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具损伤压电智能层合板的非线性主动控制和损伤监测*

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摘要: 基于 Hamilton 原理、高阶剪切变形板理论、von Kármán 型几何非线性应变-位移关系以及应变能等效原理, 考虑压电层的质量和刚度及复合材料层内的损伤效应, 建立了具损伤压电智能层合板的非线性运动方程. 采用耦合正、逆压电效应的负速度反馈控制原理, 形成闭环控制回路, 实现了对压电智能层合板的主动控制和损伤监测. 数值计算中, 以四边简支面内不可动的层合矩形板为例, 讨论了压电层位置对振动控制的影响, 以及损伤程度和损伤位置对传感层输出电压的影响, 提出一种损伤监测的方法.

关 键 词: 压电智能层合板; 非线性振动; 损伤效应; 压电效应; 主动控制; 损伤监测

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引 言

智能结构作为公认的 21 世纪最具前景的技术之一, 在航空航天、机械、建筑、医学等领域得到了广泛的应用, 常被用于结构的振动控制、形状控制、噪声控制、损伤监测及安全防护等方面. 表面粘贴或嵌入压电材料层或片作为传感器和作动器的复合材料层合结构, 是目前广泛应用的一类智能结构. 该结构利用正逆压电效应, 结合了复合材料层合结构的传统力学优点和压电材料的诸多优良特性, 成为当前研究的一个热点.

然而, 大部分有关压电智能结构的研究基于线性压电和线性弹性理论, 特别是振动控制方面[1-6], 而相关非线性问题的研究则很少. 其中, Shi 和 Atluri[7] 建立了一个空间结构的非线性振动的主动控制的分析模型; Pai 等人[8] 发展了一种用于分析含压电层的层合板的几何非线性板理论; Gao 和 Shen[9] 采用增量有限元方法, 分析了粘贴有压电作动层的层合板的几何非线性瞬态振动的主动控制.

另一方面, 损伤作为一个重要的问题, 关系到结构与系统能否正常而可靠地发挥其应有的作用和功能, 及其使用寿命和成本问题. 因此, 有必要对其进行损伤监测, 判断结构是否出现了损伤, 以及损伤的程度和位置. 但是, 有关这方面考虑损伤效应的压电智能结构的非线性动力学问题的研究, 至今尚无文献报道.

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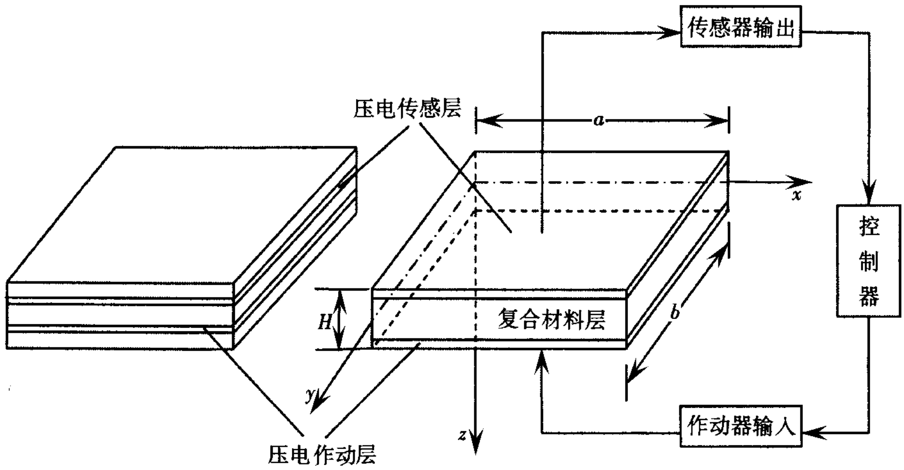
本文基于Hamilton原理、高阶剪切变形板理论、von Kârmân型几何非线性应变-位移关系以及应变能等效原理，考虑压电层的质量和刚度及复合材料层内的损伤效应，建立了具损伤压电智能层合板的非线性运动方程．通过采用耦合正、逆压电效应的负速度反馈控制原理，形成闭环控制回路，实现了对压电智能层合板的主动控制和损伤监测．

1 基本方程

考虑一嵌入或表面粘贴压电层的复合材料层合板(分别如图1(a)、(b))，其长为a，宽为b，总厚度为H，复合材料层与压电层总层数为N，其中第k层的厚度为h_k． 直角坐标系$Oxyz$位于板的中面． 假设压电层沿厚度方向极化． 在板的上表面，作用有横向动载荷$q(x,y,t)$．



(a) 嵌入压电层 (b) 粘贴压电层

图1 压电智能层合板结构与控制回路示意图

根据高阶剪切变形板理论[10-11]，板的位移场假设如下：

$$\begin{cases} u_1(x,y,z,t) = u(x,y,t) + g_1(z)\psi_x(x,y,t) - g_2(z)\partial w/\partial x, \\ u_2(x,y,z,t) = v(x,y,t) + g_1(z)\psi_y(x,y,t) - g_2(z)\partial w/\partial y, \\ u_3(x,y,z,t) = w(x,y,t), \end{cases} \tag{1}$$

式中，(u_1, u_2, u_3)分别表示板内任一点在任一时刻t沿x、y、z方向的位移，(u,v,w)为相应中面($z=0$)上点的位移，(ψ_x, ψ_y)分别为位于$z=0$的横向法线绕y和x轴的转角． 函数$g_1(z)$和$g_2(z)$取为

$$g_1(z) = z - \frac{4z^3}{3H^2}, \quad g_2(z) = \frac{4z^3}{3H^2}. \tag{2}$$

对应于式(1)的von Kârmân型几何非线性应变-位移关系为

$$\begin{cases} \varepsilon_1 = \varepsilon_x = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 + g_1\dfrac{\partial \psi_x}{\partial x} - g_2\dfrac{\partial^2 w}{\partial x^2}, \\ \varepsilon_2 = \varepsilon_y = \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 + g_1\dfrac{\partial \psi_y}{\partial y} - g_2\dfrac{\partial^2 w}{\partial y^2}, \quad \varepsilon_3 = \varepsilon_z = 0, \\ \varepsilon_4 = 2\gamma_{yz} = \dfrac{\mathrm{d}g_1}{\mathrm{d}z}\left(\psi_y + \dfrac{\partial w}{\partial y}\right), \quad \varepsilon_5 = 2\gamma_{zx} = \dfrac{\mathrm{d}g_1}{\mathrm{d}z}\left(\psi_x + \dfrac{\partial w}{\partial x}\right), \\ \varepsilon_6 = 2\gamma_{xy} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} + g_1\left(\dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x}\right) - 2g_2\dfrac{\partial^2 w}{\partial x \partial y}, \end{cases} \tag{3}$$

其中，$(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{zx}, \gamma_{xy})$ 为 Cauchy 应变，$(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6)$ 为工程应变．本文中采用后者．

包含压电效应的正交各向异性材料的本构方程为

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix}=\begin{bmatrix}c_{11}&c_{12}&c_{13}&0&0&0\\&c_{22}&c_{23}&0&0&0\\&&c_{33}&0&0&0\\&&&c_{44}&0&0\\&&&&c_{55}&0\\&&&&&c_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}-\begin{bmatrix}0&0&e_{31}\\0&0&e_{32}\\0&0&e_{33}\\0&e_{24}&0\\e_{15}&0&0\\0&0&0\end{bmatrix}\begin{Bmatrix}E_1\\E_2\\E_3\end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix}D_1\\D_2\\D_3\end{Bmatrix}=\begin{bmatrix}0&0&0&0&e_{15}&0\\0&0&0&e_{24}&0&0\\e_{31}&e_{32}&e_{33}&0&0&0\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}+\begin{bmatrix}k_{11}&0&0\\0&k_{22}&0\\0&0&k_{33}\end{bmatrix}\begin{Bmatrix}E_1\\E_2\\E_3\end{Bmatrix}, \tag{4b}$$

式中，σ_i、D_i、E_i 分别为应力、电位移、电场强度矢量的分量，c_{ij}、e_{ij}、k_{ij} 分别为材料的弹性常数、压电常数及介电常数．电场强度与电势之间的关系为

$$E_i=-\phi_{,i}. \tag{5}$$

假设压电层(位于第 k 层)中的电势可以表示为如下形式：

$$\phi^k(x,y,z,t)=\sum_{j=1}^{n}f_j^k(z)\,\varphi_j^k(x,y,t), \tag{6}$$

其中，n 是插值结点数，$f_j^k(z)\,(j=1,2,\cdots,n)$ 为 Lagrange 插值函数．

本文仅考虑复合材料层内具损伤的情况，假定板内第 k 层在局部区域$(a_1^{(k)}\leqslant x\leqslant a_2^{(k)},\ b_1^{(k)}\leqslant y\leqslant b_2^{(k)})$ 含有损伤缺陷，且定义损伤主变量为 $D_1^{(k)}$ 和 $D_2^{(k)}$，设损伤张量的主方向与材料主方向一致． 由应变能等效原理[12]，第 k 层内具损伤区域的应力与应变的本构关系为

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_6\end{Bmatrix}^{(k)}=\begin{bmatrix}c_{11}^{\mathrm d}&c_{12}^{\mathrm d}&0\\c_{12}^{\mathrm d}&c_{22}^{\mathrm d}&0\\0&0&c_{66}^{\mathrm d}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_6\end{Bmatrix}^{(k)},\quad \begin{Bmatrix}\sigma_4\\\sigma_5\end{Bmatrix}^{(k)}=\begin{bmatrix}c_{44}^{\mathrm d}&0\\0&c_{55}^{\mathrm d}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_4\\\varepsilon_5\end{Bmatrix}^{(k)}, \tag{7}$$

式中，$c_{ij}^{\mathrm d}$ 为第k 层在损伤区域内的折算弹性刚度，其表达式为

$$\begin{cases}c_{11}^{\mathrm d}=(1-D_1^{(k)})^2c_{11}^{\mathrm e},\quad c_{12}^{\mathrm d}=(1-D_1^{(k)})(1-D_2^{(k)})c_{12}^{\mathrm e},\\ c_{22}^{\mathrm d}=(1-D_2^{(k)})^2c_{22}^{\mathrm e},\quad c_{44}^{\mathrm d}=(1-D_2^{(k)})c_{44}^{\mathrm e},\quad c_{55}^{\mathrm d}=(1-D_1^{(k)})c_{55}^{\mathrm e},\\ c_{66}^{\mathrm d}=(1-D_1^{(k)})(1-D_2^{(k)})c_{66}^{\mathrm e},\end{cases} \tag{8}$$

其中，$c_{ij}^{\mathrm e}$ 为第k 层无损时的折算弹性刚度，且

$$\begin{cases}c_{11}^{\mathrm e}=E_1/(1-\mu_{12}\mu_{21}),\quad c_{12}^{\mathrm e}=\mu_{12}E_2/(1-\mu_{12}\mu_{21}),\\ c_{22}^{\mathrm e}=E_2/(1-\mu_{12}\mu_{21}),\quad c_{44}^{\mathrm e}=G_{23},\quad c_{55}^{\mathrm e}=G_{13},\quad c_{66}^{\mathrm e}=G_{12},\end{cases} \tag{9}$$

式中，E_1、E_2 为主弹性模量，μ_{12}、μ_{21} 为 Poisson 比，G_{12}、G_{23}、G_{13} 为剪切弹性模量．

为方便计算，将 $c_{ij}^{\mathrm d}$ 分写成两部分：

$$c_{ij}^{\mathrm d}=c_{ij}^{\mathrm e}-\Delta c_{ij}, \tag{10}$$

其中 Δc_{ij} 代表由损伤引起的折算弹性刚度的减小量． 对于无损区域内的应力应变本构关系，

只需令 Δc_{ij} 取为 0 即可.

基于 Hamilton 原理, 具损伤压电智能层合板的非线性运动方程将由下式导出:

$$\delta\int_{t_2}^{t_2}(K - U)\,\mathrm{d}t = 0, \tag{11}$$

式中, K 是系统的动能,

$$K = \int_V \frac{1}{2}\rho\dot{u}_i\dot{u}_i\,\mathrm{d}V,$$

其中 ρ 是材料的质量密度; U 是系统的总势能,

$$U = \int_V h(\varepsilon_{ij}, E_i)\,\mathrm{d}V - \int_A (q_i u_i - Q\phi)\,\mathrm{d}A,$$

其中 $$h = \frac{1}{2}c_{ijkl}\,\varepsilon_{ij}\,\varepsilon_{kl} - e_{kij}E_k\,\varepsilon_{ij} - \frac{1}{2}k_{ij}E_iE_j;$$

q_i、Q 分别为作用在系统的力载荷和电载荷; V、A 分别为系统的体积和表面积.

采用类似于参考文献[11]的方法, 由式(11)可得到具损伤压电智能层合板的非线性运动方程为

$$-\left[I_1\ddot{w} + \frac{\partial}{\partial x}\left(I_3\ddot{u} + I_5\ddot{\psi}_x - I_6\frac{\partial\ddot{w}}{\partial x}\right) + \frac{\partial}{\partial y}\left(I_3\ddot{v} + I_5\ddot{\psi}_y - I_6\frac{\partial\ddot{w}}{\partial y}\right)\right] + q + \frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_2}{\partial y^2} + 2\frac{\partial^2 P_6}{\partial x\partial y} + \frac{\partial Q_4}{\partial y} + \frac{\partial Q_5}{\partial x} + \left(\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y}\right)\frac{\partial w}{\partial x} + \left(\frac{\partial N_2}{\partial y} + \frac{\partial N_6}{\partial x}\right)\frac{\partial w}{\partial y} + N_1\frac{\partial^2 w}{\partial x^2} + N_2\frac{\partial^2 w}{\partial y^2} + 2N_6\frac{\partial^2 w}{\partial x\partial y} = 0, \tag{12a}$$

$$-\left(I_2\ddot{u} + I_4\ddot{\psi}_x - I_5\frac{\partial\ddot{w}}{\partial x}\right) + \frac{\partial M_1}{\partial x} + \frac{\partial M_6}{\partial y} - Q_5 = 0, \tag{12b}$$

$$-\left(I_2\ddot{v} + I_4\ddot{\psi}_y - I_5\frac{\partial\ddot{w}}{\partial y}\right) + \frac{\partial M_2}{\partial y} + \frac{\partial M_6}{\partial x} - Q_4 = 0, \tag{12c}$$

$$-\left(I_1\ddot{u} + I_2\ddot{\psi}_x - I_3\frac{\partial\ddot{w}}{\partial x}\right) + \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0, \tag{12d}$$

$$-\left(I_1\ddot{v} + I_2\ddot{\psi}_y - I_3\frac{\partial\ddot{w}}{\partial y}\right) + \frac{\partial N_2}{\partial y} + \frac{\partial N_6}{\partial x} = 0, \tag{12e}$$

式中, (N_i, M_i, P_i, Q_i) 为应力合力, $(I_1, I_2, \cdots, I_6)$ 为板的质量惯性常数, 它们分别定义如下:

$$\begin{cases} N_i = \int_{-H/2}^{H/2}\sigma_i\,\mathrm{d}z, \quad M_i = \int_{-H/2}^{H/2}\sigma_i g_1(z)\,\mathrm{d}z, \quad P_i = \int_{-H/2}^{H/2}\sigma_i g_2(z)\,\mathrm{d}z \\ \qquad\qquad (i = 1, 2, 6), \\ Q_i = \int_{-H/2}^{H/2}\sigma_i\frac{\mathrm{d}g_1}{\mathrm{d}z}\mathrm{d}z \qquad (i = 4, 5), \\ I_1 = \int_{-H/2}^{H/2}\rho\,\mathrm{d}z, \quad I_2 = \int_{-H/2}^{H/2}\rho g_1(z)\,\mathrm{d}z, \quad I_3 = \int_{-H/2}^{H/2}\rho g_2(z)\,\mathrm{d}z, \\ I_4 = \int_{-H/2}^{H/2}\rho g_1(z)g_1(z)\,\mathrm{d}z, \quad I_5 = \int_{-H/2}^{H/2}\rho g_1(z)g_2(z)\,\mathrm{d}z, \\ I_6 = \int_{-H/2}^{H/2}\rho g_2(z)g_2(z)\,\mathrm{d}z. \end{cases} \tag{13}$$

将各应力合力 (N_i, M_i, P_i, Q_i) 分解成 3 个部分:

$$\begin{cases} N_i = N_i - N_i^{\mathrm{D}} + N_i^{\mathrm{P}}, \quad M_i = M_i - M_i^{\mathrm{D}} + M_i^{\mathrm{P}}, \\ P_i = P_i - P_i^{\mathrm{D}} + P_i^{\mathrm{P}}, \quad Q_i = Q_i - Q_i^{\mathrm{D}} + Q_i^{\mathrm{P}}. \end{cases} \tag{14}$$

上面各式右边第 1 项是当压电效应及损伤均不存在时给出的分量, 第 2 项为损伤引起的合力

的减小量,第 3 项是考虑压电效应时引起的分量.

对于对称角铺设压电智能层合板,其内力分量 (N_i, M_i, P_i, Q_i) 可表示为

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix}, \tag{15a}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \\ P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 & B_{11} & B_{12} & 0 \\ & B_{22} & 0 & B_{12} & B_{22} & 0 \\ & & B_{66} & 0 & 0 & B_{66} \\ & & & D_{11} & D_{12} & 0 \\ & & & & D_{22} & 0 \\ & & & & & D_{66} \end{bmatrix} \begin{Bmatrix} \partial \psi_x / \partial x \\ \partial \psi_y / \partial y \\ \partial \psi_x / \partial y + \partial \psi_y / \partial x \\ -\partial^2 w / \partial x^2 \\ -\partial^2 w / \partial y^2 \\ -2\partial^2 w / (\partial x \partial y) \end{Bmatrix}, \tag{15b}$$

$$\begin{Bmatrix} Q_4 \\ Q_5 \end{Bmatrix} = \begin{bmatrix} F_{44} & 0 \\ 0 & F_{55} \end{bmatrix} \begin{Bmatrix} \psi_y + \partial w / \partial y \\ \psi_x + \partial w / \partial x \end{Bmatrix}, \tag{15c}$$

其中,层合板的刚度 A_{ij}、B_{ij} 等定义如下:

$$\begin{cases} A_{ij} = \displaystyle\int_{-H/2}^{H/2} Q_{ij}\,\mathrm{d}z, \quad B_{ij} = \int_{-H/2}^{H/2} Q_{ij}\, g_1(z)\, g_1(z)\,\mathrm{d}z, \\ B_{ij} = \displaystyle\int_{-H/2}^{H/2} Q_{ij}\, g_1(z)\, g_2(z)\,\mathrm{d}z, \\ D_{ij} = \displaystyle\int_{-H/2}^{H/2} Q_{ij}\, g_2(z)\, g_2(z)\,\mathrm{d}z \qquad (i, j = 1, 2, 6), \\ F_{44} = \displaystyle\int_{-H/2}^{H/2} Q_{44}\left(\frac{\mathrm{d}g_1}{\mathrm{d}z}\right)^2 \mathrm{d}z, \quad F_{55} = \int_{-H/2}^{H/2} Q_{55}\left(\frac{\mathrm{d}g_1}{\mathrm{d}z}\right)^2 \mathrm{d}z, \end{cases} \tag{16}$$

上式中, Q_{ij} 为转换的弹性系数,其与第 k 层铺设角 θ_k 的关系为

$$\begin{cases} Q_{11}^{(k)} = c_{11}^{\mathrm{e}}\cos^4\theta_k + 2(c_{12}^{\mathrm{e}} + 2c_{66}^{\mathrm{e}})\cos^2\theta_k\sin^2\theta_k + c_{22}^{\mathrm{e}}\sin^4\theta_k, \\ Q_{12}^{(k)} = c_{12}^{\mathrm{e}}\cos^4\theta_k + (c_{11}^{\mathrm{e}} + c_{22}^{\mathrm{e}} - 4c_{66}^{\mathrm{e}})\cos^2\theta_k\sin^2\theta_k + c_{12}^{\mathrm{e}}\sin^4\theta_k, \\ Q_{22}^{(k)} = c_{22}^{\mathrm{e}}\cos^4\theta_k + 2(c_{12}^{\mathrm{e}} + 2c_{66}^{\mathrm{e}})\cos^2\theta_k\sin^2\theta_k + c_{11}^{\mathrm{e}}\sin^4\theta_k, \\ Q_{44}^{(k)} = c_{44}^{\mathrm{e}}\cos^2\theta_k + c_{55}^{\mathrm{e}}\sin^2\theta_k, \\ Q_{55}^{(k)} = c_{55}^{\mathrm{e}}\cos^2\theta_k + c_{44}^{\mathrm{e}}\sin^2\theta_k, \\ Q_{66}^{(k)} = (c_{11}^{\mathrm{e}} + c_{22}^{\mathrm{e}} - 2c_{12}^{\mathrm{e}} - 2c_{66}^{\mathrm{e}})\cos^2\theta_k + \sin^2\theta_k + c_{66}^{\mathrm{e}}(\cos^4\theta_k + \sin^4\theta_k). \end{cases} \tag{17}$$

类似地,在式(15)~(17)中用 Δc_{ij} 代替 c_{ij}^{e}, ΔQ_{ij} 代替 Q_{ij}, ΔA_{ij}、ΔB_{ij}、… 代替 A_{ij}、B_{ij}、…,即可得到 $(N_i^{\mathrm{D}}, M_i^{\mathrm{D}}, P_i^{\mathrm{D}}, Q_i^{\mathrm{D}})$.

为得到压电应力分量 $(N_i^{\mathrm{P}}, M_i^{\mathrm{P}}, P_i^{\mathrm{P}}, Q_i^{\mathrm{P}})$,考虑式(6)及式(4a)中与压电效应有关的项,得到

$$\begin{cases} \sigma_1^{\mathrm{P}} = e_{31}\displaystyle\sum_{j=1}^{n}\frac{\mathrm{d}f_j^k}{\mathrm{d}z}\varphi_j^k, \quad \sigma_2^{\mathrm{P}} = e_{32}\sum_{j=1}^{n}\frac{\mathrm{d}f_j^k}{\mathrm{d}z}\varphi_j^k, \quad \sigma_6^{\mathrm{P}} = 0, \\ \sigma_4^{\mathrm{P}} = e_{24}\displaystyle\sum_{j=1}^{n} f_j^k(z)\,\frac{\partial \varphi_j^k}{\partial y}, \quad \sigma_5^{\mathrm{P}} = e_{15}\sum_{j=1}^{n} f_j^k(z)\,\frac{\partial \varphi_j^k}{\partial x}. \end{cases} \tag{18}$$

则内力分量 $(N_i^{\mathrm{P}}, M_i^{\mathrm{P}}, P_i^{\mathrm{P}}, Q_i^{\mathrm{P}})$ 可表示为

$$
\begin{cases}
N_1^{\mathrm{P}} = \sum_{k=1}^{N} e_{31}^k \left(\sum_{j=1}^{n} \eta_1^{jk} \varphi_j^k \right), \ N_2^{\mathrm{P}} = \sum_{k=1}^{N} e_{32}^k \left(\sum_{j=1}^{n} \eta_1^{jk} \varphi_j^k \right), \ N_6^{\mathrm{P}} = 0, \\
M_1^{\mathrm{P}} = \sum_{k=1}^{N} e_{31}^k \left(\sum_{j=1}^{n} \eta_2^{jk} \varphi_j^k \right), \ M_2^{\mathrm{P}} = \sum_{k=1}^{N} e_{32}^k \left(\sum_{j=1}^{n} \eta_2^{jk} \varphi_j^k \right), \ M_6^{\mathrm{P}} = 0, \\
P_1^{\mathrm{P}} = \sum_{k=1}^{N} e_{31}^k \left(\sum_{j=1}^{n} \eta_3^{jk} \varphi_j^k \right), \ P_2^{\mathrm{P}} = \sum_{k=1}^{N} e_{32}^k \left(\sum_{j=1}^{n} \eta_3^{jk} \varphi_j^k \right), \ P_6^{\mathrm{P}} = 0, \\
Q_4^{\mathrm{P}} = \sum_{k=1}^{N} e_{24}^k \left(\sum_{j=1}^{n} \Delta_4^{jk} \frac{\partial \varphi_j^k}{\partial y} \right), \ Q_5^{\mathrm{P}} = \sum_{k=1}^{N} e_{15}^k \left(\sum_{j=1}^{n} \Delta_4^{jk} \frac{\partial \varphi_j^k}{\partial x} \right),
\end{cases} \tag{19}
$$

其中

$$
\begin{cases}
\eta_1^{jk} = \int_{z_{k-1}}^{z_k} \frac{\mathrm{d}f_j^k}{\mathrm{d}z} \mathrm{d}z, \ \eta_2^{jk} = \int_{z_{k-1}}^{z_k} g_1(z) \frac{\mathrm{d}f_j^k}{\mathrm{d}z} \mathrm{d}z, \\
\eta_3^{jk} = \int_{z_{k-1}}^{z_k} g_2(z) \frac{\mathrm{d}f_j^k}{\mathrm{d}z} \mathrm{d}z, \ \Delta_4^{jk} = \int_{z_{k-1}}^{z_k} \frac{\mathrm{d}g_1}{\mathrm{d}z} f_j^k(z) \mathrm{d}z.
\end{cases} \tag{20}
$$

基于上述各应力合力的定义, 由方程(12)可以得到, 用位移和电势表示的对称角铺设的具损伤压电智能层合板的非线性运动方程为

$$
\begin{aligned}
&-(D_{11} - \Delta D_{11}) \frac{\partial^4 w}{\partial x^4} - (4D_{66} + 2D_{12} - 4\Delta D_{66} - 2\Delta D_{12}) \frac{\partial^4 w}{\partial x^2 \partial y^2} - \\
&\quad (D_{22} - \Delta D_{22}) \frac{\partial^4 w}{\partial y^4} + (F_{44} - \Delta F_{44}) \frac{\partial^2 w}{\partial y^2} + (F_{55} - \Delta F_{55}) \frac{\partial^2 w}{\partial x^2} + \\
&\quad (B_{11} - \Delta B_{11}) \frac{\partial^3 \psi_x}{\partial x^3} + (B_{12} + 2B_{66} - \Delta B_{12} - 2\Delta B_{66}) \frac{\partial^3 \psi_x}{\partial x \partial y^2} + \\
&\quad (F_{55} - \Delta F_{55}) \frac{\partial \psi_x}{\partial x} + (B_{22} - \Delta B_{22}) \frac{\partial^3 \psi_y}{\partial y^3} + (B_{12} + 2B_{66} - \Delta B_{12} - \\
&\quad 2\Delta B_{66}) \frac{\partial^3 \psi_y}{\partial x^2 \partial y} + (F_{44} - \Delta F_{44}) \frac{\partial \psi_y}{\partial y} + \sum_{k=1}^{N} e_{31}^k \left(\sum_{j=1}^{n} \eta_3^{jk} \frac{\partial^2 \varphi_j^k}{\partial x^2} \right) + \\
&\quad \sum_{k=1}^{N} e_{15}^k \left(\sum_{j=1}^{n} \Delta_4^{jk} \frac{\partial^2 \varphi_j^k}{\partial x^2} \right) + \sum_{k=1}^{N} e_{32}^k \left(\sum_{j=1}^{n} \eta_3^{jk} \frac{\partial^2 \varphi_j^k}{\partial y^2} \right) + \sum_{k=1}^{N} e_{24}^k \left(\sum_{j=1}^{n} \Delta_4^{jk} \frac{\partial^2 \varphi_j^k}{\partial y^2} \right) + \\
&\quad \left[(A_{11} - \Delta A_{11}) \left(\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) + (A_{12} - \Delta A_{12}) \left(\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right) + \right. \\
&\quad \left. \sum_{k=1}^{N} e_{31}^k \left(\sum_{j=1}^{n} \eta_1^{jk} \varphi_j^k \right) \right] \frac{\partial^2 w}{\partial x^2} + \left[(A_{12} - \Delta A_{12}) \left(\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) + \right. \\
&\quad \left. (A_{22} - \Delta A_{22}) \left(\frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \right) + \sum_{k=1}^{N} e_{32}^k \left(\sum_{j=1}^{n} \eta_1^{jk} \varphi_j^k \right) \right] \frac{\partial^2 w}{\partial y^2} + \\
&\quad 2(A_{66} - \Delta A_{66}) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) \frac{\partial^2 w}{\partial x \partial y} = \\
&\quad I_1 \ddot{w} + \frac{\partial}{\partial x} \left[I_5 \ddot{\psi}_x - I_6 \frac{\partial \ddot{w}}{\partial x} \right] + \frac{\partial}{\partial y} \left[I_5 \ddot{\psi}_y - I_6 \frac{\partial \ddot{w}}{\partial y} \right] - q,
\end{aligned} \tag{21a}
$$

$$
\begin{aligned}
&-(B_{11} - \Delta B_{11}) \frac{\partial^3 w}{\partial x^3} - (B_{12} + 2B_{66} - \Delta B_{12} - 2\Delta B_{66}) \frac{\partial^3 w}{\partial x \partial y^2} - \\
&\quad (F_{55} - \Delta F_{55}) \frac{\partial w}{\partial x} + (B_{11} - \Delta B_{11}) \frac{\partial^2 \psi_x}{\partial x^2} + (B_{66} - \Delta B_{66}) \frac{\partial^2 \psi_x}{\partial y^2} - \\
&\quad (F_{55} - \Delta F_{55}) \psi_x + (B_{12} + B_{66} - \Delta B_{12} - \Delta B_{66}) \frac{\partial^2 \psi_y}{\partial x \partial y} +
\end{aligned}
$$

$$\sum_{k=1}^{N} e_{31}^{k}\left(\sum_{j=1}^{n} \eta_{2}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial x}\right)-\sum_{k=1}^{N} e_{15}^{k}\left(\sum_{j=1}^{n} \Delta_{4}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial x}\right)=I_{4} \ddot{\psi}_{x}-I_{5} \frac{\partial \ddot{w}}{\partial x}, \tag{21b}$$

$$-(B_{22}-\Delta B_{22}) \frac{\partial^{3} w}{\partial y^{3}}-(B_{12}+2 B_{66}-\Delta B_{12}-2 \Delta B_{66}) \frac{\partial^{3} w}{\partial x^{2} \partial y}-$$
$$(F_{44}-\Delta F_{44}) \frac{\partial w}{\partial y}+(B_{22}-\Delta B_{22}) \frac{\partial^{2} \psi_{y}}{\partial y^{2}}+(B_{66}-\Delta B_{66}) \frac{\partial^{2} \psi_{y}}{\partial x^{2}}-$$
$$(F_{44}-\Delta F_{44}) \psi_{y}+(B_{12}+B_{66}-\Delta B_{12}-\Delta B_{66}) \frac{\partial^{2} \psi_{x}}{\partial x \partial y}+$$
$$\sum_{k=1}^{N} e_{32}^{k}\left(\sum_{j=1}^{n} \eta_{2}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial x}\right)-\sum_{k=1}^{N} e_{24}^{k}\left(\sum_{j=1}^{n} \Delta_{4}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial y}\right)=I_{4} \ddot{\psi}_{y}-I_{5} \frac{\partial \ddot{w}}{\partial y}, \tag{21c}$$

$$(A_{11}-\Delta A_{11}) \frac{\partial^{2} u}{\partial x^{2}}+(A_{12}+A_{66}-\Delta A_{12}-\Delta A_{66}) \frac{\partial^{2} v}{\partial x \partial y}+(A_{66}-\Delta A_{66}) \frac{\partial^{2} u}{\partial y^{2}}+$$
$$\sum_{k=1}^{N} e_{31}^{k}\left(\sum_{j=1}^{n} \eta_{1}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial x}\right)+(A_{11}-\Delta A_{11}) \frac{\partial w}{\partial x} \frac{\partial^{2} w}{\partial x^{2}}+(A_{66}-\Delta A_{66}) \frac{\partial w}{\partial x} \frac{\partial^{2} w}{\partial y^{2}}+$$
$$(A_{12}+A_{66}-\Delta A_{12}-\Delta A_{66}) \frac{\partial w}{\partial y} \frac{\partial^{2} w}{\partial x \partial y}=I_{1} \ddot{u}, \tag{21d}$$

$$(A_{22}-\Delta A_{22}) \frac{\partial^{2} v}{\partial y^{2}}+(A_{12}+A_{66}-\Delta A_{12}-\Delta A_{66}) \frac{\partial^{2} u}{\partial x \partial y}+(A_{66}-\Delta A_{66}) \frac{\partial^{2} v}{\partial x^{2}}+$$
$$\sum_{k=1}^{N} e_{32}^{k}\left(\sum_{j=1}^{n} \eta_{1}^{jk} \frac{\partial \varphi_{j}^{k}}{\partial y}\right)+(A_{22}-\Delta A_{22}) \frac{\partial w}{\partial y} \frac{\partial^{2} w}{\partial y^{2}}+(A_{66}-\Delta A_{66}) \frac{\partial w}{\partial y} \frac{\partial^{2} w}{\partial x^{2}}+$$
$$(A_{12}+A_{66}-\Delta A_{12}-\Delta A_{66}) \frac{\partial w}{\partial x} \frac{\partial^{2} w}{\partial x \partial y}=I_{1} \ddot{v}. \tag{21e}$$

本文考虑四边简支面内不可动、电学接地的层合板, 其边界条件为

$$\begin{cases} x=0, \ a: \ u=v=w=0, \ \psi_{y}=0, \ M_{x}=0, \ \Phi=0, \\ y=0, \ b: \ u=v=w=0, \ \psi_{x}=0, \ M_{y}=0, \ \Phi=0. \end{cases} \tag{22}$$

为满足上述边界条件, 设位移和电势的形式函数为

$$\begin{cases} u(x, y, t)=U_{mn}(t) \sin 2 \alpha_{m} x \sin \beta_{n} y, \\ v(x, y, t)=V_{mn}(t) \sin \alpha_{m} x \sin 2 \beta_{n} y, \\ w(x, y, t)=W_{mn}(t) \sin \alpha_{m} x \sin \beta_{n} y, \\ \psi_{x}(x, y, t)=\psi_{mn}^{x}(t) \cos \alpha_{m} x \sin \beta_{n} y, \\ \psi_{y}(x, y, t)=\psi_{mn}^{y}(t) \sin \alpha_{m} x \cos \beta_{n} y, \\ \varphi_{j}^{k}(x, y, t)=\Phi_{mn}^{jk}(t) \sin \alpha_{m} x \sin \beta_{n} y, \end{cases} \tag{23}$$

式中 $\alpha_{m}=m \pi / a, \ \beta_{m}=m \pi / b$.

假设层合板受正弦横向动载荷作用, 即

$$q(x, y, t)=q_{mn}(t) \sin \alpha_{m} x \sin \beta_{n} y. \tag{24}$$

为书写方便, 采用如下记号:

$$\begin{cases} \Pi_{1}=\cos \alpha_{m} x \sin \beta_{n} y, \quad \Pi_{2}=\cos \alpha_{m} x \cos \beta_{n} y, \quad \Pi_{3}=\sin \alpha_{m} x \sin \beta_{n} y, \\ \Pi_{4}=\sin \alpha_{m} x \cos \beta_{n} y, \quad \Pi_{1}^{21}=\cos 2 \alpha_{m} x \sin \beta_{n} y, \quad \Pi_{2}^{21}=\cos 2 \alpha_{m} x \cos \beta_{n} y, \\ \Pi_{3}^{21}=\sin 2 \alpha_{m} x \sin \beta_{n} y, \quad \Pi_{4}^{21}=\sin 2 \alpha_{m} x \cos \beta_{n} y, \quad \Pi_{1}^{12}=\cos \alpha_{m} x \sin 2 \beta_{n} y, \\ \Pi_{2}^{12}=\cos \alpha_{m} x \cos 2 \beta_{n} y, \quad \Pi_{3}^{12}=\sin \alpha_{m} x \sin 2 \beta_{n} y, \quad \Pi_{4}^{12}=\sin \alpha_{m} x \cos 2 \beta_{n} y. \end{cases} \tag{25}$$

将式(23)和(24)代入方程(21), 对结果方程施加 Galerkin 方法, 即在结果方程中的各式的两边分别依次同乘以 Π_{3}、Π_{1}、Π_{4}、Π_{3}^{21}、Π_{3}^{12}, 并在板的面积区域内及损伤区域内进行积分, 得到

$$\begin{cases} M_{11}\ddot{W}_{mn}+M_{12}\ddot{\psi}^{x}_{mn}+M_{13}\ddot{\psi}^{y}_{mn}+H_{11}W_{mn}+H_{12}\psi^{x}_{mn}+H_{13}\psi^{y}_{mn}+H^{1}_{14}U_{mn}W_{mn}+ \\ \quad H^{1}_{15}U_{mn}W_{mn}+H^{2}_{11}W^{2}_{mn}-\sum_{k=1}^{N}\sum_{j=1}^{n}L^{jk}_{1}\phi^{jk}_{mn}-4\sum_{k=1}^{N}\sum_{j=1}^{n}S^{jk}_{1}\phi^{jk}_{mn}W_{mn}=F_{11}q_{mn}, \\ M_{12}\ddot{W}_{mn}+M_{22}\ddot{\psi}^{x}_{mn}+H_{12}W_{mn}+H_{22}\psi^{x}_{mn}+H_{23}\psi^{y}_{mn}-\sum_{k=1}^{N}\sum_{j=1}^{n}L^{jk}_{2}\phi^{jk}_{mn}=0, \\ M_{13}\ddot{W}_{mn}+M_{33}\ddot{\psi}^{y}_{mn}+H_{13}W_{mn}+H_{23}\psi^{x}_{mn}+H_{33}\psi^{y}_{mn}-\sum_{k=1}^{N}\sum_{j=1}^{n}L^{jk}_{3}\phi^{jk}_{mn}=0, \\ M_{44}\ddot{U}_{mn}+H_{44}U_{mn}+H_{45}V_{mn}+H^{1}_{41}W^{2}_{mn}+\frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{n}L^{jk}_{4}\phi^{jk}_{mn}=0, \\ M_{55}\ddot{V}_{mn}+H_{45}U_{mn}+H_{45}V_{mn}+H^{1}_{51}W^{2}_{mn}+\frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{n}L^{jk}_{5}\phi^{jk}_{mn}=0, \end{cases} \tag{26}$$

式中, M_{ij}、H_{ij}、L^{jk}_{i}、S^{jk}_{i} 的表达式详见附录 A.

2 非线性反馈控制分析

为了能主动、智能地实现对层合板的振动控制与损伤监测, 必须将传感层、作动层和复合材料层组成一个整体, 以形成一个闭环控制系统. 下面基于负速度反馈控制原理, 建立了含压电传感层和作动层的压电智能层合板的主动控制和损伤监测的力电耦合模型.

压电层电势采用一个线性插值函数. 设压电传感层处于第 k 层, 根据 Liu 等人和 Reddy[13] 的工作, 其产生的电荷总量为

$$q(t)=\frac{1}{2}\left[\int_{S_2(z=z_{k-1})}D_z\mathrm{d}S+\int_{S_2(z=z_k)}D_z\mathrm{d}S\right], \tag{27}$$

其中, S_2 为传感层的有效表面电极, 具体定义见上述文献. 则传感层的输出电压可由上述电荷除以传感层电容 C_p[14] 得到

$$V_S(t)=\frac{q(t)}{C_p}=P_0(P_1W_{mn}+P_2\psi^{x}_{mn}+P_3\psi^{y}_{mn}+P_4U_{mn}+P_5V_{mn}), \tag{28}$$

式中 P_i 见附录 A.

设处于第 k 层的压电作动层上表面接地. 当采用负速度反馈控制时, 施加于作动层下表面的反馈作动电压为

$$\varphi^{k}_{2}(x,y,t)=-G_V\frac{\mathrm{d}V_s}{\mathrm{d}t}\sin\alpha_m x\sin\beta_n y=\phi^{2k}_{mn}(t)\sin\alpha_m x\sin\beta_n y, \tag{29}$$

式中, G_V 是负速度反馈控制增益. 由式(28)和(29), 可以得到

$$\phi^{2k}_{mn}(t)=-G_VP_0(P_1\dot{W}_{mn}+P_2\dot{\psi}^{x}_{mn}+P_3\dot{\psi}^{y}_{mn}+P_4\dot{U}_{mn}+P_5\dot{V}_{mn}). \tag{30}$$

将式(30)代入方程组(26), 得非线性控制方程为

$$\boldsymbol{M}_{mn}\ddot{\boldsymbol{X}}_{mn}+\boldsymbol{D}_{mn}\dot{\boldsymbol{X}}_{mn}+\boldsymbol{K}_{mn}\boldsymbol{X}_{mn}=\boldsymbol{Z}_{mn}, \tag{31}$$

式中

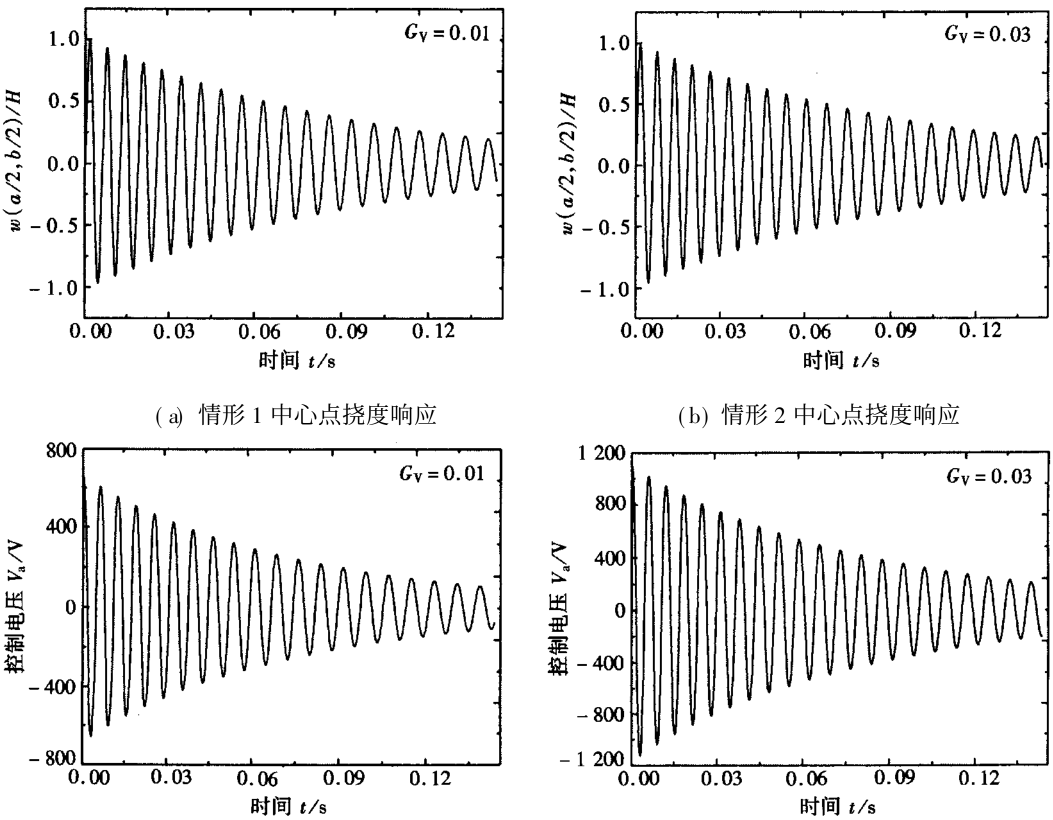
$$\begin{cases} \boldsymbol{X}_{mn}=\left\{W_{mn}(t)\quad \psi^{x}_{mn}(t)\quad \psi^{y}_{mn}(t)\quad U_{mn}(t)\quad V_{mn}(t)\right\}^{\mathrm{T}}, \\ \boldsymbol{D}_{mn}=G_VP_0\left\{L^{2N}_{1}\quad L^{2N}_{2}\quad L^{2N}_{3}\quad L^{2N}_{4}\quad L^{2N}_{5}\right\}^{\mathrm{T}}\cdot\left\{P_1\quad P_2\quad P_3\quad P_4\quad P_5\right\}, \\ K_{11}=H_{11}+H^{1}_{14}U_{mn}+H^{1}_{15}V_{mn}+H^{2}_{11}W^{2}_{mn}-4S^{11}_{1}P_0(P_1W_{mn}+P_2\psi^{x}_{mn}+ \\ \quad P_3\psi^{y}_{mn}+P_4U_{mn}+P_5V_{mn})-4S^{2N}_{1}G_VP_0(P_1\dot{W}_{mn}+P_2\dot{\psi}^{x}_{mn}+P_3\dot{\psi}^{y}_{mn}+ \\ \quad P_4\dot{U}_{mn}+P_5\dot{V}_{mn}), \end{cases} \tag{32a}$$

$$\begin{cases} K_{12}=H_{12},\ K_{13}=H_{13},\ K_{14}=K_{15}=0,\ K_{21}=H_{12},\ K_{22}=H_{22},\ K_{23}=H_{23}, \\ K_{24}=K_{25}=0,\ K_{31}=H_{13},\ K_{32}=H_{23},\ K_{33}=H_{33},\ K_{34}=K_{35}=0, \\ K_{41}=H^{1}_{41}W_{mn},\ K_{42}=K_{43}=0,\ K_{44}=H_{44},\ K_{45}=H_{45},\ K_{51}=H^{1}_{51}W_{mn}, \\ K_{52}=K_{53}=0,\ K_{54}=H_{45}, K_{55}=H_{55},\ \boldsymbol{Z}_{mn}=\begin{Bmatrix} F_{mn} & 0 & 0 & 0 & 0 \end{Bmatrix}^{\mathrm{T}}, \end{cases} \tag{32b}$$

其中, $\boldsymbol{D}_{mn}$ 是反馈增益矩阵, 即等效阻尼矩阵． 对于非线性反馈控制方程组(31), 本文采用 Newmark-β 直接积分方法求解, 且取参数 γ 和 β 分别为 0.5 和 0.25.

3 数值算例与讨论

算例中, 复合材料层与压电层分别选用石墨/环氧复合材料和压电陶瓷 PZT-5A, 各自材料常数在附录 B 中给出, 且假设各材料不显示任何材料阻尼． 除非有特别说明, 复合材料层铺设角度选为(0/90/90/0), 压电传感层与作动层分别粘贴在层合板的上下表面, 且压电层材料主方向与直角坐标系一致; 层合板的几何参数为 $a=b=0.4$ m, $a/H=100$, 各复合材料层的厚度均为 $H/5$, 压电层的厚度均为 $H/10$． 在所有算例中, 波数取 $m=n=1$.



(a) 情形 1 中心点挠度响应　　(b) 情形 2 中心点挠度响应

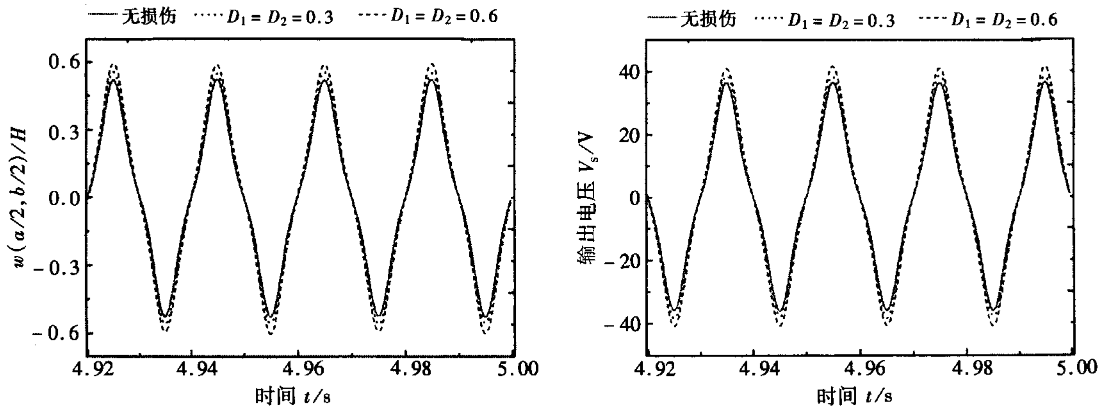
(c) 情形 1 施加的控制电压　　(d) 情形 2 施加的控制电压

图 2　压电层位置对板振动控制的影响

3.1 非线性反馈控制

为讨论压电层所在位置对层合板振动控制的影响, 研究了两种情形 (1: [P/-45/45/45/-45/P] 和 2: [-45/P/45/45/P/-45]), 所得板中心点挠度的响应曲线及作动层上所需施加的控制电压如图 2 所示． 由图 2(a)、(b) 可以看出, 情形 1 中控制增益取为 0.01 与情形 2 中控制增益取为 0.03 时两者所得到的振动抑制效果相似． 但由图 2(c)、(d) 可知, 情形 1 所需施加的

控制电压要比情形 2 的低, 即前者单位电压的控制效果优于情形 2, 其控制成本低. 因此, 压电层的位置对压电智能层合板的行为有重要的影响, 对其进行优化设计是重要的. 对本算例而言, 当压电层粘贴于层合板的上下表面时, 控制效果最好.



(a) 对振动的影响　　(b) 对输出电压的影响

图 3　损伤主变量对振动和输出电压的影响

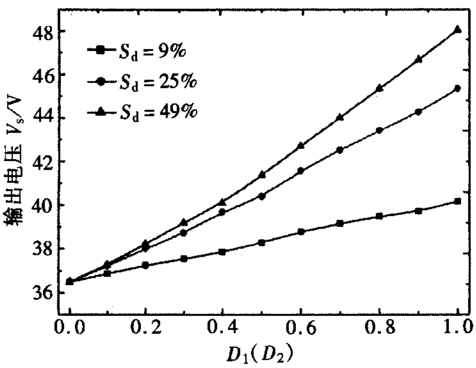
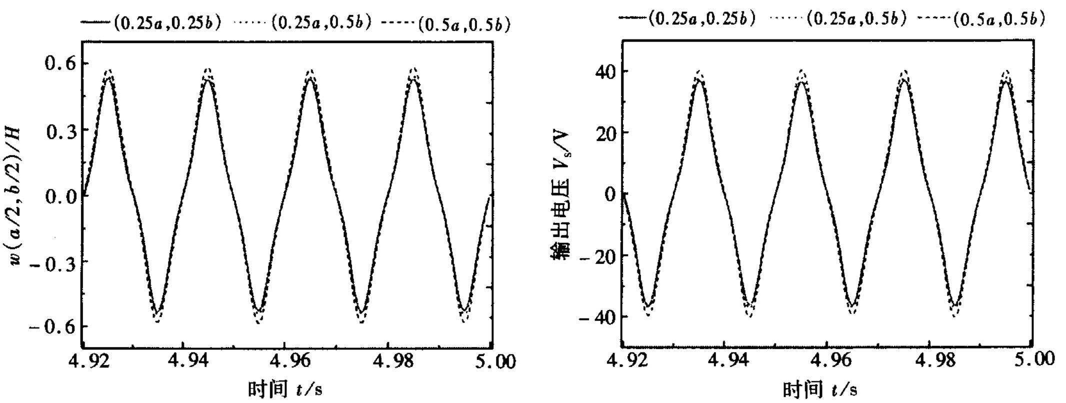


图 4　损伤程度对输出电压的影响

3.2　损伤监测

图 3(a) 显示了在一均匀分布正弦时激励下, 损伤主变量不同时, 层合板中心点挠度的稳态响应 (载荷幅值为 6 kPa, 激振频率为 50 Hz). 取控制增益为 0. 假设损伤位于中间两层的中心, 损伤面积 S_d = 25% (除以板面积以无量纲化). 相应的传感层输出电压见图 3 (b). 由图 3(a) 和图 3(b) 可知, 考虑损伤时, 板的稳态响应幅值增大, 相应的输出电压也增大.

在上述算例基础上, 图 4 显示了传感层输出电压与损伤主变量、损伤面积的关系. 可以看出,



(a) 对振动的影响　　(b) 对输出电压的影响

图 5　损伤中心位置对振动和输出电压的影响

当损伤面积给定时, 随着损伤主变量的增大, 输出电压随之近乎线性增大, 且变化较明显; 当损伤主变量不变时, 输出电压随损伤面积的增大而增大. 因此, 由图 4 可知, 传感层输出电

压随损伤程度的加剧而增大.

图4 提供了一种对结构损伤进行监测的方法, 通过对比施加相同载荷下压电传感层的输出电压, 可以大致估计损伤的程度.

在施加与图3 相同的载荷, 且设损伤位于中间两层, 损伤主变量 $D_1 = D_2 = 0.5$, 损伤面积 $S_d = 25\%$ 的条件下, 损伤中心位置不同时各自的板中心点挠度的稳态响应及输出电压见图 5 (a)、(b). 可以看出, 板的振动及传感层输出电压对位于板中心的损伤更为敏感.

4 结 论

基于 Hamilton 原理、高阶剪切变形板理论、von Kûrmûn 型几何非线性应变-位移关系以及应变能等效原理, 考虑压电层的质量和刚度及复合材料层内的损伤效应, 本文建立了具损伤压电智能层合板的非线性运动方程. 通过采用耦合了正、逆压电效应的负速度反馈控制原理, 形成闭环控制回路, 实现了对压电智能层合板的主动控制和损伤监测. 数值结果表明, 非线性效应、压电层位置对压电智能层合板的振动控制有较大影响. 同时, 本文也讨论了损伤程度及位置对传感层输出电压的影响, 提出了一种损伤监测的方法.

附 录 A

(a) 式(26) 中 M_{ij}、H_{ij}、L_i^{jk}、S_i^{jk} 的表达式:

$$M_{11} = [-I_1 - I_6(\alpha_m^2 + \beta_n^2)]\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\mathrm{d}x,\quad M_{12} = I_5\alpha_m\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\mathrm{d}x,\quad M_{13} = I_5\beta_n\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\mathrm{d}x,$$

$$M_{22} = -I_4\int_0^a\int_0^b \Pi_1\Pi_1\mathrm{d}y\mathrm{d}x,\quad M_{33} = -I_4\int_0^a\int_0^b \Pi_4\Pi_4\mathrm{d}y\mathrm{d}x,\quad M_{44} = -I_1\int_0^a\int_0^b \Pi_3^{21}\Pi_3^{21}\mathrm{d}y\mathrm{d}x,$$

$$M_{55} = -I_1\int_0^a\int_0^b \Pi_3^{12}\Pi_3^{12}\mathrm{d}y\mathrm{d}x;$$

$$H_{11} = -\alpha_m^4\left(\int_0^a\int_0^b D_{11}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta D_{11}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right) -$$
$$\alpha_m^2\beta_n^2\left[\int_0^a\int_0^b(4D_{66}+2D_{12})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(4\Delta D_{66}+2\Delta D_{12})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right] -$$
$$\beta_n^4\left(\int_0^a\int_0^b D_{22}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta D_{22}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right) - \beta_n^2\left(\int_0^a\int_0^b F_{44}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x -\right.$$
$$\left.\sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{44}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right) - \alpha_m^2\left(\int_0^a\int_0^b F_{55}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{55}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right),$$

$$H_{12} = \alpha_m^3\left(\int_0^a\int_0^b B_{11}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{11}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right) +$$
$$\alpha_m\beta_n^2\left[\int_0^a\int_0^b(B_{12}+2B_{66})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta B_{12}+2\Delta B_{66})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right] -$$
$$\alpha_m\left(\int_0^a\int_0^b F_{55}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{55}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right),$$

$$H_{13} = \beta_n^3\left(\int_0^a\int_0^b B_{22}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{22}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right) +$$
$$\alpha_m^2\beta_n\left[\int_0^a\int_0^b(B_{12}+2B_{66})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta B_{12}+2\Delta B_{66})\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right] -$$
$$\beta_n\left(\int_0^a\int_0^b F_{44}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x - \sum_{k=1}^N\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{44}\Pi_3\Pi_3\mathrm{d}y\mathrm{d}x\right),$$

$$H_{14}^{1}=-2\alpha_m^3\left(\int_0^a\int_0^b A_{11}\Pi_3\Pi_3\Pi_1^{21}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{11}\Pi_3\Pi_3\Pi_1^{21}\mathrm{d}y\mathrm{d}x\right)-$$
$$2\alpha_m\beta_n^2\left(\int_0^a\int_0^b A_{12}\Pi_3\Pi_3\Pi_1^{21}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{12}\Pi_3\Pi_3\Pi_1^{21}\mathrm{d}y\mathrm{d}x\right)+$$
$$2\alpha_m\beta_n^2\left(\int_0^a\int_0^b A_{66}\Pi_2\Pi_3\Pi_4^{21}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_2\Pi_3\Pi_4^{21}\mathrm{d}y\mathrm{d}x\right),$$

$$H_{15}^{1}=-2\beta_n^3\left(\int_0^a\int_0^b A_{22}\Pi_3\Pi_3\Pi_4^{12}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{22}\Pi_3\Pi_3\Pi_4^{12}\mathrm{d}y\mathrm{d}x\right)-$$
$$2\alpha_m^2\beta_n\left(\int_0^a\int_0^b A_{12}\Pi_3\Pi_3\Pi_4^{12}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{12}\Pi_3\Pi_3\Pi_4^{12}\mathrm{d}y\mathrm{d}x\right)+$$
$$2\alpha_m^2\beta_n\left(\int_0^a\int_0^b A_{66}\Pi_2\Pi_3\Pi_1^{12}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_2\Pi_3\Pi_1^{12}\mathrm{d}y\mathrm{d}x\right),$$

$$H_{11}^{2}=-\frac{1}{2}\alpha_m^4\left(\int_0^a\int_0^b A_{11}\Pi_3\Pi_3\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{11}\Pi_3\Pi_3\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right)-$$
$$\frac{1}{2}\alpha_m^2\beta_n^2\left(\int_0^a\int_0^b A_{12}\Pi_3\Pi_3\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{12}\Pi_3\Pi_3\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right)-$$
$$\frac{1}{2}\beta_n^4\left(\int_0^a\int_0^b A_{22}\Pi_3\Pi_3\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{22}\Pi_3\Pi_3\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x\right)-$$
$$\frac{1}{2}\alpha_m^2\beta_n^2\left(\int_0^a\int_0^b A_{12}\Pi_3\Pi_3\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{12}\Pi_3\Pi_3\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x\right)+$$
$$2\alpha_m^2\beta_n^2\left(\int_0^a\int_0^b A_{66}\Pi_1\Pi_2\Pi_3\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_1\Pi_2\Pi_3\Pi_4\mathrm{d}y\mathrm{d}x\right),$$

$$H_{22}=-\alpha_m^2\left(\int_0^a\int_0^b B_{11}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{11}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right)-\beta_n^2\left(\int_0^a\int_0^b B_{66}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\right.$$
$$\left.\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{66}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right)-\left(\int_0^a\int_0^b F_{55}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{55}\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right),$$

$$H_{23}=-\alpha_m\beta_n\left(\int_0^a\int_0^b(B_{12}+B_{66})\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta B_{12}+\Delta B_{66})\Pi_1\Pi_1\mathrm{d}y\mathrm{d}x\right),$$

$$H_{33}=-\beta_n^2\left(\int_0^a\int_0^b B_{22}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{22}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x\right)-\alpha_m^2\left(\int_0^a\int_0^b B_{66}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x-\right.$$
$$\left.\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta B_{66}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x\right)-\left(\int_0^a\int_0^b F_{44}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta F_{44}\Pi_4\Pi_4\mathrm{d}y\mathrm{d}x\right),$$

$$H_{44}=-4\alpha_m^2\left(\int_0^a\int_0^b A_{11}\Pi_3^{21}\Pi_3^{21}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{11}\Pi_3^{21}\Pi_3^{21}\mathrm{d}y\mathrm{d}x\right)-$$
$$\beta_n^2\left(\int_0^a\int_0^b A_{66}\Pi_3^{21}\Pi_3^{21}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_3^{21}\Pi_3^{21}\mathrm{d}y\mathrm{d}x\right),$$

$$H_{45}=2\alpha_m\beta_n\left(\int_0^a\int_0^b(A_{12}+A_{66})\Pi_3^{21}\Pi_2^{12}\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta A_{12}+\Delta A_{66})\Pi_3^{21}\Pi_2^{12}\mathrm{d}y\mathrm{d}x\right),$$

$$H_{41}^{1}=-\alpha_m^3\left(\int_0^a\int_0^b A_{11}\Pi_3^{21}\Pi_1\Pi_3\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{11}\Pi_3^{21}\Pi_1\Pi_3\mathrm{d}y\mathrm{d}x\right)-$$
$$\alpha_m\beta_n^2\left(\int_0^a\int_0^b A_{66}\Pi_3^{21}\Pi_1\Pi_3\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_3^{21}\Pi_1\Pi_3\mathrm{d}y\mathrm{d}x\right)+$$
$$\alpha_m\beta_n^2\left(\int_0^a\int_0^b(A_{12}+A_{66})\Pi_3^{21}\Pi_2\Pi_4\mathrm{d}y\mathrm{d}x-\sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta A_{12}+\Delta A_{66})\Pi_2^{21}\Pi_2\Pi_3\mathrm{d}y\mathrm{d}x\right),$$

$$H_{55} = -4\beta_n^2\left(\int_0^a\int_0^b A_{22}\Pi_3^{12}\Pi_3^{12}\mathrm{d}y\,\mathrm{d}x - \sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{22}\Pi_3^{12}\Pi_3^{12}\mathrm{d}y\,\mathrm{d}x\right) -$$
$$\alpha_m^2\left(\int_0^a\int_0^b A_{66}\Pi_3^{12}\Pi_3^{12}\mathrm{d}y\,\mathrm{d}x - \sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_3^{12}\Pi_3^{12}\mathrm{d}y\,\mathrm{d}x\right),$$

$$H_{51}^1 = -\beta_n^3\left(\int_0^a\int_0^b A_{22}\Pi_3^{12}\Pi_4\Pi_3\mathrm{d}y\,\mathrm{d}x - \sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{22}\Pi_3^{12}\Pi_4\Pi_3\mathrm{d}y\,\mathrm{d}x\right) -$$
$$\alpha_m^2\beta_n\left(\int_0^a\int_0^b A_{66}\Pi_3^{12}\Pi_4\Pi_3\mathrm{d}y\,\mathrm{d}x - \sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}\Delta A_{66}\Pi_3^{12}\Pi_4\Pi_3\mathrm{d}y\,\mathrm{d}x\right) +$$
$$\alpha_m^2\beta_n\left(\int_0^a\int_0^b (A_{12}+A_{66})\Pi_3^{12}\Pi_1\Pi_2\mathrm{d}y\,\mathrm{d}x - \sum_{k=1}^{N}\int_{a_1^{(k)}}^{a_2^{(k)}}\int_{b_1^{(k)}}^{b_2^{(k)}}(\Delta A_{12}+\Delta A_{66})\Pi_3^{12}\Pi_1\Pi_2\mathrm{d}y\,\mathrm{d}x\right);$$

$$L_0^{ijk} = (k_{11}T_i^{jk}\alpha_m^2 + k_{22}T_i^{jk}\beta_n^2 + k_{33}R_i^{jk})\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\,\mathrm{d}x,$$

$$L_1^{jk} = [(e_{31}^k\eta_3^{jk} + e_{15}^k\Delta_4^{jk})\alpha_m^2 + (e_{32}^k\eta_3^{jk} + e_{24}^k\Delta_4^{jk})\beta_n^2]\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\,\mathrm{d}x,$$

$$L_2^{jk} = -(e_{31}^k\eta_2^{jk} - e_{15}^k\Delta_4^{jk})\alpha_m\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\,\mathrm{d}x,\quad L_3^{jk} = -(e_{32}^k\eta_2^{jk} - e_{24}^k\Delta_4^{jk})\beta_n\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\,\mathrm{d}x,$$

$$L_4^{jk} = 2e_{31}^k\eta_1^{jk}\alpha_m\int_0^a\int_0^b \Pi_3\Pi_1^{21}\mathrm{d}y\,\mathrm{d}x,\quad L_5^{jk} = 2e_{32}^k\eta_1^{jk}\beta_n\int_0^a\int_0^b \Pi_3\Pi_4^{12}\mathrm{d}y\,\mathrm{d}x;$$

$$S_1^{jk} = \frac{1}{2}(e_{31}^k\eta_1^{jk}\alpha_m^2 + e_{32}^k\eta_1^{jk}\beta_n^2)\int_0^a\int_0^b \Pi_3\Pi_1\Pi_1\mathrm{d}y\,\mathrm{d}x;\quad F_{11} = -\int_0^a\int_0^b \Pi_3\Pi_3\mathrm{d}y\,\mathrm{d}x.$$

(b) 式(28) 中 P_i 的表达式: (其中 h_{P} 为压电层厚度)

$$P_0 = \frac{h_{\mathrm{P}}}{k_{33}ab},\quad P_1 = \frac{1}{2}(g_2(z_{k-1}) + g_2(z_k))(e_{31}\alpha_m^2 + e_{32}\beta_n^2)\int_0^a\int_0^b \Pi_3\mathrm{d}y\,\mathrm{d}x,$$

$$P_2 = -\frac{1}{2}(g_1(z_{k-1}) + g_1(z_k))e_{31}\alpha_m\int_0^a\int_0^b \Pi_3\mathrm{d}y\,\mathrm{d}x,\quad P_3 = -\frac{1}{2}(g_1(z_{k-1}) + g_1(z_k))e_{32}\beta_n\int_0^a\int_0^b \Pi_3\mathrm{d}y\,\mathrm{d}x,$$

$$P_4 = 2e_{31}\alpha_m\int_0^a\int_0^b \Pi_1^{21}\mathrm{d}y\,\mathrm{d}x,\quad P_5 = 2e_{32}\beta_n\int_0^a\int_0^b \Pi_4^{12}\mathrm{d}y\,\mathrm{d}x.$$

附 录 B

(a) 石墨/环氧复合材料参数:

$E_1 = 181$ GPa, $E_1 = E_2 = 10.3$ GPa, $G_{12} = G_{13} = 7.17$ GPa, $G_{23} = 2.87$ GPa,

$\mu_{12} = \mu_{13} = 0.28$, $\mu_{23} = 0.33$, $\rho = 1\,580$ kg/m^3.

(b) 压电陶瓷 PZT-5A 材料参数:

$E_{11} = E_{22} = 61.0$ GPa, $E_{33} = 53.2$ GPa, $\mu_{12} = 0.35$, $\mu_{13} = \mu_{23} = 0.38$, $G_{12} = 22.6$ GPa,

$G_{13} = G_{23} = 21.1$ GPa, $\rho = 7\,750$ kg/m^3, $e_{31} = e_{13} = 7.209$ C/m^2, $e_{33} = 15.118$ C/m^2,

$e_{24} = e_{15} = 12.72$ C/m^2, $\varepsilon_{11} = \varepsilon_{22} = 1.53\times10^{-8}$ F/m, $\varepsilon_{33} = 1.5\times10^{-8}$ F/m.

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Nonlinear Active Control and Damage Detection of Piezoelectric Smart Laminated Plates With Damage

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Abstract: Based on Hamilton's principle, the higher-order shear deformation plate theory, von Kûrmûn type geometrically nonlinear strain-displacement relations and the strain energy equivalence theory, considering the mass and stiffness of the piezoelectric layers and the damage effect of the composite layers, the nonlinear dynamic equations of the piezoelectric smart laminated plates with damage are derived. A negative velocity feedback control algorithm coupling the direct and converse piezoelectric effects was used to realize the active control and damage detection of the piezoelectric smart laminated plates through a closed control loop. Numerical examples for simply supported rectangular laminated plates with immovable edges were presented. And the influences of locations of the piezoelectric layers on the vibration control were investigated. Also, the effects of the degree and location of the damage on the sensor output voltage were discussed. And a way of damage detection is introduced.

Key words: piezoelectric smart laminated plates; nonlinear vibration; damage effect; piezoelectric effect; active control; damage detection