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具界面损伤压电智能层合板的非线性自由振动分析*

傅衣铭, 李 升, 姜叶洁

(湖南大学 力学与航空航天学院,长沙 410082)

(我刊编委傅衣铭来稿)

摘要: 基于广义 6 自由度板理论、应变等效原理和 Hamilton 变分原理,通过引入三维弹性平衡方程和静电平衡方程的通解来构造满足界面间力电耦合关系和各类连续条件的位移、电势分布形函数,建立了具铺设层内和层间界面处损伤效应的压电智能层合板的非线性运动控制方程组,并运用 Galerkin 方法进行求解. 数值算例中,分别讨论了,不同损伤程度、压电层厚度、厚跨比及长宽比对四边简支非理想界面压电智能层合板线性自由振动频率和非线性幅频响应曲线的影响.

关 键 词: 压电智能层合板; 非线性振动分析; 界面损伤; 损伤演化; Galerkin 方法

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引 言

压电材料由于其特殊的力电耦合性能,常作为作动器、传感器粘贴于复合材料层合结构表面,而这些结构的实际工作性能常取决于其界面间的结合情况. 因此,为了准确预计压电智能结构的工作寿命,实际分析过程中界面损伤的影响将不能忽略.

关于压电智能结构的分析理论,其准确性往往取决于对位移、电势分布形函数的选择. Pagano 等最早给出了层合板受弯时的精确理论分析;在此基础上,通过考虑压电效应,Heyliger 等[1-2]将该理论用于压电智能层合板,并给出了三维精确解[3]. 值得注意的是,这些用于压电智能层合板的精确分析理论中,其层间界面和层内材料都假设为完好无损状态;但实际工况中,由于生产制造或服役(自然原因、温度、湿度等),界面结合会出现弱化,而层内材料也存在微损伤;这些因素将改变压电智能层合板中位移和电势的分布,这也使得对考虑界面损伤压电智能层合板理论模型的建立变得尤为困难.

目前为止,由于对受损界面间复杂的力电耦合效应还很难精确描述,考虑界面损伤压电智能层合结构的研究成果还相对较少[4-5]. Seeley 和 Chattopadhyay 等[6]对考虑界面脱粘的压电层合梁进行了实验研究;Sun 和 Tong 等[7]考虑界面损伤效应,讨论了压电层合梁的振动和稳定性控制问题;近来,Soldatos 和 Shu 等[8-11]发展了一种考虑界面损伤复合材料层合板的精确分析理

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作者简介: 傅衣铭(1945—),男,湖南湘潭人,教授(E-mail: fym_2581@hnu.cn);
李升(1982—),男,湖南怀化人,博士(联系人. Tel: +86-731-8822421; E-mail: lisheng191@yahoo.com.cn).

论;并基于电学可穿透模型[12]来描述受损界面间的电学条件(电势和电位移在受损界面间仍然连续),成功地将该理论用于考虑界面剪切滑移效应压电层合梁的层间应力及电势分析[13];Geun 和 Kang 等[14]通过假设受损界面间电势不连续分布,讨论了压点层合矩形板的屈曲问题. 但是至今为止,考虑界面问题的压电层合板的理论研究还处于起步阶段,同时考虑界面损伤和层内材料损伤效应,设定受损界面间电势不连续分布,对压电智能层合板进行非线性静动力学分析还鲜有报道.

对于层间界面损伤,本文考虑界面剪切滑移现象,基于电学不可穿透模型[15-16]来描述受损界面间的电学条件(电势在受损界面间不连续),引入 3 个界面损伤变量及满足三维弹性平衡方程和静电平衡方程的通解来构造满足界面间力电耦合关系和各类连续条件的位移、电势分布形函数;对于层内材料损伤,引入两个 Kachanov[17-20]层内损伤变量来描述层内应力-应变关系,基于广义 6 自由度理论[11]和 Hamilton 变分原理,通过引入 von Kármán 型应变关系,建立了具损伤压电智能层合板的非线性运动控制方程,并运用 Galerkin 法求解,得到了许多有意义的研究成果.

1 基本方程

考虑图 1 和图 2 所示表面粘贴压电层的正交对称铺设复合材料层合板,其长为 a,宽为 b,总厚为 H. 直角坐标系 $Oxyz$ 置于板的中面,z 轴向上. 用 $z_r(r = 0,1,2,\cdots,N-1,N)$ 标定层合板中第 r 个界面的位置;对于层间界面条件,考虑界面剪切滑移现象,且假设受损界面间电势不连续[15-16].

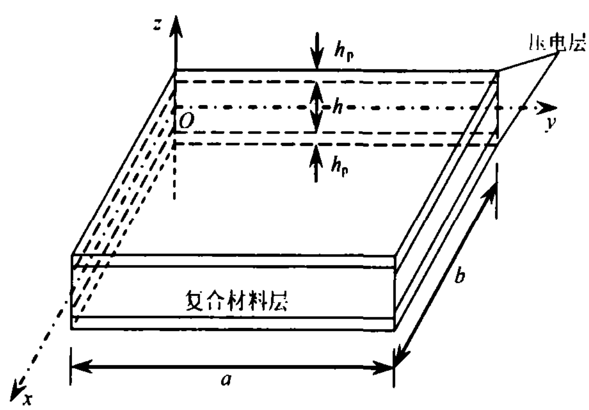

图 1 压电智能层合板的几何结构

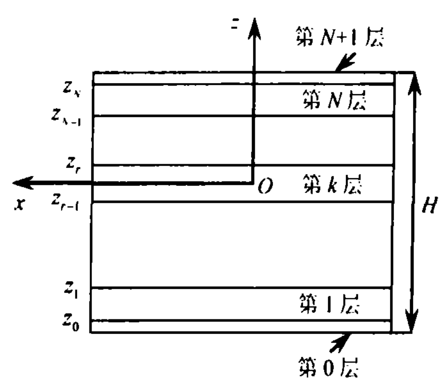

图 2 层合板横截面几何构形

在考虑界面剪切滑移的情况下,压电智能层合板内任一点的面内位移 U,V[8,10],横向挠度 W 和电势 Φ[13] 可设为

$$\begin{cases} U(x,y,z,t) = u_0(x,y,t) - zw_{0,x}(x,y,t) + \psi_1(z)u_1(x,y,t), \\ V(x,y,z,t) = v_0(x,y,t) - zw_{0,y}(x,y,t) + \psi_2(z)v_1(x,y,t), \\ W(x,y,z,t) = w_0(x,y,t),\ \Phi(x,y,z,t) = \varphi(z)\phi_1(x,y,t), \end{cases} \tag{1}$$

式中,u_0,v_0,w_0 表示板中面上相应点的位移,u_1,v_1 为板中面相应点的横向应变,ϕ_1 为板中面相应点的中面电势,$\psi_1(z),\psi_2(z)$ 和 $\varphi(z)$ 分别为位移和电势分布待定形函数,且满足以下的约束条件[13]:

$$\psi_1(0) = \psi_2(0) = 0,\ \varphi(0) = 1,\ \left.\frac{d\psi_1}{dz}\right|_{z=0} = \left.\frac{d\psi_2}{dz}\right|_{z=0} = \left.\frac{d\varphi}{dz}\right|_{z=0} = 1. \tag{2}$$

对于理想界面,面内位移分量 U,V,电势 Φ 和层间应力分量 τ_{xz},τ_{yz},电位移分量 D_z 在厚度

方向上连续[13];而对于非理想界面,面内位移分量 U, V 和电势 Φ 不连续[13]. 位移改变量 $\Delta^{(r)} = [\Delta U^{(r)} \quad \Delta V^{(r)}]$ 可以用待定形函数的不连续分布来给定,则

$$\begin{cases}\Delta\psi_1^{(r)}u_1 = [\psi_1^{(k+1)}(z_r) - \psi_1^{(k)}(z_r)]u_1 = \Delta U^{(r)},\\ \Delta\psi_2^{(r)}v_1 = [\psi_2^{(k+1)}(z_r) - \psi_2^{(k)}(z_r)]v_1 = \Delta V^{(r)},\end{cases} \tag{3a}$$

式中,上标 $r(r = 0,1,2,\cdots,N)$ 表示第 r 个界面,k 和 $k + 1$ 表示与 r 界面有关的第 k 层和第 $k + 1$ 层板.

对于界面电学条件,假设受损界面戒电势不连续分布[15-16];对于非理想界面,电势改变量 $\Delta\Phi^{(r)}$ 可以假设为

$$\Delta\varphi^{(r)}\phi_1 = [\varphi^{(k+1)}(z_r) - \varphi^{(k)}(z_r)]\phi_1 = \Delta\Phi^{(r)}. \tag{3b}$$

板内任一点的应变与位移关系为

$$\begin{cases}\varepsilon_x = u_{0,x} + w_{0,x}^2/2 - zw_{0,xx} + \psi_1 u_{1,x},\ \varepsilon_y = v_{0,y} + w_{0,y}^2/2 - zw_{0,yy} + \psi_2 v_{1,y},\\ \gamma_{xy} = u_{0,y} + v_{0,x} + w_{0,x}w_{0,y} - 2zw_{0,xy} + \psi_1 u_{1,y} + \psi_2 v_{1,x},\\ \gamma_{xz} = \psi_{1,z}u_1,\ \gamma_{yz} = \psi_{2,z}v_1.\end{cases} \tag{4}$$

定义具正交各向异性损伤的单层板在材料主方向上的损伤变量为 $D_1^{(k)}$ 和 $D_2^{(k)}$. 若 $S_{ij}^{e(k)}$ 为第 k 层板无损时的弹性常数,$S_{ij}^{d(k)}$ 为损伤后第 k 层板的折算弹性常数,且 $S_{ij}^{d(k)}$ 可表示为[17-20]

$$\begin{cases}S_{11}^{d(k)} = S_{11}^{e(k)}(1 - D_1^{(k)})^2,\ S_{12}^{d(k)} = S_{12}^{e(k)}(1 - D_1^{(k)})(1 - D_2^{(k)}),\\ S_{22}^{d(k)} = S_{22}^{e(k)}(1 - D_2^{(k)})^2,\ S_{44}^{d(k)} = S_{44}^{e(k)}(1 - D_2^{(k)}),\\ S_{55}^{d(k)} = S_{55}^{e(k)}(1 - D_1^{(k)}),\ S_{66}^{d(k)} = S_{66}^{e(k)}(1 - D_1^{(k)})(1 - D_2^{(k)}).\end{cases} \tag{5}$$

基于应变等效原理[17-18],在整体坐标系 $Oxyz$ 中,考虑复合材料铺设层内损伤效应时,层合板中第 k 层板的应力-应变关系为

$$\begin{Bmatrix}\sigma_x^{(k)}\\ \sigma_y^{(k)}\\ \tau_{xy}^{(k)}\end{Bmatrix} = \begin{bmatrix}S_{11}^{d(k)} & S_{12}^{d(k)} & 0\\ S_{12}^{d(k)} & S_{22}^{d(k)} & 0\\ 0 & 0 & S_{66}^{d(k)}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}_k,\ \begin{Bmatrix}\tau_{yz}^{(k)}\\ \tau_{xz}^{(k)}\end{Bmatrix} = \begin{bmatrix}S_{44}^{d(k)} & 0\\ 0 & S_{55}^{d(k)}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}_k \quad (k = 1,2,\cdots,N), \tag{6}$$

又考虑正交各向异性压电材料,在整体坐标系中,第 k 层压电层合板的本构关系为

$$\begin{cases}\begin{Bmatrix}\sigma_x^{(k)}\\ \sigma_y^{(k)}\\ \tau_{yz}^{(k)}\\ \tau_{xz}^{(k)}\\ \tau_{xy}^{(k)}\end{Bmatrix} = \begin{bmatrix}S_{11}^{p(k)} & S_{12}^{p(k)} & 0 & 0 & 0\\ & S_{22}^{p(k)} & 0 & 0 & 0\\ & & S_{44}^{p(k)} & 0 & 0\\ & & & S_{55}^{p(k)} & 0\\ & & & & S_{66}^{p(k)}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}_k - \begin{bmatrix}0 & 0 & e_{31}^{(k)}\\ 0 & 0 & e_{32}^{(k)}\\ 0 & e_{24}^{(k)} & 0\\ e_{15}^{(k)} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}_k\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix},\\ \begin{Bmatrix}D_x^{(k)}\\ D_y^{(k)}\\ D_z^{(k)}\end{Bmatrix} = \begin{bmatrix}0 & 0 & & e_{15}^{(k)} & 0\\ 0 & 0 & e_{24}^{(k)} & 0 & 0\\ e_{31}^{(k)} & e_{32}^{(k)} & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}_k - \begin{bmatrix}\epsilon_{11}^{(k)} & 0 & 0\\ 0 & \epsilon_{22}^{(k)} & 0\\ 0 & 0 & \epsilon_{33}^{(k)}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}\\ \qquad (k = 0, N + 1),\end{cases} \tag{7}$$

式中，D_x, D_y, D_z 为电位移分量，E_x, E_y, E_z 为电场强度分量，$S_{ij}^{p(k)}$ 为压电材料的弹性常数，$e_{ij}^{(k)}$ 为压电材料压电常数，$\epsilon_{ij}^{(k)}$ 为压电材料的介电常数，且电场强度与电势之间的关系为

$$E_x = -\Phi_{,x},\ E_y = -\Phi_{,y},\ E_z = -\Phi_{,z}, \tag{8}$$

为以后书写方便和研究的一般性，用 $S_{ij}^{(k)}$ 代表第 k 层板的弹性常数，对于复合材料层和压电层其弹性常数分别为 $S_{ij}^{d(k)}$ 和 $S_{ij}^{p(k)}$.

又假设界面劣化的初始阶段，界面具有如下线性非耦合性质[11,13-14]，则界面本构关系为

$$\Delta U^{(r)} = R_{1x}^{(r)}\tau_{xz}\mid_{z=z_r},\ \Delta V^{(r)} = R_{1y}^{(r)}\tau_{yz}\mid_{z=z_r},\ \Delta\Phi^{(r)} = -R_{e}^{(r)}D_z\mid_{z=z_r}, \tag{9}$$

式中，$R_{1x}^{(r)}, R_{1y}^{(r)}$ 是与界面剪切滑移有关的损伤分量，$R_e^{(r)}$ 是与界面间电位移和电势有关的损伤分量.

含界面损伤压电智能层合板的运动控制方程可以由 Hamilton 变分原理确定，即

$$\delta\int_{t_0}^{t_1}(\boldsymbol{K}-\boldsymbol{\Pi})\mathrm{d}t = 0, \tag{10}$$

式中，$\boldsymbol{K}$ 是系统的动能；Π 是系统总势能. 具体表达式如下：

$$\begin{cases}\boldsymbol{K} = \displaystyle\int_V \frac{1}{2}\rho\,\dot{\boldsymbol{u}}\dot{\boldsymbol{u}}\mathrm{d}V, \\ \Pi = \displaystyle\int_\Omega\int_H \frac{1}{2}(\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\varepsilon} + \boldsymbol{D}^{\mathrm{T}}\boldsymbol{E})\mathrm{d}\Omega\mathrm{d}z + \int_\Omega\sum_{r=0}^{N}[(\boldsymbol{T}^{(r)})^{\mathrm{T}}\boldsymbol{\Delta}^{(r)} - D_z^{(r)}\Delta\Phi^{(r)}]\mathrm{d}\Omega,\end{cases} \tag{11}$$

其中，Ω 为中面面积，V 为压电智能层合板的体积，ρ 是压电智能层合板单位体积的质量密度；u 和 $\dot{u}$ 分别为系统各方向的位移和速度分量；$\boldsymbol{T}^{(r)} = [\tau_{xz}\quad \tau_{yz}]_{z=z_r}^{\mathrm{T}}$.

由式(9)至(11)，得如下考虑界面损伤压电层合板的非线性运动控制方程组：

$$\begin{cases}\bar{N}_{x,x}^{c} + \bar{N}_{xy,y}^{c} = I_1\ddot{u}_0 + I_4\ddot{u}_1 - I_2\ddot{w}_{0,x},\ \bar{N}_{xy,x}^{c} + \bar{N}_{y,y}^{c} = I_1\ddot{v}_0 + I_6\ddot{v}_1 - I_2\ddot{w}_{0,y}, \\ \bar{M}_{x,xx}^{c} + 2\bar{M}_{xy,xy}^{c} + \bar{M}_{y,yy}^{c} + N_x^c w_{0,xx} + 2N_{xy}^c w_{0,xy} + N_y^c w_{0,yy} = \\ \qquad I_1\ddot{w}_0 - I_3\ddot{w}_{0,xx} + I_2\ddot{u}_{0,x} + I_5\ddot{u}_{1,x} + I_2\ddot{v}_{0,y} + I_7\ddot{v}_{1,y}, \\ \bar{M}_{x,xx}^{c} + 2\bar{M}_{xy,xy}^{c} + \bar{M}_{y,yy}^{c} + N_x^c w_{0,xx} + 2N_{xy}^c w_{0,xy} + N_y^c w_{0,yy} = \\ \qquad I_1\ddot{w}_0 - I_3\ddot{w}_{0,xx} - I_3\ddot{w}_{0,yy} + I_2\ddot{u}_{0,x} + I_5\ddot{u}_{1,x} + I_2\ddot{v}_{0,y} + I_7\ddot{v}_{1,y}, \\ \bar{M}_{yx,x}^{a} + \bar{M}_{y,y}^{a} - \bar{Q}_y^{a} - \displaystyle\sum_{r=0}^{N}\frac{(\Delta\psi_2^{(r)})^2}{R_{1y}^{(r)}}v_1 = I_6\ddot{v}_0 - I_7\ddot{w}_{0,y} + I_9\ddot{v}_1, \\ \dfrac{\partial\bar{G}_x}{\partial x} + \dfrac{\partial\bar{G}_y}{\partial y} - \bar{G}_z - \displaystyle\sum_{r=0}^{N}\frac{(\Delta\varphi^{(r)})^2}{R_e^{(r)}}\phi_1 = 0,\end{cases} \tag{12}$$

其中，$\bar{G}_x$、$\bar{G}_y$、$\bar{G}_z$ 是由于压电效应而引起的广义电位移，I_1 至 I_9 为惯性矩，它们的具体表达式为

$$\begin{cases}\bar{G}_z = \displaystyle\int_{-H/2}^{H/2} D_z\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\mathrm{d}z,\ (\bar{G}_x, \bar{G}_y) = \int_{-H/2}^{H/2}(D_x, D_y)\varphi(z)\mathrm{d}z, \\ (I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9) = \\ \qquad \displaystyle\int_{-H/2}^{H/2}(1, z, z^2, \psi_1, z\psi_1, \psi_2, z\psi_2, \psi_1\psi_1, \psi_2\psi_2)\rho\mathrm{d}z,\end{cases} \tag{13}$$

式(12)中，$\bar{N}^c, \bar{M}^c$ 为经典板理论中定义的内力和弯矩；$\bar{M}^a, \bar{Q}^a$ 为附加内力和弯矩；且 $\bar{N}^c, \bar{M}^c, \bar{M}^a, \bar{Q}^a$ 可分解成无压电效应的分量和具压电效应的分量两项之和[21]，即

$$\bar{N}^c = N^c + N^{cp},\ \bar{M}^c = M^c + M^{cp},\ \bar{M}^a = M^a + M^{ap},\ \bar{Q}^a = Q^a + Q^{ap}, \tag{14}$$

式中，N^c, M^c, M^a, Q^a 代表无压电效应的内力分量，它们的具体表达式为

$$\begin{cases}(N_x^c, N_y^c, N_{xy}^c) = \int_{-H/2}^{H/2}(\sigma_x^{(k)}, \sigma_y^{(k)}, \tau_{xy}^{(k)})\mathrm{d}z,\\ (M_x^c, M_y^c, M_{xy}^c) = \int_{-H/2}^{H/2}(\sigma_x^{(k)}, \sigma_y^{(k)}, \tau_{xy}^{(k)})z\mathrm{d}z,\\ (Q_x^a, Q_y^a) = \int_{-H/2}^{H/2}(\tau_{xz}^{(k)}\psi_{1,z}, \tau_{yz}^{(k)}\psi_{2,z})\mathrm{d}z,\\ (M_x^a, M_y^a, M_{xy}^a, M_{yx}^a) = \int_{-H/2}^{H/2}(\sigma_x^{(k)}\psi_1, \sigma_y^{(k)}\psi_2, \tau_{xy}^{(k)}\psi_1, \tau_{xy}^{(k)}\psi_2)\mathrm{d}z,\end{cases} \tag{15}$$

$N^{cp}, M^{cp}, M^{ap}, Q^{ap}$ 代表具压电效应的内力分量,具体表达形式如下:

$$\begin{cases}N_x^{cp} = \sum_{k=0}^{N+1} e_{31}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}\mathrm{d}z,\ N_y^{cp} = \sum_{k=0}^{N+1} e_{32}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}\mathrm{d}z,\ N_{xy}^{cp} = 0,\\ M_x^{cp} = \sum_{k=0}^{N+1} e_{31}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}z\mathrm{d}z,\ M_y^{cp} = \sum_{k=0}^{N+1} e_{32}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}z\mathrm{d}z,\ M_{xy}^{cp} = 0,\\ M_x^{ap} = \sum_{k=0}^{N+1} e_{31}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}\psi_1\mathrm{d}z,\ M_y^{ap} = \sum_{k=0}^{N+1} e_{32}^k\phi_1\int_{z_{k-1}}^{z_k}\frac{\mathrm{d}\varphi^k}{\mathrm{d}z}\psi_2\mathrm{d}z,\ M_{xy}^{ap} = 0,\\ Q_y^{ap} = \sum_{k=0}^{N+1} e_{24}^k\phi_{1,y}\int_{z_{k-1}}^{z_k}\varphi^k\psi_{2,z}\mathrm{d}z,\ Q_x^{ap} = \sum_{k=0}^{N+1} e_{15}^k\phi_{1,x}\int_{z_{k-1}}^{z_k}\varphi^k\psi_{1,z}\mathrm{d}z.\end{cases} \tag{16}$$

若考虑周边简支可动,电学接地的层合板,其边界条件为

$$\begin{cases}x = 0 \text{ 和 } x = a:\ N_x^c = N_{xy}^c = w_0 = M_x^c = M_x^a = M_{xy}^a = \phi_1 = 0,\\ y = 0 \text{ 和 } y = b:\ N_y^c = N_{xy}^c = w_0 = M_y^c = M_y^a = M_{xy}^a = \phi_1 = 0.\end{cases} \tag{17}$$

值得注意的是, $\phi_1 = 0$ 表示电学接地,式(12)的求解还需确定待定形函数.

2 具两种损伤模式简支压电智能层合板的形函数

在时刻 t,形函数 $\psi_1(z)$, $\psi_2(z)$ 和 $\varphi(z)$ 应满足用应力表示的三维平衡微分方程和静电平衡方程:

$$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} = 0,\ \tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} = 0,\ D_{x,x} + D_{y,y} + D_{z,z} = 0. \tag{18}$$

设定 $u_0, v_0, w_0, u_1, v_1, \phi_1$ 为如下满足简支可动边界条件的位移模式:

$$\begin{cases}(u_0, u_1) = (A_0, A_1)\cos\alpha x\sin\beta y,\\ (v_0, v_1) = (B_0, B_1)\sin\alpha x\cos\beta y,\\ (w_0, \phi_1) = (C_0, D_1)\sin\alpha x\sin\beta y,\end{cases} \tag{19}$$

其中, $\alpha = \pi/a$, $\beta = \pi/b$.

将式(19)代入式(18),并利用式(4)和(7),得

$$S_{55}^{(k)}\Psi_{1,zz}^{(k)} - (S_{11}^{(k)}\alpha^2 + S_{66}^{(k)}\beta^2)\Psi_1^{(k)} - (S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta\Psi_2^{(k)} + (e_{31}^{(k)} + e_{15}^{(k)})\alpha\Psi_{,z}^{(k)} = (S_{11}^{(k)}\alpha^2 + S_{66}^{(k)}\beta^2)(A_0 - \alpha zC_0) + (S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta(B_0 - \beta zC_0), \tag{20a}$$

$$S_{44}^{(k)}\Psi_{2,zz}^{(k)} - (S_{66}^{(k)}\alpha^2 + S_{22}^{(k)}\beta^2)\Psi_2^{(k)} - (S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta\Psi_1^{(k)} + (e_{32}^{(k)} + e_{24}^{(k)})\beta\Psi_{,z}^{(k)} = (S_{66}^{(k)}\alpha^2 + S_{22}^{(k)}\beta^2)(B_0 - \beta zC_0) + (S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta(A_0 - \alpha zC_0), \tag{20b}$$

$$\epsilon_{33}^{(k)}\Psi_{,zz}^{(k)} - (\epsilon_{11}^{(k)}\alpha^2 + \epsilon_{22}^{(k)}\beta^2)\Psi^{(k)} - (e_{15}^{(k)} + e_{31}^{(k)})\alpha\Psi_{1,z}^{(k)} - (e_{32}^{(k)} + e_{24}^{(k)})\beta\Psi_{2,z}^{(k)} =$$

$$-(e_{31}^{(k)}\alpha^2+e_{32}^{(k)}\beta^2)C_0, \tag{20c}$$

式中，$e_{ij}^{(k)}$，$\varepsilon_{ij}^{(k)}$ 分别代表第 k 层板的压电常数和介电常数，且

$$A_1\psi_1^{(k)}(z)=\Psi_1^{(k)}(z),\ B_1\psi_2^{(k)}(z)=\Psi_2^{(k)}(z),\ D_1\varphi^{(k)}(z)=\Psi^{(k)}(z), \tag{21}$$

式(20)可以分别通过消去主元 $\Psi_2^{(k)}(z)$，$\Psi^{(k)}(z)$ 和运用待定系数法得到满足齐次方程组的通解和一个满足式(20)的特解，即

$$\begin{cases}A_1\psi_1^{(k)}(z)=\overline{\Psi}_1^{(k)}(z)+\alpha zC_0-A_0,\\ B_1\psi_2^{(k)}(z)=\overline{\Psi}_2^{(k)}(z)+\beta zC_0-B_0,\\ D_1\varphi^{(k)}(z)=\overline{\Psi}^{(k)}(z)-\dfrac{e_{15}^{k}\alpha^2+e_{24}^{k}\beta^2}{\varepsilon_{11}^{k}\alpha^2+\varepsilon_{22}^{k}\beta^2}C_0,\end{cases} \tag{22}$$

其中，$\overline{\Psi}_1^{(k)}(z)$，$\overline{\Psi}_2^{(k)}(z)$，$\overline{\Psi}^{(k)}(z)$ 为式(20)的通解，其具体求解过程参见附录 A，且由附录 A 中式(A9)至(A11)，得

$$\begin{cases}\overline{\Psi}_1^{(k)}(z)=\sum_{i=1}^{6}\kappa_i^{(k)}e^{\lambda_i^{(k)}z},\\ \overline{\Psi}_2^{(k)}(z)=\sum_{i=1}^{6}o_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z},\\ \overline{\Psi}^{(k)}(z)=\sum_{i=1}^{6}p_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z},\end{cases} \tag{23}$$

式中，$\lambda_i(i=1,2,\cdots,6)$ 为式(20) 特征方程的 6 个不相等的实特征根，$o_i^{(k)}$，$p_i^{(k)}$ 为给定系数，其具体表达可参见附录 A 中式(A8)、(A10)和(A11)．

式(23)的求解过程中产生了 $6N+12$ 个待定常数 $\kappa_i^{(k)}(k=0,1,2,\cdots,N,N+1;i=1,2,\cdots,6)$，它们可以通过如下界面条件给定：层间应力 τ_{xz}，τ_{yz} 和电位移 D_z 在各层界面处的连续，确定 $3(N+1)$ 个待定系数；对于理想界面，界面位移和电势连续条件 $\Delta U=\Delta V=\Delta\Phi=0$ 或者对于非理想界面，界面位移和电势不连续，即 $\Delta U^{(r)}=R_{tx}^{(r)}\tau_{xz}|_{z=z_r}$，$\Delta V^{(r)}=R_{ty}^{(r)}\tau_{yz}|_{z=z_r}$，$\Delta\Phi^{(r)}=R_e^{(r)}D_z|_{z=z_r}$，确定 $3(N+1)$ 个待定系数；对于上下两层板面($z=\pm H/2$)，其力学条件为

$$\tau_{xz}^{(0)}|_{z=-H/2}=\tau_{yz}^{(0)}|_{z=-H/2}=0,\ \tau_{xz}^{(N+1)}|_{z=H/2}=\tau_{yz}^{(N+1)}|_{z=H/2}=0,$$

其电学条件为

$$D_z^{(0)}|_{z=-H/2}=D_z^{(N+1)}|_{z=H/2}=0,$$

确定 6 个待定系数；由此即可确定 $6N+12$ 个未知常数 $\kappa_i^{(k)}$．

将式(22)代入待定形函数的约束条件式(2)，求得 A_0，A_1，B_0，B_1，D_1 的表达式为

$$\begin{cases}A_0=\sum_{i=1}^{6}\kappa_i^{(\mathrm{mp})},\\ B_0=\sum_{i=1}^{6}o_i^{(\mathrm{mp})}\kappa_i^{(\mathrm{mp})},\\ D_1=\sum_{i=1}^{6}p_i^{(\mathrm{mp})}\kappa_i^{(\mathrm{mp})}-\dfrac{e_{15}^{(\mathrm{mp})}\alpha^2+e_{24}^{(\mathrm{mp})}\beta^2}{\varepsilon_{11}^{(\mathrm{mp})}\alpha^2+\varepsilon_{22}^{(\mathrm{mp})}\beta^2}C_0,\\ A_1=\sum_{i=1}^{6}(\lambda_i^{(\mathrm{mp})}\kappa_i^{(\mathrm{mp})}+\alpha C_0),\ B_1=\sum_{i=1}^{6}(o_i^{(\mathrm{mp})}\lambda_i^{(\mathrm{mp})}\kappa_i^{(\mathrm{mp})}+\beta C_0),\end{cases} \tag{24}$$

式中，上标"mp"表示中性层($z=0$)所在的第 k^{mp} 层板，对于压电层合板[13]，$C_0=1$．由式(22)至(24)，得以下关于形函数 $\psi_1(z)$，$\psi_2(z)$ 和 $\varphi(z)$ 的表达式：

$$\begin{cases}\psi_1^{(k)}(z) = \left(\sum_{i=1}^{6}\kappa_i^{(k)}e^{\lambda_i^{(k)}z} - \sum_{i=1}^{6}\kappa_i^{(mp)} + \alpha z\right)\Big/\left(\sum_{i=1}^{6}\lambda_i^{(mp)}\kappa_i^{(mp)} + \alpha\right), \\ \psi_2^{(k)}(z) = \left(\sum_{i=1}^{6}o_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z} - \sum_{i=1}^{6}o_i^{(mp)}\kappa_i^{(mp)} + \beta z\right)\Big/\left(\sum_{i=1}^{6}o_i^{(mp)}\lambda_i^{(mp)}\kappa_i^{(mp)} + \beta\right), \\ \varphi^{(k)}(z) = \dfrac{\sum_{i=1}^{6}p_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z} - (e_{15}^{(k)}\alpha^2 + e_{24}^{(k)}\beta^2)/(\varepsilon_{11}^{(k)}\alpha^2 + \varepsilon_{22}^{(k)}\beta^2)}{\sum_{i=1}^{6}p_i^{(mp)}\kappa_i^{(mp)} - (e_{15}^{(mp)}\alpha^2 + e_{24}^{(mp)}\beta^2)/(\varepsilon_{11}^{(mp)}\alpha^2 + \varepsilon_{22}^{(mp)}\beta^2)}.\end{cases} \tag{25}$$

3 压电智能层合板的非线性运动控制微分方程

将式(4)和(6)及(7)代入式(12),并引入如下的无量纲参数:

$$\begin{cases}\zeta = \dfrac{x}{a},\ \eta = \dfrac{y}{b},\ \xi = \dfrac{z}{H},\ \tau = t\sqrt{\dfrac{E_{22}}{I_1 H}},\ \lambda_2 = \dfrac{I_2}{I_1 H},\ \lambda_3 = \dfrac{I_3}{I_1 H^2}, \\ \lambda_4 = \dfrac{I_4}{I_1 H},\ \lambda_5 = \dfrac{I_5}{I_1 H^2},\ \lambda_6 = \dfrac{I_6}{I_1 H},\ \lambda_7 = \dfrac{I_7}{I_1 H^2},\ \lambda_8 = \dfrac{I_8}{I_1 H^2},\ \lambda_9 = \dfrac{I_9}{I_1 H^2}, \\ \bar{H} = \dfrac{H}{a},\ \bar{H}_1 = \dfrac{h}{H},\ \bar{H}_2 = \dfrac{h_p}{H},\ L = \dfrac{a}{b},\ C_1 = \dfrac{e_{31}}{E_{22}},\ C_2 = \dfrac{\varepsilon_{11}}{E_{22}}, \\ \bar{U} = \dfrac{U}{a},\ \bar{V} = \dfrac{V}{a},\ \bar{W} = \dfrac{W}{a},\ \bar{\Phi} = \dfrac{\Phi}{a},\ U_0 = \dfrac{u_0}{a},\ V_0 = \dfrac{v_0}{a},\ W_0 = \dfrac{w_0}{a}, \\ U_1 = u_1,\ V_1 = v_1,\ \Theta_1 = \phi_1,\ \Delta\bar{\psi}_1^{(r)} = \dfrac{\Delta\psi_1^{(r)}}{H},\ \Delta\bar{\psi}_2^{(r)} = \dfrac{\Delta\psi_2^{(r)}}{H}, \\ \Delta\bar{\varphi}^{(r)} = \dfrac{\Delta\varphi^{(r)}}{H},\ \bar{A}_{ij} = \dfrac{A_{ij}}{E_{22}H},\ \bar{B}_{ij} = \dfrac{B_{ij}}{E_{22}H^2},\ \bar{D}_{ij} = \dfrac{D_{ij}}{E_{22}H^3}, \\ \bar{B}_{ijl} = \dfrac{B_{ijl}}{E_{22}H^2},\ \bar{D}_{ijl} = \dfrac{D_{ijl}}{E_{22}H^3},\ \bar{D}_{ijlm} = \dfrac{D_{ijlm}}{E_{22}H^3} \quad (i,j = 1,2,6;l,m = 1,2), \\ \bar{A}_{44} = \dfrac{A_{44}}{E_{22}H},\ \bar{A}_{55} = \dfrac{A_{55}}{E_{22}H},\ \bar{R}_{tx} = \dfrac{R_{tx}E_{22}}{a},\ \bar{R}_{ty} = \dfrac{R_{ty}E_{22}}{a},\ \bar{R}_e = \dfrac{R_e e_{31}}{a}, \\ \bar{M}_{3i} = \dfrac{M_{3i}}{e_{31}H},\ \bar{I}_{3i} = \dfrac{I_{3i}}{e_{31}H^2},\ \bar{J}_{3i} = \dfrac{J_{3i}}{e_{31}H^2} \quad (i = 1,2), \\ \bar{J}_{24} = \dfrac{J_{24}}{e_{31}H^2},\ \bar{X}_{11} = \dfrac{X_1}{\varepsilon_{11}H^3},\ \bar{J}_{15} = \dfrac{J_{15}}{e_{31}H^2},\ \bar{X}_{22} = \dfrac{X_{22}}{\varepsilon_{11}H^3},\ \bar{X}_{33} = \dfrac{X_{33}}{\varepsilon_{11}H},\end{cases} \tag{26}$$

得到考虑界面损伤压电智能层合板的无量纲非线性运动控制方程为

$$\begin{aligned}&\bar{A}_{11}U_{0,\zeta\zeta} + \bar{A}_{66}L^2U_{0,\eta\eta} + (\bar{A}_{12} + \bar{A}_{66})LV_{0,\zeta\eta} + \bar{B}_{11}\bar{H}W_{0,\zeta\zeta\zeta} - (\bar{B}_{12} + 2\bar{B}_{66})\bar{H}L^2W_{0,\zeta\eta\eta} + \\ &\quad \bar{A}_{11}W_{0,\zeta}W_{0,\zeta\zeta} + (\bar{A}_{12} + \bar{A}_{66})L^2W_{0,\eta}W_{0,\zeta\eta} + \bar{A}_{66}L^2W_{0,\zeta}W_{0,\eta\eta} + \\ &\quad \bar{B}_{111}\bar{H}U_{1,\zeta\zeta} + \bar{B}_{661}L^2\bar{H}U_{1,\eta\eta} + (\bar{B}_{122} + \bar{B}_{662})\bar{H}LV_{1,\zeta\eta} + C_1\bar{M}_{31}\Theta_{1,\zeta} = \\ &\quad \bar{H}^{-2}U_{0,\tau\tau} + \lambda_4\bar{H}^{-1}U_{1,\tau\tau} - \lambda_2\bar{H}^{-1}W_{0,\zeta\tau\tau},\end{aligned} \tag{27a}$$

$$\begin{aligned}&(\bar{A}_{12} + \bar{A}_{66})U_{0,\zeta\eta} + \bar{A}_{66}L^{-1}V_{0,\zeta\zeta} + \bar{A}_{22}LV_{0,\eta\eta} - \bar{B}_{22}\bar{H}L^2W_{0,\eta\eta\eta} - (2\bar{B}_{66} + \bar{B}_{12})\bar{H}W_{0,\zeta\zeta\eta} + \\ &\quad \bar{A}_{66}W_{0,\eta}W_{0,\zeta\zeta} + (\bar{A}_{66} + \bar{A}_{12})W_{0,\zeta}W_{0,\zeta\eta} + \bar{A}_{22}L^2W_{0,\eta}W_{0,\eta\eta} + \\ &\quad (\bar{B}_{661} + \bar{B}_{121})\bar{H}U_{1,\zeta\eta} + \bar{B}_{662}\bar{H}L^{-1}V_{1,\zeta\zeta} + \bar{B}_{222}\bar{H}LV_{1,\eta\eta} + C_1\bar{M}_{32}\Theta_{1,\eta} =\end{aligned}$$

$$\bar{H}^{-2}L^{-1}V_{0,\tau\tau} + \lambda_6\bar{H}^{-1}L^{-1}V_{1,\tau\tau} - \lambda_2\bar{H}^{-1}W_{0,\eta\tau\tau}, \tag{27b}$$

$$\begin{aligned}
&\bar{B}_{11}U_{0,\zeta\zeta\zeta} + (\bar{B}_{12} + 2\bar{B}_{66})L^2U_{0,\zeta\eta\eta} + (\bar{B}_{12} + 2\bar{B}_{66})LV_{0,\zeta\zeta\eta} + \bar{B}_{22}L^3V_{0,\eta\eta\eta} - \\
&\quad 2(\bar{D}_{12} + 2\bar{D}_{66})\bar{H}L^2W_{0,\zeta\zeta\eta\eta} - \bar{D}_{11}\bar{H}W_{0,\zeta\zeta\zeta\zeta} - \bar{D}_{22}\bar{H}L^4W_{0,\eta\eta\eta\eta} + \bar{B}_{11}W_{0,\zeta\zeta}^2 + \\
&\quad \bar{B}_{11}W_{0,\zeta}W_{0,\zeta\zeta\zeta} + 2(\bar{B}_{12} + 2\bar{B}_{66})L^2W_{0,\zeta\eta}^2 + (\bar{B}_{12} + 2\bar{B}_{66})L^2W_{0,\eta}W_{0,\zeta\zeta\eta} + \\
&\quad (\bar{B}_{12} + 2\bar{B}_{66})L^2W_{0,\zeta}W_{0,\zeta\eta\eta} + \bar{B}_{22}L^4W_{0,\eta\eta}^2 + \bar{B}_{22}L^4W_{0,\eta}W_{0,\eta\eta\eta} + \\
&\quad 2\bar{B}_{66}L^2W_{0,\zeta\zeta}W_{0,\eta\eta} + D_{111}HU_{1,\zeta\zeta\zeta} + (\bar{D}_{121} + 2\bar{D}_{661})\bar{H}L^2U_{1,\zeta\eta\eta} + \bar{D}_{222}\bar{H}L^3V_{1,\eta\eta\eta} + \\
&\quad (\bar{D}_{122} + 2\bar{D}_{662})\bar{H}LV_{1,\zeta\zeta\eta} + \bar{A}_{11}\bar{H}^{-1}U_{0,\zeta}W_{0,\zeta\zeta} + A_{12}\bar{H}^{-1}LV_{0,\eta}W_{0,\zeta\zeta} + \\
&\quad \frac{\bar{A}_{11}}{2}\bar{H}^{-1}W_{0,\zeta}^2W_{0,\zeta\zeta} + \frac{\bar{A}_{12}}{2}\bar{H}^{-1}L^2W_{0,\eta}^2W_{0,\zeta\zeta} - \bar{B}_{11}W_{0,\zeta\zeta}^2 - \bar{B}_{12}L^2W_{0,\eta\eta}W_{0,\zeta\zeta} + \\
&\quad \bar{B}_{111}U_{1,\zeta}W_{0,\zeta\zeta} + \bar{B}_{122}LV_{1,\eta}W_{0,\zeta\zeta} + C_1\bar{T}_{31}\bar{H}^{-1}\Theta_1W_{0,\zeta\zeta} + 2\bar{A}_{66}\bar{H}^{-1}L^2U_{0,\eta}W_{0,\zeta\eta} + \\
&\quad 2\bar{A}_{66}\bar{H}^{-1}LV_{0,\zeta}W_{0,\zeta\eta} + 2\bar{A}_{66}\bar{H}^{-1}L^2W_{0,\zeta}W_{0,\eta}W_{0,\zeta\eta} - 4\bar{B}_{66}L^2W_{0,\zeta\eta}^2 + \\
&\quad 2\bar{B}_{661}L^2U_{1,\eta}W_{0,\zeta\eta} + 2\bar{B}_{662}LV_{1,\zeta}W_{0,\zeta\eta} + \bar{A}_{12}\bar{H}^{-1}L^2U_{0,\zeta}W_{0,\eta\eta} + \\
&\quad A_{22}\bar{H}^{-1}L^3V_{0,\eta}W_{0,\eta\eta} + \frac{\bar{A}_{12}}{2}\bar{H}^{-1}L^2W_{0,\zeta}^2W_{0,\eta\eta} + \frac{\bar{A}_{22}}{2}\bar{H}^{-1}L^4W_{0,\eta}^2W_{0,\eta\eta} - \\
&\quad \bar{B}_{22}L^4W_{0,\eta\eta}^2 - \bar{B}_{12}L^2W_{0,\eta\eta}W_{0,\zeta\zeta} + \bar{B}_{121}L^2U_{1,\zeta}W_{0,\eta\eta} + \bar{B}_{222}L^3V_{1,\eta}W_{0,\eta\eta} + \\
&\quad C_1\bar{T}_{32}\bar{H}^{-1}L^2\Theta_1W_{0,\eta\eta} + C_1\bar{I}_{31}\Theta_{1,\zeta\zeta} + C_1L^2\bar{I}_{32}\Theta_{1,\eta\eta} = \\
&\quad \bar{H}^{-3}W_{0,\tau\tau} - \lambda_3\bar{H}^{-1}W_{0,\zeta\zeta\tau\tau} + \lambda_2\bar{H}^{-2}U_{0,\zeta\tau\tau} + \lambda_5\bar{H}^{-1}U_{1,\zeta\tau\tau} + \\
&\quad \lambda_2\bar{H}^{-2}V_{0,\eta\tau\tau} + \lambda_7\bar{H}^{-1}LV_{1,\eta\tau\tau},
\end{aligned} \tag{27c}$$

$$\begin{aligned}
&\bar{B}_{111}U_{0,\zeta\zeta} + \bar{B}_{661}L^2U_{0,\eta\eta} + (\bar{B}_{121} + \bar{B}_{661})LV_{0,\zeta\eta} - \bar{D}_{111}\bar{H}W_{0,\zeta\zeta\zeta} - \\
&\quad (\bar{D}_{121} + 2\bar{D}_{661})\bar{H}L^2W_{0,\zeta\eta\eta} + \bar{B}_{111}W_{0,\zeta}W_{0,\zeta\zeta} + (\bar{B}_{121} + \bar{B}_{661})L^2W_{0,\eta}W_{0,\zeta\eta} + \\
&\quad \bar{B}_{661}L^2W_{0,\zeta}W_{0,\eta\eta} + \bar{D}_{1111}\bar{H}L^2U_{1,\zeta\zeta} + \bar{D}_{6611}\bar{H}L^2U_{1,\eta\eta} - \bar{A}_{55}\bar{H}^{-1}U_1 - \\
&\quad \sum_{r=0}^{N}\frac{(\Delta\bar{\psi}_1^{(r)})^2}{\bar{R}_{1\zeta}^{(r)}}U_1 + (\bar{D}_{1212} + \bar{D}_{6612})\bar{H}LV_{1,\zeta\eta} + C_1(\bar{J}_{31} - \bar{J}_{15})\Theta_{1,\zeta} = \\
&\quad \lambda_4\bar{H}^{-2}U_{0,\tau\tau} - \lambda_5\bar{H}^{-1}W_{0,\zeta\tau\tau} + \lambda_8\bar{H}^{-1}U_{1,\tau\tau},
\end{aligned} \tag{27d}$$

$$\begin{aligned}
&(\bar{B}_{122} + \bar{B}_{662})U_{0,\zeta\eta} + \bar{B}_{222}LV_{0,\eta\eta} + \bar{B}_{662}L^{-1}V_{0,\zeta\zeta} - \bar{D}_{222}\bar{H}L^2W_{0,\eta\eta\eta} - \\
&\quad (\bar{D}_{122} + 2\bar{D}_{662})\bar{H}W_{0,\zeta\zeta\eta} + (\bar{B}_{122} + \bar{B}_{662})W_{0,\zeta}W_{0,\zeta\eta} + \bar{B}_{222}L^2W_{0,\eta}W_{0,\eta\eta} + \\
&\quad \bar{B}_{662}W_{0,\zeta\zeta}W_{0,\eta} + (\bar{D}_{1212} + \bar{D}_{6612})\bar{H}U_{1,\zeta\eta} + \bar{D}_{2222}\bar{H}LV_{1,\eta\eta} + \bar{D}_{6622}\bar{H}L^{-1}V_{1,\zeta\zeta} - \\
&\quad L^{-1}\sum_{r=0}^{N}\frac{(\Delta\bar{\psi}_2^{(r)})^2}{\bar{R}_{1\eta}^{(r)}}V_1 - \bar{A}_{44}L^{-1}\bar{H}^{-1}V_1 + C_1(\bar{J}_{32} - \bar{J}_{24})\Theta_{1,\eta} = \\
&\quad \lambda_6\bar{H}^{-2}L^{-1}V_{0,\tau\tau} - \lambda_7\bar{H}^{-1}W_{0,\eta\tau\tau} + \lambda_9\bar{H}^{-1}L^{-1}V_{1,\tau\tau},
\end{aligned} \tag{27e}$$

$$\begin{aligned}
&-\bar{M}_{31}C_1U_{0,\zeta} - \bar{M}_{32}C_1LV_{0,\eta} + \bar{I}_{31}\bar{H}C_1W_{0,\zeta\zeta} + \bar{I}_{32}C_1\bar{H}L^2W_{0,\eta\eta} - \frac{\bar{M}_{31}}{2}C_1W_{0,\zeta}^2 - \\
&\quad \frac{\bar{M}_{32}}{2}C_1L^2W_{0,\eta}^2 - C_1(\bar{J}_{31} - \bar{J}_{15})\bar{H}U_{1,\zeta} - C_1(\bar{J}_{32} - \bar{J}_{24})\bar{H}LV_{1,\eta} - \\
&\quad \bar{X}_{33}C_2\Theta_1 + \bar{X}_{22}C_2\bar{H}^2\Theta_{1,\zeta\zeta} + \bar{X}_{11}C_2\bar{H}^2L^2\Theta_{1,\eta\eta} - C_1\bar{H}\sum_{r=0}^{N}\frac{(\Delta\bar{\varphi}^{(r)})^2}{\bar{R}_e^{(r)}}\Theta_1 = 0.
\end{aligned} \tag{27f}$$

上式中各广义刚度系数的具体表达为

$$\begin{cases}(A_{ij},B_{ij},C_{ij}) = \int_{-H/2}^{H/2} S_{ij}(1,z,z^2)\mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{ij}^{(k)}(1,z,z^2)\mathrm{d}z, \\ B_{ijl} = \int_{-H/2}^{H/2} S_{ij}\psi_l \mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{ij}^{(k)}\psi_l \mathrm{d}z, \\ D_{ijl} = \int_{-H/2}^{H/2} S_{ij} z\psi_l \mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{ij}^{(k)} z\psi_l \mathrm{d}z, \\ D_{ijlm} = \int_{-H/2}^{H/2} S_{ij}\psi_l\psi_m \mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{ij}^{(k)}\psi_l\psi_m \mathrm{d}z \qquad (i,j = 1,2,6; l,m = 1,2), \\ A_{44} = \int_{-H/2}^{H/2} S_{44}((\psi_{2,z})^2)\mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{44}^{(k)}((\psi_{2,z})^2)\mathrm{d}z, \\ A_{55} = \int_{-H/2}^{H/2} S_{55}((\psi_{1,z})^2)\mathrm{d}z = \sum_{k=0}^{N+1}\int_{z_{k-1}}^{z_k} S_{55}^{(k)}((\psi_{1,z})^2)\mathrm{d}z.\end{cases} \tag{28}$$

广义压电系数和广义介电系数的具体表达为

$$\begin{cases}M_{3i} = \int_{-H/2}^{H/2} e_{3i}\varphi_{,z}\mathrm{d}z = \sum_{k=0}^{N+1} e_{3i}^k \int_{z_{k-1}}^{z_k} \varphi_{,z}\mathrm{d}z, \\ I_{3i} = \int_{-H/2}^{H/2} e_{3i}\varphi_{,z} z\mathrm{d}z = \sum_{k=0}^{N+1} e_{3i}^k \int_{z_{k-1}}^{z_k} \varphi_{,z} z\mathrm{d}z \qquad (i = 1,2), \\ J_{3i} = \int_{-H/2}^{H/2} e_{3i}\varphi_{,z}\psi_i\mathrm{d}z = \sum_{k=0}^{N+1} e_{3i}^k \int_{z_{k-1}}^{z_k} \varphi_{,z}\psi_i\mathrm{d}z \qquad (i = 1,2), \\ J_{24} = \int_{-H/2}^{H/2} e_{24}\varphi(z)\psi_{2,z}\mathrm{d}z = \sum_{k=0}^{N+1} e_{24}^k \int_{z_{k-1}}^{z_k} \varphi(z)\psi_{2,z}\mathrm{d}z, \\ J_{15} = \int_{-H/2}^{H/2} e_{15}\varphi(z)\psi_{1,z}\mathrm{d}z = \sum_{k=0}^{N+1} e_{15}^k \int_{z_{k-1}}^{z_k} \varphi(z)\psi_{1,z}\mathrm{d}z, \\ X_{ii} = \int_{-H/2}^{H/2} \varepsilon_{ii}\varphi^2(z)\mathrm{d}z = \sum_{k=0}^{N+1} \varepsilon_{ii}^k \int_{z_{k-1}}^{z_k} \varphi^2(z)\mathrm{d}z \qquad (i = 1,2), \\ X_{33} = \int_{-H/2}^{H/2} \varepsilon_{33}\varphi_{,z}^2\mathrm{d}z = \sum_{k=0}^{N+1} \varepsilon_{33}^k \int_{z_{k-1}}^{z_k} \varphi_{,z}^2\mathrm{d}z.\end{cases} \tag{29}$$

4 损伤演化方程

对于复合材料铺设层内损伤变量 $D_1^{(k)}, D_2^{(k)}$，基于 Kachanov 连续体损伤力学理论[17]，本文采用如下无量纲化损伤演化方程：

$$\frac{\mathrm{d}D_i^{(k)}}{\mathrm{d}\tau} = \begin{cases} B_i\left(\dfrac{\sigma_{\mathrm{D}i}^{(k)}}{1 - D_i^{(k)}}\right)^{m_i}, & \sigma_i^{(k)} \geqslant \sigma_{\mathrm{D}i}^{(k)} \\ 0, & \sigma_i^{(k)} \geqslant \sigma_{\mathrm{D}i}^{(k)} \end{cases} \qquad (i = 1,2), \tag{30}$$

式中，B_i, m_i 为材料常数，这里取为 $B_1 = 3.7 \times 10^{-12}\ \mathrm{MPa}^{-m_1}$，$B_2 = 4.8 \times 10^{-11}\ \mathrm{MPa}^{-m_2}$，$m_1 = m_2 = 3$，$\sigma_{\mathrm{D}i}^{(k)}$ 为当 i 方向的损伤 $D_i^{(k)}$ 开始演化时损伤应力的门槛值，且

$$\sigma_{\mathrm{D1}}^{(k)} = \frac{1}{h^k}\int_{z_{k-1}}^{z_k} \tilde{\sigma}_{11}^{(k)}\mathrm{d}z, \quad \sigma_{\mathrm{D2}}^{(k)} = \frac{1}{h^k}\int_{z_{k-1}}^{z_k} \tilde{\sigma}_{22}^{(k)}\mathrm{d}z, \tag{31}$$

其中，$\bar{\sigma}_{11}^{(k)}$，$\bar{\sigma}_{22}^{(k)}$ 分别为第 k 层板沿着纤维方向和垂直纤维方向的有效应力分量.

对于层间界面处的损伤变量 $R_{tx}^{(r)}$，$R_{ty}^{(r)}$，$R_{e}^{(r)}$，在本文算例基于界面劣化初始阶段，给定界面损伤变量的值，研究界面损伤初期对压电智能层合板非线性力学行为的影响.

5 求解方法

对于方程组(27)寻求满足边界条件(17)的完全解析解是几乎不可能的，一般只能求其半解析解. 对于4边简支对称正交铺设的压电智能层合板，位移和电势的形式函数可设为

$$U_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos(\pi\zeta)\sin(\pi\eta)f_{u_0}(\tau), \tag{32a}$$

$$U_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos(\pi\zeta)\sin(\pi\eta)f_{u_1}(\tau), \tag{32b}$$

$$V_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin(\pi\zeta)\cos(\pi\eta)f_{v_0}(\tau), \tag{32c}$$

$$V_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin(\pi\zeta)\cos(\pi\eta)f_{v_1}(\tau), \tag{32d}$$

$$W_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin(\pi\zeta)\sin(\pi\eta)f_{w_0}(\tau), \tag{32e}$$

$$\Theta_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin(\pi\zeta)\sin(\pi\eta)f_{\phi_1}(\tau), \tag{32f}$$

将形式解(32)代入方程(27)，并施加 Galerkin 方法，且取一阶截断，则将非线性偏微分方程组化为由时间函数表示的非线性常微分方程组. 然后，应用 Newmark 方法，对时间变量的导数项在时间上离散，整个问题可以采用迭代法对它们联立求解，迭代精度取为 0.1%.

对于计算中损伤值，若在初始时刻，令板内无损伤（$D_1^{(k)}(0) = D_2^{(k)}(0) = 0$）. 在第 J 时间步时，求解式(27) 并利用式(4) 和式(5)，可以求得板内任一点的应变与应力；由式(30) 判断第 J 时间步时损伤是否发展. 记在第 J 时间步时，第 k 层板在 $i(i = 1,2)$ 方向的损伤值为 $D_i^{(k)}(J)$. 若 $\sigma_i^{(k)} \geqslant \sigma_{Di}^{(k)}$，则在 $J\Delta\tau$ 时的损伤值为

$$D_i^{(k)}(J) = D_i^{(k)}(J-1) + \dot{D}_i^{(k)}(D_i^{(k)}(J-1), \sigma_{Di}^{(k)})\Delta\tau, \tag{33}$$

若 $\sigma_i^{(k)} \leqslant \sigma_{Di}^{(k)}$，则

$$D_i^{(k)}(J) = D_i^{(k)}(J-1). \tag{34}$$

6 数值结果和讨论

6.1 线性自由振动的固有频率及验证性比较

令 $\bar{R}_{tx}^{(r)} = \bar{R}_{ty}^{(r)} = \bar{R}_{e}^{(r)} = \bar{R}^{(r)}$，取材料参数见附录 B. 定义板的无量纲线性自由振动基频和非线性自由振动基频为

$$(\bar{\omega}_0, \bar{\omega}) = \sqrt{I_1 H / E_{22}}(\omega_0, \omega), \tag{35}$$

其中，ω_0，ω 为板的有量纲线性自由振动基频和非线性自由振动基频.

取一上下表面粘贴压电层的对称正交铺设的复合材料层合方板[90°/0°/90°/0°/0°/90°/0°/90°]，弹性层材料选用石墨/环氧复合材料，压电层材料选用压电陶瓷 PZT-5A. 通过计算，不同厚跨比下压电智能层合方板的自由振动基频如表 1 所示. 从表 1 中可以看出，本文得到的数值解小于基于一阶剪切变形理论（FSDPT）[24]下得到的数值解，大于基于三维（3-D）[23]理论得

到的数值解，但三者的数据吻合很好，说明了本文分析方法的正确性和计算程序的可靠性.

表 1　不同厚跨比 (H/a) 下压电智能层合方板线性自由振动基频 (ω_0) 的比较

$H/a=0.01$			$H/a=0.1$			$H/a=0.2$		
文献[23] 3-D	文献[24] FSDPT	本文结果	文献[23] 3-D	文献[24] FSDPT	本文结果	文献[23] 3-D	文献[24] FSDPT	本文结果
268.86	283.93	276.21	2 357.7	2 516.7	2 441.5	3 648.0	3 953.2	3 871.9

下面考虑一上下表面粘结压电薄膜的 4 层正交对称铺设的具界面损伤复合材料层合板 [0°/90°/90°/0°]，复合材料层和压电层的材料分别取为石墨/环氧材料和 PVDF 压电陶瓷，相应的材料参数在附录 B 中给出，给定层内损伤值 $D_1^{(k)}$，$D_2^{(k)}$ 和界面损伤值界面损伤值为 $\bar{R}$，边界条件为简支.

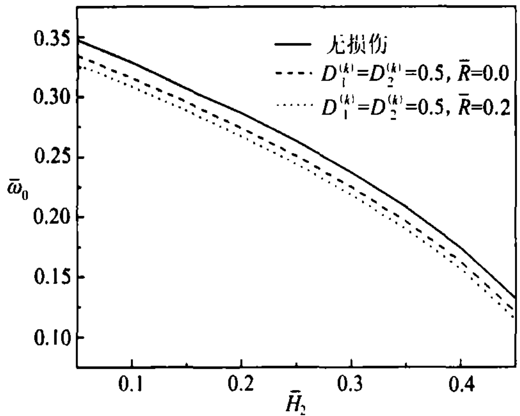

图 3　压电层厚度比 $\bar{H}_2$ 对压电智能层合板自由振动频率的影响

($L=2$, $\bar{H}=0.1$)

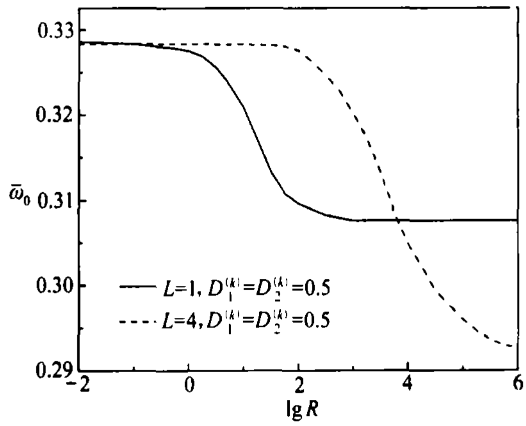

图 4　界面损伤 $\bar{R}$ 对压电智能层合板自由振动频率的影响

($\bar{H}=0.1$, $\bar{H}_1=0.8$, $\bar{H}_2=0.1$)

图 3 显示了压电层厚度与总厚度之比 $\bar{H}_2(h_p/H)$ 对压电智能层合板的无量纲自由振动频率 $\bar{\omega}_0$ 的影响. 由图 3 可知，随着压电层厚度 $\bar{H}_2$ 的增加，压电智能层合板的无量纲自由振动频率减小；层内损伤和界面损伤的出现，使得压电智能层合板的整体刚度相应地减小，其无量纲自由振动频率也变小.

图 4 显示了界面损伤 $\bar{R}$ 对压电智能层合板的无量纲自由振动频率 $\bar{\omega}_0$ 的影响. 由图 4 可知，随着界面劣化加剧($10^{-1} < \bar{R} < 10^5$)，压电智能层合板的无量纲线性自由振动频率 $\bar{\omega}_0$ 迅速减小，且其影响的范围随着长宽比的增加而增加.

6.2　不同参数对板的非线性幅频响应曲线的影响

下面仍选用以上的计算模型，但考虑层内损伤演化其初始损伤值为 $D_1^{(k)}(0)=D_2^{(k)}(0)=0$，且给定界面损伤值为 $\bar{R}^{(r)}=0.2$.

图 5 显示了厚跨比 $\bar{H}(H/a)$ 对层合板非线性自由振动幅频响应曲线的影响. 由图 5 可知，对于给定的振幅，随着厚跨比 $\bar{H}$ 的增加，板的非线性自由振动频率变大；当板厚度一定时，板的非线性自由振动频率随着振幅的增加而增大；在振幅较小时，由损伤引起的频率降低较小；但随着振幅的增大，层内损伤积累的速度加大，其非线性自由振动频率相对于无损时明显降低，且界面损伤的出现将进一步加剧这种趋势.

图 6 显示了长宽比 $L(a/b)$ 对层合板非线性自由振动幅频响应曲线的影响. 由图 6 可

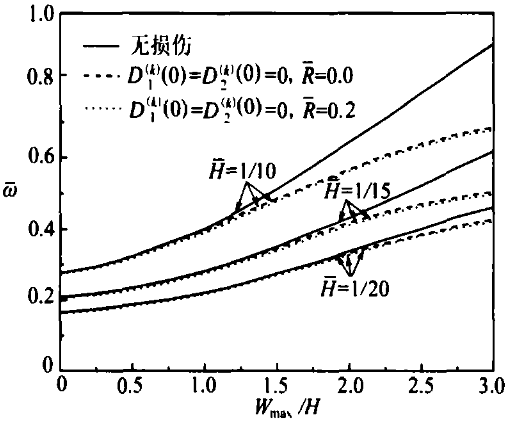

图5　厚跨比 $\bar{H}$ 对压电智能层合板非线性自由振动幅频响应曲线的影响

($L = 1, \bar{H} = 0.8$)

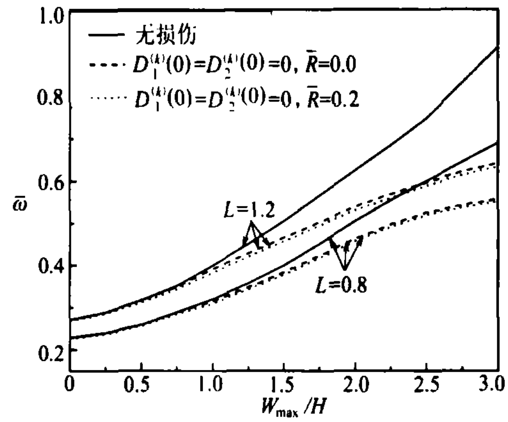

图6　长宽比 L 对压电智能层合板非线性自由振动幅频响应曲线的影响

($\bar{H} = 0.1, \bar{H}_1 = 0.8$)

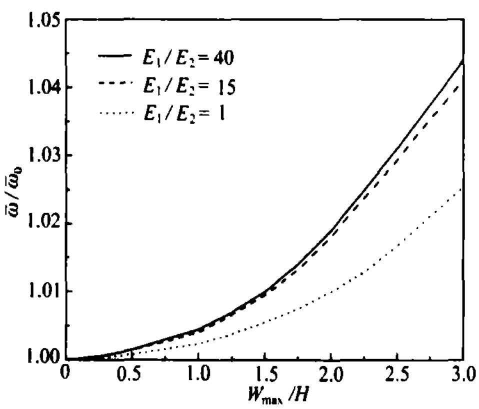

图7　材料弹性常数对压电智能层合板非线性自由振动幅频响应曲线的影响

($L = 1, \bar{H} = 0.1, \bar{H}_1 = 0.8$)

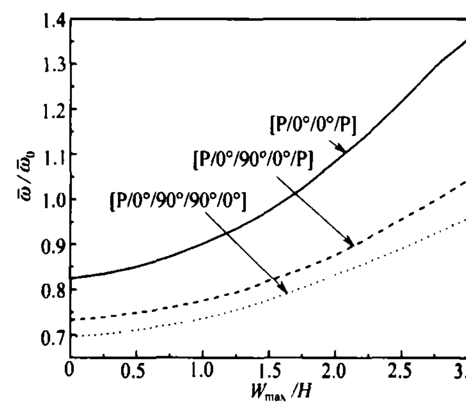

图8　铺设层数对压电智能层合板非线性自由振动幅频响应曲线的影响

($L = 1, \bar{H} = 0.1, \bar{H}_1 = 0.8$)

知，对于给定的振幅，随着长宽比 L 的增加，板的非线性自由振动频率变大；当长宽比一定时，随着振幅的增加，层内损伤积累的速度加大，其非线性自由振动频率相对于无损时明显降低.

图7显示了不同材料弹性常数对压电智能层合板非线性幅频响应曲线的影响. 其计算参数取为

$$E_2 = E_3 = 10\ \text{GPa},\ G_{12} = G_{13} = G_{23} = 0.5E_2,\ \mu_{12} = \mu_{13} = 0.25,$$

$$\mu_{23} = 0.33,\ \rho = 1\,600\ \text{kg/m}^3,\ \bar{R}^{(r)} = 0.2,\ D_1^{(k)} = D_2^{(k)} = 0.$$

由图7可以看出，当结构振幅增大时，随着材料常数 E_1/E_2 比值的增大，压电智能层合板的非线性振动频率也越大. 图8显示了在压电智能层合板总厚度 H 给定的情况下，铺设层数对压电智能层合板非线性幅频响应曲线的影响. 由图8可知，当板厚度一定时，板的铺设层数越少，则板的非线性自由振动频率随中点振幅的增加而迅速增长.

7　结　　论

1) 层内损伤和界面损伤的出现使得压电智能层合板的整体刚度相应地减小；随着界面劣化加剧，压电智能层合板的无量纲线性自由振动频率 $\bar{\omega}_0$ 迅速减小，且其影响的范围随着长宽

比的增加而增加.

2) 在振幅一定的情况下,随着板厚的增加和宽度的减小,板的非线性自由振动频率变大;当板厚和长度一定时,振幅的增大,铺设层数的减少,将引起板的非线性自由振动频率迅速增长.

3) 在振幅较小时,由损伤引起的频率降低较小;但随着振幅的增大,层内损伤积累的速度加大,且界面损伤的出现进一步加剧结构的刚度迅速下降,其非线性自由振动频率相对于无损时明显降低;而且板越厚,损伤对板的非线性自由振动频率的影响也越显著.

附 录 A

为得到式(20)的特征方程,令 $D^n = d^n/dz^n$,且设式(20)中各系数为

$$\begin{cases} a = S_{55}^{(k)},\ b = (S_{11}^{(k)}\alpha^2 + S_{66}^{(k)}\beta^2),\ c = (S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta,\ d = (e_{31}^{(k)} + e_{15}^{(k)})\alpha,\ e = S_{44}^{(k)}, \\ f = (S_{66}^{(k)}\alpha^2 + S_{22}^{(k)}\beta^2),\ g = (e_{32}^{(k)} + e_{24}^{(k)})\beta,\ h = \varepsilon_{33}^{(k)},\ i = (\varepsilon_{11}^{(k)}\alpha^2 + \varepsilon_{22}^{(k)}\beta^2). \end{cases} \tag{A1}$$

则式(20)可改写为

$$aD^2\Psi_1 - b\Psi_1 - c\Psi_2 + dD\Psi = 0, \tag{A2}$$

$$eD^2\Psi_2 - f\Psi_2 - c\Psi_1 + gD\Psi = 0, \tag{A3}$$

$$hD^2\Psi - i\Psi - dD\Psi_1 - gD\Psi_2 = 0. \tag{A4}$$

通过消去主元 $\Psi_2^{(k)}(z), \Psi$, (A2)至(A4)式可以转换为仅含 Ψ_1 的方程为

$$h_1d_1D^6\Psi_1 + (g_1d_1 - a_1e_1 - h_1c_1)D^4\Psi_1 + (b_1e_1 + a_1f_1 - c_1g_1)D^2\Psi_1 - h_1f_1\Psi_1 = 0, \tag{A5}$$

其中

$$\begin{cases} h_1 = -(ga/c),\ g_1 = (gd/c - d),\ f_1 = -i,\ e_1 = -(h - gd/c),\ d_1 = -(ceh/fg), \\ c_1 = -(d + cei/fg - cg/f),\ b_1 = -(b - c^2/f),\ a_2 = -(a + ced/fg). \end{cases} \tag{A6}$$

(A5)式可以简写为

$$m_1D^6\Psi_1 + m_2D^4\Psi_1 + m_3D^2\Psi_1 + m_4\Psi_1 = 0. \tag{A7}$$

(A7)式为仅关于 $\Psi_1^{(k)}(z)$ 的六阶常系数齐次线性微分方程,其特征方程为

$$m_1\lambda^6 + m_2\lambda^4 + m_3\lambda^2 + m_4 = 0. \tag{A8}$$

由特征方程(A8),得 $\Psi_1^{(k)}(z)$ 的通解为

$$\overline{\Psi}_1 = \sum_{i=1}^{6}\kappa_i^{(k)}e^{\lambda_i^{(k)}z}, \tag{A9}$$

其中,$\lambda_i^{(k)}(i = 1,2,\cdots,6)$ 为式(A8)的 6 个特征根,$\kappa_i^{(k)}$ 为积分常数.

将式(A9)代入式(A2)和(A3)得 $\Psi^{(k)}(z), \Psi_2^{(k)}(z)$ 的通解为

$$\begin{cases} \overline{\Psi} = \sum_{i=1}^{6}p_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z}, \\ p_i^{(k)} = \dfrac{h_1d_1(\lambda_i^{(k)})^4 - (g_1d_1 - a_1e_1)(\lambda_i^{(k)})^2 + b_1e_1}{(c_1e_1 - f_1d_1)\lambda_i^{(k)}}, \end{cases} \tag{A10}$$

$$\begin{cases} \overline{\Psi}_2 = \sum_{i=1}^{6}o_i^{(k)}\kappa_i^{(k)}e^{\lambda_i^{(k)}z}, \\ o_i^{(k)} = \dfrac{S_{55}^{(k)}(\lambda_i^{(k)})^2 + (e_{13}^{(k)} + e_{15}^{(k)})\alpha p_i^{(k)}\lambda_i^{(k)} - (S_{11}^{(k)}\alpha^2 + S_{66}^{(k)}\beta^2)}{(S_{12}^{(k)} + S_{66}^{(k)})\alpha\beta}, \end{cases} \tag{A11}$$

式(A10)和(A11)中的各系数由式(A1)和(A6)给定.

附录 B

(a) 石墨/环氧复合材料参数

$E_1 = 181$ GPa, $E_2 = E_3 = 10.3$ GPa, $G_{12} = G_{13} = 7.17$ GPa, $G_{23} = 2.87$ GPa,

$\mu_{12} = \mu_{13} = 0.28$, $\mu_{23} = 0.33$, $\rho = 1\,580$ kg/m^3.

(b) 压电陶瓷 PVDF 材料参数

$$S = \begin{bmatrix} 3.61 & 1.61 & 1.42 & 0 & 0 & 0 \\ & 3.13 & 1.31 & 0 & 0 & 0 \\ & & 1.63 & 0 & 0 & 0 \\ & & & 0.55 & 0 & 0 \\ & & & & 0.59 & 0 \\ & & & & & 0.59 \end{bmatrix} \text{GPa}, \quad d^{T} = \begin{bmatrix} 0 & 0 & 21.0 \\ 0 & 0 & 1.5 \\ 0 & 0 & -32.5 \\ 0 & -23 & 0 \\ -27 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^{-12}\ \text{C/N},$$

$$\left[\frac{\varepsilon}{\varepsilon_0}\right] = \begin{bmatrix} -6.1 & 0 & 0 \\ 0 & -7.5 & 0 \\ 0 & 0 & -6.7 \end{bmatrix}, \quad e^{T} = Sd^{T}, \ \varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2), \ \rho = 1\,800\ \text{kg/m}^3.$$

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Non-Linear Free Vibration Analysis of Piezoelastic Laminated Plates With Interfacial Damage

FU Yi-ming, LI Sheng, JIANG Ye-jie

(*College of Mechanics and Aerospace*, *Hunan University*, *Changsha* 410082, *P. R. China*)

Abstract: A nonlinear model for piezoelastic laminated plates containing the damage effect of the intra-layers and inter-laminar interface is presented, and the discontinuity of displacement and electric potential on the interfaces were depicted by three shape functions. By using the Hamilton variation principle, the three-dimensional nonlinear dynamic equations of piezoelastic laminated plates with damage effect were derived. Then, using the Galerkin method, a mathematical solution was presented. In numerical results, the effects of different damage models, the thickness of piezoelectric layer, the side-to-thickness ratio and the length-to-width ratio on the natural frequencies and non-linear amplitude-frequency response characteristics of the simply-supported piezoelastic laminated plates with interfacial imperfections were discussed.

Key words: piezoelastic laminated plates; non-linear free vibration analysis; interfacial damage; damage evolution; Galerkin method