

# 导电薄板的磁弹性组合共振分析\*

胡宇达<sup>1</sup>, 李晶<sup>1,2</sup>

(1.燕山大学 建筑工程与力学学院,河北 秦皇岛 066004;

2.唐山学院,河北 唐山 063000)

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**摘要:** 基于 Maxwell 方程,给出了导电薄板的非线性磁弹性振动方程、电动力学方程和电磁力表达式.在此基础上,研究了横向磁场中梁式导电薄板的磁弹性组合共振问题,应用 Galerkin 法导出了相应的非线性振动微分方程组.利用多尺度法进行求解,得到了系统稳态运动下的幅频响应方程,分析了组合共振激发的条件.根据 Liapunov 近似稳定性理论,对稳态解的稳定性进行了分析,得到了稳定性的判定条件.通过数值计算,给出了一、二阶模态下共振幅随调谐参数、激励幅值和磁场强度的变化规律曲线图,以及系统振动的时程响应图、相图、Poincaré 映射图和频谱图,进一步分析了电磁、机械等参量对解的稳定性及分岔特性的影响,并讨论了系统的倍周期和概周期等复杂动力学行为.

**关键词:** 磁弹性; 导电薄板; 组合共振; 稳定性; 多尺度法

**中图分类号:** O322;O442      **文献标识码:** A

## 引言

随着航空航天、核工业、磁悬浮运输、机电动力系统及大型设备等现代科技领域的快速发展,导电导磁结构件的应用愈来愈广,而对复杂电磁场环境中系统力学特性的分析,也日益引起人们的广泛关注.因问题的研究需考虑到电磁场、机械场等多场间的相互作用,从而在模型建立和方程求解方面具有一定难度,是当前研究的热点问题,具有理论和实际意义.

近几十年,电磁弹性力学理论得到了较快的发展,并取得了许多重要成果. Pao 等人<sup>[1-3]</sup>是用理性力学与公理化体系方法建立电磁弹性力学理论的奠基者,文献[4-6]分别从理论和实验的角度研究了铁磁梁的动力稳定性问题,文献[7-8]对磁场和热载荷作用下薄板的动、静态稳定性进行了研究.在导电材料板壳磁弹性振动问题的理论研究中, Амбарцумян 等人<sup>[9-10]</sup>做了一些开创性的工作,得到了很多有价值的成果.文献[11-12]分别对电流和脉冲磁场作用下导电板的动力学特性进行了分析,文献[13]研究了大型发电机端部绕组的电磁弹性共振与分岔问题.因影响系统运动的因素较多,作用形式也较为复杂,尚有许多问题有待于做进一步的研究,而磁弹性耦合系统的非线性振动、分岔以及混沌动力学问题是一个主要研究内容.

在文献[14-15]的基础上,本文首先给出了磁场环境下导电薄板的非线性磁弹性振动方程和电磁场方程,并对横向磁场中薄板的磁弹性组合共振与稳定性问题进行了研究.应用多尺

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作者简介: 胡宇达(1968—),男,黑龙江人,教授,博士(联系人. Tel: + 86-335-8074576; E-mail: huyuda03@163.com).

度法,导出了共振下的幅频响应方程,分析了解的稳定性.通过算例,给出了相应的振幅特性曲线图和动力响应图、Poincaré映射图等,并对电磁、机械等参量的影响效应进行了分析.

## 1 导电薄板的非线性磁弹性振动方程

对于图1所示外加磁场  $\mathbf{B}(B_{0x}, B_{0y}, B_{0z})$  环境中运动的导电薄板,考虑机械场与电磁场的相互作用,根据虚功原理,可建立直角坐标系  $Oxyz$  ( $xy$  为板的中面,  $z$  为法向)下薄板的非线性磁弹性振动方程<sup>[14]</sup>:

$$N_{x,x} + N_{xy,y} + F_x + P_x = \rho h u_{,t}, \quad (1)$$

$$N_{y,y} + N_{xy,x} + F_y + P_y = \rho h v_{,t}, \quad (2)$$

$$\begin{aligned} M_{x,xx} + M_{y,yy} + 2M_{xy,xy} + (N_x w_{,x} + \\ N_{xy} w_{,y})_{,x} + (N_y w_{,y} + N_{xy} w_{,x})_{,y} + \\ m_{x,x} + m_{y,y} + F_z + P_z = \\ \rho h w_{,t}, \end{aligned} \quad (3)$$

式中,  $N_x, N_y, N_{xy}$  为中面内力,  $M_x, M_y, M_{xy}$  为弯曲内力,  $P_x, P_y, P_z$  为机械载荷,  $F_x, F_y, F_z$  为电磁力,  $m_x, m_y$  为电磁力矩,  $\rho$  为材料密度,  $h$  为板厚,  $t$  为时间变量.

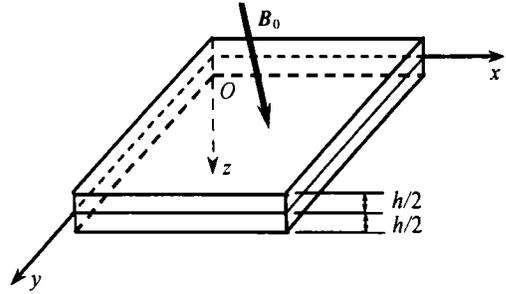


图1 磁场中的导电薄板

考虑非极化、非磁化的良导体材料运动薄板,其内部介质各电磁量满足的 Maxwell 方程为

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = 0, \quad \nabla \times \mathbf{E} = -\mathbf{B}_{,t}, \quad \nabla \times \mathbf{H} = \mathbf{J}; \quad (4)$$

所受 Lorentz 力矢量表达式为

$$\mathbf{f}(f_x, f_y, f_z) = \mathbf{J} \times \mathbf{B}; \quad (5)$$

电磁关系式为

$$\mathbf{B} = \epsilon_0 \mathbf{E}, \quad \mathbf{B} = \mu_0 \mathbf{H}, \quad \mathbf{J} = \sigma(\mathbf{E} + \mathbf{u}_{,t} \times \mathbf{B}); \quad (6)$$

式中,  $\mathbf{B}(B_x, B_y, B_z)$  为磁感应强度矢量,  $\mathbf{H}(H_x, H_y, H_z)$  为磁场强度矢量,  $\mathbf{J}(J_x, J_y, J_z)$  为电流密度矢量,  $\mathbf{D}(D_x, D_y, D_z)$  为电位移矢量,  $\mathbf{E}(E_x, E_y, E_z)$  为电场强度矢量,  $\mathbf{u}(u_x, u_y, u_z)$  为体内各点的位移矢量,  $\sigma$  为电导率,  $\epsilon_0$  为真空介电常数,  $\mu_0$  为真空磁导率, 符号  $\nabla$  为 Hamilton 算子.

当薄板在外界磁场  $\mathbf{B}_0(B_{0x}, B_{0y}, B_{0z})$  环境下运动时,体内的各电磁量可表示为<sup>[9,14]</sup>

$$\mathbf{H}(H_x, H_y, H_z) = \mathbf{H}_0(H_{0x}, H_{0y}, H_{0z}) + \mathbf{h}(h_x, h_y, h_z), \quad (7a)$$

$$\mathbf{B}(B_x, B_y, B_z) = \mathbf{B}_0(B_{0x}, B_{0y}, B_{0z}) + \mathbf{b}(b_x, b_y, b_z), \quad (7b)$$

$$\mathbf{E}(E_x, E_y, E_z) = \mathbf{e}(e_x, e_y, e_z), \quad (7c)$$

$$\mathbf{D}(D_x, D_y, D_z) = \mathbf{d}(d_x, d_y, d_z), \quad (7d)$$

式中,  $\mathbf{h}, \mathbf{b}, \mathbf{e}, \mathbf{d}$  为扰动后激发产生的各电磁矢量. 而  $\mathbf{B}_0$  满足静磁场方程:

$$\nabla \cdot \mathbf{B}_0 = 0, \quad \nabla \times \mathbf{B}_0 = 0. \quad (8)$$

这样,在薄体磁弹性基本假设<sup>[9]</sup>基础上,联立方程(4)~(6),并对  $z$  从  $-h/2$  到  $h/2$  进行积分,可推得如下磁场作用下导电薄板的电动力学方程和电磁力表达式:

电动力学方程

$$h_{z,x} + \sigma(e_y - u_{,t} B_{0z} + w_{,t} B_{0x}) = (h_x^+ - h_x^-)/h, \quad (9)$$

$$h_{z,y} - \sigma(e_x + v_{,t} B_{0z} - w_{,t} B_{0y}) = (h_y^+ - h_y^-)/h, \quad (10)$$

$$e_{y,x} - e_{x,y} + \mu_0 h_{z,t} = 0; \quad (11)$$

电磁力(力矩)表达式

$$F_x = \sigma h B_{0z} (e_y - u_{,t} B_{0z} + w_{,t} B_{0x}), \quad (12)$$

$$F_y = -\sigma h B_{0z}(e_x + v_{,t} B_{0z} - w_{,t} B_{0y}), \quad (13)$$

$$F_z = \sigma h B_{0y}(e_x + v_{,t} B_{0z} - w_{,t} B_{0y}) - \sigma h B_{0x}(e_y - u_{,t} B_{0z} + w_{,t} B_{0x}), \quad (14)$$

$$m_x = \sigma h^3 B_{0z}^2 w_{,xx}/12, \quad m_y = \sigma h^3 B_{0z}^2 w_{,yy}/12; \quad (15)$$

上面各式中,  $u, v, w$  分别为沿坐标  $x, y, z$  方向的中面位移分量,  $h_x^+ = h_x(x, y, h/2, t), h_x^- = h_x(x, y, -h/2, t)$  为板面处的扰动磁感应强度分量。

## 2 横向磁场中导电薄板的组合共振问题

研究恒定横向磁场  $B_0(0, 0, B_{0z})$  中受法向动载  $P_z$  作用的两对边 ( $x = 0, l$ ) 简支梁式导电金属薄板, 并设  $P_z = P_0[\sin(\pi x/l) + \sin(2\pi x/l)]\cos\Omega t$ 。因各量与  $y$  无关, 则上面给出的振动方程及电磁方程可得到相应的简化, 这样可推得如下仅关于挠度  $w$  的非线性磁弹性振动方程:

$$D_M w_{,xxxx} + \rho h w_{,tt} - 3D_N (w_{,x})^2 w_{,xx}/2 - \sigma h^3 B_{0z}^2 w_{,xxx}/12 - P_0[\sin(\pi x/l) + \sin(2\pi x/l)]\cos\Omega t = 0, \quad (16)$$

式中,  $D_N = Eh/(1-\nu^2)$  为拉伸刚度,  $D_M = Eh^3/(1-\nu^2)/12$  为弯曲刚度,  $E$  为弹性模量,  $\nu$  为 Poisson 系数,  $P_0$  为激励幅值,  $\Omega$  为激励频率。

在考虑一、二阶模态情况下, 可将满足边界条件的位移解展为如下形式:

$$w = P_1(t)\sin(\pi x/l) + P_2(t)\sin(2\pi x/l).$$

将上式代入方程(16)中, 并应用 Galerkin 积分法, 可推得如下以位移形式表示的无量纲化两自由度振动微分方程组:

$$\dot{q}_1 + q_1 = -\varepsilon\eta_1 \dot{q}_1 - \varepsilon\eta_{12} q_1 q_2^2 - \varepsilon\eta_2 q_1^3 + \eta_3 \cos\Omega_0 \tau, \quad (17)$$

$$\dot{q}_2 + 16q_2 = -4\varepsilon\eta_1 \dot{q}_2 - \varepsilon\eta_{12} q_1^2 q_2 - 16\varepsilon\eta_2 q_2^3 + \eta_3 \cos\Omega_0 \tau, \quad (18)$$

式中,  $\eta_1 = \sigma h^2 B_{0z}^2 (\pi/l)^2 / (12\rho\omega_n^2 \varepsilon)$ ,  $\eta_2 = 3D_N h (\pi/l)^4 / (8\rho\omega_n^3 \varepsilon)$ ,  $\eta_3 = P_0 / (\rho h^2 \omega_n^2)$ ,  $\eta_{12} = 3D_N h (\pi/l)^4 / (\rho\omega_n^2 \varepsilon)$ ,  $\omega_n = (\pi/l)^2 \sqrt{D_M/\rho h}$ ; 无量纲量  $\tau = \omega_n t$ ,  $q_1 = P_1/h$ ,  $q_2 = P_2/h$ ,  $\Omega_0 = \Omega/\omega_n$ ;  $\dot{q}_1, \ddot{q}_1$  和  $\dot{q}_2, \ddot{q}_2$  分别表示对  $\tau$  的一阶和二阶导数,  $\varepsilon$  为小参数。

当频率之间满足如下关系式时, 将会激发起系统的组合共振,

$$2\Omega_0 = \omega_1 + \omega_2 + \varepsilon\delta = 5 + \varepsilon\delta, \quad (19)$$

这里,  $\omega_1 = 1$  和  $\omega_2 = 4$  为对应式(17)、(18)的一、二阶无量纲化线性固有频率,  $\delta$  为调谐参数。

下面利用多尺度法<sup>[16-17]</sup>进行近似解析求解。在考虑1阶近似情况下, 设

$$q_m(\tau, \varepsilon) = q_{m0}(T_0, T_1) + \varepsilon q_{m1}(T_0, T_1), \quad (20)$$

这里, 引入的时间尺度  $T_0 = \tau, T_1 = \varepsilon\tau, m = 1, 2$  (下同)。

将式(20)代入方程(17)、(18)中, 令  $\varepsilon$  的同次幂项系数相等, 得到

$$D_0^2 q_{10} + q_{10} = \eta_3 \cos\Omega_0 T_0, \quad (21)$$

$$D_0^2 q_{20} + 16q_{20} = \eta_3 \cos\Omega_0 T_0, \quad (22)$$

$$D_0^2 q_{11} + q_{11} = -2D_0 D_1 q_{10} - \eta_1 D_0 q_{10} - \eta_2 q_{10}^3 - \eta_{12} q_{10} q_{20}^2, \quad (23)$$

$$D_0^2 q_{21} + 16q_{21} = -2D_0 D_1 q_{20} - 4\eta_1 D_0 q_{20} - 16\eta_2 q_{20}^3 - \eta_{12} q_{10}^2 q_{20}, \quad (24)$$

式中,  $D_0 = \partial/\partial T_0, D_1 = \partial/\partial T_1, D_0^2 = \partial^2/\partial T_0^2$ 。

方程(21)、(22)的解可写为如下复数形式:

$$q_{10} = A_1(T_1)\exp(iT_0) + \Lambda_1 \exp(i\Omega_0 T_0) + \text{c.c.}, \quad (25)$$

$$q_{20} = A_2(T_1)\exp(4iT_0) + \Lambda_2 \exp(i\Omega_0 T_0) + \text{c.c.}, \quad (26)$$

式中,  $\Lambda_1 = \eta_3(1 - \Omega_0^2)^{-1}/2, \Lambda_2 = \eta_3(16 - \Omega_0^2)^{-1}/2, A_1, A_2$  为待定的复函数,  $i^2 = -1, \text{c.c.}$  代表

前面各项的共轭复数项.

将表达式(25)、(26)代入方程(23)、(24)中,根据消除久期项的条件,整理后得到  $A_1$  和  $A_2$  满足的关系式为

$$i(2A_1' + \eta_1 A_2) + 3\eta_2 A_1^2 \bar{A}_1 + 6\eta_2 A_1 \Lambda_1^2 + 2\eta_{12} A_1 \Lambda_2^2 + 2\eta_{12} A_1 A_2 \bar{A}_2 + 2\eta_{12} \bar{A}_2 \Lambda_1 \Lambda_2 \exp(i\delta T_1) = 0, \quad (27)$$

$$i(8A_2' + 16\eta_1 A_2) + 48\eta_2 A_2^2 \bar{A}_2 + 96\eta_2 A_2 \Lambda_2^2 + 2\eta_{12} A_2 \Lambda_1^2 + 2\eta_{12} A_1 \bar{A}_1 A_2 + 2\eta_{12} \bar{A}_1 \Lambda_1 \Lambda_2 \exp(i\delta T_1) = 0, \quad (28)$$

式中的“'”表示对  $T_1$  的一阶导数(下同).

再令  $A_m = a_m \exp(i\beta_m)/2$ , 并将其代入方程(27)、(28)中,进行实部和虚部分离后得到

$$a_1' = -\eta_1 a_1/2 - \eta_{12} \Lambda_1 \Lambda_2 a_2 \sin \gamma, \quad (29)$$

$$a_2' = -2\eta_1 a_2 - \eta_{12} \Lambda_1 \Lambda_2 a_1 \sin \gamma/4, \quad (30)$$

$$a_1 \beta_1' = 3\eta_2 a_1^3/8 + 3\eta_2 \Lambda_1^2 a_1 + \eta_{12} \Lambda_2^2 a_1 + \eta_{12} a_1 a_2^2/4 + \eta_{12} \Lambda_1 \Lambda_2 a_2 \cos \gamma, \quad (31)$$

$$a_2 \beta_2' = 3\eta_2 a_2^3/2 + 12\eta_2 \Lambda_2^2 a_2 + \eta_{12} \Lambda_1^2 a_2/4 + \eta_{12} a_1^2 a_2/16 + \eta_{12} \Lambda_1 \Lambda_2 a_1 \cos \gamma/4, \quad (32)$$

这里,  $\gamma = \delta T_1 - \beta_1 - \beta_2$ .

方程(29)~(32)可进一步改写为如下形式:

$$a_1' = -\eta_1 a_1/2 - 4Fa_2 \sin \gamma, \quad (33)$$

$$a_2' = -2\eta_1 a_2 - Fa_1 \sin \gamma, \quad (34)$$

$$\gamma' = \delta - (3\eta_2/8 + \eta_{12}/16)a_1^2 - (3\eta_2/2 + \eta_{12}/4)a_2^2 - F(4a_2/a_1 + a_1/a_2)\cos \gamma - H_1 - H_2, \quad (35)$$

式中  $F = \eta_{12} \Lambda_1 \Lambda_2/4$ ,  $H_1 = 3\eta_2 \Lambda_1^2 + \eta_{12} \Lambda_2^2$ ,  $H_2 = 12\eta_2 \Lambda_2^2 + \eta_{12} \Lambda_1^2/4$ .

在分析稳态响应时,令  $a_1' = a_2' = 0, \gamma' = 0$ ,则在方程(33)~(35)的基础上消去  $\gamma$ ,可得到如下关于稳态解  $a_1$  和  $a_2$  的代数方程组:

$$a_1 = 4a_2, \quad (36)$$

$$25\eta_1^2/4 + [\delta - (15\eta_2/2 + 5\eta_{12}/4)a_2^2 - (H_1 + H_2)]^2 = 25F^2. \quad (37)$$

若激励项为单个正弦加载的作用形式,通过分析可知,组合共振情形不会被激发.

### 3 稳定性分析

下面对薄板在磁弹性组合共振下稳态解的稳定性进行分析. 根据 Liapunov 稳定性近似理论,稳态运动下定常解在微小扰动下的稳定性,将依赖于方程(33)~(35)的一阶近似系统右端系数矩阵的特征值. 为此,通过相应的 Jacobi 矩阵形式并考虑到稳态解满足  $a_1' = a_2' = 0, \gamma' = 0$  的条件,可得到如下判定系统稳态解稳定性的特征方程:

$$\begin{vmatrix} -\eta_1/2 - \lambda & 2\eta_1 & -4a_2 G_1/5 \\ \eta_1/2 & -2\eta_1 - \lambda & -4a_2 G_1/5 \\ G_2 - 3G_1/(20a_2) & G_2 + 3G_1/(5a_2) & -5\eta_1/2 - \lambda \end{vmatrix} = 0, \quad (38)$$

式中  $G_1 = \delta - H_1 - H_2 - (15\eta_2/2 + 5\eta_{12}/4)a_2^2$ ,  $G_2 = -(3\eta_2 + \eta_{12}/2)a_2$ .

对式(38)进行展开,进一步得到

$$\lambda^3 + c_1 \lambda^2 + c_2 \lambda + c_3 = 0, \quad (39)$$

式中,  $c_i (i = 1, 2, 3)$  为相应的系数表达式.

根据稳定性理论可知,只有当全部特征根均有负实部时,解才是稳定的;否则,只要有一个特征值的实部取正值,解就是不稳定的.

### 4 数值结果分析

对于铝制材料薄板, 给定相应的参数<sup>[14-15]</sup>, 通过编程计算, 图 2~图 5 分别给出了对应一、二阶模态下共振振幅随调谐参数、激励幅值和磁感应强度的变化特性曲线图(实线代表稳定解, 虚线代表非稳定解)。

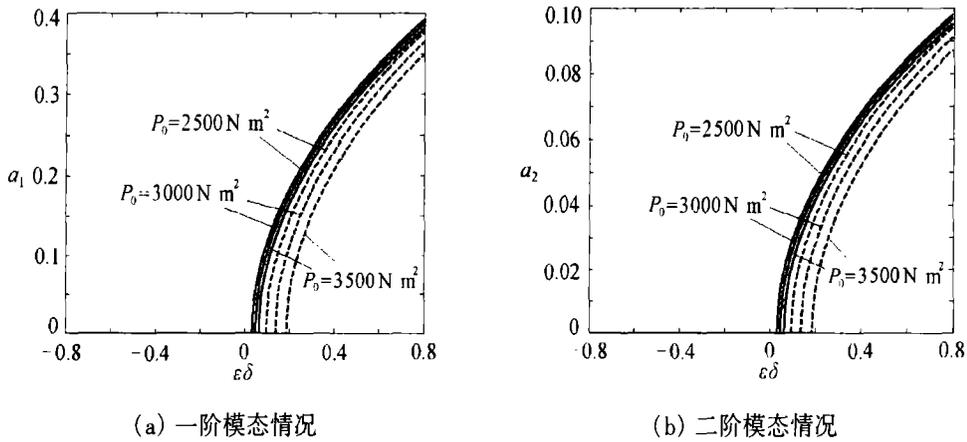


图 2 幅频特性图 ( $B_{0z} = 0.5 \text{ T}, h = 4 \text{ mm}$ )

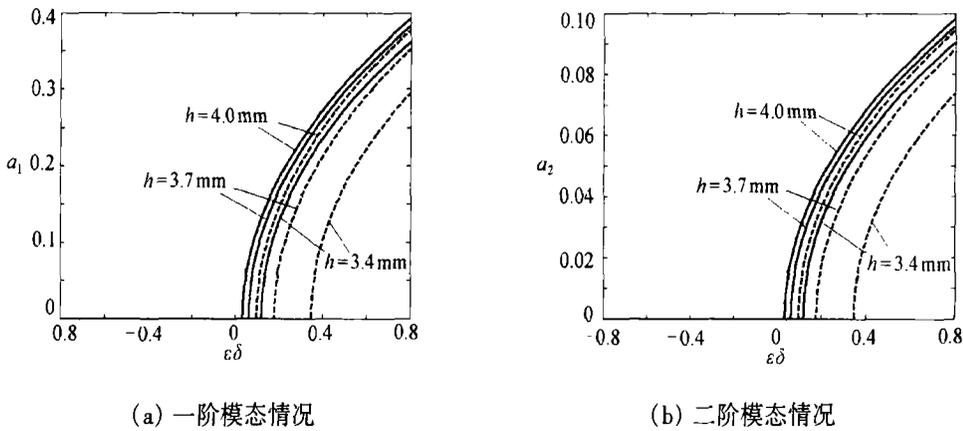


图 3 幅频特性图 ( $B_{0z} = 0.5 \text{ T}, P_0 = 2500 \text{ N/m}^2$ )

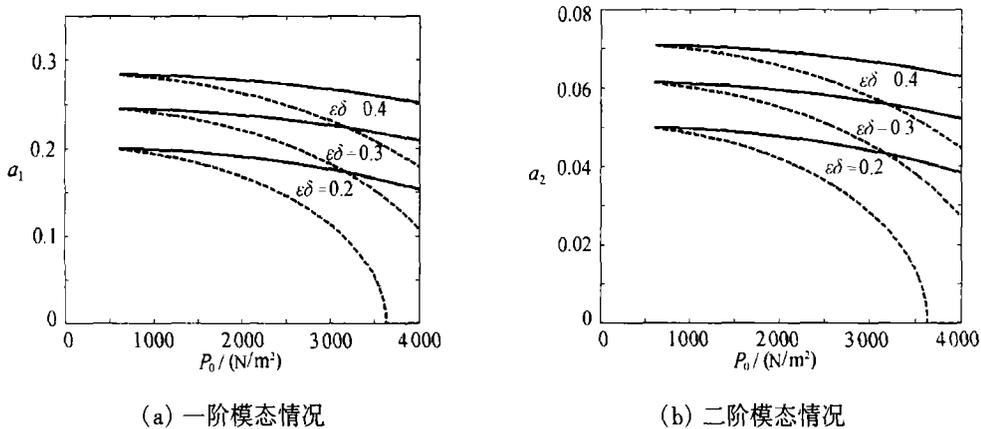


图 4 振幅特性图 ( $B_{0z} = 0.5 \text{ T}, h = 4 \text{ mm}$ )

由图 2 和图 3 的幅频图可见, 共振曲线呈现硬特性, 而随调谐参数由左至右逐渐增大并达到一定值后, 平凡解失去稳定性, 同时激发了稳定的非平凡共振解, 发生超临界叉型分岔; 当调谐参数继续增大并穿越第 2 个分岔点后, 系统又增加了 1 个不稳定的非平凡解, 呈现多值

现象。同时亦见,共振曲线相对于调协参数的零点 ( $\varepsilon\delta = 0$ ) 向右均呈现不同程度的偏移,且随激励幅值的增大(图 2)或板厚的减小(图 3)偏移程度加大。

由图 4 中曲线可见,只有激励幅值达到一定值的硬激励情况下,才会激发起系统的组合共振,并产生稳定(上支)和不稳定(下支)的两个非平凡解,而每种情况对应的分岔点几乎相同。比较图中 3 条曲线亦可看出,对应的共振振幅随调协参数的增大而增大。

由图 5 中曲线可见,在共振区域内,系统亦存在稳定和不安定的两个非平凡解。而当磁感应强度增大并达到一定值时,产生鞍结分岔,共振现象随之消失,说明可通过磁场强度的适当调整而达到控制系统共振的目的。同时从图 2~图 5 中可见,一、二阶模态下的分岔点是相同

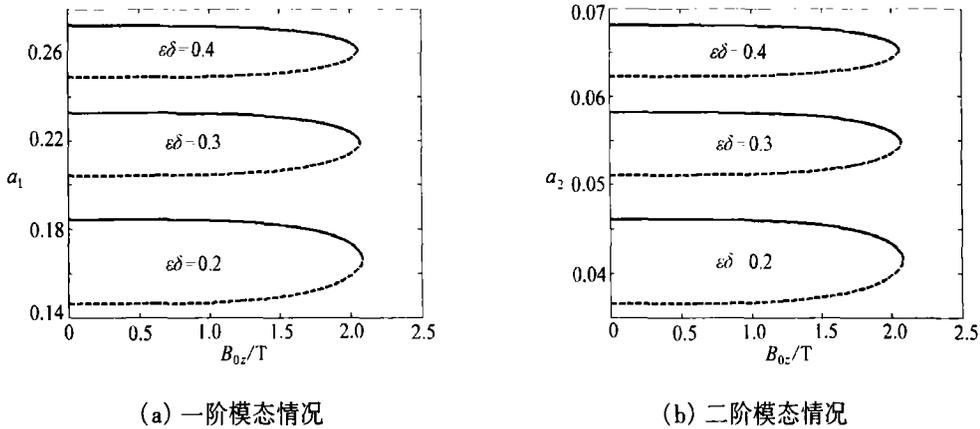
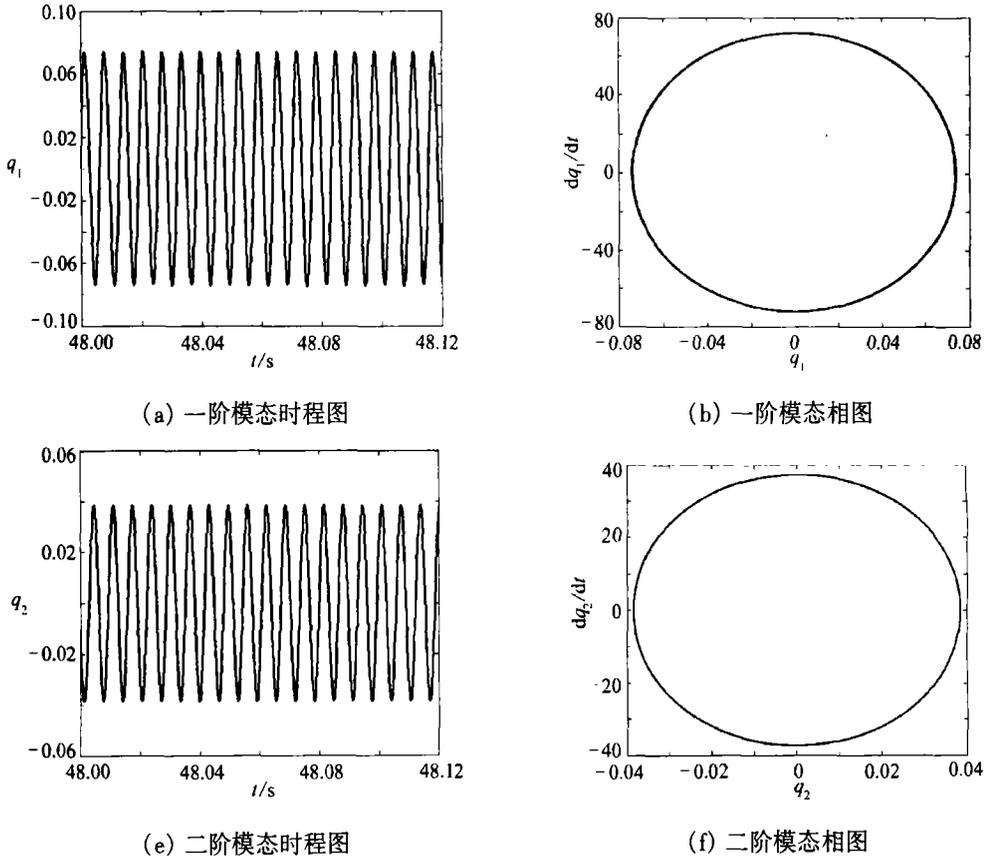
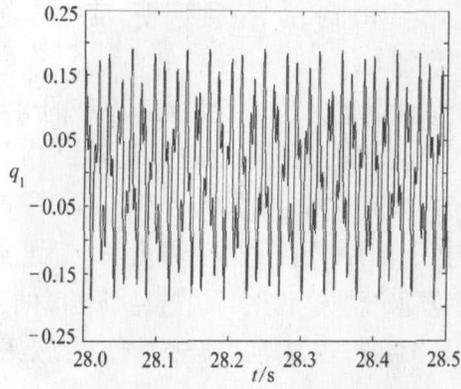


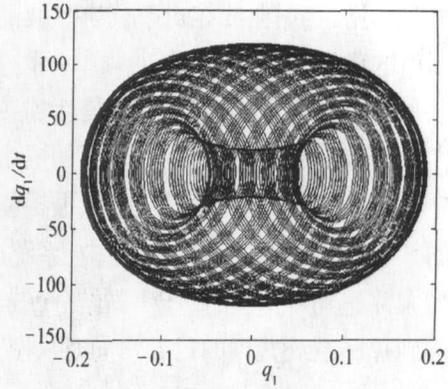
图 5 振幅特性图 ( $P_0 = 2\ 500\ \text{N/m}^2, h = 4\ \text{mm}$ )



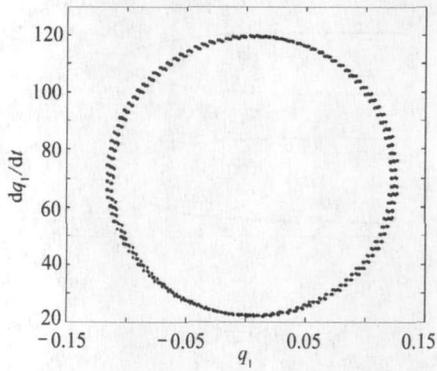
(i)  $\varepsilon\delta = 0$



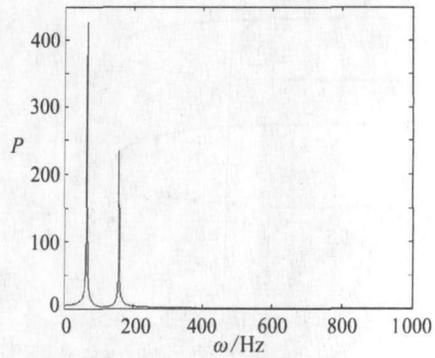
(a) 一阶模态时程图



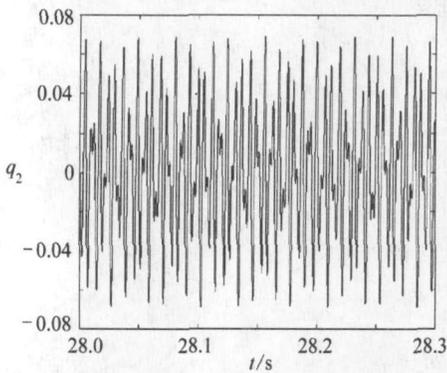
(b) 一阶模态相图



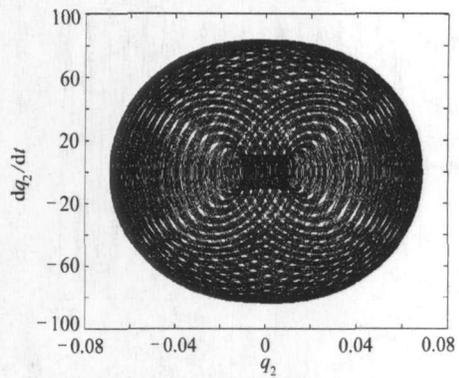
(c) 一阶模态 Poincaré 图



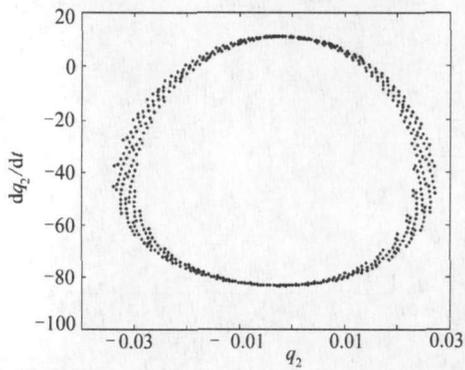
(d) 一阶模态频谱图



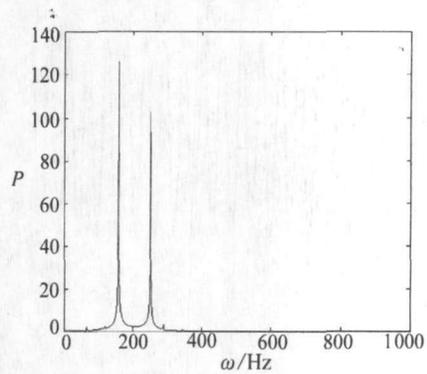
(e) 二阶模态时程图



(f) 二阶模态相图

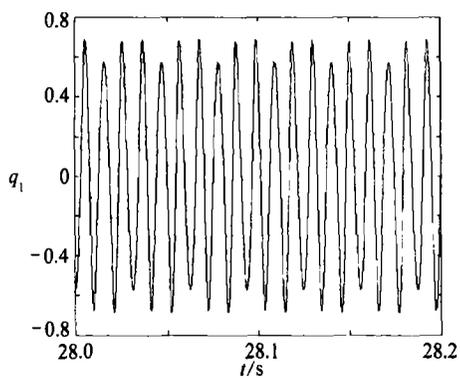


(g) 二阶模态 Poincaré 图

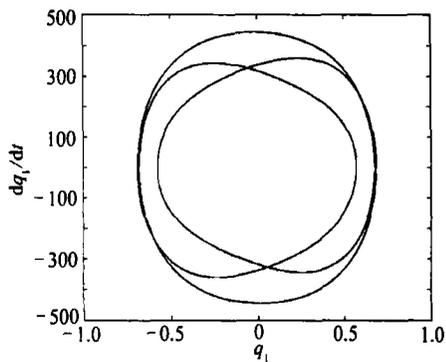


(h) 二阶模态频谱图

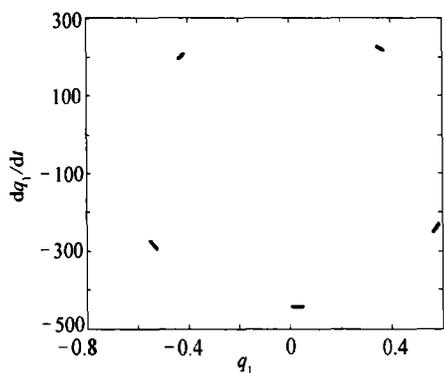
(ii)  $\epsilon\delta = 0.1$



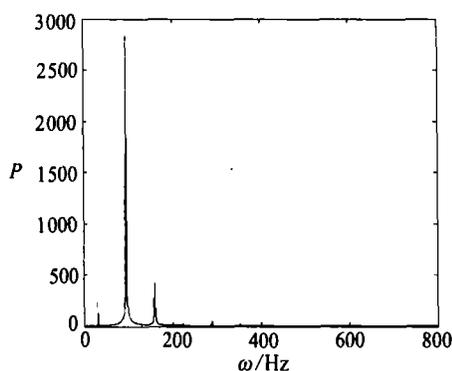
(a) 一阶模态时程图



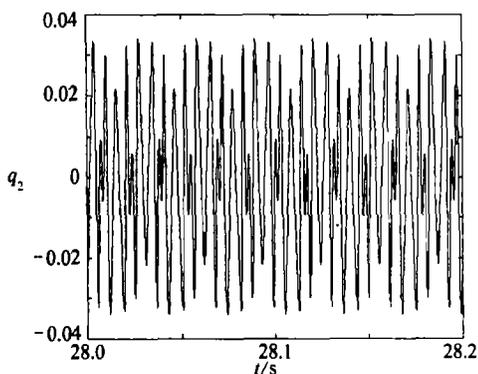
(b) 一阶模态相图



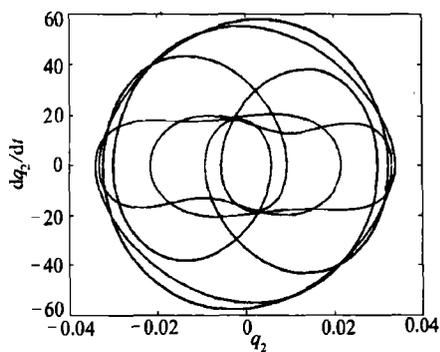
(c) 一阶模态 Poincaré 图



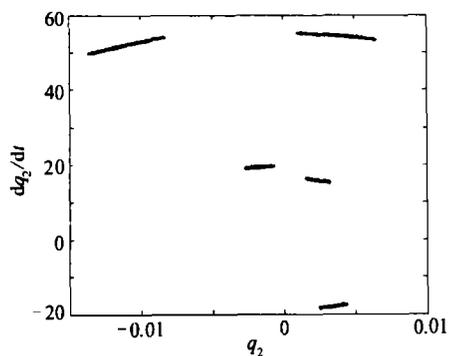
(d) 一阶模态频谱图



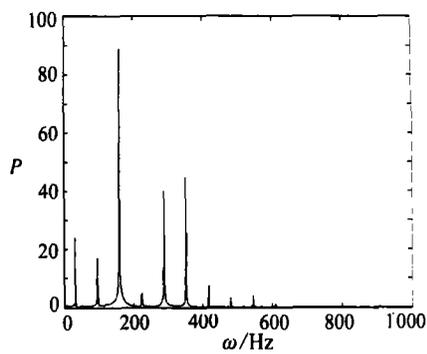
(e) 二阶模态时程图



(f) 二阶模态相图

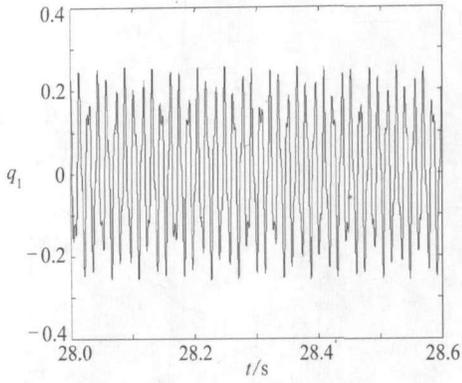


(g) 二阶模态 Poincaré 图

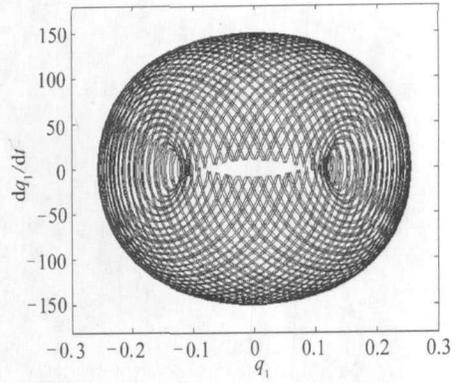


(h) 二阶模态频谱图

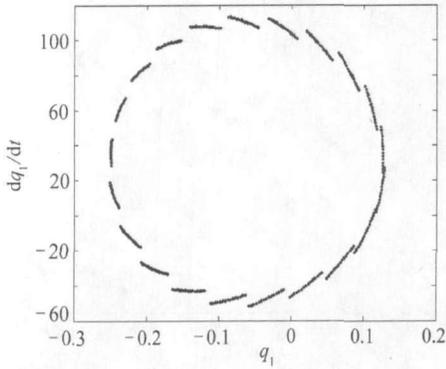
(iii)  $\epsilon\delta = 0.16$



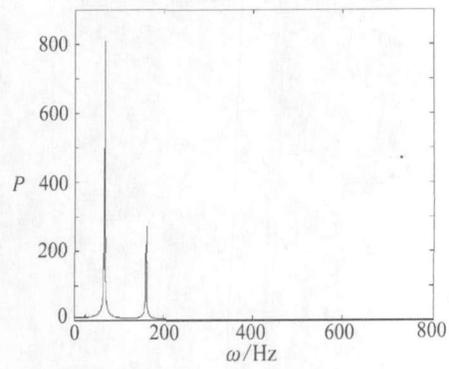
(a) 一阶模态时程图



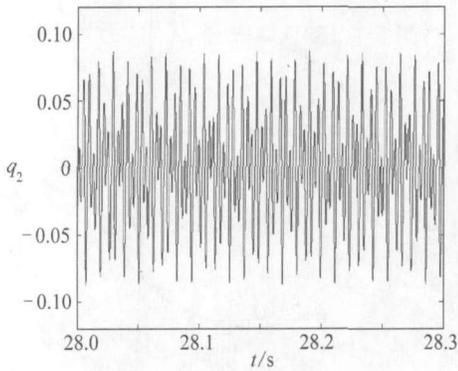
(b) 一阶模态相图



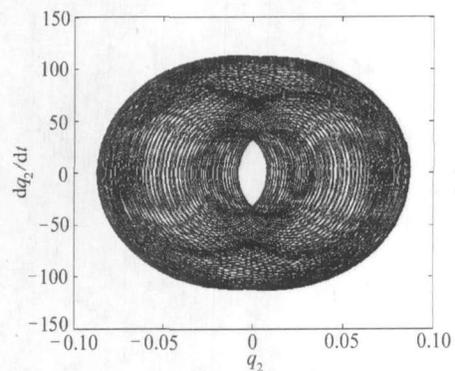
(c) 一阶模态 Poincaré 图



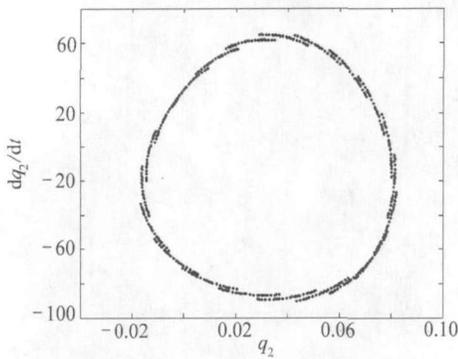
(d) 一阶模态频谱图



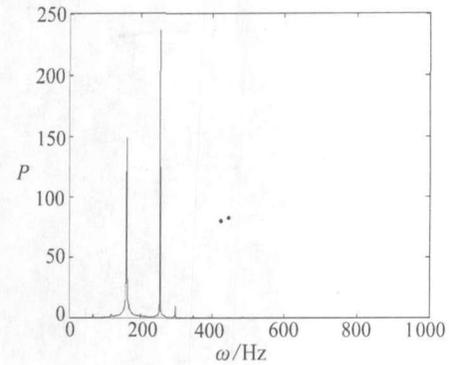
(e) 二阶模态时程图



(f) 二阶模态相图

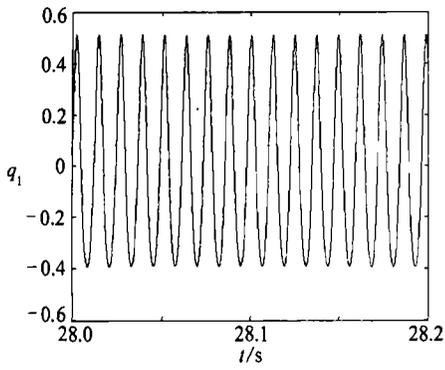


(g) 二阶模态 Poincaré 图

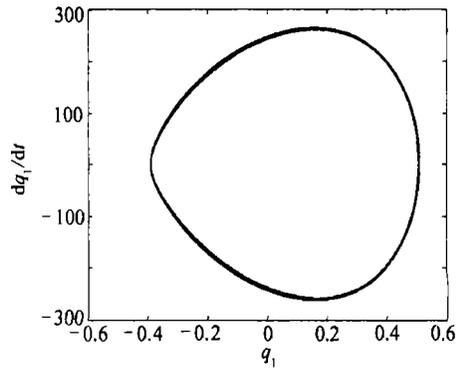


(h) 二阶模态频谱图

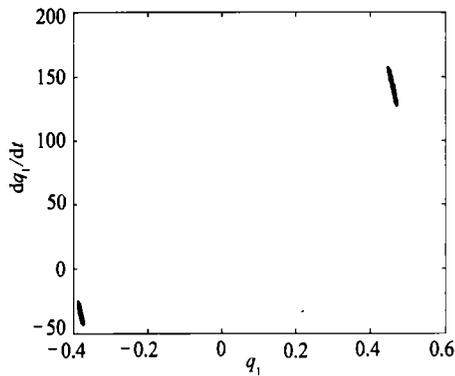
(IV)  $\epsilon\delta = 0.2$



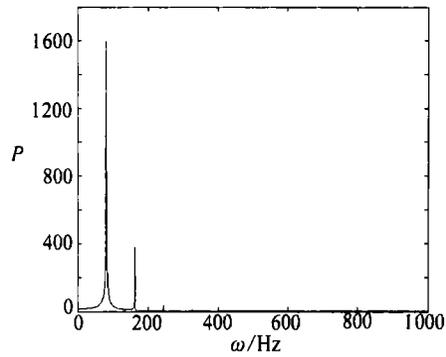
(a) 一阶模态时程图



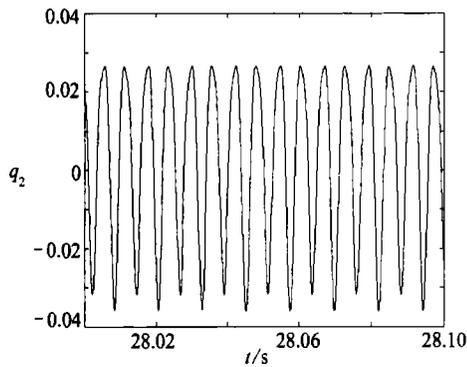
(b) 一阶模态相图



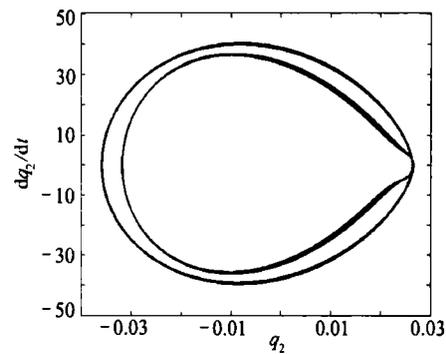
(c) 一阶模态 Poincaré 图



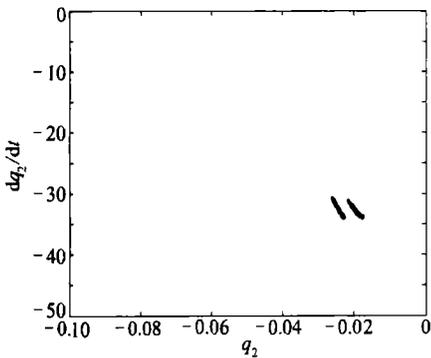
(d) 一阶模态频谱图



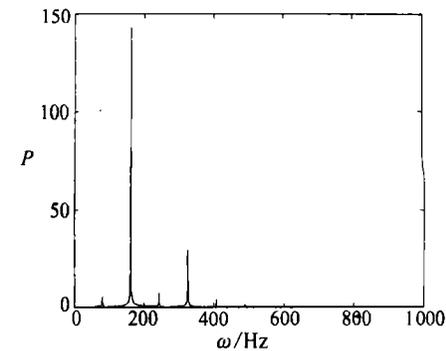
(e) 二阶模态时程图



(f) 二阶模态相图

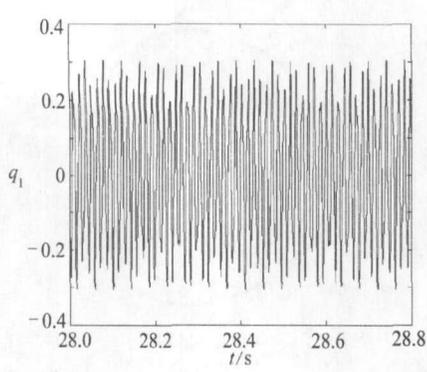


(g) 二阶模态 Poincaré 图

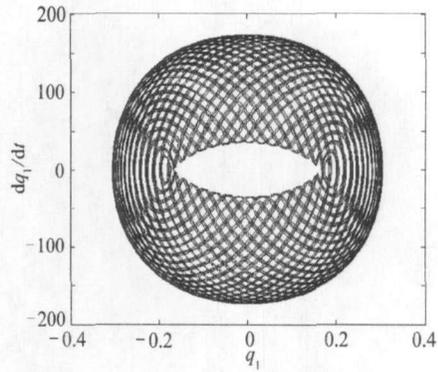


(h) 二阶模态频谱图

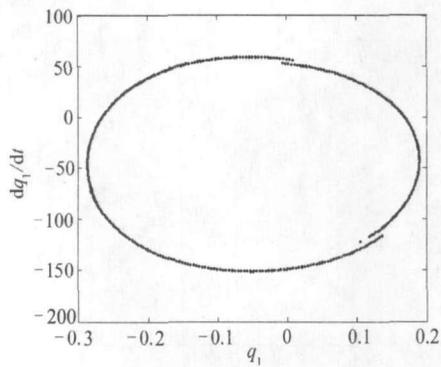
(V)  $\epsilon\delta = 0.24$



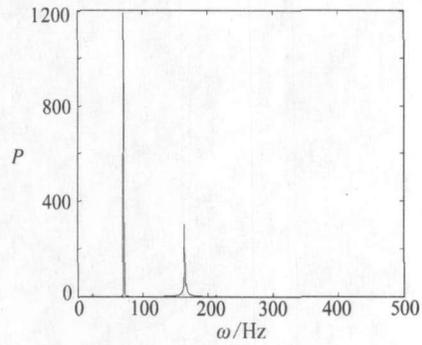
(a) 一阶模态时程图



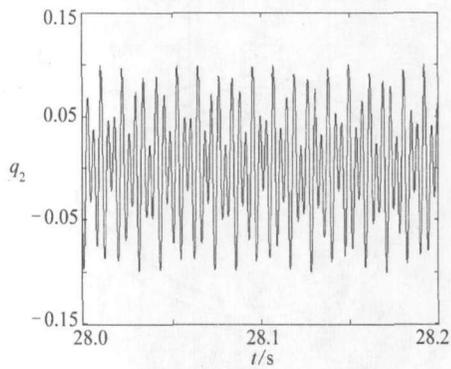
(b) 一阶模态相图



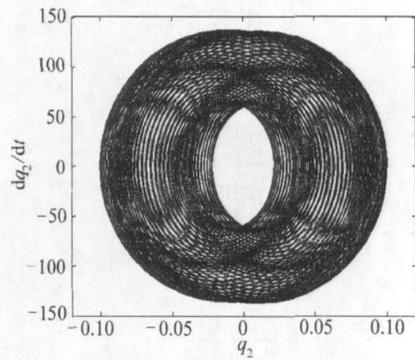
(c) 一阶模态 Poincaré 图



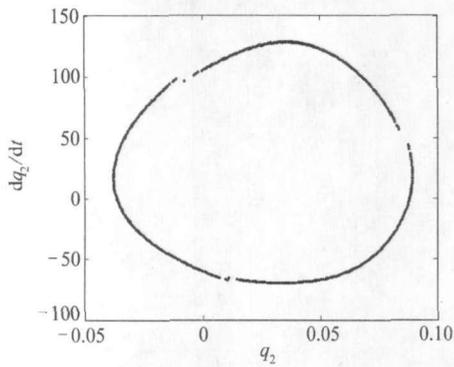
(d) 一阶模态频谱图



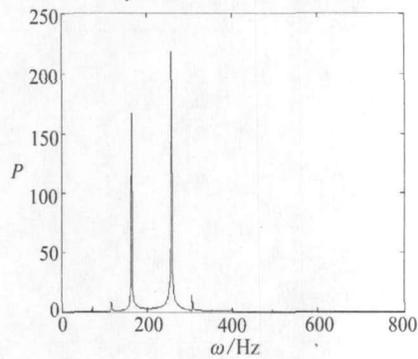
(e) 二阶模态时程图



(f) 二阶模态相图



(g) 二阶模态 Poincaré 图



(h) 二阶模态频谱图

(vi)  $\epsilon\delta = 0.3$

图6 不同协调参数下的振动特性图

的,但幅值相差4倍,这与式(36)、(37)描述的情况是一致的,即随确定参量的变化,一、二阶模态下所对应系统运动性态的改变是同时发生的。

在给定激励幅值( $P_0 = 2500 \text{ N/m}^2$ )和板厚( $h = 4 \text{ mm}$ )情况下,图6分别给出了不同调协参数下的时程响应图、相图、Poincaré映射图和频谱图。由图可见( $B_{0z} = 0.5 T$ ),当 $\epsilon\delta = 0$ 时,共振没有被激发,系统做外激励引起的稳定单频周期运动,相图为1条封闭的圆环。当 $\epsilon\delta = 0.1$ 时,共振被激发,并与强迫激励响应叠加,对应一、二阶模态下系统存在2个频率成分,Poincaré映射图上呈现环状的点集,系统表现为概周期运动形式。当 $\epsilon\delta = 0.16$ 时,一阶模态下的主要频率成分变为3个,二阶模态下存在更多的频率成分,相图呈现多圈相套的封闭曲线,为倍周期运动形式。之后,随调协参数的增加,出现概周期( $\epsilon\delta = 0.2, 0.3$ )和倍周期( $\epsilon\delta = 0.24$ )交替变化的运动形式。

## 5 结 论

本文以磁场环境中的导电薄板为研究对象,对其非线性磁弹性组合共振及稳定性问题进行了研究,通过数值算例,分析了共振幅值及其倍周期、概周期等复杂动力学行为的变化规律,说明了磁场环境、激励条件等因素的确定,能够达到抑制或激发系统组合共振的目的,所得结果具有理论和实际意义。在本文工作基础上,可进一步对复杂电磁场环境中弹性结构的多维磁弹性动力学稳定性、分岔及混沌问题开展更深入的研究工作。

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## Magneto-Elastic Combination Resonances Analysis of Current-Conducting Thin Plate

HU Yu-da<sup>1</sup>, LI Jing<sup>1,2</sup>

- (1. *School of Civil Engineering and Mechanics, Yanshan University,  
Qinhuangdao, Hebei 066004, P. R. China;*  
2. *Tangshan College, Tangshan, Hebei 063000, P. R. China*)

**Abstract:** Based on Maxwell equations, the nonlinear magneto-elastic vibration equations of a thin plate were derived. The electrodynamic equations and expressions of electromagnetic forces were also derived. In addition, the magneto-elastic combination resonances and stabilities of the thin beam-plate subjected to mechanical loadings in a constant transverse magnetic field were studied. By means of the Galerkin Method, the corresponding nonlinear vibration differential equations were derived. The amplitude frequency response equation of the system in steady motion was obtained by the method of multiple scales. The excitation condition of combination resonances was analyzed. Based on the Liapunov stability theory, the stabilities of steady solutions were analyzed and the critical conditions of stability were also obtained. Through the numerical calculation, the curves which resonance-amplitudes changing with detuning parameters, excitation amplitudes and magnetic intensity in the first and the second order modality were obtained respectively. The time history response plots, the phase charts, the Poincaré mapping charts and the spectrum plots of vibrations were also obtained. The effect of electro-magnetic and mechanical parameters for the stabilities of solutions and the bifurcation are further analyzed. Some complex dynamic performances such as the period-doubling motion and the quasi-period motion were discussed.

**Key words:** magneto-elastic; current-conducting thin plate; combination resonance; stability; method of multiple scales