

# 磁振子压电能量采集器的多尺度分析\*

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**摘要:** 根据磁振子压电能量采集器实验系统的数学模型,基于系统静平衡位形,引入坐标变换,建立相对位移的标准控制方程.利用 Taylor 级数展开法处理磁力非线性项,运用多尺度法近似解析分析,通过消除长期项获得可解性条件,并由此推导出稳态响应时的幅频关系.四阶 Runge-Kutta 方法用于数值计算受迫振动时间历程,数值算例给出了系统前两阶主共振下的稳态幅频响应关系及其失稳区域.结果表明,多尺度方法所得到的一致有效解具有较高精度,可以为优化设计磁振子压电能量采集器提供理论依据.

**关键词:** 受迫振动; 非线性; 多尺度方法; 能量采集

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## 引 言

能量采集器可以将环境中的废弃能量转换成电能,近年来迅速成为学者们的研究热点.振动能量采集器因其具有绿色、环保、可持续使用的特点,正受到越来越广泛地重视<sup>[1-4]</sup>.

Williams 等<sup>[5]</sup>借助于单自由度质量弹簧系统,最早提出了将振动能转化为电能的机制.研究表明在外界激励频率与能量采集器的固有频率相接近,系统阻尼较小的情况下,能量转化效率更高.但由于单自由度模型具有固有频率单一、仅在外激励频率和系统固有频率接近时有效工作、响应频带窄的局限性,国内外诸多学者提议使用具有多个模态的能量采集器,以拓宽频带宽度.Kim 等<sup>[6]</sup>建立了两自由度压电能量采集器模型,实验发现对比单自由度模型,两自由度能量采集模型具有两个不同的共振频率,拓宽了能量采集频带宽度.Shahruz<sup>[7]</sup>设计了一组悬臂梁组成的带通滤波装置,通过改变梁的长度和质量块质量,有效拓宽了共振频率范围并对其进行了最优化设计.Ferrari 等<sup>[8]</sup>则在 3 个尺寸材质完全相同的悬臂梁的自由端分别固定不同的质量块,不同程度地改变悬臂梁的共振频率,使能量采集器在 3 个不同的共振频率点获得稳定,从而有效拓宽工作频带.Yang 等<sup>[9]</sup>建立了双梁压电耦合振动能量采集器模型,研究发现,

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选择适当参数可以拓宽能量采集器采集带宽,但会降低输出功率大小。

除此之外,研究人员发现非线性的引入可以有效地改进能量采集办法。Daqaq 等<sup>[10]</sup>综述了振动能量采集概念和主要问题,对比线性能量采集,指出非线性能量采集具有更宽的稳态频率带宽,能够提高环境振动中能量采集效率。Cottone 等<sup>[11]</sup>研究了单稳态压电能量采集系统在随机激励下的响应,指出非线性可以提高能量采集器的效率。Wu 等<sup>[12]</sup>利用贴有质量块的主梁(外梁)和副梁(内梁)建立了两自由度的多频响应压电能量采集器,通过实验对比了单自由度模型和单梁的两自由度模型,发现该结构可以有效拓宽共振频带。随后 Wu 等<sup>[13]</sup>又通过内梁和基座中间安装磁铁引入磁相互作用,建立两自由度的非线性多频响应压电能量采集器,数值和实验研究发现选取合适的结构参数和磁铁间距可以将系统两共振峰调至相近,从而有效地拓宽能量采集工作频带。对比线性模型发现,在有电容的情况下,两自由度非线性压电能量采集工作频带更宽、输出功率更高。

Tang 等<sup>[14]</sup>通过引入磁振子模型,设计了两自由度的非线性振动能量采集器,数值计算显示多频响应非线性能量采集工作频带更宽,输出功率也大幅提高,改进效果显著,并得到实验验证。但至今未见有关理论分析的报道。Daqaq 等<sup>[15]</sup>利用多尺度法研究了参激振动下软弹簧特性的压电悬臂梁式能量采集器,得到了系统电压和功率的表达式,讨论了机电耦合和负载电阻对输出电压和功率的影响,并通过实验验证该结构可以拓宽共振频带。本文基于磁振子压电能量采集器实验模型,通过 Taylor 级数展开法处理磁力非线性项,利用多尺度法近似解析分析,拟研究前两阶主共振条件下的非线性受迫振动,得到稳态响应时输出电压和功率的幅频响应曲线;并辅以数值计算时程,定量验证理论结果。

## 1 控制方程

考虑磁振子压电能量采集器实验模型,如图 1 所示。悬臂梁自由端固定磁铁块,在其正上方引入由弹簧悬挂的另一磁铁块,并将压电片安装在悬臂梁根部,外接电容和电阻构成电流回路。通过非线性磁相互作用,以实现改进能量采集效果,力学模型如图 2 所示。

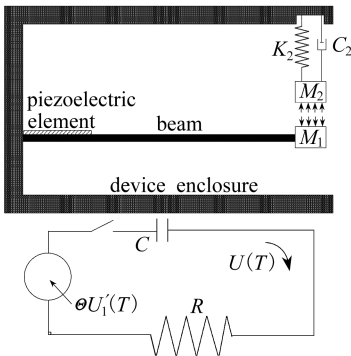


图 1 磁振子非线性压电能量采集器

Fig. 1 A nonlinear PEH with a magnetic oscillator

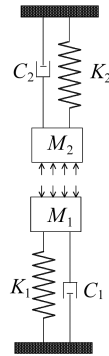


图 2 磁振子非线性压电能量采集器力学模型

Fig. 2 The mechanical model for the nonlinear PEH with a magnetic oscillator

假设两磁铁初始间距为  $D_0$ 。如果用  $U_1(T)$  和  $U_2(T)$  分别表示不同时刻悬臂梁尖端磁铁和磁振子的位移,  $U(T)$  表示输出电压,并以竖直向下为正方向,建立数学模型:

$$\begin{cases} M_1 U_1'' + C_1 U_1' + K_1 U_1 - \frac{\zeta}{(D_0 + U_1 - U_2)^4} - \Theta U = F_1 \sin(\Omega T), \\ M_2 U_2'' + C_2 U_2' + K_2 U_2 + \frac{\zeta}{(D_0 + U_1 - U_2)^4} = F_2 \sin(\Omega T), \\ CU' + \frac{1}{R} U + \Theta U' = 0, \end{cases} \quad (1)$$

式中, 右上标“'”代表对时间  $T$  求导。悬臂梁长为  $L_b$ ,  $M_1$  为悬臂梁及其尖端磁铁的等效质量,  $K_1$  为其等效刚度,  $C_1$  为机械阻尼系数,  $M_2$  为磁振子质量,  $K_2$  为弹簧刚度,  $C_2$  为磁振子阻尼系数,  $\zeta$  表示磁力系数,  $D_0$  表示磁铁初始间距,  $\Theta$  表示机电耦合系数,  $C$  表示外接电容,  $R$  表示外接电阻。

考虑单位电压  $e$  (1 V), 引入无量纲参数:

$$\begin{cases} d_0 = \frac{D_0}{L_b}, u_1 = \frac{U_1}{L_b}, u_2 = \frac{U_2}{L_b}, f_1 = \frac{F_1}{K_1 L_b}, f_2 = \frac{F_2}{K_1 L_b}, \omega_0^2 = \frac{K_2 M_1}{K_1 M_2}, \\ \eta = \frac{M_2}{M_1}, \mu_1 = \frac{C_1}{\sqrt{M_1 K_1}}, \mu_2 = \frac{C_2}{\sqrt{M_1 K_1}}, \omega = \Omega \sqrt{\frac{M_1}{K_1}}, t = T \sqrt{\frac{K_1}{M_1}}, \\ c_0 = \frac{\zeta}{K_1 L_b^5}, v = \frac{U}{e}, c = \frac{e^2 C}{K_1 L_b^2}, \kappa = \frac{e^2}{R L_b^2 \sqrt{K_1^3}}, \theta = \frac{e \Theta}{K_1 L_b}, \end{cases} \quad (2)$$

可得标准控制方程:

$$\begin{cases} \ddot{u}_1 + \mu_1 \dot{u}_1 + u_1 - \frac{c_0}{(d_0 + u_1 - u_2)^4} - \theta v = f_1 \sin(\omega t), \\ \eta \ddot{u}_2 + \mu_2 \dot{u}_2 + \eta \omega_0^2 u_2 + \frac{c_0}{(d_0 + u_1 - u_2)^4} = f_2 \sin(\omega t), \\ c \dot{v} + \kappa v + \theta \dot{u}_1 = 0, \end{cases} \quad (3)$$

这里上标“ $\cdot$ ”代表对时间  $t$  求导。 $c_0$  代表无量纲后的磁力系数, 当  $c_0 > 0$  时表示悬臂梁右端受排斥力作用; 如果添加一个负号使得  $c_0 < 0$ , 则表示悬臂梁右端受吸引力作用; 当  $c_0 = 0$  时, 悬臂梁右端将不受磁力作用。

在控制方程中如果不考虑随时间变化的所有项, 则可得到关于静平衡位形满足的代数方程组。此时不会产生电压,  $v = 0$ 。如果以  $d_1, d_2$  分别代表上下两磁铁的静平衡位移, 可知它们满足如下方程组:

$$d_1 - \frac{c_0}{(d_0 + d_1 - d_2)^4} = 0, \quad \eta \omega_0^2 d_2 + \frac{c_0}{(d_0 + d_1 - d_2)^4} = 0. \quad (4)$$

引入坐标变换:  $z_1 = u_1 - d_1; z_2 = u_2 - d_2$ , 得到相对位移的标准控制方程:

$$\begin{cases} \ddot{z}_1 + \mu_1 \dot{z}_1 + z_1 + d_1 - \frac{c_0}{(d + z_1 - z_2)^4} - \theta v = f_1 \sin(\omega t), \\ \eta \ddot{z}_2 + \mu_2 \dot{z}_2 + \eta \omega_0^2 (z_2 + d_2) + \frac{c_0}{(d + z_1 - z_2)^4} = f_2 \sin(\omega t), \\ c \dot{v} + \kappa v + \theta \dot{z}_1 = 0, \end{cases} \quad (5)$$

式中  $d = d_0 + d_1 - d_2$  表示平衡时磁铁间距。对于幅值较小的振动, 相对位移均为小量, 应用 Taylor 级数展开法处理非线性项可得

$$\frac{c_0}{(d+z_1-z_2)^4} - \frac{c_0}{d^4} = \sum_{j=1}^{\infty} c_j (z_1 - z_2)^j, \quad c_j = \frac{(j+3)!}{j!} \cdot \frac{(-1)^j c_0}{6d^{j+4}},$$

$$j = 1, 2, 3, \dots \quad (6)$$

## 2 多尺度分析

引入多个时间尺度:  $T_j = \varepsilon^j t$ ,  $j = 0, 1, 2, \dots$ , 由于系统存在平方非线性项, 为了研究第  $n$  阶主谐波共振, 引入解谐参数  $\sigma_n$ , 以标示激励频率与固有频率之间的偏离程度:  $\omega = \omega_n + \varepsilon^2 \sigma_n$ , 从而可知, 系统在微幅受迫振动作用下的稳态响应将与时间尺度  $T_1$  无关, 于是可以假定控制方程(5)的形式解为

$$v(t) = \varepsilon V_{01}(T_0, T_2) + \varepsilon^2 V_{02}(T_0, T_2) + \varepsilon^3 V_{03}(T_0, T_2) + O(\varepsilon^4), \quad (7)$$

$$z_m(t) = \varepsilon Z_{m1}(T_0, T_2) + \varepsilon^2 Z_{m2}(T_0, T_2) + \varepsilon^3 Z_{m3}(T_0, T_2) + O(\varepsilon^4), \quad m = 1, 2. \quad (8)$$

重新标度压电耦合系数、阻尼和外激励振幅

$$\mu_1 \leftrightarrow \varepsilon^2 \mu_1, \quad \mu_2 \leftrightarrow \varepsilon^3 \mu_2, \quad f_1 \leftrightarrow \varepsilon^3 f_1, \quad f_2 \leftrightarrow \varepsilon^3 f_2, \quad \theta \leftrightarrow \varepsilon \theta.$$

代入控制方程(5)分离变量可得  $\varepsilon^1$  阶方程为

$$\begin{cases} Z_{11, T_0 T_0} + Z_{11} - c_1(Z_{11} - Z_{21}) = 0, \\ \eta Z_{21, T_0 T_0} + \eta \omega_0^2 Z_{21} + c_1(Z_{11} - Z_{21}) = 0, \quad cV_{01, T_0} + \kappa V_{01} = 0. \end{cases} \quad (9)$$

$\varepsilon^2$  阶方程为

$$\begin{cases} Z_{12, T_0 T_0} + Z_{12} - c_1(Z_{12} - Z_{22}) = c_2(Z_{11} - Z_{21})^2 + \theta V_{01}, \\ \eta Z_{22, T_0 T_0} + \eta \omega_0^2 Z_{22} + c_1(Z_{12} - Z_{22}) = -c_2(Z_{11} - Z_{21})^2, \\ cV_{02, T_0} + \kappa V_{02} = -\theta Z_{11, T_0}. \end{cases} \quad (10)$$

$\varepsilon^3$  阶方程为

$$\begin{cases} Z_{13, T_0 T_0} + Z_{13} - c_1(Z_{13} - Z_{23}) = f_1 \sin(\omega t) - \mu_1 Z_{11, T_0} - 2Z_{11, T_0 T_2} + \\ \quad [2c_2(Z_{11} - Z_{21})(Z_{12} - Z_{22}) + c_3(Z_{11} - Z_{21})^3] + \theta V_{02}, \\ \eta Z_{23, T_0 T_0} + \eta \omega_0^2 Z_{23} + c_1(Z_{13} - Z_{23}) = f_2 \sin(\omega t) - \mu_2 Z_{21, T_0} - 2\eta Z_{21, T_0 T_2} - \\ \quad [2c_2(Z_{11} - Z_{21})(Z_{12} - Z_{22}) + c_3(Z_{11} - Z_{21})^3], \\ cV_{03, T_0} + \kappa V_{03} = -cV_{01, T_2} - \theta Z_{12, T_0}. \end{cases} \quad (11)$$

根据自由振动及频率分析可知, 对于  $\varepsilon^1$  阶方程(9)的稳态解可写为

$$Z_{11} = Y_{0n}(T_2) e^{i\omega_n T_0} + cc, \quad Z_{21} = q_n Y_{0n}(T_2) e^{i\omega_n T_0} + cc, \quad V_{01} = 0, \quad (12)$$

式中,  $cc$  代表等式右边位于它前面这一项的复数共轭,

$$\omega_{1,2} = \sqrt{\frac{1 + \omega_0^2 - c_1}{2} - \frac{c_1}{2\eta}} \pm \sqrt{\frac{c_1^2(1 + \eta)^2}{4\eta^2} + \frac{(1 - \omega_0^2)^2}{4} + \frac{c_1(1 - \eta)(1 - \omega_0^2)}{2\eta}}, \quad (13)$$

$$q_n = 1 + \frac{\omega_n^2 - 1}{c_1}, \quad Y_{0n}(T_2) = \frac{1}{2} a_n(T_2) e^{i[\sigma_n T_2 - \varphi_n(T_2)]}. \quad (14)$$

将式(12)代入  $\varepsilon^2$  阶方程, 可知其形式解为

$$\begin{cases} Z_{m2} = 2c_2(1 - q_n)^2 (\alpha_{mn} Y_{0n} \bar{Y}_{0n} + \beta_{mn} Y_{0n}^2 e^{2i\omega_n T_0}) + cc, \\ V_{02} = -\frac{i\theta\omega_n}{\kappa + ic\omega_n} Y_{0n} e^{i\omega_n T_0} + cc, \end{cases} \quad m = 1, 2, \quad (15)$$

式中

$$\begin{cases} \alpha_{1n} = \frac{\eta\omega_0^2}{2\eta\omega_0^2(1-c_1) - 2c_1}, \beta_{1n} = \frac{\eta(\omega_0^2 - 4\omega_n^2)}{2(1-4\omega_n^2 - c_1)(\eta\omega_0^2 - 4\eta\omega_n^2 - c_1) - 2c_1^2}, \\ \alpha_{2n} = \frac{-1}{2\eta\omega_0^2(1-c_1) - 2c_1}, \beta_{2n} = \frac{4\omega_n^2 - 1}{2(1-4\omega_n^2 - c_1)(\eta\omega_0^2 - 4\eta\omega_n^2 - c_1) - 2c_1^2}. \end{cases} \quad (16)$$

将  $\varepsilon^1$  阶方程解(12) 和  $\varepsilon^2$  阶方程解(15) 代入  $\varepsilon^3$  阶方程(11), 一致有效解可写作如下形式:

$$Z_{m3}(T_0, T_2) = Y_{mn}(T_2) e^{i\omega_n T_0} + cc + NST, \quad m = 1, 2, \quad (17)$$

式中, NST 代表不会产生发散解的其余各项. 在形式解中, 与慢时间尺度  $T_2$  有关的两个新函数满足如下方程组:

$$\begin{cases} (1 - \omega_n^2 - c_1)Y_{1n} + c_1Y_{2n} = \frac{f_1}{2i} e^{i\sigma_n T_2} - i\mu_1\omega_n Y_{0n} - 2i\omega_n Y'_{0n} - \frac{i\theta^2\omega_n}{\kappa + ic\omega_n} Y_{0n} + \\ (1 - q_n)^3 [8c_2(\alpha_{1n} - \alpha_{2n}) + 4c_2^2(\beta_{1n} - \beta_{2n}) + 3c_3] Y_{0n}^2 \bar{Y}_{0n}, \\ c_1Y_{1n} + (\eta\omega_0^2 - \eta\omega_n^2 - c_1)Y_{2n} = \frac{f_2}{2i} e^{i\sigma_n T_2} - i\mu_2\omega_n q_n Y_{0n} - 2i\eta\omega_n q_n Y'_{0n} - \\ (1 - q_n)^3 [8c_2(\alpha_{1n} - \alpha_{2n}) + 4c_2^2(\beta_{1n} - \beta_{2n}) + 3c_3] Y_{0n}^2 \bar{Y}_{0n}. \end{cases} \quad (18)$$

由于该方程组系数矩阵行列式为 0, 必须满足相容性条件, 化简可得可解性条件:

$$\Lambda_{1n} Y'_{0n} + \Lambda_{2n} Y_{0n} - i\Lambda_{3n} Y_{0n} - 4i\Lambda_{4n} Y_{0n}^2 \bar{Y}_{0n} + 0.5\Lambda_{5n} e^{i\sigma_n T_2} = 0, \quad (19)$$

式中,  $\Lambda_{1n}$  表示矩阵  $\Lambda$  中第 1 行第  $n$  列元素

$$\begin{cases} \Lambda_{1n} = 2\omega_n(1 + \eta q_n^2), \Lambda_{2n} = \omega_n(\mu_1 + q_n^2 \mu_2) + \frac{\kappa\omega_n\theta^2}{\kappa^2 + c^2\omega_n^2}, \Lambda_{3n} = \frac{c\theta^2\omega_n^2}{\kappa^2 + c^2\omega_n^2}, \\ \Lambda_{4n} = -(1 - q_n)^4 [0.75c_3 + (2\alpha_{1n} - 2\alpha_{2n} + \beta_{1n} - \beta_{2n})c_2^2], \Lambda_{5n} = f_1 + q_n f_2. \end{cases} \quad (20)$$

将极坐标形式解代入后, 分离实部和虚部, 消去相角可得稳态响应时的频响方程:

$$\sigma_n = \frac{\Lambda_{3n} + \Lambda_{4n} a_n^2}{\Lambda_{1n}} \pm \frac{1}{a_n \Lambda_{1n}} \sqrt{\Lambda_{5n}^2 - \Lambda_{2n}^2 a_n^2}. \quad (21)$$

根据 Routh-Hurwitz 判据可知其失稳边界为

$$\tilde{\sigma}_n = \frac{2\Lambda_{4n} \tilde{a}_n^2 + \Lambda_{3n} \pm \sqrt{\Lambda_{4n}^2 \tilde{a}_n^4 - \Lambda_{2n}^2}}{\Lambda_{1n}}. \quad (22)$$

将极坐标形式解代入稳态响应解可知

$$z_1(t) = \varepsilon a_n \cos(\omega t - \varphi_n) + c_2 \varepsilon^2 a_n^2 \alpha_{1n} (1 - q_n)^2 + c_2 \varepsilon^2 a_n^2 \beta_{1n} (1 - q_n)^2 \cos(2\omega t - 2\varphi_n) + O(\varepsilon^3), \quad (23)$$

$$z_2(t) = \varepsilon a_n q_n \cos(\omega t - \varphi_n) + c_2 \varepsilon^2 a_n^2 \alpha_{2n} (1 - p_n)^2 + c_2 \varepsilon^2 a_n^2 \beta_{2n} (1 - q_n)^2 \cos(2\omega t - 2\varphi_n) + O(\varepsilon^3). \quad (24)$$

电压近似解为

$$v(t) = \varepsilon^2 b_n \sin\left(\omega t - \varphi_n - \arccos \frac{\kappa}{\sqrt{\kappa^2 + c^2\omega_n^2}}\right) + O(\varepsilon^3), \quad b_n = \frac{\theta\omega_n a_n}{\sqrt{\kappa^2 + c^2\omega_n^2}}. \quad (25)$$

得到第  $n$  阶主共振下悬臂梁振幅  $A_n(z_1) = \varepsilon a_n$ , 磁振子振幅  $A_n(z_2) = \varepsilon a_n q_n$ , 能量采集器输出电压幅值  $A_n(v) = \varepsilon^2 b_n$ , 其中  $n = 1, 2$ . 从而可以根据物理参数近似求得输出的平均功率:

$$P_n = \frac{1}{T} \int_0^T \frac{U^2}{R} dt \approx \frac{\varepsilon^4 b_n^2}{2R}. \quad (26)$$

### 3 数值算例

以排斥力相互作用为例,选取如下物理参数<sup>[14]</sup>:

$$\begin{aligned} L_b &= 0.1 \text{ m}, K_1 = 100 \text{ N}\cdot\text{m}^{-1}, M_1 = 0.01 \text{ kg}, C_1 = 0.01 \text{ N}\cdot\text{s}\cdot\text{m}^{-1}, M_2 = 0.04 \text{ kg}, \\ K_2 &= 400 \text{ N}\cdot\text{m}^{-1}, C_2 = 0.01 \text{ N}\cdot\text{s}\cdot\text{m}^{-1}, \zeta = 8.0 \times 10^{-9} \text{ N}\cdot\text{m}^4, D_0 = 0.014 \text{ m}, \\ \Theta &= 1.0 \times 10^{-4} \text{ N}\cdot\text{V}^{-1}, C = 1.0 \times 10^{-8} \text{ F}, R = 1\ 000 \text{ k}\Omega. \end{aligned}$$

无量纲化为

$$d_0 = 0.14, c_0 = 8.0 \times 10^{-6}, \theta = 1.0 \times 10^{-5}, c = \kappa = 1.0 \times 10^{-8}, \mu_1 = \mu_2 = 0.01.$$

假定无量纲激励振幅分别为  $f_1 = 0.000\ 5$ ,  $f_2 = 0$ , 此时系统固有频率  $\omega_1 = 1$ ,  $\omega_2 = 1.320\ 5$ ;  $q_1 = 1$ ,  $q_2 = -0.25$ , 选取零初始条件, 以 0.001 作为时间步长, 利用四阶 Runge-Kutta 方法可计算出关于相对位移的控制方程(5), 从而可得到位移和瞬时电压  $v(t)$  的时间历程, 以时间历程中位于(1 900, 2 000) s 之间的位移作为稳态响应, 并记录该时段位移。

图 3 和图 4 给出了悬臂梁和磁振子前两阶主共振幅频特性曲线及失稳边界, 并与数值结果作对比。外激励频率  $\omega$  作为控制参数, 实线代表稳态幅频响应, 虚线代表失稳边界, 圆点表示数值解, 可以看出在排斥力作用下固有频率  $\omega_1 = 1$  的一阶主共振附近非线性消失, 振幅随着幅频响应曲线缓慢增加并在共振点处达到峰值然后逐渐减小。二阶主共振附近幅频响应曲线体现硬特性, 出现共振点附近的多解和跳跃现象等非线性特性。对比发现, 悬臂梁在二阶主共振附近的受迫振动振幅明显增大。

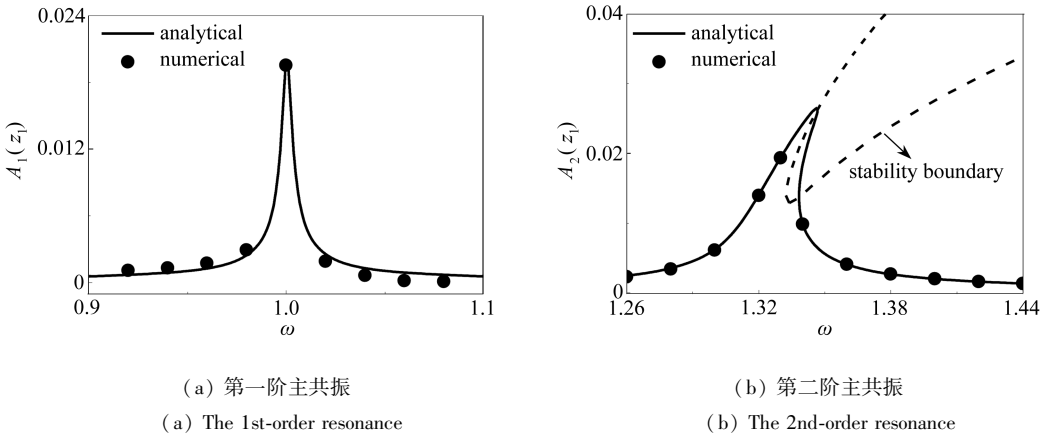
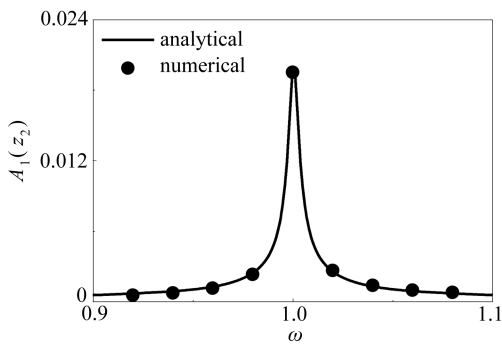


图 3 悬臂梁幅频特性曲线

Fig. 3 Amplitude-frequency curves of the cantilever beam

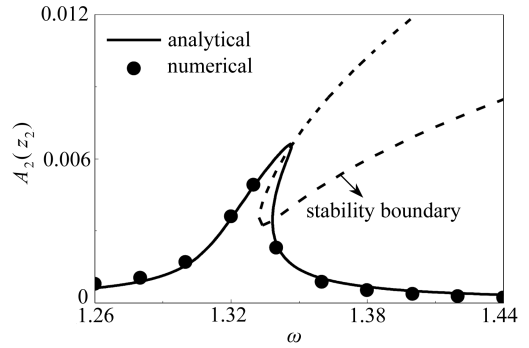
结合实际物理参数,图 5 和图 6 给出了能量采集器输出电压和功率在前两阶主共振附近的幅频响应。可以看出:一阶主共振附近能量采集器输出电压及功率随着幅频响应曲线缓慢增加并在共振点处达到峰值然后逐渐减小;二阶主共振附近能量采集器输出电压及功率出现跳跃现象,随着外激励频率逐渐增加沿幅频响应曲线缓慢增加至失稳边界,后随着外激励频率继续增加输出电压和功率幅值出现突变,再缓慢减小。

由图 3 至图 6 可知,数值方法所得到的结果与近似解析结果吻合得较好,定量和定性地证实了跳跃现象,并验证了多尺度方法的有效性。



(a) 第一阶主共振

(a) The 1st-order resonance

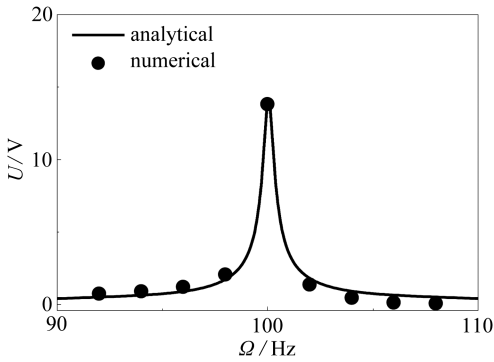


(b) 第二阶主共振

(b) The 2nd-order resonance

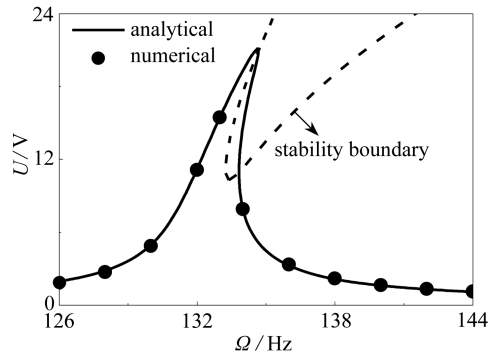
图4 磁振子幅频特性曲线

Fig. 4 Amplitude-frequency curves of the magnetic oscillator



(a) 第一阶主共振

(a) The 1st-order resonance

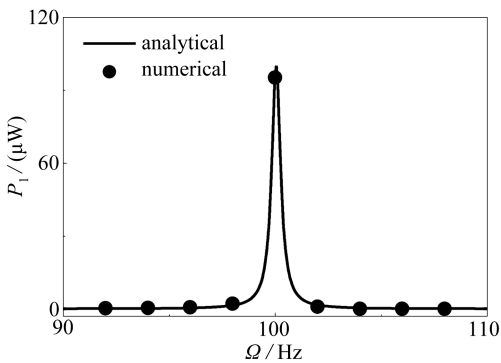


(b) 第二阶主共振

(b) The 2nd-order resonance

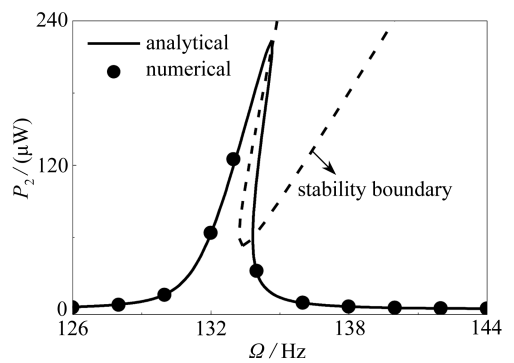
图5 输出电压幅频特性曲线

Fig. 5 Amplitude-frequency curves of the output voltage



(a) 第一阶主共振

(a) The 1st-order resonance



(b) 第二阶主共振

(b) The 2nd-order resonance

图6 输出功率幅频特性曲线

Fig. 6 Amplitude-frequency curves of the output power

## 4 结 论

本文根据磁振子能量采集器实验模型,基于静平衡位形,引入坐标变换,建立了相对位移的标准控制方程.利用 Taylor 级数展开法处理了磁力非线性项,运用多尺度方法进行了近似解析分析,研究了能量采集器微幅受迫振动的稳态响应.并利用四阶 Runge-Kutta 方法数值求解控制方程,得到以下结论:

- 1) 系统在二阶主共振附近幅频响应曲线共振峰右移,出现跳跃现象;
- 2) 在排斥力作用下当系统固有频率为 1 时,一阶主共振附近幅频响应曲线非线性特性消失,退化为线性特征;
- 3) 定量验证了近似解析结果,证实了多尺度法的有效性.

### 参考文献(References):

- [1] 杨志安, 贾尚帅. RLC 串联电路与微梁耦合系统 1:2 内共振分析[J]. 应用力学学报, 2010, **27**(1): 80-85.(YANG Zhi-an, JIA Shang-shuai. Analysis of the 1:2 internal resonance of coupled RLC circuit and microbeam system[J]. *Chinese Journal of Applied Mechanics*, 2010, **27**(1): 80-85.(in Chinese))
- [2] 李海涛, 秦卫阳. 双稳态压电能量获取系统的分岔混沌阈值[J]. 应用数学和力学, 2014, **35**(6): 652-662.(LI Hai-cao, QIN Wei-yang. Bifurcation and chaos thresholds of bistable piezoelectric vibration energy harvesting systems[J]. *Applied Mathematics and Mechanics*, 2014, **35**(6): 652-662.(in Chinese))
- [3] 郭抗抗, 曹树谦. 压电发电悬臂梁的非线性动力学建模及响应分析[J]. 动力学与控制学报, 2014, **12**(1): 18-23.(GUO Kang-kang, CAO Shu-qian. Nonlinear modeling and analysis of piezoelectric cantilever energy harvester[J]. *Journal of Dynamics and Control*, 2014, **12**(1): 18-23.(in Chinese))
- [4] 杨德庆, 刘正兴. 自由端受集中力作用下压电悬臂梁弯曲问题解析解[J]. 力学季刊, 2003, **24**(3): 327-333.(YANG De-qing, LIU Zheng-xing. Analytical solution for bending of a piezoelectric cantilever beam under an end load[J]. *Chinese Quarterly of Mechanics*, 2003, **24**(3): 327-333.(in Chinese))
- [5] Williams C B, Yates R B. Analysis of a micro-electric generator for microsystems[J]. *Sensors and Actuators A: Physical*, 1996, **52**(1/3): 8-11.
- [6] Kim I H, Jung H J, Lee B M, Jang S J. Broadband energy-harvesting using a two degree-of-freedom vibrating body[J]. *Applied Physics Letters*, 2011, **98**(21): 214102.
- [7] Shahruz S M. Design of mechanical band-pass filters with large frequency bands for energy scavenging[J]. *Mechatronics*, 2006, **16**(9): 523-531.
- [8] Ferrari M, Ferrari V, Guizzetti M, Marioli D, Taroni A. Piezoelectric multifrequency energy converter for power harvesting in autonomous microsystems[J]. *Sensors and Actuators A: Physical*, 2008, **142**(1): 329-335.
- [9] YANG Zeng-tao, YANG Jia-shi. Connected vibrating piezoelectric bimorph beams as a wide-band piezoelectric power harvester[J]. *Journal of Intelligent Material Systems and Structures*, 2009, **20**(5): 569-574.
- [10] Daqaq M F, Masana R, Erturk A, Quinn D D. On the role of nonlinearities in vibratory energy harvesting: a critical review and discussion[J]. *Applied Mechanics Reviews*, 2014, **66**(4): 040801.



- [11] Cottone F, Vocca H, Gammaitoni L. Nonlinear energy harvesting[J]. *Physical Review Letters*, 2009, **102**(8): 080601.
- [12] WU Hao, TANG Li-hua, YANG Yao-wen, SOH Chee-kiong. A novel two-degrees-of-freedom piezoelectric energy harvester[J]. *Journal of Intelligent Material Systems and Structures*, 2013, **24**(3): 357-368. doi: 10.1177/1045389X12457254.
- [13] WU Hao, TANG Li-hua, YANG Yao-wen, SOH Chee-kiong. Development of a broadband nonlinear two-degree-of-freedom piezoelectric energy harvester[J]. *Journal of Intelligent Material Systems and Structures*, 2014, **25**(14): 1875-1889. doi: 10.1177/1045389X14541494.
- [14] TANG Li-hua, YANG Yao-wen. A nonlinear piezoelectric energy harvester with magnetic oscillator[J]. *Applied Physics Letters*, 2012, **101**(9): 094102.
- [15] Daqaq M F, Stabler C, Qaroush Y, Seuaciuc-Osório T. Investigation of power harvesting via parametric excitations[J]. *Journal of Intelligent Material Systems and Structures*, 2009, **20**(5): 545-557.

## Multi-Scale Analysis of Piezoelectric Energy Harvesters With Magnetic Oscillators

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**Abstract:** A piezoelectric energy harvester with a magnetic oscillator was studied. The dynamic equation was derived via introduction of coordinate transform based on the equilibrium configuration. The Taylor series expansion method was employed to deal with the nonlinear function of the magnetic force. The multi-scale method was applied to obtain the steady-state periodic solutions of the system. The solvability condition and the amplitude-frequency relationship were derived through elimination of the secular terms. Then the Runge-Kutta method was used to numerically calculate the system's forced vibration time history and give the amplitude-frequency response characteristics and instability boundary of the 1st 2 primary resonance cases. The results show that the multi-scale analysis yields uniformly valid solutions of high accuracy, and provides a theoretic base for the optimal design of piezoelectric energy harvesters with magnetic oscillators.

**Key words:** forced vibration; nonlinearity; multi-scale method; energy harvesting

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