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横观各向同性电磁弹性固体耦合方程的一般解*

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摘要: 横观各向同性电磁弹性固体的耦合特征由 5 个关于弹性位移、电位和磁位的二阶偏微分方程控制• 基于势函数理论, 耦合的方程组被简化为 5 个非耦合的关于势函数的广义 Laplace 方程• 弹性场和电磁场由势函数表示, 这构成了横观各向同性电磁弹性固体的一般解•

关 键 词: 电磁弹性固体; 一般解; 势函数
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引 言

同时拥有力_电_磁耦合效应的电磁弹性材料在新兴的智能材料与结构中呈现出良好的应用前景[1], 因此这类材料的力学和物理问题最近引起了人们的重视• 已有的一些研究工作包括: 1) 各种边界条件半无限大各向异性电磁弹性介质中表面波的存在性[2, 3]; 2) Green 函数[4~7]; 3) 电磁弹性复合材料的夹杂和细观力学问题[8~13]• 尽管大多数电磁弹性复合材料是横观各向同性的[8~13], 但据作者所知这类材料耦合方程的一般解未见报导• 本文得到了横观各向同性电磁弹性材料耦合方程的一般解, 这对于研究电磁弹性固体的许多问题是重要的, 这些问题包括缺陷产生的扰动场、Green 函数、有效性能预报和断裂性能等•

1 基本方程

横观各向同性电磁弹性介质的本构方程是:

$$\left\{\begin{aligned}\sigma_{xx} &= c_{11}\frac{\partial u_x}{\partial x}+(c_{11}-2c_{66})\frac{\partial u_y}{\partial y}+c_{13}\frac{\partial u_z}{\partial z}+e_{31}\frac{\partial \phi}{\partial z}+h_{31}\frac{\partial \varphi}{\partial z},\\ \sigma_{xx} &= (c_{11}-2c_{66})\frac{\partial u_x}{\partial x}+c_{11}\frac{\partial u_y}{\partial y}+c_{13}\frac{\partial u_z}{\partial z}+e_{31}\frac{\partial \phi}{\partial z}+h_{31}\frac{\partial \varphi}{\partial z},\\ \sigma_{zz} &= c_{13}\frac{\partial u_x}{\partial x}+c_{13}\frac{\partial u_y}{\partial y}+c_{33}\frac{\partial u_z}{\partial z}+e_{33}\frac{\partial \phi}{\partial z}+h_{33}\frac{\partial \varphi}{\partial z},\end{aligned}\right. \qquad (1a)_{1,2,3}$$

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$$\begin{cases} \sigma_{xy} = c_{66}\left(\dfrac{\partial u_x}{\partial y} \dfrac{\partial u_y}{\partial x}\right), \quad \sigma_{xz} = c_{44}\left(\dfrac{\partial u_x}{\partial z} + \dfrac{\partial u_z}{\partial x}\right) + e_{15}\dfrac{\partial \phi}{\partial x} + h_{15}\dfrac{\partial \varphi}{\partial x}, \\ \sigma_{yz} = c_{44}\left(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}\right) + e_{15}\dfrac{\partial \phi}{\partial y} + h_{15}\dfrac{\partial \varphi}{\partial y}, \end{cases} \tag{1a}_{4,5,6}$$

$$\begin{cases} D_x = e_{15}\left(\dfrac{\partial u_x}{\partial z} + \dfrac{\partial u_z}{\partial x}\right) - \kappa_{11}\dfrac{\partial \phi}{\partial x} - \alpha_{11}\dfrac{\partial \varphi}{\partial x}, \\ D_y = e_{15}\left(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}\right) - \kappa_{11}\dfrac{\partial \phi}{\partial y} - \alpha_{11}\dfrac{\partial \varphi}{\partial y}, \\ D_z = e_{31}\left(\dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y}\right) + e_{33}\dfrac{\partial u_z}{\partial z} - \kappa_{33}\dfrac{\partial \phi}{\partial z} - \alpha_{33}\dfrac{\partial \varphi}{\partial z}, \end{cases} \tag{1b}$$

$$\begin{cases} B_x = h_{15}\left(\dfrac{\partial u_x}{\partial z} + \dfrac{\partial u_z}{\partial x}\right) - \alpha_{11}\dfrac{\partial \phi}{\partial x} - \mu_{11}\dfrac{\partial \varphi}{\partial x}, \\ B_y = h_{15}\left(\dfrac{\partial u_y}{\partial z} + \dfrac{\partial u_z}{\partial y}\right) - \alpha_{11}\dfrac{\partial \phi}{\partial y} - \mu_{11}\dfrac{\partial \varphi}{\partial y}, \\ B_z = h_{31}\left(\dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y}\right) + h_{33}\dfrac{\partial u_z}{\partial z} - \alpha_{33}\dfrac{\partial \phi}{\partial z} - \mu_{33}\dfrac{\partial \varphi}{\partial z}, \end{cases} \tag{1c}$$

其中 $u_i(i = x, y, z)$ 是弹性位移, σ_{ij} 是应力分量; D_i 是电位移, ϕ 是电位, 而电场为 $-\partial\phi/\partial i$; B_i 为磁感强度, φ 为磁位, 磁场由 $-\partial\varphi/\partial i$ 得到• c_{ij}、e_{ij}、h_{ij}、α_{ij} 分别为弹性、压电、压磁和电磁常数; κ_{ij} 和 μ_{ij} 分别为介电常数和磁通率• 平衡方程为

$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial x} + \dfrac{\partial \sigma_{xy}}{\partial y} + \dfrac{\partial \sigma_{xz}}{\partial z} = 0, \quad \dfrac{\partial \sigma_{xy}}{\partial x} + \dfrac{\partial \sigma_{yy}}{\partial y} + \dfrac{\partial \sigma_{yz}}{\partial z} = 0, \\ \dfrac{\partial \sigma_{xz}}{\partial x} + \dfrac{\partial \sigma_{yz}}{\partial y} + \dfrac{\partial \sigma_{zz}}{\partial z} = 0, \end{cases} \tag{2a}$$

$$\dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z} = 0, \quad \dfrac{\partial B_x}{\partial x} + \dfrac{\partial B_y}{\partial y} + \dfrac{\partial B_z}{\partial z} = 0 \tag{2b, c}$$

将(1)代入(2)得到

$$\begin{cases} c_{11}\dfrac{\partial^2 u_x}{\partial x^2} + c_{66}\dfrac{\partial^2 u_x}{\partial y^2} + c_{44}\dfrac{\partial^2 u_x}{\partial z^2} + (c_{11} - c_{66})\dfrac{\partial^2 u_y}{\partial x\partial y} + \\ \qquad \dfrac{\partial^2}{\partial x\partial z}(c_s u_z + e_s\phi + h_s\varphi) = 0, \\ c_{66}\dfrac{\partial^2 u_y}{\partial x^2} + c_{11}\dfrac{\partial^2 u_y}{\partial y^2} + c_{44}\dfrac{\partial^2 u_y}{\partial z^2} + (c_{11} - c_{66})\dfrac{\partial^2 u_x}{\partial x\partial y} + \\ \qquad \dfrac{\partial^2}{\partial y\partial z}(c_s u_z + e_s\phi + h_s\varphi) = 0, \\ c_{44}\Delta u_z + c_{33}\dfrac{\partial^2 u_z}{\partial z^2} + c_s\left(\dfrac{\partial^2 u_x}{\partial x\partial z} + \dfrac{\partial^2 u_y}{\partial y\partial z}\right) + e_{15}\Delta\phi + e_{33}\dfrac{\partial^2 \phi}{\partial z^2} + \\ \qquad h_{15}\Delta\varphi + h_{33}\partial^2\varphi/\partial z^2 = 0, \\ e_{15}\Delta u_z + e_{33}\dfrac{\partial^2 u_z}{\partial z^2} + e_s\left(\dfrac{\partial^2 u_x}{\partial x\partial z} + \dfrac{\partial^2 u_y}{\partial y\partial z}\right) - \kappa_{11}\Delta\phi - \kappa_{33}\dfrac{\partial^2 \phi}{\partial z^2} - \\ \qquad \alpha_{11}\Delta\varphi - \alpha_{33}\partial^2\varphi/\partial z^2 = 0, \\ h_{15}\Delta u_z + h_{33}\dfrac{\partial^2 u_z}{\partial z^2} + h_s\left(\dfrac{\partial^2 u_x}{\partial x\partial z} + \dfrac{\partial^2 u_y}{\partial y\partial z}\right) - \alpha_{11}\Delta\phi - \alpha_{33}\dfrac{\partial^2 \phi}{\partial z^2} - \\ \qquad \mu_{11}\Delta\varphi - \mu_{33}\partial^2\varphi/\partial z^2 = 0, \end{cases} \tag{3}$$

式中, $c_s = c_{13} + c_{44}$, $e_s = e_{15} + e_{31}$, $h_s = h_{13} + h_{31}$ 和 $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$•

2 一 般 解

本节使用求解横观各向同性弹性材料的势函数理论[14]来得到方程(3)的解, 文献[15~18]已经扩展该理论得到了横观各向同性压电材料的一般解• 定义复位移 $u = u_x + \mathrm{i}u_y$ 和它的共轭 $\bar{u} = u_x - \mathrm{i}u_y$, 以及 $\Lambda = \partial/\partial x + \mathrm{i}\partial/\partial y$, 方程(3)成为

$$\begin{cases} \frac{1}{2}(c_{11} + c_{66})\Delta u + c_{44}\frac{\partial^2 u}{\partial z^2} + \frac{1}{2}(c_{11} - c_{66})\Lambda^2 \bar{u} + \Lambda\frac{\partial}{\partial z}(c_s u_z + e_s \phi + h_s \varphi) = 0, \\ c_s\frac{\partial}{\partial z}\mathrm{Re}(\bar{\Lambda} u) + \Delta[c_{44}u_z + e_{15}\phi + h_{15}\varphi] + \frac{\partial^2}{\partial z^2}[c_{33}u_z + e_{33}\phi + h_{33}\varphi] = 0, \\ e_s\frac{\partial}{\partial z}\mathrm{Re}(\bar{\Lambda} u) + \Delta[e_{15}u_z - \kappa_{11}\phi - \alpha_{11}\varphi] + \frac{\partial^2}{\partial z^2}[e_{33}u_z - \kappa_{33}\phi - \alpha_{33}\varphi] = 0, \\ h_s\frac{\partial}{\partial z}\mathrm{Re}(\bar{\Lambda} u) + \Delta[h_{15}u_z - \alpha_{11}\phi - \mu_{11}\varphi] + \frac{\partial^2}{\partial z^2}[h_{33}u_z - \alpha_{33}\phi - \mu_{33}\varphi] = 0, \end{cases} \tag{4}$$

式中 Re 表示复数的实部•

方程(4)是关于 u_i, ϕ 和 φ 的耦合的二阶偏微分方程组, 很难通过直接积分的方法得到它的解• 为了使方程(4)解耦并得到它的一般解, 假定

$$u = \Lambda(F + \mathrm{i}G), \quad u_z = l\frac{\partial F}{\partial z}, \quad \phi = m\frac{\partial F}{\partial z}, \quad \varphi = n\frac{\partial F}{\partial z}, \tag{5}$$

式中 $F(x, y, z)$ 和 $G(x, y, z)$ 是势函数, l, m 和 n 是待定常数, 将(5)代入(4), 得到下面方程

$$\Delta G + \frac{c_{44}}{c_{66}}\frac{\partial^2 G}{\partial z^2} = 0, \tag{6}$$

$$\begin{cases} \Delta F + \frac{c_{44} + c_s l + e_s m + h_s n}{c_{11}}\frac{\partial^2 F}{\partial z^2} = 0, \\ \Delta F + \frac{c_{33}l + e_{33}m + h_{33}n}{c_{44}l + e_{15}m + h_{15}n + c_s}\frac{\partial^2 F}{\partial z^2} = 0, \\ \Delta F + \frac{e_{33}l - \kappa_{33}m - \alpha_{33}n}{e_{15}l - \kappa_{11}m - \alpha_{11}n + e_s}\frac{\partial^2 F}{\partial z^2} = 0, \\ \Delta F + \frac{h_{33}l - \alpha_{33}m - \mu_{33}n}{h_{15}l - \alpha_{11}m - \mu_{11}n + h_s}\frac{\partial^2 F}{\partial z^2} = 0• \end{cases} \tag{7}$$

方程(7)的非零解要求 $\partial^2 F/\partial z^2$ 前的系数满足下面的相容条件:

$$\frac{c_{33}l + e_{33}m + h_{33}n}{c_{44}l + e_{15}m + h_{15}n + c_s} = \frac{e_{33}l - \kappa_{33}m - \alpha_{33}n}{e_{15}l - \kappa_{11}m - \alpha_{11}n + e_s} = \frac{h_{33}l - \alpha_{33}m - \mu_{33}n}{h_{15}l - \alpha_{11}m - \mu_{11}n + h_s} = \frac{c_{44} + c_s l + e_s m + h_s n}{c_{11}} = \lambda, \tag{8}$$

消去上述等式中的 l, m 和 n, 得到关于 λ 的四次方程为:

$$A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0, \tag{9}$$

式中, A_k 仅与材料常数有关, 其具体表达式在附录中给出• 用 $\lambda_j (j = 1, 2, 3, 4)$ 表示方程(9)的四个根, 则对于稳定的材料, 所有的 λ_j 是实数或两对共轭的复数• 对于四个 λ_j, 存在四个势函数 $F_j (j = 1, 2, 3, 4)$, 每个势函数满足

$$\Delta F_j + \lambda_j\frac{\partial^2 F_j}{\partial z^2} = 0• \tag{10}$$

引入新的变量 $z_j = z/\sqrt{\lambda_j} (j = 1, 2, 3, 4, 5)$, 这里 $\lambda_5 = c_{44}/c_{66}$, 则方程(6)和(7)可以写为

$$\Delta F_j + \frac{\partial^2 F_j}{\partial z_j^2} = 0 \qquad (j = 1, 2, 3, 4, 5), \tag{11}$$

此式为 $F_j(x, y, z_j)$ 的广义 Laplace 方程, 其中 $F_5 = G$·

基于上面的推导, 弹性位移、电位和磁位可以用势函数表示为:

$$\begin{cases} u = \Lambda\left(\sum_{j=1}^{4} F_j + \mathrm{i}F_5\right), \quad u_z = \sum_{j=1}^{4} l_j \dfrac{\partial F_j}{\partial z}, \\ \phi = \sum_{j=1}^{4} m_j \dfrac{\partial F_j}{\partial z}, \quad \varphi = \sum_{j=1}^{4} n_j \dfrac{\partial F_j}{\partial z}, \end{cases} \tag{12}$$

式中 l_j, m_j 和 n_j 可以由(8)式解出如下:

$$l_j = \Pi_l / \Pi, \quad m_j = \Pi_m / \Pi, \quad n_j = \Pi_n / \Pi, \tag{13}$$

其中

$$\begin{cases} \Pi = \begin{vmatrix} c_s & e_s & h_s \\ e_{33} - e_{15}\lambda_j & \kappa_{11}\lambda_j - \kappa_{33} & \alpha_{11}\lambda_j - \alpha_{33} \\ h_{33} - h_{15}\lambda_j & \alpha_{11}\lambda_j - \alpha_{33} & \mu_{11}\lambda_j - \mu_{33} \end{vmatrix}, \\ \Pi_l = \begin{vmatrix} c_{11}\lambda_j - c_{44} & e_s & h_s \\ e_s\lambda_j & \kappa_{11}\lambda_j - \kappa_{33} & \alpha_{11}\lambda_j - \alpha_{33} \\ h_s\lambda_j & \alpha_{11}\lambda_j - \alpha_{33} & \mu_{11}\lambda_j - \mu_{33} \end{vmatrix}, \\ \Pi_m = \begin{vmatrix} c_s & c_{11}\lambda_j - c_{44} & h_s \\ e_{33} - e_{15}\lambda_j & e_s\lambda_j & \alpha_{11}\lambda_j - \alpha_{33} \\ h_{33} - h_{15}\lambda_j & h_s\lambda_j & \mu_{11}\lambda_j - \mu_{33} \end{vmatrix}, \\ \Pi_n = \begin{vmatrix} c_s & e_s & c_{11}\lambda_j - c_{44} \\ e_{33} - e_{15}\lambda_j & \kappa_{11}\lambda_j - \kappa_{33} & e_s\lambda_j \\ h_{33} - h_{15}\lambda_j & \alpha_{11}\lambda_j - \alpha_{33} & h_s\lambda_j \end{vmatrix}. \end{cases} \tag{14}$$

使用(1)和(12)两式, 便可得到用势函数 F_j 表示的应力 σ_{ij}、电位移 D_i 和 B_i· 为使表达简法、紧凑, 定义

$$\sigma_1 = \sigma_{xx} + \sigma_{yy}, \quad \sigma_2 = \sigma_{xx} - \sigma_{yy} + 2\mathrm{i}\sigma_{xy}, \quad \tau_z = \sigma_{xz} + \mathrm{i}\sigma_{yz},$$
$$D = D_x + \mathrm{i}D_y, \quad B = B_x + \mathrm{i}B_y. \tag{15}$$

利用(15), (1)式中的十二个本构方程缩减为下面的八个方程:

$$\begin{cases} \sigma_1 = 2(c_{11} - c_{66})\mathrm{Re}(\Lambda u) + 2\dfrac{\partial}{\partial z}(c_{13}u_z + e_{31}\phi + h_{31}\varphi), \\ \sigma_2 = 2c_{66}\Lambda u, \\ \sigma_{zz} = c_{13}\mathrm{Re}(\Lambda u) + \dfrac{\partial}{\partial z}(c_{33}u_z + e_{31}\phi + h_{31}\varphi), \\ \tau_z = c_{44}\left(\partial u/\partial z + \Lambda u_z\right) + \Lambda(e_{15}\phi + h_{15}\varphi), \\ D_z = e_{31}\mathrm{Re}(\Lambda u) + \dfrac{\partial}{\partial z}(e_{33}u_z - \kappa_{33}\phi - \alpha_{33}\varphi), \\ D = e_{15}\left(\partial u/\partial z + \Lambda u_z\right) - \Lambda(\kappa_{11}\phi + \alpha_{11}\varphi), \\ B_z = h_{31}\mathrm{Re}(\Lambda u) + \dfrac{\partial}{\partial z}(h_{33}u_z - \alpha_{33}\phi - \mu_{33}\varphi), \\ B = h_{15}\left(\partial u/\partial z + \Lambda u_z\right) - \Lambda(\alpha_{11}\phi + \mu_{11}\varphi). \end{cases} \tag{16}$$

将(12)代入(16)并对每一个 λ_j 利用关系式(8), 得到

$$\begin{cases}\sigma_1 = 2\dfrac{\partial^2}{\partial z^2}\sum_{j=1}^{4}\left\{c_{66}[\lambda_j-(1+l_j)\lambda_5]-e_{15}m_j-h_{15}n_j\right\}F_j,\\ \sigma_2 = 2c_{66}\Lambda\left[\sum_{j=1}^{4}F_j+\mathrm{i}F_5\right],\\ \sigma_{zz} = \dfrac{\partial^2}{\partial z^2}\sum_{j=1}^{4}\left[c_{44}(1+l_j)+e_{15}m_j+h_{15}n_j\right]\lambda_j F_j = \\ \qquad -\Delta\sum_{j=1}^{4}\left[c_{44}(1+l_j)+e_{15}m_j+h_{15}n_j\right]F_j,\\ \tau_z = \Lambda\dfrac{\partial}{\partial z}\left\{\sum_{j=1}^{4}\left[c_{44}(1+l_j)+e_{15}m_j+h_{15}n_j\right]F_j+\mathrm{i}c_{44}F_5\right\},\end{cases}\tag{17a}$$

$$\begin{cases}D_z = \dfrac{\partial^2}{\partial z^2}\sum_{j=1}^{4}\left[e_{15}(1+l_j)-\kappa_{11}m_j-\alpha_{11}n_j\right]\lambda_j F_j = \\ \qquad -\Delta\sum_{j=1}^{4}\left[e_{15}(1+l_j)-\kappa_{11}m_j-\alpha_{11}n_j\right]F_j,\\ D = \Lambda\dfrac{\partial}{\partial z}\left\{\sum_{j=1}^{4}\left[e_{15}(1+l_j)-\kappa_{11}m_j-\alpha_{11}n_j\right]F_j+\mathrm{i}e_{15}F_5\right\},\end{cases}\tag{17b}$$

$$\begin{cases}B_z = \dfrac{\partial^2}{\partial z^2}\sum_{j=1}^{4}\left[h_{15}(1+l_j)-\alpha_{11}m_j-\mu_{11}n_j\right]\lambda_j F_j = \\ \qquad -\Delta\sum_{j=1}^{4}\left[h_{15}(1+l_j)-\alpha_{11}m_j-\mu_{11}n_j\right]F_j,\\ B = \Lambda\dfrac{\partial}{\partial z}\left\{\sum_{j=1}^{4}\left[h_{15}(1+l_j)-\alpha_{11}m_j-\mu_{11}n_j\right]F_j+\mathrm{i}h_{15}F_5\right\},\end{cases}\tag{17c}$$

从上面的推导可以看出, 对于给定的边值问题, 所需要做的就是确定势函数 $F_j(x, y, z_j)$ $(j = 1\sim 5)$•

3 结 束 语

基于横观各向同性弹性介质的势函数理论, 推导了横观各向同性电磁弹性介质耦合方程的一般解, 所有的场变量是用五个势函数 $F_j(x, y, z_j)$ 表示的, 每个势函数满足三维 Laplace 方程• 这种形式的一般解使我们能够得到与裂纹、夹杂和 Green 函数相关的一些基本问题的显示解•

附 录

$$A_0 = c_{44}\left[c_{33}(\alpha_{33}^2-\kappa_{33}\mu_{33})+e_{33}(h_{33}\alpha_{33}-e_{33}\mu_{33})+h_{33}(e_{33}\alpha_{33}-h_{33}\kappa_{33})\right]• \tag{A1}$$

$$\begin{aligned}A_1 = {}& c_{33}\left[c_{11}(\kappa_{33}\mu_{33}-\alpha_{33}^2)+\mu_{33}(c_{44}\kappa_{11}+e_s^2)+\kappa_{33}(c_{44}\mu_{11}+h_s^2)-2(e_sh_s+c_{44}\alpha_{11})a_{33}\right]+\\ & h_{33}\left[h_{33}(c_{11}\kappa_{33}+e_s^2)+(c_se_s-c_{44}e_{15})\alpha_{33}-(c_sh_s-c_{44}h_{15})\kappa_{33}-(c_{11}\alpha_{33}+h_se_s)e_{33}+\right.\\ & \left.c_{44}(h_{33}\kappa_{11}-e_{33}\alpha_{11})\right]+e_{33}\left[e_{33}(c_{11}\mu_{33}+h_s^2)+(c_sh_s-c_{44}h_{15})\alpha_{33}-(c_se_s-c_{44}e_{15})\mu_{33}-\right.\\ & \left.(c_{11}\alpha_{33}+h_se_s)h_{33}+c_{44}(e_{33}\mu_{11}-h_{33}\alpha_{11})\right]-c_s\left[c_s(\kappa_{33}\mu_{33}-\alpha_{33}^2)+(h_{33}\kappa_{33}-e_{33}\alpha_{33})h_s+\right.\\ & \left.(e_{33}\mu_{33}-h_{33}\alpha_{33})e_s\right]+c_{44}^2(\kappa_{33}\mu_{33}-\alpha_{33}^2)+e_{15}c_{44}(e_{33}\mu_{33}-h_{33}\alpha_{33})•\end{aligned} \tag{A2}$$

$$\begin{aligned}A_2 = {}& -c_{33}\left[c_{44}(\kappa_{11}\mu_{11}-\alpha_{11}^2)+\mu_{11}(c_{11}\kappa_{33}+e_s^2)+\kappa_{11}(c_{11}\mu_{33}+h_s^2)-2\alpha_{11}(e_sh_s+c_{11}\alpha_{33})\right]+\\ & c_{44}\left[c_{11}(\kappa_{33}\mu_{33}-\alpha_{33}^2)+\mu_{33}(c_{44}\kappa_{11}+e_s^2)+\kappa_{33}(c_{44}\mu_{11}+h_s^2)-2\alpha_{33}(e_sh_s+c_{44}\alpha_{11})\right]+\\ & e_{33}\left[h_{15}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+\mu_{11}(c_se_s-c_{44}e_{15}-c_{11}e_{33})-e_{15}(c_{11}\mu_{33}+h_s^2)+\right.\end{aligned}$$

$$\alpha_{11}(c_{11}h_{33}-c_sh_s)]+h_{33}[e_{15}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+\kappa_{11}(c_sh_s-c_{44}h_{15}-c_{11}h_{33})-$$
$$h_{15}(c_{11}\kappa_{33}+e_s^2)+\alpha_{11}(c_{11}e_{33}-c_se_s)]+e_{15}[h_{33}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+$$
$$\mu_{33}(c_se_s-c_{44}e_{15}-c_{11}e_{33})-e_{33}(c_{44}\mu_{11}+h_s^2)+\alpha_{33}(c_{44}h_{15}-c_sh_s)]+$$
$$h_{15}[e_{33}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+\kappa_{33}(c_sh_s-c_{44}h_{15}-c_{11}h_{33})-h_{33}(c_{44}\kappa_{11}+e_s^2)+$$
$$\alpha_{33}(c_{44}e_{15}-c_se_s)]-c_s[e_s(h_{15}\alpha_{33}+h_{33}\alpha_{11}-e_{15}\mu_{33}-e_{33}\mu_{11})+$$
$$h_s(e_{15}\alpha_{33}+e_{33}\alpha_{11}-h_{15}\kappa_{33}-h_{33}\kappa_{11})+c_s(2\alpha_{11}\alpha_{33}-\kappa_{11}\mu_{33}-\kappa_{33}\mu_{11})]\cdot \tag{A3}$$

$$A_3=c_{11}[c_{33}(\kappa_{11}\mu_{11}-\alpha_{11}^2)+e_{33}(e_{15}\mu_{11}-h_{15}\alpha_{11})+h_{33}(h_{15}\kappa_{11}-e_{15}\alpha_{11})]+$$
$$e_{15}[h_{15}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+\mu_{11}(c_se_s-c_{44}e_{15}-c_{11}e_{33})-e_{15}(c_{11}\mu_{33}+h_s^2)+$$
$$\alpha_{11}(c_{11}h_{33}-c_sh_s)]+h_{15}[e_{15}(e_sh_s+c_{11}\alpha_{33}+c_{44}\alpha_{11})+\kappa_{11}(c_sh_s-c_{44}h_{15}-c_{11}h_{33})-$$
$$h_{15}(c_{11}\kappa_{33}+e_s^2)+\alpha_{11}(c_{11}e_{33}-c_se_s)]+c_{44}[\kappa_{11}(c_{11}\mu_{33}+h_s^2)+\mu_{11}(c_{11}\kappa_{33}+e_s^2)+$$
$$c_{44}(\kappa_{11}\mu_{11}-\alpha_{11}^2)-2\alpha_{11}(e_sh_s+c_{11}\alpha_{33})]-c_2[c_s(\kappa_{11}\mu_{11}-\alpha_{11}^2)+$$
$$e_{15}(e_s\mu_{11}-h_s\alpha_{11})+h_{15}(h_s\kappa_{11}-e_s\alpha_{11})]\cdot \tag{A4}$$

$$A_4=c_{11}[2h_{15}e_{15}\alpha_{11}-\mu_{11}e_{15}^2-\kappa_{11}h_{15}^2-c_{44}(\kappa_{11}\mu_{11}-\alpha_{11}^2)]\cdot \tag{A5}$$

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General Solution for the Coupled Equations of Transversely Isotropic Magnetoelectroelastic Solids

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Abstract: The coupling feature of transversely isotropic magnetoelectroelastic solids are governed by a system of five partial differnetial equations with respect to the elastic displacements, the electric potential and the magnetic potential. Based on the potential theory, the coupled equations are reduced to the five uncoupled generalized Laplace equations with respect to five potential functions. Further, the elastic fields and electromagnetic fields are expressed in terms of the potential functions. These expressions constitute the general solution of transversely isotropic magnetoelectroelastic media.

Key words: magnetoelectroelastic solids; general solution; potential function