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轴向运动导电导磁梁的磁弹性振动方程*

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摘要: 针对磁场环境中轴向运动导电导磁梁磁弹性耦合振动的理论建模问题进行研究.基于 Timoshenko(铁木辛柯)梁理论并考虑几何非线性因素,给出轴向运动弹性梁在横向双向振动下的形变势能、动能计算式以及电磁力和机械力的虚功表达式.应用 Hamilton(哈密顿)变分原理,推得磁场中轴向运动 Timoshenko 梁的非线性磁弹性耦合振动方程,并给出了简化形式的 Euler-Bernoulli(欧拉-伯努利)梁磁弹性振动方程.根据电磁理论和相应的电磁本构关系,得到载流导电弹性梁所受电磁力的表达式,基于磁偶极子-电流环路模型给出铁磁弹性梁所受磁体力和磁体力偶的表述形式.通过算例,分析了轴向运动导电弹性梁的奇点分布及其稳定性问题.

关 键 词: 导电导磁梁; 磁弹性; 振动; 轴向运动; Hamilton 原理
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引　　言

轴向运动结构及器件在航空航天、交通运输、动力传动、机电系统等工程领域中应用较广,其中所表现出的动力学行为及复杂场环境下的耦合振动等问题是影响结构安全运行的主要因素之一,同时引起研究者的关注并开展了深入的理论研究.针对轴向运动梁结构的振动问题,Chen 等[1-2]研究了轴向运动粘弹性梁的非线性强迫振动问题,分析了轴向速度、边界条件等参数对结构振动频率及动力稳定性的影响;Chen 等[3-4]采用多元 L-P 方法和增量谐波平衡法对轴向运动梁的非线性内共振与分岔特性进行了研究;Ghayesh 和 Pakdemirli 等[5-6]分析了轴向变速运动弹性梁的参数振动及稳定性,讨论了临界速度问题;Ozkaya 等[7]应用多尺度法对刚度为小量时加速运动梁的动力稳定性进行了研究.另一方面,在结构磁弹性振动问题的研究中,Wu[8]研究了横向磁场和热载荷作用下铁磁梁的大幅振动及动态稳定性问题;Pratiher 等[9-10]分析了时变磁场中受周期载荷作用下悬臂梁的非线性共振问题;Hu 等[11-12]建立了磁场中轴向运动导电薄板的磁弹性耦合振动方程,研究了系统的共振问题.总之,针对复杂运动条件下结构的多场耦合动力学问题进行研究具有理论和实际意义.本文在同时考虑磁弹性效应和轴向运动条件下,应用 Hamilton 变分原理并基于电磁理论,建立轴向运动导电导磁梁的磁弹性耦合振动方程,给出载流导电弹性梁和铁磁弹性梁所受电磁力的表述形式,分析轴向运动磁弹性梁的动力稳定性问题.

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1 形变势能

图 1 示出处于外加磁场环境中的轴向运动弹性梁,梁长为 l,横截面积为 A,轴向运动速度为 $\boldsymbol{V}_0$,$\boldsymbol{B}_{0y}$ 和 $\boldsymbol{B}_{0z}$ 分别表示沿坐标轴方向的外加磁感应强度,x 轴为形心轴.本文基于 Timoshenko 梁理论建立梁的基本动力学方程.

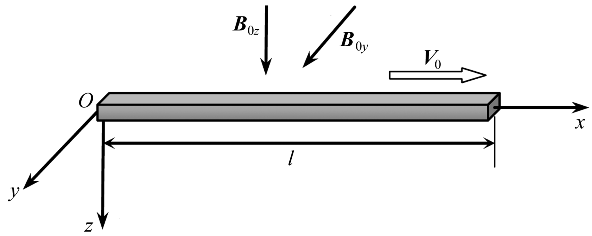

图 1 磁场中轴向运动梁

Fig. 1 The axially moving beam in a magnetic field

当研究轴向运动弹性梁的横向双向振动问题时,可将体内任意点的变形位移表示为

$$\begin{cases} u_1(x,y,z,t) = u(x,t) + y\psi(x,t) + z\varphi(x,t), \\ v_1(x,y,z,t) = v(x,t),\ w_1(x,y,z,t) = w(x,t), \end{cases} \tag{1}$$

式中, u,v,w 为轴线 x 上点的位移,ψ 和 φ 分别为由弯矩产生的截面绕中性轴 z 和 y 的转角.

考虑剪切变形的影响,描述应变分量的非线性几何方程为

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} + y\dfrac{\partial \psi}{\partial x} + z\dfrac{\partial \varphi}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2, \\ \gamma_{xz} = \dfrac{\partial w}{\partial x} + \varphi,\ \gamma_{xy} = \dfrac{\partial v}{\partial x} + \psi. \end{cases} \tag{2}$$

物理方程为

$$\begin{cases} N_x = \int_A \sigma_x \mathrm{d}A = \int_A E\varepsilon_x \mathrm{d}A = EA\left[\dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}\right)^2 + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2\right], \\ M_y = \int_A \sigma_x z\mathrm{d}A = \int_A E\varepsilon_x z\mathrm{d}A = EI_y\dfrac{\partial \varphi}{\partial x}, \\ M_z = \int_A \sigma_x y\mathrm{d}A = \int_A E\varepsilon_x y\mathrm{d}A = EI_z\dfrac{\partial \psi}{\partial x}, \\ Q_y = \int_A \tau_{xy}\mathrm{d}A = \int_A Gk\gamma_{xy}\mathrm{d}A = kGA\left(\dfrac{\partial v}{\partial x} + \psi\right), \\ Q_z = \int_A \tau_{xz}\mathrm{d}A = \int_A Gk\gamma_{xz}\mathrm{d}A = kGA\left(\dfrac{\partial w}{\partial x} + \varphi\right), \end{cases} \tag{3}$$

式中, N_x 为轴向内力;M_y,M_z 为弯矩;Q_y,Q_z 为剪力;σ_x 为截面正应力;τ_{xy},τ_{xz} 为剪应力;$G = E/(2(1+\mu))$ 为切变模量;E 为弹性模量; μ 为 Poisson(泊松) 比;k 为截面形状修正系数;$I_y = \int_A z^2\mathrm{d}A$ 和 $I_z = \int_A y^2\mathrm{d}A$ 为横截面对 y 轴和 z 轴的惯性矩.

依据弹性理论,弹性梁因变形产生的形变势能为

$$U = \frac{1}{2}\int_V (\sigma_x\varepsilon_x + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz})\mathrm{d}V, \tag{4}$$

这里 V 表示体积.形变势能(4)的变分为

$$\delta U=\int_V(\sigma_x\delta\varepsilon_x+\tau_{xy}\delta\gamma_{xy}+\tau_{xz}\delta\gamma_{xz})\mathrm{d}V=$$
$$\int_0^l\left[N_x\delta\left(\frac{\partial u}{\partial x}\right)+M_z\delta\left(\frac{\partial\psi}{\partial x}\right)+M_y\delta\left(\frac{\partial\varphi}{\partial x}\right)+N_x\frac{\partial v}{\partial x}\delta\left(\frac{\partial v}{\partial x}\right)+N_x\frac{\partial w}{\partial x}\delta\left(\frac{\partial w}{\partial x}\right)+\right.$$
$$\left.Q_y\delta\left(\frac{\partial v}{\partial x}\right)+Q_y\delta\psi+Q_z\delta\left(\frac{\partial w}{\partial x}\right)+Q_z\delta\varphi\right]\mathrm{d}x. \tag{5}$$

两端轴向拉力 N_0 引起的形变势能为

$$U_0=\int_0^l N_0\varepsilon_x^0\mathrm{d}x=\int_0^l N_0\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]\mathrm{d}x, \tag{6}$$

其变分为

$$\delta U_0=\int_0^l N_0\left[\delta\left(\frac{\partial u}{\partial x}\right)+\frac{\partial v}{\partial x}\delta\left(\frac{\partial v}{\partial x}\right)+\frac{\partial w}{\partial x}\delta\left(\frac{\partial w}{\partial x}\right)\right]\mathrm{d}x. \tag{7}$$

2 动能和外力功

弹性梁在轴向运动过程中体内各点的绝对速度分量为

$$\begin{cases}V_x=V_0+\dfrac{\mathrm{d}u_1}{\mathrm{d}t}=V_0+\dfrac{\mathrm{d}u}{\mathrm{d}t}+y\dfrac{\mathrm{d}\psi}{\mathrm{d}t}+z\dfrac{\mathrm{d}\varphi}{\mathrm{d}t},\\ V_y=\dfrac{\mathrm{d}v_1}{\mathrm{d}t}=\dfrac{\mathrm{d}v}{\mathrm{d}t},\ V_z=\dfrac{\mathrm{d}w_1}{\mathrm{d}t}=\dfrac{\mathrm{d}w}{\mathrm{d}t},\end{cases} \tag{8}$$

式中的绝对导数

$$\frac{\mathrm{d}}{\mathrm{d}t}=V_0\frac{\partial}{\partial x}+\frac{\partial}{\partial t}.$$

由式(8)进一步可得到系统总动能的表达式为

$$T=\int_V\frac{\rho}{2}(V_x^2+V_y^2+V_z^2)\mathrm{d}V=\frac{\rho A}{2}\int_0^l\left[\left(V_0+\frac{\mathrm{d}u}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)^2\right]\mathrm{d}x+$$
$$\frac{\rho I_z}{2}\int_0^l\left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\right)^2\mathrm{d}x+\frac{\rho I_y}{2}\int_0^l\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2\mathrm{d}x, \tag{9}$$

式中，ρ 为体密度.动能(9)的变分为

$$\delta T=\rho A\int_0^l\left[\left(V_0+\frac{\mathrm{d}u}{\mathrm{d}t}\right)\delta\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)+\frac{\mathrm{d}v}{\mathrm{d}t}\delta\left(\frac{\mathrm{d}v}{\mathrm{d}t}\right)+\frac{\mathrm{d}w}{\mathrm{d}t}\delta\left(\frac{\mathrm{d}w}{\mathrm{d}t}\right)\right]\mathrm{d}x+$$
$$\rho I_z\int_0^l\frac{\mathrm{d}\psi}{\mathrm{d}t}\delta\left(\frac{\mathrm{d}\psi}{\mathrm{d}t}\right)\mathrm{d}x+\rho I_y\int_0^l\frac{\mathrm{d}\varphi}{\mathrm{d}t}\delta\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)\mathrm{d}x. \tag{10}$$

对于处于电磁场环境中的轴向运动弹性梁，将受到相应电磁力和机械载荷的作用，这些力在虚位移上所做的虚功为

$$\delta W_1=\int_0^l(F_x\delta u+F_y\delta v+F_z\delta w+m_y\delta\varphi+m_z\delta\psi)\mathrm{d}x, \tag{11}$$

$$\delta W_2=\int_0^l(P_x\delta u+P_y\delta v+P_z\delta w)\mathrm{d}x, \tag{12}$$

式中，F_x,F_y,F_z 为梁所受单位长度电磁力分量；m_y,m_z 为单位长度电磁力矩分量；P_x,P_y,P_z 为梁所受单位长度机械力分量.

3 应用 Hamilton 原理建立振动方程

对于本文研究的轴向运动弹性梁，所满足的 Hamilton 变分原理[13]一般表达式为

$$\int_{t_1}^{t_2}[\delta T-\delta(U+U_0)+\delta(W_1+W_2)]\mathrm{d}t=0,\tag{13}$$

式中 t_1 和 t_2 为两个固定时刻.

将式(5),(7),(10),(11),(12)代入式(13)中,令 $\delta u,\delta v,\delta w,\delta\varphi,\delta\psi$ 为相互独立变分量,且在固定时刻 t_1 和 t_2 时满足 $\delta u=\delta v=\delta w=\delta\psi=\delta\varphi=0$ 的条件.最终,经过变分和分部积分运算,推得如下轴向运动 Timoshenko 梁的磁弹性耦合非线性振动方程:

$$\begin{cases}\dfrac{\partial N_x}{\partial x}+F_x+P_x=\rho A\left(\dfrac{\mathrm{d}V_0}{\mathrm{d}t}+\dfrac{\mathrm{d}^2u}{\mathrm{d}t^2}\right),\\ \dfrac{\partial}{\partial x}\left(N_x\dfrac{\partial v}{\partial x}\right)+\dfrac{\partial Q_y}{\partial x}+N_0\dfrac{\partial^2 v}{\partial x^2}+F_y+P_y=\rho A\dfrac{\mathrm{d}^2v}{\mathrm{d}t^2},\\ \dfrac{\partial}{\partial x}\left(N_x\dfrac{\partial w}{\partial x}\right)+\dfrac{\partial Q_z}{\partial x}+N_0\dfrac{\partial^2 w}{\partial x^2}+F_z+P_z=\rho A\dfrac{\mathrm{d}^2w}{\mathrm{d}t^2},\\ \dfrac{\partial M_z}{\partial x}-Q_y+m_z=\rho I_z\dfrac{\mathrm{d}^2\psi}{\mathrm{d}t^2},\ \dfrac{\partial M_y}{\partial x}-Q_z+m_y=\rho I_y\dfrac{\mathrm{d}^2\varphi}{\mathrm{d}t^2},\end{cases}\tag{14}$$

式中的绝对导数

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}=V_0^2\frac{\partial^2}{\partial x^2}+2V_0\frac{\partial}{\partial x\partial t}+\frac{\mathrm{d}V_0}{\mathrm{d}t}\frac{\partial}{\partial x}+\frac{\partial^2}{\partial t^2}.$$

若略去剪切变形影响及转动惯性力项,由方程(14)可化简得到下面 Euler-Bernoulli 型弹性梁的磁弹性耦合振动方程:

$$\frac{\partial N_x}{\partial x}+F_x+P_x=\rho A\left(\frac{\mathrm{d}V_0}{\mathrm{d}t}+\frac{\mathrm{d}^2u}{\mathrm{d}t^2}\right),\tag{15a}$$

$$\frac{\partial}{\partial x}\left(N_x\frac{\partial v}{\partial x}\right)+\frac{\partial^2 M_z}{\partial x^2}+N_0\frac{\partial^2 v}{\partial x^2}+F_y+P_y+\frac{\partial m_z}{\partial x}=\rho A\frac{\mathrm{d}^2v}{\mathrm{d}t^2},\tag{15b}$$

$$\frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x}\right)+\frac{\partial^2 M_y}{\partial x^2}+N_0\frac{\partial^2 w}{\partial x^2}+F_z+P_z+\frac{\partial m_y}{\partial x}=\rho A\frac{\mathrm{d}^2w}{\mathrm{d}t^2}.\tag{15c}$$

这里,因略去剪切变形的影响(但剪力是维持平衡必需的),有 $\gamma_{xz}=\gamma_{xy}=0$, 因此式(15)中的弯矩表达式应为

$$M_y=-EI_y\frac{\partial^2 w}{\partial x^2},\ M_z=-EI_z\frac{\partial^2 v}{\partial x^2}.$$

4　电磁力的表述

4.1　载流导电弹性梁

对于磁场环境中的载电流导电材料弹性梁,设沿轴线方向所载电流体密度为 $\boldsymbol{J}_0$, 则梁在运动时体内总的电流密度为

$$\boldsymbol{J}=\boldsymbol{J}_0+\sigma(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B}_0),\tag{16}$$

式中, $\boldsymbol{E}$ 为感应电场强度矢量, $\boldsymbol{V}$ 为绝对速度矢量, $\boldsymbol{B}_0(0,B_{0y},B_{0z})$ 为外加磁感应强度矢量, σ 为电导率.

若考虑到细长梁内 y 和 z 向感应电流为 0 的条件,则由式(16)可得梁在运动变形时体内电流密度的分量式为

$$
\begin{cases}
J_x \approx J_0 + \sigma\left(E_x + \dfrac{\mathrm{d}v}{\mathrm{d}t}B_{0z} - \dfrac{\mathrm{d}w}{\mathrm{d}t}B_{0y}\right), \\
J_y \approx J_0\dfrac{\partial v}{\partial x},\ J_z \approx J_0\dfrac{\partial w}{\partial x}.
\end{cases} \tag{17}
$$

这样，根据 Lorentz（洛仑兹）体积力 $\boldsymbol{f}=\boldsymbol{J}\times\boldsymbol{B}_0$ 的表达形式，沿横截面进行积分，可得到梁所受单位长度的电磁力及力矩为

$$
\begin{cases}
F_x = J_0 A\left(\dfrac{\partial v}{\partial x}B_{0z} - \dfrac{\partial w}{\partial x}B_{0y}\right), \\
F_y = -\left[J_0 + \sigma\left(E_x + \dfrac{\mathrm{d}v}{\mathrm{d}t}B_{0z} - \dfrac{\mathrm{d}w}{\mathrm{d}t}B_{0y}\right)\right]AB_{0z}, \\
F_z = \left[J_0 + \sigma\left(E_x + \dfrac{\mathrm{d}v}{\mathrm{d}t}B_{0z} - \dfrac{\mathrm{d}w}{\mathrm{d}t}B_{0y}\right)\right]AB_{0y}, \\
m_y = 0,\ m_z = 0.
\end{cases} \tag{18}
$$

4.2 铁磁弹性梁

研究处于外加横向磁场 $\boldsymbol{B}_0(0, B_{0y}, B_{0z})$ 中的轴向运动铁磁材料弹性梁，此时需考虑磁化效应．根据磁偶极子-电流环路模型[14]，磁介质内所受磁体力和磁体力偶为

$$\boldsymbol{f} = \boldsymbol{J}\times\boldsymbol{B} + (\nabla\boldsymbol{B})\cdot\boldsymbol{M}_c, \tag{19}$$

$$\boldsymbol{m}_c = \boldsymbol{M}_c\times\boldsymbol{B}, \tag{20}$$

式中，$\boldsymbol{M}_c$ 为磁介质内的磁化强度矢量，$\boldsymbol{J}$ 为感应电流密度矢量，磁感应强度矢量 $\boldsymbol{B}=\boldsymbol{B}_0+\boldsymbol{b}$，$\boldsymbol{b}$ 为小的扰动磁感应强度矢量，∇为 Hamilton 算子．

基于上面的电磁理论模型（19），（20），可有以下两种简化模型：

4.2.1 磁体力模型

对于各向同性线性铁磁材料，其电磁本构关系为

$$
\begin{cases}
\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{V}\times\boldsymbol{B}_0), \\
\boldsymbol{B} = \mu_c\boldsymbol{H} = \mu_0\mu_r\boldsymbol{H} = \mu_0(1+\chi_m)\boldsymbol{H}, \\
\boldsymbol{M}_c = \chi_m\boldsymbol{H},
\end{cases} \tag{21}
$$

式中，μ_c 为材料磁导率，μ_0 为真空磁导率，$\mu_r = 1+\chi_m$ 为相对磁导率，χ_m 为材料磁化率，$\boldsymbol{H}$ 为磁场强度矢量．

这样，由式（19），（20）可得磁体力和磁体力偶的计算式为

$$\boldsymbol{f} = \boldsymbol{J}\times\boldsymbol{B} + \mu_0\mu_r\chi_m(\nabla\boldsymbol{H})\cdot\boldsymbol{H}, \tag{22}$$

$$\boldsymbol{m}_c = \boldsymbol{0}. \tag{23}$$

4.2.2 磁体力偶模型

对于长宽比很大的细梁结构，可认为铁磁梁内的磁感应强度近似等于外加磁感应强度 $\boldsymbol{B}_0$，且在梁体内为不随空间坐标变化的均匀磁场，则有$\nabla\boldsymbol{B}=\nabla\boldsymbol{B}_0=0$．但产生磁化效应的磁场应理解为总磁场 $\boldsymbol{B}$ 所引起的，即磁体力偶不为 0．因此有

$$\boldsymbol{f} = \boldsymbol{J}\times\boldsymbol{B}_0, \tag{24}$$

$$\boldsymbol{m}_c = \boldsymbol{M}_c\times\boldsymbol{B}_0, \tag{25}$$

式中 $\boldsymbol{M}_c = \chi_m\boldsymbol{H} = \chi_m\boldsymbol{B}/\mu_c$．

最终，联立以上振动方程和电磁力表达式，即构成了较完备地描述轴向运动导电导磁梁磁弹性耦合动力学行为的理论模型．

5　算例　平行导线磁场中轴向运动导电梁的动力稳定性

作为算例,研究磁场环境中轴向运动导电弹性梁的动力稳定性问题,并设空间磁场是由在 Oxz 面内与轴向运动梁平行放置的单根载流导线感生. 依据电磁场理论,通电导线在弹性梁所在位置产生磁场的磁感应强度为

$$B_{0y} = \frac{\mu_0 I}{2\pi(d - w)}, \tag{26}$$

上式中, μ_0 为真空磁导率, I 为载流导线的电流值, d 为梁与导线间的距离, w 为梁的横向振动位移.

这样,应用式(18)并代入方程(15c)中,考虑小变形情况并略去感应电场强度的影响,可得到 Euler-Bernoulli 型轴向运动导电梁的磁弹性横向自由振动方程:

$$\rho A\frac{\partial^2 w}{\partial t^2} + 2\rho A V_0 \frac{\partial^2 w}{\partial x \partial t} + \rho A V_0^2 \frac{\partial^2 w}{\partial x^2} - N_0 \frac{\partial^2 w}{\partial x^2} + EI_y \frac{\partial^4 w}{\partial x^4} = - \sigma A \frac{\partial w}{\partial t}\left[\frac{\mu_0 I}{2\pi(d - w)}\right]^2. \tag{27}$$

下面针对两端铰支边界约束弹性梁的动力稳定性问题进行分析. 设振动系统(27)满足边界条件的位移解为如下模态展开形式:

$$w = q(t)\sin(\pi x/l). \tag{28}$$

将式(28)代入振动方程(27)中,同时将方程(27)的等号右端项进行 Taylor 级数展开,然后采用 Galerkin(伽辽金)积分法,可推得如下非保守系统自由振动微分方程:

$$\ddot{q}(t) + g_1\dot{q}(t)\left[1 + g_2 q(t) + g_3 q^2(t)\right] + kq(t) = 0, \tag{29}$$

式中

$$g_1 = \frac{\sigma\mu_0^2 I^2}{4\rho\pi^2 d^2},\ g_2 = \frac{16}{3\pi d},\ g_3 = \frac{9}{4d^2},$$

$$k = \frac{1}{\rho A}\left[-\rho A V_0^2\left(\frac{\pi}{l}\right)^2 + N_0\left(\frac{\pi}{l}\right)^2 + EI_y\left(\frac{\pi}{l}\right)^4\right].$$

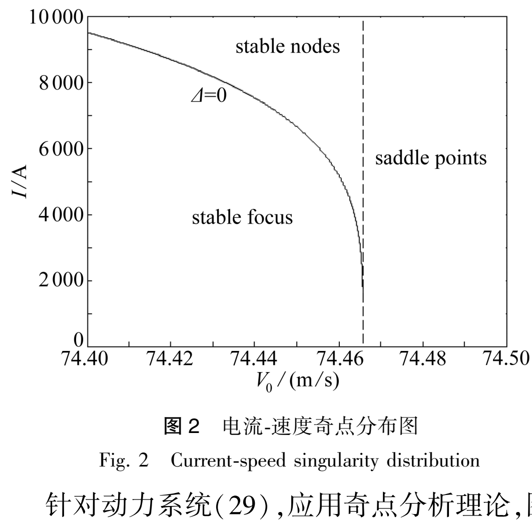

图 2　电流-速度奇点分布图

Fig. 2　Current-speed singularity distribution

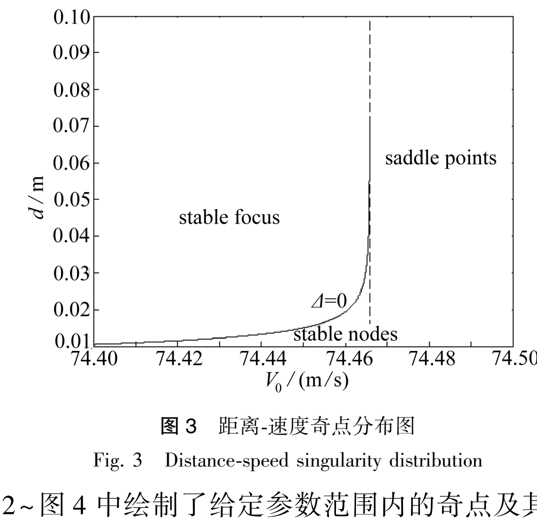

图 3　距离-速度奇点分布图

Fig. 3　Distance-speed singularity distribution

针对动力系统(29),应用奇点分析理论,图 2~图 4 中绘制了给定参数范围内的奇点及其稳定性区域划分图. 其中,边域判别式 $\Delta = (g_1)^2 - 4k$,选取的主要参数:密度 $\rho = 2\ 670\ \mathrm{kg/m^3}$,弹性模量 $E = 71$ GPa,电导率 $\sigma = 3.63 \times 10^7\ (\Omega\cdot\mathrm{m})^{-1}$,杆长 $l = 1$ m,轴向力 $N_0 = 500$ N. 由图

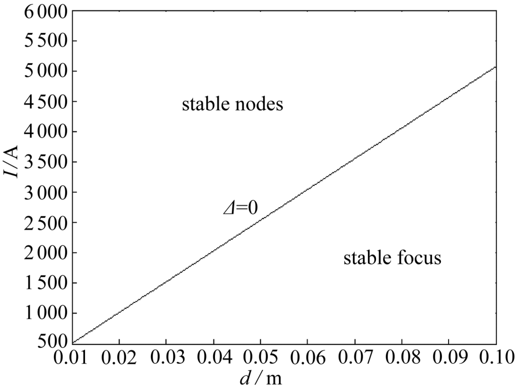

图 4 电流-距离奇点分布图

Fig. 4 Current-distance singularity distribution

2 可知,当通电导线与轴向运动梁间的距离为定值时(取 d = 0.05 m),随着轴向速度的增大,稳定焦点与稳定结点间分界线对应的电流值呈逐渐减小趋势,并且当速度达到一定值后奇点均转变为不稳定的鞍点;同时表明,奇点的分布对速度的依赖非常敏感.图 3 则表明,当通电导线的电流为定值时(取 I = 2 000 A),随着轴向运动速度的增大,稳定焦点与稳定结点间分界线对应的距离值呈逐渐增大趋势,并且当速度达到一定值后奇点也均转变为不稳定的鞍点.由图 4 可知,当给定轴向速度时(取 V_0 = 74.465 5 m/s),奇点被一直线划分为稳定结点和稳定焦点两部分,且分界线上所对应的电流值 I 与距离 d 间呈正比增大关系.

6 结 论

考虑力、运动、电磁等场量效应间的相互耦合作用,基于 Timoshenko 梁理论、电磁场理论并应用 Hamilton 变分原理,建立了磁场中轴向运动导电导磁梁的磁弹性耦合振动基本方程,得到了载流导电弹性梁和铁磁弹性梁所受电磁力的表述形式.结果表明,所建理论模型呈现多场耦合下的复杂动力学系统,轴向运动速度和电流强度对系统的奇点及其动力稳定性有显著影响.所得结果可为轴向运动结构磁弹性问题的进一步研究提供理论参考.

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Magneto-Elastic Vibration Equations for Axially Moving Conductive and Magnetic Beams

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Abstract: The magneto-elastic coupled vibration theoretical model for axially moving conductive and magnetic beams in magnetic field environment was studied. Based on the Timoshenko beam theory and with the geometric nonlinearity considered, the expressions for the deformation potential energy, kinetic energy, electromagnetic force and the virtual work of mechanical force of the elastic beam in axial motion and lateral bidirectional vibration were gained. Then the Hamilton variational principle was applied to get the nonlinear magneto-elastic coupled vibration equations for the axially moving Timoshenko beam in a magnetic field, and get those for the simplified Euler-Bernoulli beam. Based on the electromagnetic theory and the constitutive relation of the corresponding electromagnetism, the expressions for the electromagnetic force of the current-conducting elastic beam, and for the magnet force and magnet force couple of the magneto-elastic beam based on the magnetic dipole-current loop model, were derived. Through the numerical example, the singularity distribution and stability of the conductive and elastic beam in axial movement were analyzed.

Key words: conductive and magnetic beam; magneto-elastic; vibration; axially moving; Hamilton principle

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