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考虑横向剪切的对称圆柱正交异性层合扁球壳的热屈曲*

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(我刊编委刘人怀来稿)

摘要: 研究了对称层合圆柱正交异性扁球壳在温度场和均布压力联合作用下的热屈曲问题. 考虑横向剪切的影响, 应用修正迭代法获得了温度场影响下的临界荷载的解析表达式, 讨论了剪切刚度和温度场对临界荷载的影响.

关 键 词: 修正迭代法; 温度场; 热屈曲; 层合扁球壳
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引 言

复合材料作为一种比较理想的结构材料和功能材料, 具有许多独特的优点和重要的实用价值, 其在工程中应用多为板壳的形式. 复合材料层合扁球壳在工程中应用广泛, 且受力情况复杂, 其力学性能的研究日益受到重视. 对复合材料板壳热屈曲问题的研究起源于 19 世纪 70 年代, 1971 年, Whitney 和 Ashton[1] 运用能量方程和瑞兹方法确定了固定铺设角的复合材料层合板的临界屈曲温度. 1978 年, Flagg 和 Vinson[2] 用同样的方法分析了温度和湿度对复合材料层合板屈曲荷载的影响, 表明湿热效应能够降低临界屈曲荷载. 迄今, 对复合材料壳体热屈曲的研究多集中在柱壳方面, 所涉及到的温度场度多是均匀的, 或者面内不均匀而沿着壁厚是均匀的, 所采用的研究方法多为数值方法[3]. 由于非线性数学问题求解的困难, 复合材料层合扁球壳的屈曲问题则无人研究, 为此, 我们开展了一些工作[4-10], 但在其热屈曲问题方面, 一直无人触及.

本文研究了对称层合圆柱正交异性扁球壳在温度场和均布压力联合作用下的非线性热屈曲问题, 考虑了横向剪切的影响, 应用修正迭代法获得了温度场影响下的临界荷载的解析表达式, 讨论了剪切刚度和温度场对临界荷载的影响, 为复合材料层合扁球壳在工程中的应用提供

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一些有价值的参考.

1 基本方程

考虑图1所示的层合扁球壳, 其厚度为 h, 半径为 a, 曲率半径为 R, 拱高为 f, 该扁球壳同时受温度场 T 和均布压力 q 的作用,

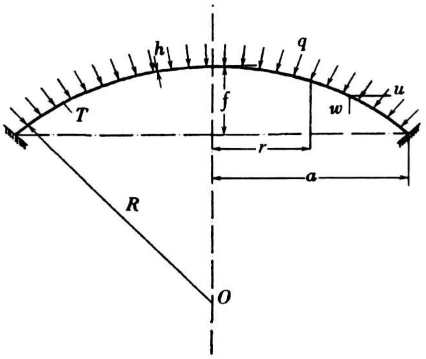

图1 复合材料层合扁球壳

$$\begin{cases} T = T_0 + T_d z/h, \quad T_0 = (T^+ + T^-)/2, \\ T_d = T^+ - T^-, \end{cases} \tag{1}$$

其中 T^+ 和 T^- 分别是壳体上下表面的温度, T_0 是上下表面温度的平均值, T_d 是上下表面的温度差.

以壳体的中曲面作为坐标曲面, 这里, r 为径向坐标, θ 为环向坐标.

由参考文献[5], 可得如下的控制方程:

$$Gr\frac{dw}{dr} + rN_r\left(\frac{r}{R} + \frac{dw}{dr}\right) + Gr\phi + \frac{1}{2}qr^2 = 0, \tag{2}$$

$$D_{11}\frac{d}{dr}r\frac{d\phi}{dr} - \left(Gr + \frac{D_{22}}{r}\right)\phi - Gr\frac{dw}{dr} = (\alpha_r - \alpha_\theta)T_d, \tag{3}$$

$$A_3\frac{d}{dr}r\frac{d(rN_r)}{dr} - A_1N_r + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 + \frac{r}{R}\frac{dw}{dr} = [(A_1 + A_2)\alpha_r - (A_2 + A_3)\alpha_\theta]T_0, \tag{4}$$

其中

$$\alpha_r = \sum_{k=1}^{n}(h_k - h_{k-1})(Q_{11}^{(k)}\alpha_r + Q_{12}^{(k)}\alpha_\theta), \quad \alpha_\theta = \sum_{k=1}^{n}(h_k - h_{k-1})(Q_{12}^{(k)}\alpha_r + Q_{22}^{(k)}\alpha_\theta),$$

$$\alpha_r = \sum_{k=1}^{n}\frac{(h_k^3 - h_{k-1}^3)(Q_{11}^{(k)}\alpha_r + Q_{12}^{(k)}\alpha_\theta)}{3h},$$

$$\alpha_\theta = \sum_{k=1}^{n}\frac{(h_k^3 - h_{k-1}^3)(Q_{12}^{(k)}\alpha_r + Q_{22}^{(k)}\alpha_\theta)}{3h},$$

其中 h_k、h_{k-1}、$Q_{11}^{(k)}$、$Q_{12}^{(k)}$ 和 $Q_{22}^{(k)}$ 同参考文献[5], α_r 和 α_θ 是壳体的径向和环向线膨胀系数.

在夹紧固定边界条件下, 有

$$\text{在 } r = a \text{ 时}, w = 0, \phi = 0, u = 0, \tag{5}$$

$$\text{在 } r = 0 \text{ 时}, \phi = 0, N_r \text{ 有限}. \tag{6}$$

采用以下无量纲参数

$$y = r/a, \quad W = w/h, \quad \Phi = ky + dW/dy, \quad \Psi = a\phi/h, \quad S = arN_r/D_{11},$$

$$P = a^4q/(2D_{11}h), \quad k = a^2/(Rh), \quad m = a^2G/D_{11}, \quad \beta_1^2 = D_{22}/D_{11}, \quad \beta_2^2 = A_1/A_3,$$

$$\beta_3 = h^2/(A_3D_{11}), \quad \beta_4 = A_2/A_3, \quad \beta_5 = D_{12}/D_{11}, \quad \lambda_1 = a^2(\alpha_r - \alpha_\theta)T_d/(hD_{11}),$$

$$\lambda_2 = a^2\beta_3[2A_1\alpha_r - (A_2 + A_3)\alpha_\theta]T_0/h^2,$$

其中 λ_1 和 λ_2 是无量纲化的热弯矩和热膜力.

则边值问题(2)~(6)成为以下无量纲的形式

$$my\Phi = -[S\Phi + my\Psi + (P - km)y^2], \tag{7}$$

$$L_1(y^{\beta_1}\Psi) = my(\Psi + \Phi - ky) + \lambda_1, \tag{8}$$

$$L_2(y^{\beta_2}S) = -\frac{1}{2}\beta_3(\Phi^2 - k^2y^2) + \lambda_2, \tag{9}$$

在 $y = 1$ 时, $W = 0, \Psi = 0, \frac{dS}{dy} - \beta_4 \frac{S}{y} = 0,$ (10)

在 $y = 0$ 时, $\Psi = 0, S = 0,$ (11)

其中

$$L_1 = y^{\beta_1} \frac{d}{dy} y^{1-\beta_1} \frac{d}{dy}(\cdots), \quad L_2 = y^{\beta_2} \frac{d}{dy} y^{1-\beta_2} \frac{d}{dy}(\cdots).$$

2 解 析 解

采用修正迭代法求解无量纲非线性边值问题(7)~(11),选取无量纲中心挠度 $W_m = -\int_0^1(\Phi - ky)dy$ 作为迭代参数,经过 2 次迭代,便得对称圆柱正交异性层合扁球壳的非线性特征关系式如下:

$$P = e_1 W_m^3 + (e_2\lambda_1 + ke_3) W_m^2 + (e_4\lambda_1^2 + ke_5\lambda_1 + k^2 e_6 + e_7\lambda_2 + e_8) W_m + (ke_9\lambda_1^2 + k^2 e_{10}\lambda_1 + e_{11}\lambda_1^3 + e_{12}\lambda_1\lambda_2 + ke_{13}\lambda_2 + e_{14}\lambda_1), \tag{12}$$

其中 $e_i(i = 1, 2, \cdots, 8)$ 是多项表达式的系数,其公式给在附录中.

为了得到壳体的临界荷载 P^*, 应用以下极值条件

$$\frac{dP}{dW_m} = 0, \tag{13}$$

于是,得到屈曲发生时的壳体的无量纲临界中心挠度

$$W_m^* = -\frac{(e_2\lambda_1 + ke_3) + \sqrt{(e_2\lambda_1 + ke_3)^2 - 3e_1(e_5\lambda_1^2 + ke_5\lambda_1 + k^2 e_6 + e_7\lambda_2 + e_8)}}{3e_1}. \tag{14}$$

将式(14)代入(12),得无量纲的临界荷载表达式如下:

$$P^* = e_1 W_m^{*3} + (e_2\lambda_1 + ke_3) W_m^{*2} + (e_4\lambda_1^2 + ke_5\lambda_1 + k^2 e_6 + e_7\lambda_2 + e_8) W_m^* + (ke_9\lambda_1^2 + k^2 e_{10}\lambda_1 + e_{11}\lambda_1^3 + e_{12}\lambda_1\lambda_2 + ke_{13}\lambda_2 + e_{14}\lambda_1). \tag{15}$$

当式(13)的判别式为 0 时,可得用来区分壳体屈曲与否的临界几何参数 k_0 的公式

$$k_0 = \left\{-(2e_2e_3 - 3e_1e_5)\lambda_1 + \sqrt{(2e_2e_3 - 3e_1e_5)^2\lambda_1^2 - 4(e_3^2 - 3e_1e_6)[(e_2^2 - 3e_1e_4)\lambda_1^2 - 3e_1e_7\lambda_2 - 3e_1e_8]}\right\} / \left\{2(e_3^2 - 3e_1e_6)\right\}. \tag{16}$$

3 数 值 算 例

考虑对称层合圆柱正交异性扁球壳,为使计算方便,又不失一般性,假设壳体各层具有相同的厚度和弹性常数,且有 $E_\theta/E_r = 1.5$, $\nu_{r\theta} = 0.2$, $\alpha_\theta/\alpha_r = 0.05$, 则得到 $\beta_1^2 = \beta_2^2 = 1.5$, $\beta_3 = 16.92$, $\beta_4 = \beta_5 = 0.3$.

按照式(12)~式(16),其数值结果给在图 4~图 7 中,这里,我们仅给出了有实际意义的上临界荷载的曲线.

由图形,可看到:

1) 从荷载挠度曲线(图 2)可见,当 k 值较小时,荷载挠度曲线单调增加,表明壳体不会发生屈曲;而当 k 值较大时,荷载挠度曲线成为 形线状,壳体发生了屈曲,$k = k_0$ 为临界点.

2) 从剪切刚度对临界荷载的影响曲线(图 3)可见,稳定曲线单调上升.

3) 从热弯矩对临界荷载的影响曲线(图 4)可见,当热弯矩从负值到正值变化时,临界荷载先增大,后减小. 从热膜力对临界荷载的影响曲线(图 5)可见,$P^* - \lambda_2$ 曲线单调上升,临界荷

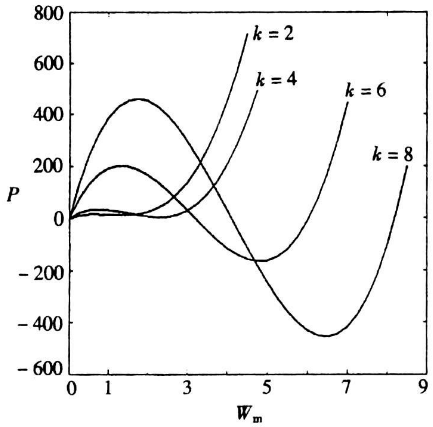

图 2　荷载挠度曲线 ($\lambda_1 = \lambda_2 = 0; m = 10$)

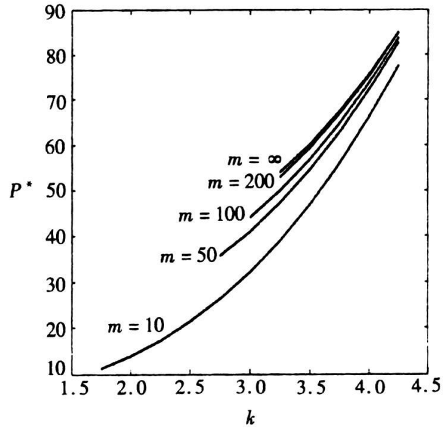

图 3　剪切刚度对临界荷载的影响 ($\lambda_1 = \lambda_2 = 0$)

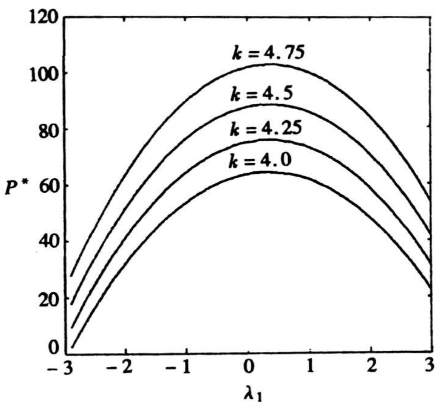

图 4　热弯矩 λ_1 对临界荷载的影响 ($\lambda_2 = -2; m = 10$)

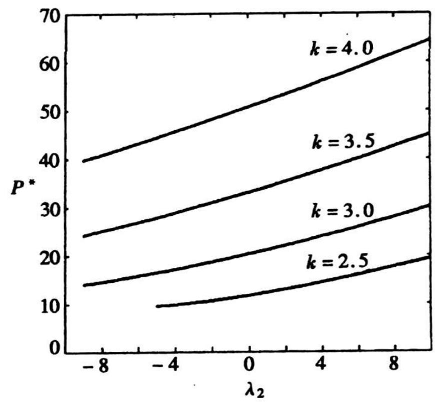

图 5　热膜力 λ_2 对临界荷载的影响 ($\lambda_2 = 2; m = 10$)

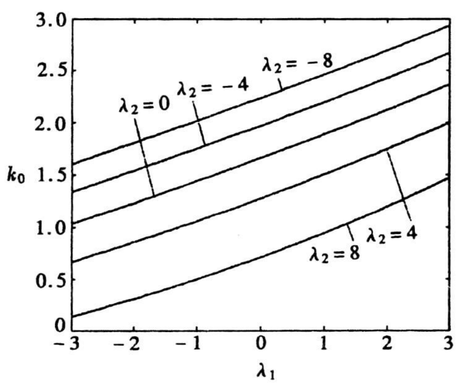

图 6　热弯矩 λ_1 对临界几何参数的影响 ($m = 10$)

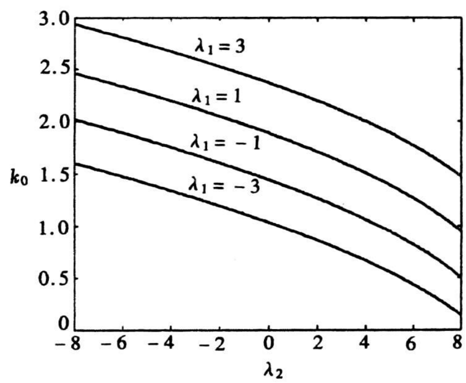

图 7　热膜力 λ_2 对临界几何参数的影响 ($m = 10$)

载随热膜力的增大而增大.

4) 从热弯矩对临界几何参数的影响曲线(图 6) 可见, 当热弯矩逐渐增大时, 在不同的剪切刚度下, 临界几何参数逐渐增加. 从热膜力对临界几何参数的影响曲线(图 7) 可见, k_0-λ_2 曲线单调下降, 当热弯矩逐渐增大时, 临界几何参数逐渐减小.

4 结　　论

本文求解了夹紧固定边界条件下对称圆柱正交异性层合扁球壳的热屈曲问题, 通过分析数值结果, 可得以下结论:

1) 壳体的临界荷载随着无量纲剪切刚度 m 的增加而增加.

2) 壳体的临界荷载随着热弯矩的增加, 先增大, 后减小, 随着热膜力的增大而增大. 如果壳体上下表面的温度差为 0 时, 本文的结果就是均匀升温场中复合材料层合扁球壳的热屈曲问题.

3) 扁球壳的临界几何参数随着热弯矩的增大而增大, 而随着热膜力的增大而减小.

4) 当不存在温度场时, 本文的非线性荷载-挠度关系式(12)、临界中心挠度关系式(14)、临界荷载表达式(15) 和临界几何参数表达式(16) 就退化到文献[5] 的结果, 其剪切刚度对临界荷载的影响曲线也与文献[5] 相同.

附　　录

$$\begin{cases} a_1 = -1/(9-\beta_1^2),\ a_2 = -1/(1-\beta_1^2),\ a_3 = 1/(a_1/4 - a_1/(1+\beta_1) + 0.5/m, \\ a_4 = -a_3(a_2/2 - a_2/(1+\beta_1)),\ a_5 = a_1 a_3,\ a_6 = a_3/m,\ a_7 = a_1 a_4, \\ a_8 = -a_2 - a_1 a_4,\ a_9 = a_4/m + a_2, \end{cases} \tag{A1}$$

$$\begin{cases} b_1 = \dfrac{a_5^2}{49-\beta_2^2},\ b_2 = \dfrac{2a_5a_6}{25-\beta_2^2},\ b_3 = \dfrac{a_6^2}{9-\beta_2^2},\ b_4 = \dfrac{-2a_5a_6}{(2+\beta_1)^2-\beta_2^2},\ b_5 = -\dfrac{2a_5^2}{(4+\beta_1)^2-\beta_2^2}, \\ b_6 = \dfrac{a_5^2}{(1+2\beta_1)^2-\beta_2^2},\ b_7 = \dfrac{a_5a_7}{49-\beta_2^2},\ b_8 = \dfrac{a_5a_9+a_6a_7}{25-\beta_2^2},\ b_9 = \dfrac{a_6a_9}{9-\beta_2^2},\ b_{10} = \dfrac{a_6a_8-a_5a_9}{(2+\beta_1)^2-\beta_2^2}, \\ b_{11} = \dfrac{a_5a_8-a_5a_7}{(4+\beta_1)^2-\beta_2^2},\ b_{12} = \dfrac{-a_5a_8}{(1+2\beta_1)^2-\beta_2^2},\ b_{13} = \dfrac{a_5}{25-\beta_2^2},\ b_{14} = \dfrac{-a_5}{(2+\beta_1)^2-\beta_2^2}, \\ b_{15} = \dfrac{a_6}{9-\beta_2^2},\ b_{16} = -\dfrac{a_7}{25-\beta_2^2},\ b_{17} = \dfrac{a_8}{(2+\beta_1)^2-\beta_2^2},\ b_{18} = \dfrac{a_9}{9-\beta_2^2},\ b_{19} = \dfrac{a_7^2}{49-\beta_2^2}, \\ b_{20} = \dfrac{2a_7a_9}{25-\beta_2^2},\ b_{21} = \dfrac{a_9^2}{9-\beta_2^2},\ b_{22} = \dfrac{2a_8a_9}{(2+\beta_1)^2-\beta_2^2},\ b_{23} = \dfrac{2a_7a_8}{(4+\beta_1)^2-\beta_2^2}, \\ b_{24} = \dfrac{a_8^2}{(1+2\beta_1)^2-\beta_2^2},\ b_{25} = \dfrac{1}{1-\beta_2^2}, \\ b_{26} = [(7-\beta_4)b_1 + (5-\beta_4)b_2 + (3-\beta_4)b_3 + (2+\beta_1-\beta_4)b_4 + (4+\beta_1-\beta_4)b_5 + \\ \quad (1+2\beta_1-\beta_4)b_6]/(\beta_2-\beta_4), \\ b_{27} = [(7-\beta_4)b_7 + (5-\beta_4)b_8 + (3-\beta_4)b_9 + (2+\beta_1-\beta_4)b_{10} + (4+\beta_1-\beta_4)b_{11} + \\ \quad (1+2\beta_1-\beta_4)b_{12}]/(\beta_2-\beta_4), \\ b_{28} = [(5-\beta_4)b_{13} + (2+\beta_1-\beta_4)b_{14} + (3-\beta_4)b_{15}]/(\beta_2-\beta_4), \\ b_{29} = [(5-\beta_4)b_{16} + (2+\beta_1-\beta_4)b_{17} + (3-\beta_4)b_{18}]/(\beta_2-\beta_4), \\ b_{30} = [(7-\beta_4)b_{19} + (5-\beta_4)b_{20} + (3-\beta_4)b_{21} + (2+\beta_1-\beta_4)b_{22} + \\ \quad (4+\beta_1-\beta_4)b_{23} + (1+2\beta_1-\beta_4)b_{24}]/(\beta_2-\beta_4), \\ b_{31} = (1-\beta_4)b_{25}/(\beta_2-\beta_4), \end{cases} \tag{A2}$$

$$
\left\{
\begin{aligned}
& e_1 = -d_1/d_{14},\ e_2 = -d_2/d_{14},\ e_3 = -d_3/d_{14},\ e_4 = -d_4/d_{14},\ e_5 = -d_5/d_{14}, \\
& e_6 = -d_6/d_{14},\ e_7 = -d_7/d_{14},\ e_8 = 1/d_{14},\ e_9 = -d_8/d_{14},\ e_{10} = -d_9/d_{14}, \\
& e_{11} = -d_{10}/d_{14},\ e_{12} = -d_{11}/d_{14},\ e_{13} = -d_{12}/d_{14},\ e_{14} = -d_{13}/d_{14},
\end{aligned}
\right.
\tag{A3}
$$

其中

$$
\left\{
\begin{aligned}
d_1 = {} & \frac{\beta_3}{2}\Bigg[\frac{1}{m}\Bigg(\frac{f_1}{10}+\frac{f_2}{8}+\frac{f_3}{6}+\frac{f_4}{4}+\frac{f_5}{5+\beta_1}+\frac{f_6}{7+\beta_1}+\frac{f_7}{4+2\beta_1}+\frac{f_8}{3+\beta_1}+\frac{f_9}{2+2\beta_1}+ \\
& \frac{f_{10}}{1+3\beta_1}+\frac{f_{11}}{\beta_1+\beta_2}+\frac{f_{12}}{1+\beta_2}+\frac{f_{13}}{3+\beta_2}\Bigg)-\Bigg(\frac{c_1}{12}+\frac{c_2}{10}+\frac{c_3}{8}+\frac{c_4}{6}+\frac{c_5}{7+\beta_1}+\frac{c_6}{9+\beta_1}+ \\
& \frac{c_7}{6+2\beta_1}+\frac{c_8}{5+\beta_1}+\frac{c_9}{4+2\beta_1}+\frac{c_{10}}{3+3\beta_1}+\frac{c_{11}}{2+\beta_2+\beta_1}+\frac{c_{12}}{3+\beta_2}+\frac{c_{13}}{5+\beta_2}-\frac{c_{102}}{1+\beta_1}\Bigg)\Bigg], \\
d_2 = {} & \beta_3\Bigg[\frac{1}{m}\Bigg(\frac{f_{14}}{10}+\frac{f_{15}}{8}+\frac{f_{16}}{6}+\frac{f_{17}}{4}+\frac{f_{18}}{5+\beta_1}+\frac{f_{19}}{7+\beta_1}+\frac{f_{20}}{4+2\beta_1}+\frac{f_{21}}{3+\beta_1}+\frac{f_{22}}{2+2\beta_1}+ \\
& \frac{f_{23}}{1+3\beta_1}+\frac{f_{24}}{\beta_1+\beta_2}+\frac{f_{25}}{1+\beta_2}+\frac{f_{26}}{3+\beta_2}\Bigg)-\Bigg(\frac{c_{14}}{12}+\frac{c_{15}}{10}+\frac{c_{16}}{8}+\frac{c_{17}}{6}+\frac{c_{18}}{7+\beta_1}+\frac{c_{19}}{9+\beta_1}+ \\
& \frac{c_{20}}{6+2\beta_1}+\frac{c_{21}}{5+\beta_1}+\frac{c_{22}}{4+2\beta_1}+\frac{c_{23}}{3+3\beta_1}+\frac{c_{24}}{2+\beta_2+\beta_1}+\frac{c_{25}}{3+\beta_2}+\frac{c_{26}}{5+\beta_2}-\frac{c_{103}}{1+\beta_1}\Bigg)\Bigg], \\
d_3 = {} & \beta_3\Bigg[-\frac{1}{m}\Bigg(\frac{f_{27}}{8}+\frac{f_{28}}{6}+\frac{f_{29}}{4}+\frac{f_{30}}{5+\beta_1}+\frac{f_{31}}{3+\beta_1}+\frac{f_{32}}{2+2\beta_1}+\frac{f_{33}}{3+\beta_2}+\frac{f_{34}}{1+\beta_2}+\frac{f_{35}}{\beta_1+\beta_2}\Bigg)+ \\
& \Bigg(\frac{c_{27}}{10}+\frac{c_{28}}{8}+\frac{c_{29}}{6}+\frac{c_{30}}{7+\beta_1}+\frac{c_{31}}{5+\beta_1}+\frac{c_{32}}{4+2\beta_1}+\frac{c_{33}}{5+\beta_2}+\frac{c_{34}}{3+\beta_2}+\frac{c_{35}}{2+\beta_2+\beta_1}-\frac{c_{104}}{1+\beta_1}\Bigg)\Bigg], \\
d_4 = {} & \frac{1}{m}\Bigg(\frac{f_{36}}{10}+\frac{f_{37}}{8}+\frac{f_{38}}{6}+\frac{f_{39}}{4}+\frac{f_{40}}{5+\beta_1}+\frac{f_{41}}{7+\beta_1}+\frac{f_{42}}{4+2\beta_1}+\frac{f_{43}}{3+\beta_1}+\frac{f_{44}}{2+2\beta_1}+ \\
& \frac{f_{45}}{1+3\beta_1}+\frac{f_{46}}{\beta_1+\beta_2}+\frac{f_{47}}{1+\beta_2}+\frac{f_{48}}{3+\beta_2}\Bigg)-\Bigg(\frac{c_{36}}{12}+\frac{c_{37}}{10}+\frac{c_{38}}{8}+\frac{c_{39}}{6}+\frac{c_{40}}{7+\beta_1}+\frac{c_{41}}{9+\beta_1}+ \\
& \frac{c_{42}}{6+2\beta_1}+\frac{c_{43}}{5+\beta_1}+\frac{c_{44}}{4+2\beta_1}+\frac{c_{45}}{3+3\beta_1}+\frac{c_{46}}{2+\beta_2+\beta_1}+\frac{c_{47}}{3+\beta_2}+\frac{c_{48}}{5+\beta_2}-\frac{c_{105}}{1+\beta_1}\Bigg), \\
d_5 = {} & \beta_3\Bigg[-\frac{1}{m}\Bigg(\frac{f_{49}}{8}+\frac{f_{50}}{6}+\frac{f_{51}}{4}+\frac{f_{52}}{3+\beta_1}+\frac{f_{53}}{5+\beta_1}+\frac{f_{54}}{2+2\beta_1}+\frac{f_{55}}{3+\beta_2}+\frac{f_{56}}{1+\beta_2}+\frac{f_{57}}{\beta_1+\beta_2}\Bigg)+ \\
& \Bigg(\frac{c_{49}}{10}+\frac{c_{50}}{8}+\frac{c_{51}}{6}+\frac{c_{52}}{5+\beta_1}+\frac{c_{53}}{7+\beta_1}+\frac{c_{54}}{4+2\beta_1}+\frac{c_{55}}{5+\beta_2}+\frac{c_{56}}{3+\beta_2}+\frac{c_{57}}{2+\beta_2+\beta_1}-\frac{c_{106}}{1+\beta_1}\Bigg)\Bigg], \\
d_6 = {} & \beta_3\Bigg[\frac{1}{m}\Bigg(\frac{b_{13}}{6}+\frac{b_{14}}{3+\beta_1}+\frac{b_{15}}{4}-\frac{b_{28}}{1+\beta_2}\Bigg)- \\
& \Bigg(\frac{c_{58}}{8}+\frac{c_{59}}{5+\beta_1}+\frac{c_{60}}{6}+\frac{c_{61}}{3+\beta_2}-\frac{c_{107}}{1+\beta_1}\Bigg)\Bigg], \\
d_7 = {} & -\frac{1}{m}\Bigg(\frac{f_{58}}{4}+\frac{f_{59}}{2}+\frac{f_{60}}{1+\beta_1}+\frac{f_{61}}{3+\beta_2}+\frac{f_{62}}{1+\beta_2}+\frac{f_{63}}{\beta_1+\beta_2}\Bigg)+ \\
& \Bigg(\frac{c_{62}}{6}+\frac{c_{63}}{4}+\frac{c_{64}}{3+\beta_1}+\frac{c_{65}}{5+\beta_2}+\frac{c_{66}}{3+\beta_2}+\frac{c_{67}}{2+\beta_2+\beta_1}-\frac{c_{108}}{1+\beta_1}\Bigg), \\
d_8 = {} & \frac{1}{m}\Bigg[\Bigg(\frac{f_{64}}{8}+\frac{f_{65}}{6}+\frac{f_{66}}{5}+\frac{f_{67}}{3+\beta_1}+\frac{f_{68}}{5+\beta_1}+\frac{f_{69}}{3+2\beta_1}+\frac{f_{70}}{3+\beta_2}+\frac{f_{71}}{\beta_1+\beta_2}+\frac{f_{72}}{1+\beta_2}\Bigg)- \\
& \Bigg(\frac{c_{68}}{10}+\frac{c_{69}}{8}+\frac{c_{70}}{6}+\frac{c_{71}}{5+\beta_1}+\frac{c_{72}}{7+\beta_1}+\frac{c_{73}}{4+2\beta_1}+\frac{c_{74}}{5+\beta_2}+\frac{c_{75}}{2+\beta_2+\beta_1}+\frac{c_{76}}{3+\beta_2}-\frac{c_{109}}{1+\beta_1}\Bigg)\Bigg], \\
d_9 = {} & \beta_3\Bigg[\frac{1}{m}\Bigg(\frac{b_{16}}{6}+\frac{b_{17}}{3+\beta_1}+\frac{b_{18}}{4}-\frac{b_{29}}{1+\beta_2}\Bigg)- \\
& \Bigg(\frac{c_{77}}{8}+\frac{c_{78}}{5+\beta_1}+\frac{c_{79}}{6}+\frac{c_{80}}{3+\beta_2}-\frac{c_{110}}{1+\beta_1}\Bigg)\Bigg],
\end{aligned}
\right.
\tag{A4a}
$$

$$\begin{cases} d_{10} = -\dfrac{1}{m}\left(\dfrac{f_{73}}{10} + \dfrac{f_{74}}{8} + \dfrac{f_{75}}{6} + \dfrac{f_{76}}{4} + \dfrac{f_{77}}{5+\beta_1} + \dfrac{f_{78}}{7+\beta_1} + \dfrac{f_{79}}{4+2\beta_1} + \dfrac{f_{80}}{3+\beta_1} + \dfrac{f_{81}}{2+2\beta_1} + \right. \\ \quad \left. \dfrac{f_{82}}{1+3\beta_1} + \dfrac{f_{83}}{\beta_1+\beta_2} + \dfrac{f_{84}}{1+\beta_2} + \dfrac{f_{85}}{3+\beta_2}\right) + \left(\dfrac{c_{81}}{12} + \dfrac{c_{82}}{10} + \dfrac{c_{83}}{8} + \dfrac{c_{84}}{6} + \dfrac{c_{85}}{7+\beta_1} + \dfrac{c_{86}}{9+\beta_1} + \right. \\ \quad \left. \dfrac{c_{87}}{6+2\beta_1} + \dfrac{c_{88}}{5+\beta_1} + \dfrac{c_{89}}{4+2\beta_1} + \dfrac{c_{90}}{3+3\beta_1} + \dfrac{c_{91}}{2+\beta_2+\beta_1} + \dfrac{c_{92}}{3+\beta_2} + \dfrac{c_{93}}{5+\beta_2} - \dfrac{c_{111}}{1+\beta_1}\right), \\ d_{11} = -\dfrac{1}{m}\left[\left(\dfrac{f_{86}}{4} + \dfrac{f_{87}}{2} + \dfrac{f_{88}}{1+\beta_1} + \dfrac{f_{89}}{3+\beta_2} + \dfrac{f_{90}}{1+\beta_2} + \dfrac{f_{91}}{\beta_1+\beta_2}\right) + \right. \\ \quad \left. \left(\dfrac{c_{94}}{6} + \dfrac{c_{95}}{4} + \dfrac{c_{96}}{3+\beta_1} + \dfrac{c_{97}}{5+\beta_2} + \dfrac{c_{98}}{3+\beta_2} + \dfrac{c_{99}}{2+\beta_2+\beta_1} - \dfrac{c_{112}}{1+\beta_1}\right)\right], \\ d_{12} = -\dfrac{1}{m}\left[\left(\dfrac{b_{25}}{2} - \dfrac{b_{31}}{1+\beta_2}\right) + \left(\dfrac{c_{100}}{4} + \dfrac{c_{101}}{3+\beta_2} - \dfrac{c_{113}}{1+\beta_1}\right)\right], \\ d_{13} = \dfrac{1}{2(1-\beta_1^2)} - \dfrac{1}{(1+\beta_1)(1-\beta_1^2)}, \quad d_{14} = -\dfrac{1}{4(9-\beta_1^2)} + \dfrac{1}{(1+\beta_1)(9-\beta_1^2)} + \dfrac{1}{2m}; \end{cases} \tag{A4b}$$

$$\begin{cases} f_1 = a_5b_1,\ f_2 = a_5b_2 + a_6b_1,\ f_3 = a_5b_3 + a_6b_2,\ f_4 = a_6b_3,\ f_5 = a_5b_4 - a_5b_2 + a_6b_5, \\ f_6 = a_5b_5 - a_5b_1,\ f_7 = a_5b_6 - a_5b_5,\ f_8 = -a_5b_3 + a_6b_4,\ f_9 = -a_5b_4 + a_6b_6,\ f_{10} = -a_5b_6, \\ f_{11} = a_5b_{26},\ f_{12} = -a_6b_{26},\ f_{13} = -a_5b_{26},\ f_{14} = a_7b_1/2 + a_5b_7, \\ f_{15} = a_7b_2/2 + a_9b_1/2 + a_5b_8 + a_6b_7,\ f_{16} = a_7b_3/2 + a_9b_2/2 + a_5b_9 + a_6b_8, \\ f_{17} = a_9b_3/2 + a_6b_9,\ f_{18} = a_7b_4/2 + a_8b_2/2 + a_9b_5/2 + a_5b_{10} - a_5b_8 + a_6b_{11}, \\ f_{19} = a_7b_5/2 + a_8b_1/2 + a_5b_{11} - a_5b_7,\ f_{20} = a_7b_6/2 + a_8b_5/2 + a_5b_{12} - a_5b_{11}, \\ f_{21} = a_8b_3/2 + a_9b_4/2 - a_5b_9 + a_6b_{10},\ f_{22} = a_8b_4/2 + a_9b_6/2 - a_5b_{10} + a_6b_{12}, \\ f_{23} = a_8b_6/2 - a_5b_{12},\ f_{24} = -a_8b_{26}/2 + a_5b_{27},\ f_{25} = -(a_9b_{26}/2 + a_6b_{27}), \\ f_{26} = -(a_7b_{26}/2 + a_5b_{27}),\ f_{27} = b_1/2 + a_5b_{13},\ f_{28} = b_2/2 + a_5b_{15} + a_6b_{13},\ f_{29} = b_3/2 + a_6b_{15}, \\ f_{30} = a_4b_{14} - a_5b_{13},\ f_{31} = b_4/2 - a_5b_{15} + a_6b_{14},\ f_{32} = b_6/2 - a_5b_{14},\ f_{33} = -a_5b_{28}, \\ f_{34} = -b_{26}/2 - a_6b_{28},\ f_{35} = a_5b_{28},\ f_{36} = \beta_3a_7b_7 - a_5b_{19},\ f_{37} = \beta_3(a_7b_8 + a_9b_7) - (a_5b_{20} + a_6b_{19}), \\ f_{38} = \beta_3(a_7b_9 + a_9b_8) - (a_5b_{23} + a_6b_{20}),\ f_{39} = \beta_3a_9b_9 - a_6b_{21}, \\ f_{40} = \beta_3(a_7b_{10} + a_8b_8 + a_9b_{11}) - (a_5b_{22} - a_5b_{20} + a_6b_{23}),\ f_{41} = \beta_3(a_7b_{11} + a_8b_7) - (a_5b_{23} - a_5b_{19}), \\ f_{42} = \beta_3(a_7b_{12} + a_8b_{11}) - (a_5b_{24} - a_5b_{23}),\ f_{43} = \beta_3(a_8b_9 + a_9b_{10}) - (-a_5b_{21} + a_6b_{22}), \\ f_{44} = \beta_3(a_8b_{10} + a_9b_{12}) - (-a_5b_{22} + a_6b_{24}),\ f_{45} = \beta_3a_8b_{12} + a_5b_{24},\ f_{46} = \beta_3a_8b_{27} - a_5b_{30}, \\ f_{47} = \beta_3a_8b_{12} + a_5b_{24},\ f_{48} = \beta_3a_7b_{27} + a_5b_{30},\ f_{49} = b_7 + a_7b_{13} + a_5b_{16}, \\ f_{50} = b_8 + a_7b_{15} + a_7b_{13} + a_8b_{18} + a_6b_{16},\ f_{51} = b_9 + a_9b_{15} + a_6b_{18}, \\ f_{52} = b_{10} + a_8b_{15} + a_9b_{14} - a_5b_{18} + a_6b_{17},\ f_{53} = a_7b_{14} + a_8b_{13} + a_5b_{17} - a_5b_{16} + b_{11}, \\ f_{54} = b_{12} + a_8b_{14} - a_5b_{17},\ f_{55} = -(a_7b_{28} + a_5b_{29}),\ f_{56} = -(b_{27} + a_9b_{28} + a_6b_{29}), \\ f_{57} = -(a_8b_{28} - a_5b_{29}),\ f_{58} = a_5b_{25},\ f_{59} = a_6b_{25},\ f_{60} = -a_5b_{25},\ f_{61} = -a_5b_{31},\ f_{62} = -a_6b_{31}, \\ f_{63} = a_5b_{31},\ f_{64} = b_{19} - \beta_3a_7b_{16},\ f_{65} = b_{20} - \beta_3(a_7b_{18} + a_9b_{16}),\ f_{66} = b_{21} - \beta_3a_9b_{18}, \\ f_{67} = b_{22} - \beta_3(a_8b_{18} + a_9b_{17}),\ f_{68} = b_{23} - \beta_3(a_7b_{17} + a_8b_{16}),\ f_{69} = b_{24} - \beta_3a_8b_{17},\ f_{70} = \beta_3a_7b_{29}, \\ f_{71} = \beta_3a_8b_{29},\ f_{72} = -b_{30} + \beta_3a_9b_{29},\ f_{73} = a_7b_{19},\ f_{74} = a_7b_{20} + a_9b_{19},\ f_{75} = a_7b_{21} + a_9b_{20}, \\ f_{76} = a_9b_{21},\ f_{77} = a_7b_{22} + a_8b_{20} + a_9b_{23},\ f_{78} = a_7b_{23} + a_8b_{19},\ f_{79} = a_7b_{24} + a_8b_{23}, \\ f_{80} = a_8b_{21} + a_9b_{22},\ f_{81} = a_8b_{22} + a_9b_{24},\ f_{82} = a_8b_{24},\ f_{83} = -a_8b_{30},\ f_{84} = -a_9b_{30},\ f_{85} = -a_7b_{30}, \\ f_{86} = a_7b_{25},\ f_{87} = a_9b_{25},\ f_{88} = a_8b_{25},\ f_{89} = -a_7b_{31},\ f_{90} = -a_9b_{31},\ f_{91} = -a_8b_{31}, \end{cases}$$

(A5)

$$
\left\{
\begin{aligned}
& c_1=\frac{f_1}{121-\beta_1^2},\ c_2=\frac{f_2}{81-\beta_1^2},\ c_3=\frac{f_3}{49-\beta_1^2},\ c_4=\frac{f_4}{25-\beta_1^2},\ c_5=\frac{f_5}{12(3+\beta_1)},\ c_6=\frac{f_6}{16(4+\beta_1)},\\
& c_7=\frac{f_7}{(5+\beta_1)(5+3\beta_1)},\ c_8=\frac{f_8}{8(2+\beta_1)},\ c_9=\frac{f_9}{3(1+\beta_1)(3+\beta_1)},\ c_{10}=\frac{f_{10}}{4(1+\beta_1)(1+2\beta_1)},\\
& c_{11}=\frac{f_{11}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{12}=\frac{f_{12}}{(2+\beta_2)^2-\beta_1^2},\ c_{13}=\frac{f_{13}}{(4+\beta_2)^2-\beta_1^2},\ c_{14}=\frac{f_{14}}{121-\beta_1^2},\\
& c_{15}=\frac{f_{15}}{81-\beta_1^2},\ c_{16}=\frac{f_{16}}{49-\beta_1^2},\ c_{17}=\frac{f_{17}}{25-\beta_1^2},\ c_{18}=\frac{f_{18}}{12(3+\beta_1)},\ c_{19}=\frac{f_{19}}{16(4+\beta_1)},\\
& c_{20}=\frac{f_{20}}{(5+\beta_1)(5+3\beta_1)},\ c_{21}=\frac{f_{21}}{8(2+\beta_1)},\ c_{22}=\frac{f_{22}}{3(1+\beta_1)(3+\beta_1)},\ c_{23}=\frac{f_{23}}{4(1+\beta_1)(1+2\beta_1)},\\
& c_{24}=\frac{f_{24}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{25}=\frac{f_{25}}{(2+\beta_2)^2-\beta_1^2},\ c_{26}=\frac{f_{26}}{(4+\beta_2)^2-\beta_1^2},\ c_{27}=\frac{f_{27}}{81-\beta_1^2},\\
& c_{28}=\frac{f_{28}}{49-\beta_1^2},\ c_{29}=\frac{f_{29}}{25-\beta_1^2},\ c_{30}=\frac{f_{30}}{12(3+\beta_1)},\ c_{31}=\frac{f_{31}}{8(2+\beta_1)},\ c_{32}=\frac{f_{32}}{3(1+\beta_1)(3+\beta_1)},\\
& c_{33}=\frac{f_{33}}{(4+\beta_2)^2-\beta_1^2},\ c_{34}=\frac{f_{34}}{(2+\beta_2)^2-\beta_1^2},\ c_{35}=\frac{f_{35}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{36}=\frac{f_{36}}{121-\beta_1^2},\\
& c_{37}=\frac{f_{37}}{81-\beta_1^2},\ c_{38}=\frac{f_{38}}{49-\beta_1^2},\ c_{39}=\frac{f_{39}}{25-\beta_1^2},\ c_{40}=\frac{f_{40}}{12(3+\beta_1)},\ c_{41}=\frac{f_{41}}{16(4+\beta_1)},\\
& c_{42}=\frac{f_{42}}{(5+\beta_1)(5+3\beta_1)},\ c_{43}=\frac{f_{43}}{8(2+\beta_1)},\ c_{44}=\frac{f_{44}}{(3+\beta_1)(3+3\beta_1)},\ c_{45}=\frac{f_{45}}{(2+2\beta_1)(2+4\beta_1)},\\
& c_{46}=\frac{f_{46}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{47}=\frac{f_{47}}{(2+\beta_2)^2-\beta_1^2},\ c_{48}=\frac{f_{48}}{(4+\beta_2)^2-\beta_1^2},\ c_{49}=\frac{f_{49}}{81-\beta_1^2},\\
& c_{50}=\frac{f_{50}}{49-\beta_1^2},\ c_{51}=\frac{f_{51}}{25-\beta_1^2},\ c_{52}=\frac{f_{52}}{8(2+\beta_1)},\ c_{53}=\frac{f_{53}}{5(5+2\beta_1)},\ c_{54}=\frac{f_{54}}{3(1+\beta_1)(3+\beta_1)},\\
& c_{55}=\frac{f_{55}}{(4+\beta_2)^2-\beta_1^2},\ c_{56}=\frac{f_{56}}{(2+\beta_2)^2-\beta_1^2},\ c_{57}=\frac{f_{57}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{58}=\frac{b_{13}}{49-\beta_1^2},\\
& c_{59}=\frac{b_{14}}{8(2+\beta_1)},\ c_{60}=\frac{b_{15}}{25-\beta_1^2},\ c_{61}=-\frac{b_{28}}{(2+\beta_2)^2-\beta_1^2},\ c_{62}=\frac{f_{58}}{25-\beta_1^2},\ c_{63}=\frac{f_{59}}{9-\beta_2^2},\\
& c_{64}=\frac{f_{60}}{4(1+\beta_1)},\ c_{65}=\frac{f_{61}}{(4+\beta_2)^2-\beta_1^2},\ c_{66}=-\frac{f_{62}}{(2+\beta_2)^2-\beta_1^2},\ c_{67}=\frac{f_{63}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\\
& c_{68}=\frac{f_{64}}{81-\beta_1^2},\ c_{69}=\frac{f_{65}}{49-\beta_1^2},\ c_{70}=\frac{f_{66}}{25-\beta_1^2},\ c_{71}=\frac{f_{67}}{8(2+\beta_1)},\ c_{72}=\frac{f_{68}}{12(3+\beta_1)},\\
& c_{73}=\frac{f_{69}}{3(1+\beta_1)(3+\beta_1)},\ c_{74}=\frac{f_{70}}{(4+\beta_2)^2-\beta_1^2},\ c_{75}=\frac{f_{71}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\\
& c_{76}=\frac{f_{72}}{(2+\beta_2)^2-\beta_1^2},\ c_{77}=\frac{b_{16}}{49-\beta_1^2},\ c_{78}=\frac{b_{17}}{8(2+\beta_1)},\ c_{79}=\frac{b_{18}}{25-\beta_1^2},\ c_{80}=-\frac{b_{29}}{(2+\beta_2)^2-\beta_1^2},\\
& c_{81}=\frac{f_{73}}{121-\beta_1^2},\ c_{82}=\frac{f_{74}}{81-\beta_1^2},\ c_{83}=\frac{f_{75}}{49-\beta_1^2},\ c_{84}=\frac{f_{76}}{25-\beta_1^2},\ c_{85}=\frac{f_{77}}{12(3+\beta_1)},\\
& c_{86}=\frac{f_{78}}{16(4+\beta_1)},\ c_{87}=\frac{f_{79}}{(5+\beta_1)(5+3\beta_1)},\ c_{88}=\frac{f_{80}}{8(2+\beta_1)},\ c_{89}=\frac{f_{81}}{3(1+\beta_1)(3+\beta_1)},\\
& c_{90}=\frac{f_{82}}{4(1+\beta_1)(1+2\beta_1)},\ c_{91}=\frac{f_{83}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{92}=-\frac{f_{84}}{(2+\beta_2)^2-\beta_1^2},\\
& c_{93}=\frac{f_{85}}{(4+\beta_2)^2-\beta_1^2},\ c_{94}=\frac{f_{86}}{25-\beta_1^2},\ c_{95}=\frac{f_{87}}{9-\beta_1^2},\ c_{96}=\frac{f_{88}}{4(1+\beta_1)},\ c_{97}=\frac{f_{89}}{(4+\beta_2)^2-\beta_1^2},\\
& c_{98}=\frac{f_{90}}{(2+\beta_2)^2-\beta_1^2},\ c_{99}=\frac{f_{91}}{(1+\beta_2)(1+\beta_2+2\beta_1)},\ c_{100}=\frac{b_{25}}{9-\beta_1^2},\ c_{101}=\frac{b_{31}}{(2+\beta_2)^2-\beta_1^2},
\end{aligned}
\right.
\tag{A6a}
$$

$$\begin{cases} c_{102} = \sum_{i=1}^{13} c_i, \quad c_{103} = \sum_{i=14}^{26} c_i, \quad c_{104} = \sum_{i=27}^{35} c_i, \quad c_{105} = \sum_{i=36}^{48} c_i, \quad c_{106} = \sum_{i=49}^{57} c_i, \\ c_{107} = \sum_{i=58}^{61} c_i, \quad c_{108} = \sum_{i=62}^{67} c_i, \quad c_{109} = \sum_{i=68}^{76} c_i. \end{cases} \tag{A6b}$$

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Thermal Buckling of Axisymmetrically Laminated Cylindrically Orthotropic Shallow Spherical Shells Including Transverse Shear

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Abstract: The nonlinear thermal buckling of symmetrically laminated cylindrically orthotropic shallow spherical shell under temperature field and uniform pressure including transverse shear is studied by using the modified iteration method. And the analytic formulas for determining the critical buckling loads under different temperature fields were obtained. The effect of transverse shear deformation and different temperature fields on critical buckling loads were discussed in a numerical example.

Key words: modified iteration method; temperature field; buckling; laminated shallow spherical shell