应用数学和力学, 第 29 卷 第 2 期
2008 年 2 月 15 日出版
Applied Mathematics and Mechanics
Vol. 29, No. 2, Feb. 15, 2008

文章编号: 1000-0887(2008) 02-0217-11


# 海洋波导的可穿透障碍物散射场远场分布的若干性质*

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(黄永念推荐)

摘要: 研究了海洋波导中可穿透目标时谐声波散射传播远场分布的性质, 构造了透射问题解的集合, 使得所构造解的集合在边界上的限制在某个 Hilbert 空间中是稠密的, 确定了传播远场分布的集合在某个 Hilbert 空间中是完备的. 这些性质对研究海洋波导中的逆透射问题有重要的应用.

关 键 词: 海洋波导; 可穿透目标; 传播远场分布; 完备性; 稠密性
中图分类号: O175 文献标识码: A

## 引 言

在声散射问题的研究中, 由于远场分布包含了散射物的几乎所有物理和几何信息, 如散射物的位置、形状等, 因而远场分布起着极为重要的作用. 而逆声散射原理就是提取这些远场信息反演散射体的形状. 在自由空间中, 有关远场分布的一些性质已有一些研究[1-3]. 由于这些性质, 一些成功的数值方法, 如非线性优化对偶空间方法和线性抽样方法, 被构造出来求解逆声散射问题. 具体参考文献[4-5]和其中的引用文献.

需要指出的是, 无论是优化对偶空间方法和线性抽样方法, 都需要散射场远场分布的全孔径入射和散射数据, 但是这在实际应用中受到了很大的限制. 近来, You 和 Miao 等人发展了一种指示器样本方法, 避免了这一缺陷[6-16]. 本文作者将该方法应用到海洋波导中, 表明了即使在有限孔径的数据时也能得到较好的重构.

另外, 这些方法都只考虑了自由空间. 研究表明声波在海洋中传播时, 海底和海面对声传播的多重散射的影响是不可忽视的[17-18]. 文献[19-22]研究了海洋波导中不可穿透目标的远场分布, 表明了海洋波导中由于海底和海面的相互作用, 其远场分布与自由空间是根本不相同的.

由于海洋中目标的材料性能可能不同于海水介质, 这时目标是可穿透的, 因而本文研究海洋波导中可穿透目标时谐声波散射传播远场分布的性质.

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* 收稿日期: 2007-08-14; 修订日期: 2008-01-03
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# 1 正透射问题

假设海洋波导为由各向同性且均匀的介质组成的, 海面是压力释放的, 海底为刚性的, 在柱坐标系下将它记为 $R_h^3 = \left\{(\boldsymbol{x}, z) = (x_1, x_2, z) \mid \boldsymbol{x} = (x_1, x_2) \in R^2, 0 \leqslant z \leqslant h\right\}$ (除非特别说明, 本文黑体字符均表示向量).

设 $D$ 为 $R_h^3$ 中的一个有界连通区域, $D$ 的边界 $\partial D \in C^{2,\sigma}$ ($0 < \sigma \leqslant 1$, $\sigma$ 为 Hölder 指数), $\nu$ 为 $\partial D$ 的单位外法向量, $D_e = R_h^3 \setminus D$ 为 $D$ 的外部区域. 在 $R_h^3$ 中有一时谐射声波 $u^i$

$$u^i(\boldsymbol{x}, z; \boldsymbol{d}, z_0) = \sum_{n=1}^{N(k)} \phi_n(z)\phi_n(z_0)e^{ik_n \boldsymbol{x}\cdot\boldsymbol{d}}, \qquad z_0 \in [0, h], \tag{1}$$

其中, $N(k) = \mathrm{INT}[kh/\pi + 1/2]$ 为模态总数(INT 为取整算子), $\phi_n(z) = \sqrt{2/h}\sin(\gamma_n z)$ 为垂向特征函数, $\gamma_n = (2n-1)\pi/(2h)$ 为第 $n$ 个模态的垂向波数, $k_n = \sqrt{k^2 - \gamma_n^2}$ $(k \neq \gamma_n)$ 为第 $n$ 个模态的水平波数, $k = \omega/c_e$ 为波数, $c_e$ 为海水中的声速, $\boldsymbol{d} = (\cos\theta, \sin\theta)$ 为入射波的方向, $\theta$ 为入射角, $z_0$ 为入射波所在的位置, $\hat{\boldsymbol{x}} = \boldsymbol{x}/|\boldsymbol{x}|$ 为 $\boldsymbol{x}$ 的单位矢量. 如图 1.

设 $D$ 为声学上的可穿透障碍物, 当入射声波(1)遇到障碍物 $D$ 时, 将在外部区域 $D_e$ 产生一个散射场 $u^s$, 而在 $D$ 内产生一个透射场 $u_i$, 记外部区域的总场为 $u_e = u^i + u^s$. 那么, 正透射问题就是求 $u_e \in C^2(D_e) \cap C(\overline{D_e})$ 和透射场 $u_i \in C^2(D) \cap C(\overline{D})$, 使它们满足

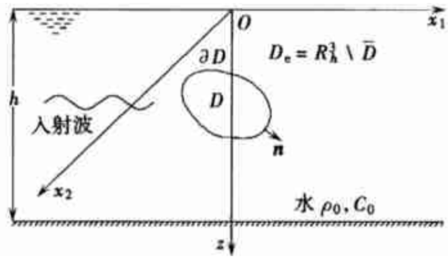


图 1 三维海洋波导

$$\Delta u_e + k^2 u_e = 0, \qquad \text{在 } D_e \text{ 中}, \tag{2}$$

$$\Delta u_i + \kappa^2 u_i = 0, \qquad \text{在 } D \text{ 中}, \tag{3}$$

$$\mu_e u_e = \mu_i u_i, \qquad \text{在 } \partial D \text{ 上}, \tag{4}$$

$$\frac{\partial u_e}{\partial \nu} = \frac{\partial u_i}{\partial \nu}, \qquad \text{在 } \partial D \text{ 上}, \tag{5}$$

$$u_e|_{z=0} = 0, \quad \frac{\partial u_e}{\partial z}\Big|_{z=h} = 0, \tag{6}$$

其中, $k$、$\kappa$ 为不相同的实常数, $\kappa = \omega/c_i$ 为散射体内部的波数, $c_i$ 为声波在散射体内部中的传播速度, $\mu_e$、$\mu_i$ 是不相等的常数.

对散射场 $u^s$ 有简正模态表示

$$u^s = \sum_{n=0}^{+\infty} \phi_n(z) u_n^s(\boldsymbol{x}), \tag{7}$$

其中, $u_n^s$ 为 $u^s$ 的第 $n$ 个传播模态, 满足广义的 Sommerfield 辐射条件

$$\lim_{r\to\infty}\left(\frac{\partial u_n^s}{\partial r} - ik_n u_n^s\right) = 0, \qquad n = 1, 2, \cdots, +\infty, \quad r = |\boldsymbol{x}|. \tag{8}$$

# 2 远场分布的互易关系和完备性

对外部区域 $D_e$ 应用 Green 定理, 可得

$$u^s(\boldsymbol{x}, z; \boldsymbol{d}, z_0) = \int_{\partial D}\left[u_e(\xi, \eta; \boldsymbol{d}, z_0)\frac{\partial G_k(z, \eta, |\boldsymbol{x} - \xi|)}{\partial \nu(\xi, \eta)} - G_k(z, \eta, |\boldsymbol{x} - \xi|)\frac{\partial u_e(\xi, \eta; \boldsymbol{d}, z_0)}{\partial \nu(\xi, \eta)}\right] ds(\xi, \eta), \tag{9}$$

其中, $G$ 为波导 $R_h^3$ 的 Green 函数

$$G(z,\eta,|x-\xi|)=\frac{\mathrm{i}}{2h}\sum_{n=1}^{\infty}\phi_n(z)\phi_n(\eta)\mathrm{H}_0^{(1)}(k_n|x-\xi|). \tag{10}$$

由 Hankel 函数 $\mathrm{H}_0^{(1)}$ 的渐近性质可得

$$u^{s}(x,z)=\frac{\mathrm{i}\mathrm{e}^{-\mathrm{i}\pi/4}}{2h}\sum_{n=1}^{N(k)}\sqrt{\frac{2}{\pi k_n r}}\mathrm{e}^{\mathrm{i}k_n r}u_n^{\infty}(\hat{x},z;\boldsymbol{d},z_0)+O\left(\frac{1}{r^{3/2}}\right), \tag{11}$$

其中

$$u_n^{\infty}(\hat{x},z;\boldsymbol{d},z_0)=\phi_n(z)\int_{\partial D}\left[u_{\mathrm{e}}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial(\mathrm{e}^{-\mathrm{i}k_n\hat{x}\cdot\xi}\phi_n(\eta))}{\partial\nu(\xi,\eta)}-\right.$$
$$\left.(\mathrm{e}^{-\mathrm{i}k_n\hat{x}\cdot\xi}\phi_n(\zeta))\frac{\partial u_{\mathrm{e}}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}\right]\mathrm{d}s(\xi,\eta), \tag{12}$$

$$u^{\infty}(\hat{x},z;\boldsymbol{d},z_0)=\sum_{n=1}^{N(k)}u_n^{\infty}(\hat{x},z;\boldsymbol{d},z_0). \tag{13}$$

函数 $u^{\infty}(\hat{x},z;\boldsymbol{d},z_0)$ 称为散射场 $u^{s}$ 的传播远场分布, 而 $u_n^{\infty}(\hat{x},z;\boldsymbol{d},z_0)$ 为散射场 $u^{s}$ 的第 $n$ 个传播模态 $u_n^{s}$ 的传播远场分布, 它们都是定义在单位柱 $\Omega=\partial B\times[0,h]$ 上的函数, $\partial B$ 为单位圆.

定理 2. 1(远场互易关系) 对任意的 $(\hat{x},z)$ 和 $(\boldsymbol{d},z_0)\in\Omega$, 由式(13) 定义的远场分布 $u^{\infty}(\hat{x},z;\boldsymbol{d},z_0)$ 满足如下的互易关系

$$u^{\infty}(x,z;\boldsymbol{d},z_0)=u^{\infty}(-\boldsymbol{d},z_0;-\hat{x},z). \tag{14}$$

证明 将入射场 $u^{\mathrm{i}}$ 表示为

$$u^{\mathrm{i}}(x,z;\boldsymbol{d},z_0)=\sum_{n=1}^{N(k)}u_n^{\mathrm{i}}(x,z;\boldsymbol{d},z_0), \tag{15}$$

其中, $u_n^{\mathrm{i}}$ 为入射场的第 $n$ 个传播模态

$$u_n^{\mathrm{i}}(x,z;\boldsymbol{d},z_0)=\phi_n(z)\phi_n(z_0)\mathrm{e}^{\mathrm{i}k_n x\cdot\boldsymbol{d}}. \tag{16}$$

记 $u_{\mathrm{e},n}$ 和 $u_{\mathrm{i},n}$ 为由入射场的第 $n$ 个传播模态 $u_n^{\mathrm{i}}$ 产生的在外部区域的总场和在内部区域的透射场. 由 Green 定理可得

$$\int_{\partial D}\left\{u_{\mathrm{i},n}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_{\mathrm{i},n}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}-\right.$$
$$\left.u_{\mathrm{i},n}(\xi,\eta;-\hat{x},z)\frac{\partial u_{\mathrm{i},n}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=0, \tag{17}$$

$$\int_{\partial D}\left\{u_n^{s}(\xi,\eta;-\hat{x},z)\frac{\partial u_n^{s}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}-\right.$$
$$\left.u_n^{s}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^{s}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=0, \tag{18}$$

$$\int_{\partial D}\left\{u_n^{\mathrm{i}}(\xi,\eta;-\hat{x},z)\frac{\partial u_n^{\mathrm{i}}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}-\right.$$
$$\left.u_n^{\mathrm{i}}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^{\mathrm{i}}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=0. \tag{19}$$

由式(12) 和式(17)~(19) 可得

$$u_n^{\infty}(\hat{x},z;\boldsymbol{d},z_0)=\int_{\partial D}\left\{u_{\mathrm{e},n}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^{\mathrm{i}}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}-\right.$$
$$\left.u_n^{\mathrm{i}}(\xi,\eta;-\hat{x},z)\frac{\partial u_{\mathrm{e},n}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=$$

$$
\begin{aligned}
&\int_{\partial D}\left\{\frac{\mu_i}{\mu_e}u_{i,n}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial(u_{i,n}-u_n^s)}{\partial\nu(\xi,\eta)}(\xi,\eta;-\hat{x},z)-\right.\\
&\left.\left(\frac{\mu_e}{\mu_i}u_{i,n}-u_n^s\right)(\xi,\eta;-\hat{x},z)\frac{\partial u_{i,n}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&\int_{\partial D}\left\{u_n^s(\xi,\eta;-\hat{x},z)\frac{\partial u_{i,n}(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}-\right.\\
&\left.\frac{\mu_i}{\mu_e}u_{i,n}(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^s(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&\int_{\partial D}\left\{u_n^s(\xi,\eta;-\hat{x},z)\frac{\partial(u_n^s+u_n^i)}{\partial\nu(\xi,\eta)}(\xi,\eta;\boldsymbol{d},z_0)-\right.\\
&\left.(u_n^s+u_n^i)(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^s(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&\int_{\partial D}\left\{u_n^s(\xi,\eta;-\hat{x},z)\frac{\partial u_n^i(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}-\right.\\
&\left.u_n^i(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_n^s(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&\int_{\partial D}\left\{u_{e,n}(\xi,\eta;-\hat{x},z)\frac{\partial u_n^i(\xi,\eta;\boldsymbol{d},z_0)}{\partial\nu(\xi,\eta)}-\right.\\
&\left.u_n^i(\xi,\eta;\boldsymbol{d},z_0)\frac{\partial u_{e,n}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&\int_{\partial D}\left\{u_{e,n}(\xi,\eta;-\hat{x},z)\frac{\partial(\phi_n(\eta)\phi_n(z_0)\mathrm{e}^{-\mathrm{i}k_n\xi\cdot(-\boldsymbol{d})})}{\partial\nu(\xi,\eta)}-\right.\\
&\left.(\phi_n(\eta)\phi_n(z_0)\mathrm{e}^{-\mathrm{i}k_n\xi\cdot(-\boldsymbol{d})})\frac{\partial u_{e,n}(\xi,\eta;-\hat{x},z)}{\partial\nu(\xi,\eta)}\right\}\mathrm{d}s(\xi,\eta)=\\
&u_n^\infty(-\boldsymbol{d},z_0;-\hat{x},z),
\end{aligned}\tag{20}
$$

则

$$
\begin{aligned}
u^\infty(\hat{x},z;\boldsymbol{d},z_0)&=\sum_{n=1}^{N(k)}u_n^\infty(\hat{x},z;\boldsymbol{d},z_0)=\\
&\sum_{n=1}^{N(k)}u_n^\infty(-\boldsymbol{d},z_0;-\hat{x},z)=u^\infty(-\boldsymbol{d},z_0;-\hat{x},z).
\end{aligned}\tag{21}
$$

定义函数

$$
v(\boldsymbol{x},z)=\int_\Omega g(\hat{\xi},\eta)\sum_{n=1}^{N}\phi_n(\eta)\phi_n(z)\mathrm{e}^{-\mathrm{i}k_n\boldsymbol{x}\cdot\hat{\xi}}\mathrm{d}s(\hat{\xi},\eta),\tag{22}
$$

其中 $\hat{\xi}=\xi/|\xi|$. 很明显的, 这样定义的函数满足 Helmholtz 方程

$$
\Delta v+k^2v=0,\qquad 在\ R_h^3\ 中\tag{23}
$$

和波导的边界条件

$$
v|_{z=0}=0,\quad \left.\frac{\partial v}{\partial z}\right|_{z=h}=0.\tag{24}
$$

称满足 Helmholtz 方程(23) 和边界条件(24) 的函数(22) 为波导的广义的 Herglotz 波函数, $g\in L^2(\Omega)$ 称为广义 Herglotz 波函数的核.

设 $v\in C^2(D)\cap C^1(D)$, $w\in C^2(D)\cap C^1(D)$ 是非齐次内透射问题

$$
\Delta v+k^2v=0,\qquad 在\ D\ 中,\tag{25}
$$

$$
\Delta w+\kappa^2w=0,\qquad 在\ D\ 中,\tag{26}
$$

$$\mu_e v - \mu_i w = \mu_e G_0, \qquad 在\ \partial D\ 上, \tag{27}$$

$$\frac{\partial v}{\partial \nu} - \frac{\partial w}{\partial \nu} = \frac{\partial G_0}{\partial \nu}, \qquad 在\ \partial D\ 上 \tag{28}$$

的解，其中 $G_0 = G_k(\bullet; \xi_0, \eta_0)$，$(\xi_0, \eta_0)$ 为 $D$ 中固定的点. 由文献[1]可知，当 $k^2$ 不是内透射问题的特征值时，方程(25)~(28)存在唯一解.

定义集合

$$S = \left\{ u^\infty(\hat{x}, z; \boldsymbol{d}_m, z_l) - u^\infty(\hat{x}, z; \boldsymbol{d}_1, z_1) : m, l = 1, 2, \cdots \right\}, \tag{29}$$

$\left\{(\boldsymbol{d}_m, z_l)\right\}_{m,l=1}^{\infty} \subset \Omega$ 为 $\Omega$ 上一组稠密的点集.

**定理 2.2** 设 $k^2$ 不是内透射问题的特征值，$(v, w)$ 是内透射方程(25)~(28)的唯一解.

1) 如果 $v$ 是其核为 $g$ 的 Herglotz 波函数，则 $S^\perp = \mathrm{span}\{g\}$；

2) 如果 $v$ 不是 Herglotz 波函数，则 $S^\perp = \mathrm{span}\{0\}$.

证明 设 $g \in L^2(\Omega)$ 并使得

$$\int_\Omega g(\hat{x}, z)[u^\infty(\hat{x}, z; \boldsymbol{d}_m, z_l) - u^\infty(\hat{x}, z; \boldsymbol{d}_1, z_1)]\mathrm{d}s(\hat{x}, z) = 0 \quad (m, l = 1, 2, \cdots). \tag{30}$$

由式(12)和(13)，当 $(\boldsymbol{d}, \eta) = (\boldsymbol{d}_m, z_l)\ (m, l = 1, 2, \cdots)$ 时，

$$\int_\Omega u^\infty(\hat{x}, z; \boldsymbol{d}, \eta) g(\hat{x}, z)\mathrm{d}s(\hat{x}, z) = \int_{\partial D}\left(u_e \frac{\partial v}{\partial \nu} - v\frac{\partial u_e}{\partial \nu}\right)\mathrm{d}s, \tag{31}$$

式中 $v$ 是Herglotz波函数(22)，设 $u_e^{(m,l)} = u_e(\boldsymbol{x}, z; \boldsymbol{d}_m, z_l)$，$u_i^{(m,l)} = u_i(\boldsymbol{x}, z; \boldsymbol{d}_m, z_l)$，$w_e^{(m,l)} = u_e^{(m,l)} - u_e^{(1,1)}$，$w_i^{(m,l)} = u_i^{(m,l)} - u_i^{(1,1)}$，其中 $u_e^{(m,l)}$，$u_i^{(m,l)}$ 是内透射问题(2)~(6)相应于入射波 $u^i = \sum_{n=1}^{N(k)} \phi_n(z)\phi_n(z_l)\mathrm{e}^{\mathrm{i}k_n \boldsymbol{x}\cdot\boldsymbol{d}_m}$ 的解，由式(30)和(31)可知

$$0 = \mu_e \int_{\partial D}\left(w_e^{(m,l)}\frac{\partial v}{\partial \nu} - v\frac{\partial w_e^{(m,l)}}{\partial \nu}\right)\mathrm{d}s = \int_{\partial D}\left(\mu_i w_i^{(m,l)}\frac{\partial v}{\partial \nu} - \mu_e v\frac{\partial w_i^{(m,l)}}{\partial \nu}\right)\mathrm{d}s, \tag{32}$$

另一方面，由式(32)和正透射问题以及 Green 定理可得

$$\begin{aligned}
&\int_{\partial D}\left(\mu_i w_i^{(m,l)}\frac{\partial G_0}{\partial \nu} - \mu_e G_0\frac{\partial w_i^{(m,l)}}{\partial \nu}\right)\mathrm{d}s = \\
&\int_{\partial D}\left[\mu_i w_i^{(m,l)}\left(\frac{\partial v}{\partial \nu} - \frac{\partial w}{\partial \nu}\right) - (\mu_e v - \mu_i w)\frac{\partial w_i^{(m,l)}}{\partial \nu}\right]\mathrm{d}s = \\
&\int_{\partial D}\left[\mu_i w_i^{(m,l)}\frac{\partial v}{\partial \nu} - \mu_e v\frac{\partial w_i^{(m,l)}}{\partial \nu}\right]\mathrm{d}s - \\
&\int_{\partial D}\left[\mu_i w_i^{(m,l)}\frac{\partial w}{\partial \nu} - \mu_e w\frac{\partial w_i^{(m,l)}}{\partial \nu}\right]\mathrm{d}s = 0.
\end{aligned} \tag{33}$$

定义集合 $V$ 和 $V'$ 分别为

$$V(k, \kappa, \mu_i, \mu_e) = \mathrm{span}\left\{\left(\mu_i u_i, \frac{\partial u_i}{\partial \nu}\right)\Big|_{\partial D} : (u_i, u_e);\ 解方程(2)\sim(6)\right.$$
$$\left. u^i(\boldsymbol{x}, z) = \sum_{n=1}^{N(k)} \phi_n(z)\phi_n(z_l)\mathrm{e}^{\mathrm{i}k_n \boldsymbol{x}\cdot\boldsymbol{d}_m}, m, l = 1, 2, \cdots\right\}, \tag{34}$$

$$V'(\kappa, \mu_i) = \left\{\left(\frac{\partial u}{\partial \nu} - \mu_i u\right)\Big|_{\partial D} : u \in C^2(D) \cap C^1(D),\right.$$

$$\text{和}\ \Delta u+\kappa^2u=0,\ \text{在}\ D\ \text{中}\Big\},\tag{35}$$

则由文献[15],有

$$L^2(\partial D)\times L^2(\partial D)=V\oplus\overline{V}^{\perp}.\tag{36}$$

下面证明 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 在集合 $V$ 中. 假设 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 不在 $V$ 中而在 $V^{\perp}$ 中,则对 $(\boldsymbol{x},z)\in\partial D$ 有

$$\frac{\partial G_0}{\partial\nu}=\frac{\partial u}{\partial\nu},\quad -\mu_eG_0=-\mu_iu,\qquad \text{在}\ \partial D\ \text{上}.\tag{37}$$

当 $u\in C^2(D)\cap C^1(\overline{D})$,使得 $\Delta u+\kappa^2u=0$ 成立. 对式(37)的两边取虚部,令 $v_0=\mathrm{Im}G_0$, $w_0=\mathrm{Im}u$,由于

$$v_0=-\frac{1}{4}\sum_{n=1}^{\infty}\phi_n(z)\phi_n(\eta_0)\mathrm{J}_0(k_n|\boldsymbol{x}-\boldsymbol{\xi}_0|),\tag{38}$$

则当 $(\boldsymbol{x},z)\in\partial D$ 时,有

$$\frac{\partial v_0}{\partial\nu}=\frac{\partial w}{\partial\nu},\quad \mu_ev_0=\mu_iw_0,\tag{39}$$

则 $(v_0,w_0)$ 为齐次内透射问题的非平凡解,这与 $k^2$ 不是内透射问题的特征值矛盾. 还需要排除 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}\in V^{\perp}$ 的可能性. 为此,假设可能性成立,设 $(\partial u_j/\partial\nu,-\mu_iu_j)|_{\partial D}$ 为 $V^{\perp}$ 中的 Cauchy 序列,使得 $j\to\infty$ 时有

$$\left(\frac{\partial u_j}{\partial\nu},-\mu_iu_j\right)\Big|_{\partial D}\to\left(\frac{\partial G_0}{\partial\nu},-\mu_eG_0\right)\Big|_{\partial D},$$

由 Green 定理可得

$$\int_{\partial D}\left[u_j(\xi,\eta)\frac{\partial G_\kappa}{\partial\nu(\xi,\eta)}(\boldsymbol{x},z;\xi,\eta)-G_\kappa(\boldsymbol{x},z;\xi,\eta)\frac{\partial u_j}{\partial\nu(\xi,\eta)}(\xi,\eta)\right]\mathrm{d}s(\xi,\eta)=0,\qquad(\boldsymbol{x},z)\in D_e,\tag{40}$$

$$\int_{\partial D}\left[G_\kappa(\boldsymbol{x},z;\xi,\eta)\frac{\partial u_j}{\partial\nu(\xi,\eta)}(\xi,\eta)-u_j(\xi,\eta)\frac{\partial G_\kappa}{\partial\nu(\xi,\eta)}(\boldsymbol{x},z;\xi,\eta)\right]\mathrm{d}s(\xi,\eta)=u_j(\boldsymbol{x},z),\qquad(\boldsymbol{x},z)\in D.\tag{41}$$

令 $j\to+\infty$,则有

$$\int_{\partial D}\left[\frac{\mu_e}{\mu_i}G_0(\boldsymbol{x},z;\xi,\eta)\frac{\partial G_\kappa}{\partial\nu}(\boldsymbol{x},z;\xi,\eta)-G_\kappa(\boldsymbol{x},z;\xi,\eta)\frac{\partial G_0}{\partial\nu}(\boldsymbol{x},z;\xi,\eta)\right]\mathrm{d}s(\xi,\eta)=0,\qquad(\boldsymbol{x},z)\in D_e,\tag{42}$$

$$\int_{\partial D}\left[G_\kappa(\boldsymbol{x},z;\xi,\eta)\frac{\partial G_0}{\partial\nu}(\boldsymbol{x},z;\xi,\eta)-\frac{\mu_e}{\mu_i}G_0(\boldsymbol{x},z;\xi,\eta)\frac{\partial G_\kappa}{\partial\nu}(\boldsymbol{x},z;\xi,\eta)\right]\mathrm{d}s(\xi,\eta)=u(\boldsymbol{x},z),\qquad(\boldsymbol{x},z)\in D,\tag{43}$$

其中 $u(\boldsymbol{x},z)=\lim\limits_{j\to\infty}u_j(\boldsymbol{x},z)$, $(\boldsymbol{x},z)\in D$. 由式(42)和(43)以及单层势和双层势的正则性,有 $u\in C^2(D)\cap C^1(\overline{D})$ 并且满足 Helmholtz 方程 $\Delta u+\kappa^2u=0$;同时由式(42)和(43)以及单层势和双层势的不连续性,$u$ 满足边界条件(37),由上面的证明,这是不可能的,因而 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 不在 $V^{\perp}$ 中,由此得 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 在 $V$ 中.

记 $\mathscr{M}$ 为 $(\mu_iw_i^{(m,l)},\partial w_i^{(m,l)}/\partial\nu)|_{\partial D}$ 在 $L^2(\partial D)\times L^2(\partial D)$ 所张成的空间的闭包,则 $\mathscr{M}\subset V$,但由式(33)和(36)知,$(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 正交于 $\mathscr{M}$,且 $(\partial G_0/\partial\nu,-\mu_eG_0)|_{\partial D}$ 在 $V^{\perp}$ 中.

同时, 设$\mathscr{N}$为由$(\mu_i u_i^{(1,1)}, \partial u_i^{(1,1)}/\partial\nu)|_{\partial D}$所张成的空间, 则$V = \mathscr{M}+\mathscr{N}$, 因而, $\mathscr{M}^{\perp}$ 的维数为1, 即

$$\mathscr{M}^{\perp} = \mathrm{span}\left(\frac{\partial G_0}{\partial\nu}, -\mu_e G_0\right)\Big|_{\partial D}. \tag{44}$$

由式(32), 则有

$$\left(\frac{\partial v}{\partial\nu}, -\mu_e v\right) = c\left(\frac{\partial G_0}{\partial\nu}, -\mu_e G_0\right)\Big|_{\partial D} + \left(\frac{\partial w}{\partial\nu}, -\mu_i w\right)\Big|_{\partial D}, \tag{45}$$

其中 $c$ 为常数, $(\partial w/\partial\nu, -\mu_i w)\in V'$. 由于 $v$ 是波导中满足Helmholtz方程的解, 则$(\partial v/\partial\nu, -\mu_e v)\in V'$. 因而, 对$(\boldsymbol{x}, z)\in\partial D$,

$$\frac{\partial v}{\partial\nu} = c\frac{\partial G_0}{\partial\nu} + \frac{\partial w}{\partial\nu}, \quad -\mu_e v = -c\mu_e G_0 - \mu_i w. \tag{46}$$

如果 $g\neq 0$, 由于 $k^2$ 不是内透射问题的特征值, 则 $c\neq 0$. 因而

$$\mu_i\left(\frac{1}{c}w\right) = \mu_e\left(\frac{1}{c}v\right) - \mu_e G_0, \quad \frac{\partial}{\partial\nu}\left(\frac{1}{c}w\right) = \frac{\partial}{\partial\nu}\left(\frac{1}{c}v\right) - \frac{\partial G_0}{\partial\nu}, \tag{47}$$

从而问题得证.

# 3 传播远场分布的稠密性

定义函数空间

$$H(k, D_e) = \left\{u: u\in C^2(D_e)\cap C^1(\bar{D}_e), u \text{ 满足方程(2)、(6) 和(8)}\right\}, \tag{48}$$

$$A(k, R_h^3) = \left\{u: u(\boldsymbol{x}, z) = \int_\Omega g(\boldsymbol{d}, z_0)\left(\sum_{n=1}^{N(k)}\phi_n(z)\phi_n(z_0)e^{ik_n\boldsymbol{x}\cdot\boldsymbol{d}}\right)ds(\boldsymbol{d}, z_0),\right.$$
$$\left.(\boldsymbol{x}, z)\in R_h^3, g\in L^2(\Omega)\right\}, \tag{49}$$

$$H^2(k, \partial D) = \left\{u, \frac{\partial u}{\partial\nu}\Big|_{\partial D}: u\in H(k, D_e)\right\}, \tag{50}$$

$$A^2_{\mu_e,\mu_i}(\kappa, \partial D) = \left\{\left(\mu_i u, \mu_e\frac{\partial u}{\partial\nu}\right)\Big|_{\partial D}: u\in A(\kappa, R_h^3)\right\}, \tag{51}$$

则有下面的定理

定理 3.1 1) $H^2(k, \partial D) + A^2_{\mu_e,\mu_i}(\kappa, \partial D)$ 在 $L^2(\partial D)\times L^2(\partial D)$ 中是稠密的.

2) $H^2(k, \partial D)\cap A^2_{\mu_e,\mu_i}(\kappa, \partial D) = \{0, 0\}$.

证明 设 $\psi_n^m(\xi, \eta) = \phi_n(\eta)H_m^{(1)}(k_n r_\xi)e^{im\theta_\xi}$, $\gamma_n^m(\xi, \eta) = \phi_n(\eta)J_m(\kappa_n r_\xi)e^{im\theta_\xi}$, 其中 $r_\xi = |\xi - \xi_0|$, $(r_\xi, \theta_\xi)$ 是 $\xi$的极坐标. 由于 $\psi_n^m\in H(k, D_e)$, $\gamma_n^m\in A(\kappa, R_h^3)$, 考虑

$$\int_{\partial D}\left(g\psi_n^m + f\frac{\partial\psi_n^m}{\partial\nu}\right)ds = 0, \tag{52}$$

$$\int_{\partial D}\left(\mu_i g\gamma_n^m + \mu_e f\frac{\partial\gamma_n^m}{\partial\nu}\right)ds = 0, \tag{53}$$

式中 $n = 1, 2, \cdots; m = 0, 1, 2, \cdots; (f, g)\in L^2(\partial D)$. 则要证的结果为$f = 0$, $g = 0$.

由文献[1]知Hankel函数有展开式

$$H_0^{(1)}(k_n|\boldsymbol{x} - \xi|) = \sum_{m=0}^{\infty} i^m H_m^{(1)}(k_n r_{\boldsymbol{x}})J_m(k_n r_\xi)e^{im(\theta_{\boldsymbol{x}} - \theta_\xi)}, \tag{54}$$

其中 $r_{\boldsymbol{x}} = |\boldsymbol{x} - \xi_0|$, $r_\xi < r_{\boldsymbol{x}}$ 和$(r_{\boldsymbol{x}}, \theta_{\boldsymbol{x}})$ 是 $\boldsymbol{x}$ 的极坐标. 定义函数

$$w(\boldsymbol{x}, z) = \int_{\partial D}\left[g(\xi, \eta)G_k(\boldsymbol{x}, z; \xi, \eta) + f(\xi, \eta)\frac{\partial G_k(\boldsymbol{x}, z; \xi, \eta)}{\partial\nu}\right]ds(\xi, \eta), \tag{55}$$

$$v(\boldsymbol{x}, z) = \int_{\partial D}\Big[\mu_i g(\xi, \eta) G_\kappa(\boldsymbol{x}, z; \xi, \eta) + \mu_e f(\xi, \eta)\frac{\partial G_\kappa(\boldsymbol{x}, z; \xi, \eta)}{\partial \nu}\Big] ds(\xi, \eta), \tag{56}$$

则由式(52)和(53)可得,由式(55)和(56)定义的函数 $w$ 和 $v$ 在 $D_e$ 和 $D$ 中恒为 0. 由于 $w$ 是式(2)中代替 $u_e$ 在 $R_h^3 \setminus \partial D$ 中的解, $v$ 是式(3)代替 $u_i$ 在 $D$ 中的解,由单层势和双层势的连续性和边界上的跳越性,有

$$w_+ = 2f, \quad \left(\frac{\partial w}{\partial \nu}\right)_+ = -2g, \qquad 在\ \partial D\ 上, \tag{57}$$

$$v_- = -2\mu_e f, \quad \left(\frac{\partial v}{\partial \nu}\right)_- = 2\mu_i g, \qquad 在\ \partial D\ 上, \tag{58}$$

下标+表示从边界外取极限, −表示从边界内取极限. 设 $u = -\mu_i w$, 则

$$\mu_e u_+ - \mu_i v_- = 0, \quad \left(\frac{\partial u}{\partial \nu}\right)_+ - \left(\frac{\partial v}{\partial \nu}\right)_- = 0, \qquad 在\ \partial D\ 上, \tag{59}$$

即当 $u \in D_e$, $v \in D$ 时都满足齐次透射边界条件, 因而, $u = 0$, $v = 0$, 则 $f = 0$, $g = 0$.

2) 假设 $(\alpha, \beta) \in H^2(k, \partial D) \cap A^2_{\mu_e, \mu_i}(\kappa, \partial D)$, 则存在函数 $u \in H^2(k, \partial D)$ 和函数 $v \in A(\kappa, \partial D)$ 使得在 $\partial D$ 上 $\alpha = u = \mu_i v$, $\beta = \partial u/\partial \nu = \mu_e(\partial v/\partial \nu)$, 即 $u$ 和 $\mu_e v$ 满足齐次透射边界条件. 因而, 由解的解析和延拓性有, 在 $D_e$ 和 $D$ 中 $u = v = 0$, 即 $(\alpha, \beta) = (0, 0)$.

下面考虑传播远场算子. 定义函数空间 $V_N = L^2(\partial B) \times \mathrm{span}\{\phi_1, \phi_2, \cdots, \phi_N\}$, 对 $f \in H^{1/2}(\partial D)$, $h \in H^{-1/2}(\partial D)$, 定义传播远场算子 $F_T: H^{1/2}(\partial D) \times H^{-1/2}(\partial D) \to V_N$, 使得 $F_T(f, h)$ 为透射问题

$$\Delta u_e + k^2 u_e = 0, \qquad 在\ D_e\ 中, \tag{60}$$

$$\Delta u_i + \kappa^2 u_i = 0, \qquad 在\ D\ 中, \tag{61}$$

$$\mu_i u_i - \mu_e u_e = f, \qquad 在\ \partial D\ 上, \tag{62}$$

$$\frac{\partial u_i}{\partial \nu} - \frac{\partial u_e}{\partial \nu} = h, \qquad 在\ \partial D\ 上, \tag{63}$$

$$u_e|_{z=0} = 0, \quad \left.\frac{\partial u_e}{\partial z}\right|_{z=h} = 0 \tag{64}$$

的解为 $(u_e, u_i)$ 的远场分布, 并且 $u_e$ 满足辐射条件(8).

定理 3.2 1) $F_T^*$ 是内射的, 且

$$F_T^* g = \left(-\frac{1}{\mu_i}\frac{\partial}{\partial \nu} w_i, w_i\right)\Big|_{\partial D},$$

其中 $F_T^*$ 为 $F_T$ 的共扼, $v$ 为 Herglotz 波函数, 其核 $g \in L^2(\Omega)$, $(w_i, w_e)$ 为透射问题

$$\Delta w_e + k^2 w_e = 0, \qquad 在\ D_e\ 中, \tag{65}$$

$$\Delta w_i + \kappa^2 w_i = 0, \qquad 在\ D\ 中, \tag{66}$$

$$w_i - w_e = v, \qquad 在\ \partial D\ 上, \tag{67}$$

$$\mu_e \frac{\partial w_i}{\partial \nu} - \mu_i \frac{\partial w_e}{\partial \nu} = \mu_i \frac{\partial v}{\partial \nu} \qquad 在\ \partial D\ 上, \tag{68}$$

$$w_e|_{z=0} = 0, \quad \left.\frac{\partial w_e}{\partial z}\right|_{z=h} = 0, \tag{69}$$

且 $w_e$ 满足辐射条件(8)的解, 其中 $v$ 为式(22)定义的广义的 Herglotz 波函数, 其核 $g \in V_N$.

2) 集合 $F_T(H^{1/2}(\partial D) \times H^{-1/2}(\partial D))$ 在 $L^2(V_N)$ 中是稠密的, 其中 $H^{1/2}$ 为 Sobolev 空间, $H^{-1/2}$ 为 $H^{1/2}$ 的对偶空间.

证明　对 $f\in H^{1/2}(\partial D)$, $h\in H^{-1/2}(\partial D)$, 由式(12)和(13)可得

$$(F_T(f,h))(\hat{x},z)=\sum_{n=1}^{N(k)}\phi_n(z)\int_{\partial D}\Big[u_e(\xi,\eta)\frac{\partial(e^{-ik_n\hat{x}\cdot\xi}\phi_n(\eta))}{\partial\nu(\xi,\eta)}-$$
$$(e^{-ik_n\hat{x}\cdot\xi}\phi_n(\zeta))\frac{\partial u_e(\xi,\eta)}{\partial\nu(\xi,\eta)}\Big]ds(\xi,\eta),\tag{70}$$

则对 $g\in V_N$ 有

$$(F_T(f,h),g)=\int_{\partial D}\Big[u_e\frac{\partial v}{\partial\nu}-v\frac{\partial u_e}{\partial\nu}\Big]ds=$$
$$\int_{\partial D}\Big[u_e\Big(\frac{\mu_e}{\mu_i}\frac{\partial w_i}{\partial\nu}-\frac{\partial w_e}{\partial\nu}\Big)-(w_i-w_e)\frac{\partial u_e}{\partial\nu}\Big]ds=$$
$$\int_{\partial D}\Big[\frac{\mu_e}{\mu_i}u_e\frac{\partial w_i}{\partial\nu}-w_i\frac{\partial u_e}{\partial\nu}\Big]ds=$$
$$\int_{\partial D}\Big[\Big(u_i-\frac{1}{\mu_i}f\Big)\frac{\partial w_i}{\partial\nu}-w_i\Big(\frac{\partial u_i}{\partial\nu}-h\Big)\Big]ds=\int_{\partial D}\Big[hw_i-\frac{1}{\mu_i}f\frac{\partial w_i}{\partial\nu}\Big]ds,\tag{71}$$

其中 $v$ 为式(22)定义的核 $g\in V_N$ 的 Herglotz 波函数. 因而, 其对偶算子 $F_T^*$ 可以由

$$F_T^*g=\Big(-\frac{1}{\mu_i}\frac{\partial}{\partial\nu}w_i|_{\partial D},w_i|_{\partial D}\Big)\tag{72}$$

特征化. 设 $F_T^*g=0$, $g\in L^2(V_N)$, 由其表达式, 在边界上有 $w_i=0$, $\partial w_i/\partial\nu=0$, 则由式(67)和(68)在边界 $\partial D$ 上有 $w_e=-v$, $\partial w_e/\partial\nu=-\partial v/\partial\nu$. 由解的解析性 $w_e$ 可以延拓到 $D$ 中, 因而 $w_e$ 为波导中满足 Helmholtz 方程和辐射条件(8)的整解. 这是唯一可能的, 如果 $w_e=0$, 意味着 $v=0$, 则有 $g=0$.

由 Jacobi-Anger 展开

$$e^{ik_nx\cdot\hat{\xi}}=e^{ik_nr_x\cos(\theta_{\hat{x}}-\theta_{\hat{\xi}})}=\sum_{m=-\infty}^{+\infty}i^mJ_m(k_nr_x)e^{im(\theta_{\hat{x}}-\theta_{\hat{\xi}})},\tag{73}$$

可得

$$v(\boldsymbol{x},z)=\sum_{m=-\infty}^{+\infty}\sum_{n=1}^{N(k)}i^m\phi_n(z)J_m(k_nr_{\boldsymbol{x}})e^{im\theta_{\boldsymbol{x}}}\times$$
$$\int_0^h d\eta\int_0^{2\pi}g(\hat{\xi},\eta)\phi_n(\eta)e^{-im\theta_{\hat{\xi}}}d\theta_{\hat{\xi}}=0,\qquad(\boldsymbol{x},z)\in R_h^3.\tag{74}$$

由于 $\{\phi_n(z)e^{im\theta_{\boldsymbol{x}}}: n=1,2,\dots,N(k),m=0,\pm1,\pm2,\dots\}$ 在 $V_N$ 中是完备的, 则

$$\int_0^h d\eta\int_0^{2\pi}g(\hat{\xi},\eta)\phi_n(\eta)e^{-im\theta_{\hat{\xi}}}d\theta_{\hat{\xi}}=0,\tag{75}$$

其中 $n=1,2,\dots,N(k)$, $m=0,\pm1,\pm2,\dots$. 同时由 $\{\phi_n(\eta)e^{-im\theta_{\hat{\xi}}}: n=1,2,\dots,N(k),m=0,\pm1,\pm2,\dots\}$ 在 $V_N$ 的完备性可得 $g=0$, 即因而 $F_T^*$ 在 $L^2(\Omega)$ 中是内射的.

2) 由证明 1) 知, $F_T$ 的值域即 $R(F_T)$ 在 $L^2(\Omega)$ 中是稠密的.

定理 3.2 表明了传播远场算子的对偶是内射的, 但传播远场算子并不是内射的, 下面给出透射问题传播远场算子不是内射的充分条件.

**定理 3.3**　对 $u^i\in A(k,R_H^3)$, 透射问题(2)～(8)如果存在不恒等于0的 $v\in A(k,R_H^3)$, 使得

$$\begin{pmatrix}\partial v/\partial\nu\\-u\end{pmatrix}\perp A^2_{\mu_e,\mu_i}(\kappa,\partial D),\tag{76}$$

则透射边界值问题的远场分布在 $V_N$ 不是稠密的.

证明 由远场分布的表达式(12)有

$$F_{\mathrm{T}}u^{\mathrm{i}}(\hat{x}, z) = \sum_{n=1}^{N(k)}\phi_n(z)\int_{\partial D}\left[u_{\mathrm{e}}(\xi, \eta)\frac{\partial(\mathrm{e}^{-\mathrm{i}k_n\hat{x}\cdot\xi}\phi_n(\eta))}{\partial\nu(\xi, \eta)} - (\mathrm{e}^{-\mathrm{i}k_n\hat{x}\cdot\xi}\phi_n(\eta))\frac{\partial u_{\mathrm{e}}(\xi, \eta)}{\partial\nu(\xi, \eta)}\right]\mathrm{d}s(\xi, \eta). \tag{77}$$

对 $u^{\mathrm{i}} \in A(k, R_h^3)$，由式(77)可得

$$\int_{\Omega}\overline{g(\xi, \eta)}F_{\mathrm{T}}u_n^{\mathrm{i}}(\hat{x}, z)\mathrm{d}s(\xi, \eta) = 0, \qquad g \in V_N. \tag{78}$$

则证明的结果为要证存在一个不恒等于0的 $g$，使得

$$\int_{\partial D}\left[u_{\mathrm{e}}(\xi, \eta)\frac{\partial\overline{v(\xi, \eta)}}{\partial\nu(\xi, \eta)} - \frac{\partial u_{\mathrm{e}}(\xi, \eta)}{\partial\nu(\xi, \eta)}\overline{v(\xi, \eta)}\right]\mathrm{d}s(\xi, \eta) = 0 \tag{79}$$

成立．其中 $v$ 为式(22)定义的核 $g \in V_N$ 的Herglotz波函数．由透射边界条件(4)和(5)可得

$$\int_{\partial D}\left[\mu_{\mathrm{i}}u_{\mathrm{i}}(\xi, \eta)\frac{\partial\overline{v(\xi, \eta)}}{\partial\nu(\xi, \eta)} - \mu_{\mathrm{e}}\frac{\partial u_{\mathrm{i}}(\xi, \eta)}{\partial\nu(\xi, \eta)}\overline{v(\xi, \eta)}\right]\mathrm{d}s(\xi, \eta) = 0. \tag{80}$$

由假设知, 存在一个不恒为0的 $g \in V_N$，使得

$$\begin{pmatrix}\partial u/\partial\nu \\ -u\end{pmatrix} \perp A^2_{\mu_{\mathrm{e}}, \mu_{\mathrm{i}}}(k_{\mathrm{i}}, \partial D) \tag{81}$$

成立．由定理3.1知, 当 $g \in V_N$, 对所有的 $u^{\mathrm{i}} \in A(k, R_h^3)$，式(79)和(80)是成立的, 因而其传播远场分布在 $V_N$ 不是稠密的．

致谢 本文作者感谢武汉理工大学博士基金(471-38650283)和大学生创新基金(A145), 对本文的资助.

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# Some Properties on the Far-Field Pattern of Scattering by a Penetrable Obstacle in an Ocean Waveguide

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**Abstract:** The properties of propagation far-field patterns corresponding to the scattering of time harmonic acoustic waves by a bounded penetrable obstacle in an ocean waveguide were concerned with. The sets of solutions to the transmission problem were constructed such that the restriction of these solutions to the boundary of the penetrable obstacle is dense in a Hilbert space. Then conditions under which a set of propagation far-field patterns is complete in a Hilbert space were determined. These properties are important in investigating inverse transmission problems in an ocean waveguide.

**Key words:** oceanic waveguide; penetrable object; far-field pattern; completeness; denseness