new difference schemes were proved to be stable as long as r was more than 1/6 with the Fourier analysis method. Finally, a numerical experiment was conduted on comparison of accuracy between the exact solutions, results of the new difference schemes and those of the other schemes with the same order truncation errors, as well as comparison of computational efficiency between the new schemes and the classical implicit difference schemes. The results demonstrate the high accuracy and efficiency of the presented difference schemes.

Key words: one-dimensional parabolic equation; implicit difference scheme; truncation error

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