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扁薄锥壳在周边弯矩和横向载荷共同作用下的非线性振动*

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(我刊编委叶开沅来稿)

摘要: 从问题的变分方程和协调方程出发, 选取扁锥壳中心最大振幅为摄动参数, 采用摄动变分法, 对周边简支的扁薄锥壳在周边弯矩和横向载荷共同作用下的非线性振动问题进行了求解• 一次近似得到了扁薄锥壳在静载荷作用下的线性固有频率, 二次近似得到了扁薄锥壳在静载荷作用下的精确度较高的非线性固有频率• 并给出了小变形时固有频率与周边弯矩、横向载荷、振幅以及锥底角之间非线性关系的三次近似解析表达式, 数值结果的图形反映了在一定范围内固有频率和各参数之间非线性关系的复杂性和规律性•

关 键 词: 摄动变分法; 非线性振动; 固有频率; 扁薄锥壳
中图分类号: O405 文献标识码: A

引 言

非线性振动问题的研究不论在理论方面还是在应用方面都具有重要意义; 锥壳在建筑、航海、航天等工程中都占有重要地位, 而且被广泛应用于仪表中的弹性元件• 板壳的非线性振动问题是个复杂问题, 是个难度较大的力学和数学问题, 若再考虑外载荷(静载、热载、磁场) 作用问题难度更大[1~7], 就单从静载作用下的非线性振动问题也引起不同的新意[8], 结构非线性振动时考虑外载的作用主要原因是非线性振动时引起的中面张力, 即使在大变形下线性振动时也得考虑静载影响, 这是大变形时引起的中面张力所致• 本文是在小变形情况下, 选取了扁锥壳中心最大振幅为摄动参数, 采用摄动变分法[9, 10], 研究了静载对锥壳非线性振动时的固有频率的影响•

1 扁薄锥壳的混合边值问题

考虑周边简支, 承受横向载荷 q 和周边弯矩 m 作用的扁薄锥壳, 如图 1 所示• 锥底的半径为 a, 厚度为 h, 锥底角为 θ, 则扁薄锥壳周边简支时的变分方程为

$$\int_{t_2}^{t_1}\int_0^a\left\{DL_1(w)-q-\frac{1}{r}\frac{\partial}{\partial r}\left[rN_r\left(\theta+\frac{\partial w}{\partial r}\right)\right]+\rho\frac{\partial^2 w}{\partial t^2}\right\}\delta w r\,\mathrm{d}r\,\mathrm{d}t=0; \tag{1}$$

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协调方程:

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}(rN_r)=-Eh\frac{\partial w}{\partial r}\left(\theta+\frac{1}{2}\frac{\partial w}{\partial r}\right); \qquad (2)$$

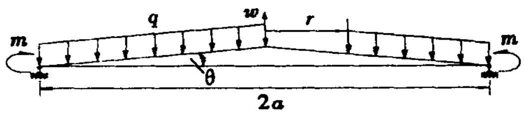

图 1　结构受力简图

边界条件:

$$r=a\text{ 时}, w=0,\ D\left(\frac{\partial^2 w}{\partial r^2}+\frac{\nu}{r}\frac{\partial w}{\partial r}\right)=-m,\ N_r=0, \qquad (3a, b, c)$$

$$r=0\text{ 时}, w,\ \frac{\partial w}{\partial r},\ N_r\text{ 有限}; \qquad (4a, b, c)$$

初始条件:

$$t=0\text{ 时}, w=w(0,r),\ \frac{\partial w(0,r)}{\partial t}=0; \qquad (5a, b)$$

其中, 算子

$$L_1=\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r},$$

$$D=\frac{Eh^3}{12(1-\nu^2)}$$

为壳的抗弯刚度, ρ为壳的面密度, w 表示横向位移, N_r 表示径向内力• E、ν分别为材料的弹性模量和泊松比•

2　问题的求解

为了便于求解和结果的讨论, 引入下列无量纲量

$$x=\frac{r}{a},\ y=\sqrt{12(1-\nu^2)}\frac{w}{h},\ s=\frac{ar}{D}N_r,\ \tau=t\omega,\ \Omega^2=\frac{a^4\rho}{D}\omega^2,$$

$$K=\sqrt{12(1-\nu^2)}\frac{a}{h}\theta,\ Q=\sqrt{12(1-\nu^2)}\frac{a^4}{Dh}q,\ M=\sqrt{12(1-\nu^2)}\frac{a^2}{Dh}m,$$

并且取

$$y=y_0+y\cos\tau, \qquad (6)$$

$$s=s_1\cos\tau+s_2\cos^2\tau, \qquad (7)$$

其中 y_0 为小挠度的解• 则根据边界条件可求得

$$y_0=\frac{Q}{64}(1-x^2)^2+\frac{1}{2(1+\nu)}\left(\frac{Q}{8}+M\right)(1-x^2), \qquad (8)$$

$$\varphi=\frac{dy_0}{dx}=\frac{Q}{16}(x^2-1)x-\frac{1}{(1+\nu)}\left(\frac{Q}{8}+M\right)x• \qquad (9)$$

把(6)~(9)式代入(1)~(4)式中, 可得如下边值问题

$$\int_0^1\left\{L(y)-\frac{1}{x}\frac{d}{dx}\left[s_1(K+\varphi)+\frac{3}{4}s_2\frac{dy}{dx}\right]-\Omega^2 y\right\}\delta yx\,dx=0, \qquad (10)$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_1)=-(K+\varphi)\frac{dy}{dx}, \qquad (11)$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xs_2)=-\frac{1}{2}\left(\frac{dy}{dx}\right)^2; \qquad (12)$$

边界条件:

$$x=1\text{ 时}, y=0,\ \frac{d^2y}{dx^2}+\nu\frac{dy}{dx}=0,\ s_1=s_2=0, \qquad (13a, b, c, d)$$

$$x = 0 \text{ 时}, y, \frac{dy}{dx}, s_1, s_2 \text{ 有限};\tag{14a, b, c, d}$$

其中

$$L = \frac{1}{x}\frac{d}{dx}x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}x\frac{d}{dx}.$$

取振动时的最大振幅 y_0 为摄动参数，且

$$y = \sum_{i=1}^{\infty} y_i y_0^i, \quad s_1 = \sum_{i=1}^{\infty} s_{1i} y_0^i, \quad s_2 = \sum_{i=1}^{\infty} s_{2i} y_0^i, \quad \Omega^2 = \sum_{i=1}^{\infty} \Omega_i^2 y_0^i.\tag{15}$$

将(15)式代入(11)~(14)式，根据 y_0 的同次幂项可得一系列边值问题.

2.1 一次近似边值问题

$$\int_0^1\left\{L(y_1) - \frac{1}{x}\frac{d}{dx}\left[s_{11}(K+\varphi)\right] - \Omega_0^2 y_1\right\} x y_1 dx = 0,\tag{16}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x s_{11}) = -(K+\varphi)\frac{dy_1}{dx},\tag{17}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x s_{21}) = 0;\tag{18}$$

边界条件：

$$x = 1 \text{ 时}, y_1 = 0, \frac{d^2 y_1}{dx^2} + \nu\frac{dy_1}{dx} = 0, s_{11} = s_{21} = 0,\tag{19a, b, c, d}$$

$$x = 0 \text{ 时}, y_1 = 1, \frac{dy_1}{dx}, s_{11}, s_{21} \text{ 有限}.\tag{20a, b, c, d}$$

2.2 二次近似边值问题

$$\int_0^1\left\{L(y_2) - \frac{1}{x}\frac{d}{dx}\left[s_{12}(K+\varphi) + \frac{3}{4}s_{21}\frac{dy_1}{dx}\right] - \Omega_1^2 y_1 - \Omega_0^2 y_2\right\} x y_1 dx + 2\int_0^1\left\{L(y_1) - \frac{1}{x}\frac{d}{dx}\left[s_{11}(K+\varphi)\right] - \Omega_0^2 y_1\right\} x y_2 dx = 0,\tag{21}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x s_{12}) = -(K+\varphi)\frac{dy_2}{dx},\tag{22}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x s_{22}) = -\frac{1}{2}\left(\frac{dy_1}{dx}\right)^2;\tag{23}$$

边界条件：

$$x = 1 \text{ 时}, y_2 = 0, \frac{d^2 y_2}{dx^2} + \nu\frac{dy_2}{dx} = 0, s_{12} = s_{22} = 0,\tag{24a, b, c, d}$$

$$x = 0 \text{ 时}, y_2 = 0, \frac{dy_2}{dx}, s_{12}, s_{22} \text{ 有限}.\tag{25a, b, c, d}$$

2.3 三次近似边值问题

$$\int_0^1\left\{L(y_3) - \frac{1}{x}\frac{d}{dx}\left[s_{13}(K+\varphi) + \frac{3}{4}\left(s_{21}\frac{dy_2}{dx} + s_{22}\frac{dy_1}{dx}\right)\right] - \Omega_2^2 y_1 - \Omega_1^2 y_2 - \Omega_0^2 y_3\right\} x y_1 dx + 2\int_0^1\left\{L(y_2) - \frac{1}{x}\frac{d}{dx}\left[s_{12}(K+\varphi) + \frac{3}{4}s_{21}\frac{dy_1}{dx}\right] - \Omega_1^2 y_1 - \Omega_0^2 y_2\right\} x y_2 dx + 3\int_0^1\left\{L(y_1) - \frac{1}{x}\frac{d}{dx}\left[s_{11}(K+\varphi)\right] - \Omega_0^2 y_1\right\} x y_3 dx = 0,\tag{26}$$

$$x\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x s_{13}) = -(K+\varphi)\frac{dy_3}{dx},\tag{27}$$

$$x\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}(xs_{23})=-\frac{\mathrm{d}y_1}{\mathrm{d}x}\frac{\mathrm{d}y_2}{\mathrm{d}x};\tag{28}$$

边界条件:

$$x=1\text{ 时}, y_3=0,\ \frac{\mathrm{d}^2y_3}{\mathrm{d}x^2}+\nu\frac{\mathrm{d}y_3}{\mathrm{d}x}=0,\ s_{13}=s_{23}=0,\tag{29a, b, c, d}$$

$$x=0\text{ 时}, y_3=0,\ \frac{\mathrm{d}y_3}{\mathrm{d}x},\ s_{13}, s_{23}\text{ 有限}.\tag{30a, b, c, d}$$

…….

为了求解上述 3 个边值问题, 下面给出符合边界条件的位移形式

$$y_1=\frac{(x^2-1)[(\nu+1)x^2-(\nu+5)]}{(\nu+5)},$$

$$y_2=y_1(2x^3-3x^2),$$

$$y_3=y_4=\cdots=0.$$

求解一次近似边值问题(16) ~ (20), 可得

$$\begin{cases}s_{11}=(A_1x^7+A_2x^5+A_3x^3+A_4x)Q+A_5x^5+A_6x^4+A_7x^3+A_8x^2+A_9x,\\ s_{21}=0,\\ \Omega_0^2=\alpha_1Q^2+\alpha_2QM+\alpha_3Q+\alpha_4M^2+\alpha_5M+\alpha_6,\end{cases}\tag{31}$$

其中

$$A_1=-\frac{1}{192}\frac{\nu+1}{\nu+5},\ A_2=\frac{1}{48}\frac{\nu+3}{\nu+5},\ A_3=-\frac{1}{32}\frac{(\nu+3)^2}{(\nu+1)(\nu+5)},$$

$$A_4=\frac{1}{192}\frac{3\nu^2+22\nu+43}{(\nu+1)(\nu+5)},\ A_5=\frac{M}{6(\nu+5)},\ A_6=-\frac{4K}{15}\frac{\nu+1}{\nu+5},$$

$$A_7=-\frac{M}{2}\frac{\nu+3}{(\nu+1)(\nu+5)},\ A_8=\frac{4}{3}\frac{\nu+3}{\nu+5},$$

$$A_9=-\frac{8K}{15}\frac{2\nu+7}{\nu+5}+\frac{M}{3}\frac{\nu+4}{(\nu+1)(\nu+5)},\ g_0=3\nu^2+36\nu+113,$$

$$\alpha_1=\frac{5}{21\,504}\frac{9\nu^4+148\nu^3+950\nu^2+2\,836\nu+3\,369}{(\nu+1)^2g_0},$$

$$\alpha_2=\frac{5}{48}\frac{(\nu+5)(\nu^2+8\nu+19)}{(\nu+1)^2g_0},$$

$$\alpha_3=-\frac{K}{693}\frac{142\nu^3+1\,757\nu^2+7\,576\nu+11\,505}{(\nu+1)g_0},$$

$$\alpha_4=\frac{2}{3}\frac{2\nu^2+19\nu+47}{(\nu+1)^2g_0},\ \alpha_5=-\frac{32}{63}\frac{10\nu^2+89\nu+205}{(\nu+1)g_0},$$

$$\alpha_6=\frac{4}{15}\frac{1\,200(\nu+7)(\nu+1)+(19\nu^2+158\nu+339)K^2}{g_0}.$$

求解二次近似边值问题(21) ~ (25), 可得

$$\begin{cases}s_{12}=(B_1x^{10}+B_2x^9+B_3x^8+B_4x^7+B_5x^6+B_6x^5+B_7x^4+B_8x^3)Q+\\ \qquad B_9x^8+B_{10}x^7+B_{11}x^6+B_{12}x^5+B_{13}x^4+B_{14}x^3+B_{15}x^2,\\ s_{22}=C_1x^7+C_2x^5+C_3x^3+C_4x,\\ \Omega_1^2=\beta_1Q^2+\beta_2QM+\beta_3Q+\beta_4M^2+\beta_5M+\beta_6,\end{cases}\tag{32}$$

其中

$$B_1 = -\frac{7}{792}\frac{\nu+1}{\nu+5},\quad B_2 = \frac{9}{640}\frac{\nu+1}{\nu+5},\quad B_3 = \frac{17}{504}\frac{\nu+3}{\nu+5},$$

$$B_4 = -\frac{7}{128}\frac{\nu+3}{\nu+5},\quad B_5 = -\frac{1}{280}\frac{13\nu^2+78\nu+105}{(\nu+1)(\nu+5)},\quad B_6 = \frac{1}{64}\frac{5\nu^2+30\nu+41}{(\nu+1)(\nu+5)},$$

$$B_7 = \frac{1}{40}\frac{\nu+3}{\nu+1},\quad B_8 = -\frac{3}{64}\frac{\nu+3}{\nu+1},\quad B_9 = \frac{2M}{9(\nu+5)},$$

$$B_{10} = -\frac{7K}{24}\frac{\nu+1}{\nu+5} - \frac{3M}{8(\nu+5)},\quad B_{11} = \frac{18K}{35}\frac{\nu+1}{\nu+5} - \frac{4M}{7}\frac{\nu+3}{(\nu+1)(\nu+5)},$$

$$B_{12} = \frac{5K}{6}\frac{\nu+3}{\nu+5} + M\frac{\nu+3}{(\nu+1)(\nu+5)},\quad B_{13} = -\frac{8K}{5}\frac{\nu+3}{\nu+5} + \frac{2M}{5(\nu+1)},$$

$$B_{14} = -\frac{3K}{4} - \frac{3M}{4(\nu+1)},\quad B_{15} = 2K,\quad C_1 = -\frac{1}{6}\frac{(\nu+1)^2}{(\nu+5)^2},$$

$$C_2 = \frac{2}{3}\frac{(\nu+3)(\nu+1)}{(\nu+5)^2},\quad C_3 = -\frac{(\nu+3)^2}{(\nu+5)^2},\quad C_4 = \frac{1}{6}\frac{3\nu^2+22\nu+43}{(\nu+5)^2},$$

$$g_1 = \frac{5\,937\nu^2+80\,064\nu+288\,037}{2\,002g_0},$$

$$\beta_1 = g_1\alpha_1 + \frac{9\,519\nu^4+502\,448\nu^3+2\,261\,878\nu^2-6\,089\,608\nu-37\,730\,685}{209\,104\,896(\nu+1)^2g_0},$$

$$\beta_2 = g_1\alpha_2 - \frac{10\,375\nu^3+102\,601\nu^2+520\,397\nu+1\,306\,763}{384\,384(\nu+1)^2g_0},$$

$$\beta_3 = g_1\alpha_3 - \frac{K(49\,853\nu^3+1\,133\,607\nu^2+5\,180\,143\nu+5\,125\,989)}{1\,921\,920(\nu+1)g_0},$$

$$\beta_4 = g_1\alpha_4 - \frac{8\,881\nu^2+70\,724\nu+171\,667}{12\,012(\nu+1)^2g_0},$$

$$\beta_5 = g_1\alpha_5 + \frac{K(4\,129\nu^2+5\,244\nu-18\,685)}{4\,620(\nu+1)g_0},$$

$$\beta_6 = g_1\alpha_6 - \frac{16(\nu+1)(187\nu+1\,555)}{7g_0} + \frac{2K^2(247\nu^2+3\,926\nu+10\,719)}{385g_0}.$$

求解三次近似边值问题(26)~(30),可得

$$\begin{cases} s_{13} = 0, \\ s_{23} = D_1x^{10} + D_2x^9 + D_3x^8 + D_4x^7 + D_5x^6 + D_6x^5 + D_7x^4 + D_8x^3 + D_9x, \\ \Omega_2^2 = \gamma_1Q^2 + \gamma_2QM + \gamma_3Q + \gamma_4M^2 + \gamma_5M + \gamma_6, \end{cases} \tag{33}$$

其中

$$D_1 = -\frac{56}{99}\frac{(\nu+1)^2}{(\nu+5)^2},\quad D_2 = \frac{9}{10}\frac{(\nu+1)^2}{(\nu+5)^2},\quad D_3 = \frac{136}{63}\frac{(\nu+1)(\nu+3)}{(\nu+5)^2},$$

$$D_4 = -\frac{7}{2}\frac{(\nu+1)(\nu+3)}{(\nu+5)^2},\quad D_5 = -\frac{8}{35}\frac{13\nu^2+78\nu+105}{(\nu+5)^2},$$

$$D_6 = \frac{5\nu^2+30\nu+41}{(\nu+5)^2},\quad D_7 = \frac{8}{5}\frac{\nu+3}{\nu+5},\quad D_8 = -\frac{3(\nu+3)}{\nu+5},$$

$$D_9 = \frac{1}{3\,465}\frac{1\,311\nu^2+12\,907\nu+26\,644}{(\nu+5)^2},\quad g_2 = -\frac{955\nu^2+14\,090\nu+56\,191}{1\,001g_0},$$

$$\gamma_1 = g_2\alpha_1 + g_1\beta_1 + [1\,030\,840\,989\nu^4 + 20\,352\,776\,719\nu^3 + 160\,525\,308\,329\nu^2 + 578\,430\,621\,073\nu + 848\,591\,658\,642]/[3\,441\,605\,207\,040(\nu+1)^2g_0],$$

$$\gamma_2 = g_2\alpha_2 + g_1\beta_2 + [33\,407\,551\nu^3 + 513\,131\,233\nu^2 + 3\,073\,131\,653\nu + 6\,529\,909\,475]/[2\,058\,376\,320(\nu+1)^2g_0],$$

$$\gamma_3 = g_2\alpha_3 + g_1\beta_3 - [K(25\,600\,885\nu^3 + 404\,680\,635\nu^2 + 2\,296\,247\,447\nu + 4\,279\,696\,593)]/[686\,125\,440(\nu+1)g_0],$$

$$\gamma_4 = g_2\alpha_4 + g_1\beta_4 + \frac{2\,219\,233\nu^2 + 23\,355\,890\nu + 79\,026\,385}{7\,567\,560(\nu+1)^2 g_0},$$

$$\gamma_5 = g_2\alpha_5 + g_1\beta_5 - \frac{K(1\,160\,903\nu^2 + 12\,995\,886\nu + 46\,578\,055)}{1\,261\,260(\nu+1)g_0},$$

$$\gamma_6 = g_2\alpha_6 + g_1\beta_6 + [92\,439\nu^4 + 1\,881\,996\nu^3 + 14\,145\,066\nu^2 + 46\,516\,940\nu + 56\,436\,695]/[154(\nu+1)^2 g_0] + \frac{K^2(94\,651\nu^2 + 1\,137\,158\nu + 3\,966\,155)}{76\,440 g_0}.$$

由此可得三次近似的非线性固有频率

$$\Omega^2 = \Omega_0^2 + \Omega_1^2 y_0 + \Omega_2^2 y_0^2. \tag{34}$$

3 结果分析

根据(34)式可画出非线性固有频率 Ω^2 与 Q、M、K、y_0 之间的关系曲线. 这里我们不妨取 $\nu = 0.3$, 从(34)式得

$$\Omega^2 = \Omega_0^2 + \Omega_1^2 y_0 + \Omega_2^2 y_0^2 =$$
$$0.834\,174\,256K^2 + 0.004\,778\,712\,90Q^2 + 0.056\,583\,207\,9QM + 0.168\,130\,576M^2 - 0.124\,713\,094KQ - 0.732\,502\,32KM + 24.476\,505\,2 +$$
$$(1.548\,819\,44K^2 + 0.005\,116\,527\,22Q^2 + 0.052\,938\,754\,8QM + 0.134\,685\,102M^2 - 0.178\,828\,495KQ - 0.944\,285\,811KM - 7.782\,658\,7)y_0 +$$
$$(1.997\,799\,32K^2 + 0.005\,548\,057\,02Q^2 + 0.056\,430\,071\,1QM + 0.141\,931\,932M^2 - 0.209\,523\,098KQ - 1.080\,127\,93KM + 111.903\,979)y_0^2.$$

下面我们分别计算了固有频率 Ω^2 与 Q、M、K、y_0 之间的非线性关系.

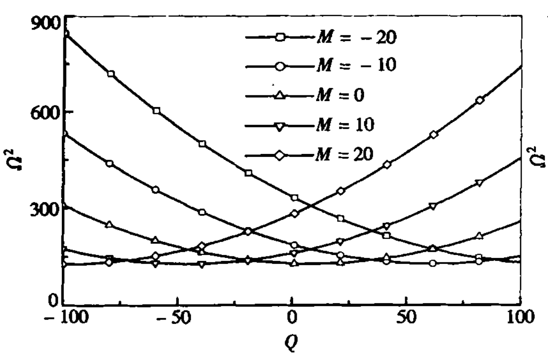

图2 $K = 0.5$, $y_0 = 1.0$ 时 Ω^2 与 Q 之间的关系

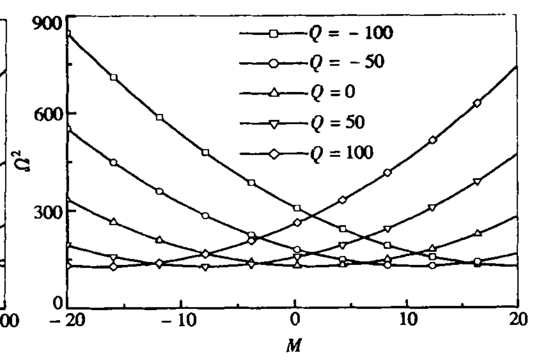

图3 $K = 0.5$, $y_0 = 1.0$ 时 Ω^2 与 M 之间的关系

图2、图3为 $K = 0.5$, $y_0 = 1.0$ 时 Ω^2 与 Q、M 之间的关系. 从图中可以看出, 随着 M 或 Q 的小范围增加, 固有频率均呈非线性增加趋势. 当 M 或 Q 小范围负增加时, 固有频率亦呈非线性增加的趋势. 当 M 或 Q 的变化范围较大时, 固有频率的变化是复杂的.

图4、图5为 $Q = 50$, $M = 10$ 时 Ω^2 与 K、y_0 之间的关系. 从图中可以看出, 扁锥壳的非

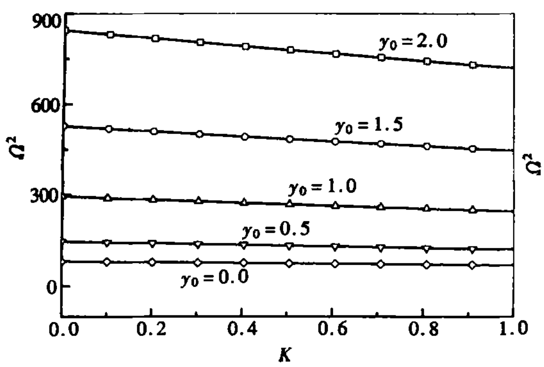

图 4 $Q=50, M=10$ 时 Ω^2 与 K 之间的关系

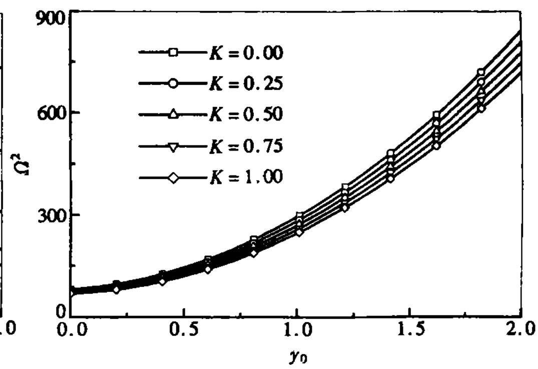

图 5 $Q=50, M=10$ 时 Ω^2 与 y_0 之间的关系

线性固有频率随 K 值增大而减小,随着挠度的增大非线性固有频率也增大,这和文[3, 5, 6, 9]的结果是一致的,和物理概念是吻合的•

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Nonlinear Vibration of Thin Shallow Conic Shells Under Combined Action of Peripheral Moment and Transverse Loads

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Abstract: Based on the variation and harmonic equations and by taking the maximum amplitude of the shell center as the perturbation parameter, nonlinear vibration of thin shallow conic shells under combined action of peripheral moment and transverse loads was solved. The linear natural frequency can be got by the first_order approximation and the more accurate nonlinear frequency is got by the second_order approximation under the action of static loads. Meanwhile the third_order approximate analytic expression is given for describing the nonlinear relation between nature frequency and peripheral moment, transverse loads, amplitude, base angle under the small deformation. Within some range, the complex and regularity of the nonlinear relation can be directly observed from the numeric results.

Key words: perturbation variation method; nonlinear vibration; natural frequency; thin shallow conical shell