应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号:1000-0887(2003)04-0391-07

一类最小相高阶串联系统的几乎干扰解耦问题*

毕卫萍, 慕小武, 林 澜

(郑州大学 数学系,郑州 450052)

(郭兴明推荐)

摘要: 讨论了一类最小相高阶串联系统的几乎干扰解耦问题的反馈设计,对一般的由 L_{2m}-L_{2mp} 所定义的非线性增益指标,利用加幂积分器和 Back-stepping 方法,我们构造性的给出了一种光滑控制律的设计方法,保证闭环系统在内稳定的基础上,使系统达到干扰衰减.

关 键 词: 几乎干扰解耦; 内稳定性; 非线性增益
中图分类号: O231.2 **文献标识码:** A

引 言

1981~1982 年,Willems 在[1,2]中首先提出并解决了线性系统的几乎干扰解耦问题(ADD),但对非线性系统,已知的几乎干扰解耦问题(ADD)的大部分结果都是基于被控系统是反馈线性化的系统(或部分反馈线性化的系统)获得的([3~5]). 对于本质非线性系统(或高阶的非线性系统),甚至通常刻画 ADD 问题的 L_2- 增益指标都是不适定的,在[6] 中,利用 L_2-L_{2p} 增益给出了高阶非线性系统 ADD 问题的适定提法,并且结合加指数积分器的技巧,构造性的给出了保证系统内稳定的 ADD 问题的解. 文[5]对于可以部分反馈线性化的系统,把[4]中的结果推广到一大类具有最小相的非线性系统.

本文讨论了一类具有最小相的高阶串联系统的几乎干扰解耦问题的反馈设计,对一般的由 L_{2m}-L_{2mp} 所定义的非线性增益指标,我们构造性的给出了一种光滑控制律的设计方法,保证闭环系统在内稳定的基础上,使系统达到干扰衰减.

1 问题的提法与基本假设

在本文中,我们考虑如下一类具有最小相的高阶串联系统的几乎干扰解耦问题:

* 收稿日期: 2002-01-21; 修订日期: 2003-01-14
作者简介: 毕卫萍(1963—),女,副教授,博士,现在河南师范大学数学系,主要研究方向为非线性系统和稳定性理论(E-mail:wpbi888@xinhuanet.com).

.net

$$\begin{cases} \dot{z} = f_0(z,x_1) + g_0(z,x_1)w, \\ \dot{x}_1 = x_2^{p_1} + f_1(z,\boldsymbol{x},t) + g_1(z,\boldsymbol{x},t)w, \\ \cdots\cdots, \\ \dot{x}_i = x_{i+1}^{p_i} + f_i(z,\boldsymbol{x},t) + g_i(z,\boldsymbol{x},t)w, \\ \cdots\cdots, \\ \dot{x}_r = u^{p_r} + f_r(z,\boldsymbol{x},t) + g_r(z,\boldsymbol{x},t)w, \\ y = h(z,x_1), \end{cases} \tag{1}$$

这儿 $\boldsymbol{x} = (x_1,\cdots,x_r)^{\mathrm{T}}, u \in \mathbf{R}, y \in \mathbf{R}$ 和 $w \in R^q$ 分别是系统的状态、输入、输出和干扰．$f_i(\cdot)$ 和 $g_i(\cdot), i = 1,\cdots,r$ 是未知的光滑函数，而 $f_0(\cdot)$、$g_0(\cdot)$ 和 $h(\cdot)$ 是已知的光滑函数，且 $h(0,0) = 0$.

由假定，存在光滑函数 $h_j(z,x_1), j = 1,2$，使得：

$$y = z^{\mathrm{T}}h_1(z,x_1) + x_1h_2(z,x_1). \tag{2}$$

几乎干扰解耦问题(ADD)：

$\forall \lambda > 0$，我们要找一个光滑的状态反馈 u

$$u = u_\lambda(z,\boldsymbol{x}) \qquad u_\lambda(0,0) = 0 \tag{3}$$

使得闭环系统(1)、(3)满足：

1) 当 $w = 0$ 时，闭环系统在平衡点 $\boldsymbol{x} = 0$ 是一致渐近稳定的.

2) $\forall \boldsymbol{w}(t) \in L_{2m}$，闭环系统(1)、(3) 从 $\boldsymbol{x}(0) = 0$ 出发的解满足：

$$\int_0^t |y(s)|^{2mp_0}\mathrm{d}s \leqslant \lambda^2\int_0^t \|\boldsymbol{w}(s)\|^{2m}\mathrm{d}s \qquad \forall t \geqslant 0. \tag{4}$$

设 $$L_p = \left\{z(t)\,\middle|\,\int_0^\infty \|z(s)\|^p\mathrm{d}s < \infty, p \geqslant 1\right\}.$$

为了解以上的几乎干扰解耦问题，我们做如下假设：

假设 1 对 $i = 1,\cdots,r$,

$$|f_i(z,\boldsymbol{x},t)| \leqslant (\|z\|^{p_i} + |x_1|^{p_i} + \cdots + |x_i|^{p_i})\rho_i(z,x_1,\cdots,x_i),$$
$$|g_i(z,\boldsymbol{x},t)| \leqslant \varphi_i(z,x_1,\cdots,x_i),$$
$$|f_0(z,x_1)| \leqslant (\|z\|^{p_1} + |x_1|^{p_1})\rho_0(z,x_1),$$

这儿 $\rho_0(\cdot)$、$\rho_i(\cdot)$ 和 $\varphi_i(\cdot)$ 是已知的非负、光滑有界函数.

假设 2 $p_0 \geqslant p_1 \geqslant \cdots \geqslant p_r \geqslant 1$ 是奇整数.

下面我们引进一个非常有用的引理，是作为[1]中相应结果的自然推广：

引理 设 a, b 和 $c_i, i = 1,\cdots,l$ 是实数，假设 $d_j: R^{l+1} \to \mathbf{R}, j = 1,2$，是两个光滑函数，则，任给正整数 m, n 及实数 $N > 0$，存在两个非负的光滑函数 $d_3: R^{l+2} \to \mathbf{R}$, $d_4: R^{l+1} \to \mathbf{R}$，使得：

1) $|a^m[(b + ad_1(c_1,\cdots,c_l,a))^n - (ad_1(c_1,\cdots,c_l,a))^n]| \leqslant$
$|a|^{m+n}/N + |b|^{m+n}d_3(c_1,\cdots,c_l,a,b);$

2) $|b^n(c_1^m + \cdots + c_l^m + b^m)d_2(c_1,\cdots,c_l,b)| \leqslant$
$(|c_1|^{m+n} + \cdots + |c_l|^{m+n})/N + |b|^{m+n}d_4(c_1,\cdots,c_l,b).$

2 主要结果

本节我们将反复利用加指数积分器的技巧，对系统(1)构造 ADD 问题的解，并使闭环系统达到内稳定.

定理 假设存在光滑正定的 Liapunov 函数 $V(z)$,满足:

$$\left|\frac{\partial V}{\partial z}g_0(z,x_1)\right| \leqslant (\|z\|^{mp_0}+|x_1|^{mp_0})h_0(z,x_1),\ h_0(z,x_1)\geqslant 0, \tag{5}$$

$$\left\|\frac{\partial V}{\partial z}\frac{\partial f_0(z,x_1)}{\partial x_1}\right\| \leqslant (\|z\|^{2mp_0-1}+|x_1|^{2mp_0-1})\tilde{h}_0(z,x_1),\ \tilde{h}_0(z,x_1)\geqslant 0, \tag{6}$$

$$\frac{\partial V}{\partial z}f_0(z,x_1^*)+\left(1-\frac{1}{2m}\right)\left(\left(\frac{\partial V}{\partial z}g_0(z,x_1^*)\right)^{2m}\Big/2m\beta\right)^{1/(2m-1)}+h^{2mp_0}(z,x_1^*)\leqslant -\|z\|^{2mp_0}W(z) \quad (\text{其中 } W \text{ 为光滑函数且 } W(z)>0), \tag{7}$$

这儿 $x_1^*(z)$ 是光滑函数并满足 $x_1^*(0)=0$,β 是正整数. 则,在假设 1 和假设 2 下,对系统(1)的几乎干扰解耦问题是可解的.

证明 我们将构造性的给出状态反馈的设计方法.

第一步 考虑系统(1)的 (z,x_1)- 子系统

令

$$\xi_1 = x_1 - x_1^*(z),$$

$$V_1(z,x_1) = V(z)+\frac{(x_1-x_1^*)^{2mp_0-p_1+1}}{2mp_0-p_1+1}. \tag{8}$$

显然,存在光滑函数 $\alpha(z)$ 使得:

$$x_1^*(z) = z^{\mathrm{T}}\alpha(z). \tag{9}$$

对任意的 $\beta>0$,有:

$$\begin{aligned}\dot{V}_1(z,x_1)+y^{2mp_0}-2\beta\|w\|^{2m} = {}&\frac{\partial V}{\partial z}f_0(z,x_1^*)+y^{2mp_0}(z,x_1^*)+\frac{\partial V}{\partial z}g_0(z,x_1^*)w-\\ &\beta\|w\|^{2m}+\frac{\partial V}{\partial z}(f_0(z,x_1)-f_0(z,x_1^*))+y^{2mp_0}(z,x_1)-y^{2mp_0}(z,x_1^*)+\\ &(\xi_1)^{2mp_0-p_1}\left(f_1(z,x,t)-\frac{\partial x_1^*}{\partial z}f_0(z,x_1)\right)+\xi_1^{2mp_0-p_1}x_2^{p_1}+\left[\frac{\partial V}{\partial z}(g_0(z,x_1)-\right.\\ &\left.g_0(z,x_1^*))+\xi_1^{2mp_0-p_1}\left(g_1(z,x,t)-\frac{\partial x_1^*}{\partial z}g_0(z,x)\right)\right]w-\beta\|w\|^{2m}.\end{aligned} \tag{10}$$

由假设 1、假设 2、(2)、(9)及引理,存在光滑非负函数 $s_j(\cdot),\bar{s}_j(\cdot),j=0,1$ 和 $\psi_1(\cdot)$,满足:

$$\begin{cases}\left|\frac{\partial V}{\partial z}(f_0(z,x_1)-f_0(z,x_1^*))\right|\leqslant\frac{\|z\|^{2mp_0}}{8}W(z)+\xi_1^{2mp_0}s_0(z,\xi_1),\\ |y^{2mp_0}(z,x_1)-y^{2mp_0}(z,x_1^*)|\leqslant\frac{\|z\|^{2mp_0}}{8}W(z)+\xi_1^{2mp_0}\bar{s}_0(z,\xi_1),\\ \left|(\xi_1)^{2mp_0-p_1}\left(f_1(z,x,t)-\frac{\partial x_1^*}{\partial z}f_0(z,x_1)\right)\right|\leqslant\frac{\|z\|^{2mp_0}}{8}+\xi_1^{2mp_0}s_1(z,\xi_1),\\ \frac{\partial V}{\partial z}(g_0(z,x_1)-g_0(z,x_1^*))+\\ \quad\xi_1^{2mp_0-p_1}\left(g_1(z,x,t)-\frac{\partial x_1^*}{\partial z}g_0(z,x_1)\right)w-\beta^2\|w\|^{2m}\leqslant\\ \quad\frac{\|z\|^{2mp_0}}{8}W(z)+\xi_1^{2mp_0}\bar{s}_1(z,\xi_1)+\left(1-\frac{1}{2m}\right)\left(\frac{\xi_1^{2m(p_0-p_1)}\psi_1^{2m}(\cdot)}{2m\beta}\right)^{1/(2m-1)}\xi_1^{2mp_0}.\end{cases} \tag{11}$$

由(11)和(10)我们得到:

$$\dot{V}_1(z,x_1)+y^{2mp_0}-2\beta\|w\|^{2m}\leqslant$$

.net

$$
-\frac{1}{2}\|z\|^{2mp_0}W(z)+\xi_1^{2mp_0-p_1}(x_2^{p_1}-x_2^{*\,p_1})+
$$

$$
\xi_1^{2mp_0-p_1}\Big[x_2^{*\,p_1}+\xi_1^{p_1}\Big(s_0(\cdot)+\tilde{s}_0(\cdot)+s_1(\cdot)+\tilde{s}_1(\cdot)+
$$

$$
\Big(1-\frac{1}{2m}\Big)\Big(\frac{\xi_1^{2m(p_0-p_1)}\psi_1^{2m}(\cdot)}{2m\beta}\Big)^{1/(2m-1)}\Big)\Big]. \tag{12}
$$

令

$$
x_2^*(z,x_1)=-\xi_1\alpha_1(z,\xi_1)=-\xi_1\Big[rW(z)+s_0(\cdot)+\tilde{s}_0(\cdot)+
$$

$$
s_1(\cdot)+\tilde{s}_1(\cdot)+\Big(1-\frac{1}{2m}\Big)\Big(\frac{\xi_1^{2m(p_0-p_1)}\psi_1^{2m}(\cdot)}{2m\beta}\Big)^{1/(2m-1)}\Big]^{1/p_1}, \tag{13}
$$

由 $\alpha_1(z,\xi_1)>0$ 我们有：

$$
\dot{V}_1(z,x_1)+y^{2mp_0}-2\beta\|w\|^{2m}\leqslant
$$

$$
-\Big(\frac{\|z\|^{2mp_0}}{2}+r\xi_1^{2mp_0}\Big)W(z)+\xi_1^{2mp_0-p_1}(x_2^{p_1}-x_2^{*\,p_1}). \tag{14}
$$

第二步 考虑系统(1)的 (z,x_1,x_2) 子系统

令 $\xi_2=x_2-x_2^*(z,x_1)$,

则有 $\dot{\xi}_2=x_3^{p_2}+F_2(z,\boldsymbol{x},t)+G_2(z,\boldsymbol{x},t)w$, (15)

这儿

$$
F_2(z,\boldsymbol{x},t)=f_2(z,\boldsymbol{x},t)-\frac{\partial x_2^*}{\partial z}f_0(z,x_1)-\frac{\partial x_2^*}{\partial x_1}(x_2^{p_1}+f_1(z,\boldsymbol{x},t)),
$$

$$
G_2(z,\boldsymbol{x},t)=g_2(z,\boldsymbol{x},t)-\frac{\partial x_2^*}{\partial z}g_0(z,x_1)-\frac{\partial x_2^*}{\partial x_1}g_1(z,\boldsymbol{x},t),
$$

由假设1和假设2,存在两个光滑非负的函数 $s_2(\cdot)$ 和 $\psi_2(\cdot)$ 满足：

$$
|F_2(z,\boldsymbol{x},t)|\leqslant(\|z\|^{p_2}+|\xi_1|^{p_2}+|\xi_2|^{p_2})s_2(z,\xi_1,\xi_2), \tag{16}
$$

$$
|G_2(z,\boldsymbol{x},t)|\leqslant\psi_2(z,\xi_1,\xi_2). \tag{17}
$$

令

$$
V_2(z,\xi_1,\xi_2)=V_1(z,\xi_1)+\frac{\xi_2^{2mp_0-p_2+1}}{2mp_0-p_2+1}, \tag{18}
$$

$\forall\beta>0$, 有：

$$
\begin{aligned}
&\dot{V}_2(z,\xi_1,\xi_2)+y^{2mp_0}-3\beta\|w\|^{2m}=\\
&\dot{V}_1(z,\xi_1)+y^{2mp_0}-2\beta\|w\|^{2m}+\xi_2^{2mp_0-p_2}\dot{\xi}_2-\beta\|w\|^{2m}\leqslant\\
&-\Big(\frac{\|z\|^{2mp_0}}{2}+r\xi_1^{2mp_0}\Big)W(z)+\xi_1^{2mp_0-p_1}[(\xi_2+x_2^*)^{p_1}-x_2^{*\,p_1}]+\\
&\xi_2^{2mp_0-p_2}(x_3^{p_2}+F_2(\cdot)+G_2(\cdot)w)-\beta\|w\|^{2m}.
\end{aligned} \tag{19}
$$

由(16)、(17)和引理,我们有

$$
\begin{cases}
\left|\xi_1^{2mp_0-p_1}[(\xi_2+x_2^*)^{p_1}-x_2^{*\,p_1}]\right|\leqslant\dfrac{3\xi_1^{2mp_0}}{4}W(z)+\xi_2^{2mp_0}\tilde{s}_2(z,\xi_1,\xi_2),\\
\left|\xi_2^{2mp_0-p_2}F_2(z,\boldsymbol{x},t)\right|\leqslant\dfrac{\|z\|^{2mp_0}+\xi_1^{2mp_0}}{4}W(z)+\xi_2^{2mp_0}\hat{s}_2(z,\xi_1,\xi_2),\\
\xi_2^{2mp_0-p_2}G_2(z,\boldsymbol{x},t)w-\beta\|w\|^{2m}\leqslant\\
\qquad\Big(1-\dfrac{1}{2m}\Big)\xi_2^{2mp_0}\Big(\dfrac{\xi_2^{2m(p_0-p_2)}\psi_2^{2m}(\cdot)}{2m\beta}\Big)^{1/(2m-1)},
\end{cases} \tag{20}
$$

这儿 $\tilde{s}_2(\cdot)$ 和 $\hat{s}_2(\cdot)$ 是非负的光滑函数,由(20)和(19)得到：

$$\dot{V}_2(z,\xi_1,\xi_2) + y^{2mp_0} - 3\beta\|w\|^{2m} \leqslant -\left(\frac{\|z\|^{2mp_0}}{2^2} + (r-1)\xi_1^{2mp_0}\right)W(z) + \xi_2^{2mp_0-p_2}(x_3^{p_2} - x_3^{*p_2}) + \xi_2^{2mp_0-p_2}\Big[x_3^{*p_2} + \xi_2^{p_2}\Big(\bar{s}_2(\cdot) + \hat{s}_2(\cdot) + \left(1-\frac{1}{2m}\right)\left(\frac{\xi_2^{2m(p_0-p_2)}\psi_2^{2m}(\cdot)}{2m\beta}\right)^{1/(2m-1)}\Big)\Big]. \tag{21}$$

令

$$x_3^*(z,x_1,x_2) = -\xi_2\alpha_2(z,\xi_1,\xi_2) = -\xi_2\Big[(r-1)W(z) + \bar{s}_2(\cdot) + \hat{s}_2(\cdot) + \left(1-\frac{1}{2m}\right)\left(\frac{\xi_2^{2m(p_0-p_2)}\psi_2^{2m}(\cdot)}{2m\beta}\right)^{1/(2m-1)}\Big]^{1/p_2}, \tag{22}$$

由 $\alpha_2(z,\xi_1,\xi_2) > 0$，得到：

$$\dot{V}_2(z,\xi_1,\xi_2) + y^{2mp_0} - 3\beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^2} + (r-1)(\xi_1^{2mp_0} + \xi_2^{2mp_0})\right]W(z) + \xi_2^{2mp_0-p_2}(x_3^{p_2} - x_3^{*p_2}). \tag{23}$$

由此下去，假定在第 k 步，我们得到了一系列光滑函数

$$x_1^* = 0,\ x_j^*(z,x_1,\cdots,x_{j-1}) \text{ 满足 } x_j^*(0,\cdots,0) = 0 \quad (j = 2,\cdots,k),$$

$$\begin{cases}\xi_1 = x_1 - x_1^*(z),\ \xi_2 = x_2 - x_2^*(z,x_1),\cdots,\\ \xi_k = x_k - x_k^*(z,x_1,\cdots,x_{k-1})\end{cases} \tag{24}$$

以及 $$x_{k+1}^*(z,x_1,\cdots,x_k) = -\xi_k\alpha_k(z,\xi_1,\cdots,\xi_k), \tag{25}$$

满足 $\alpha_k(\cdot) > 0$ 使得：

$$\dot{V}_k(z,\xi_1,\cdots,\xi_k) + y^{2mp_0} - (k+1)\beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^k} + (r-k+1)(\xi_1^{2mp_0} + \cdots + \xi_k^{2mp_0})\right]W(z) + \xi_k^{2mp_0-p_k}(x_{k+1}^{p_k} - x_{k+1}^{*p_k}). \tag{26}$$

这儿 $$V_k(z,\xi_1,\cdots,\xi_k) = V(z) + \sum_{j=1}^{k}\frac{\xi_j^{2mp_0-p_j+1}}{2mp_0 - p_j + 1}.$$

令 $$\xi_{k+1} = x_{k+1} - x_{k+1}^*(z,x_1,\cdots,x_k),$$

由此 $$\dot{\xi}_{k+1} = x_{k+2}^{p_{k+1}} + F_{k+1}(z,x,t) + G_{k+1}(z,x,t)w, \tag{27}$$

这儿

$$F_{k+1}(z,x,t) = f_{k+1}(z,x,t) - \frac{\partial x_{k+1}^*}{\partial z}f_0(z,x_1) - \sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}(x_{j+1}^{p_j} + f_j(z,x,t)),$$

$$G_{k+1}(z,x,t) = g_{k+1}(z,x,t) - \frac{\partial x_{k+1}^*}{\partial z}g_0(z,x_1) - \sum_{j=1}^{k}\frac{\partial x_{k+1}^*}{\partial x_j}g_j(z,x,t).$$

注意到坐标变换(24)及 $x_j(0,\cdots,0) = 0, j = 1,\cdots,k$. 类似于第二步，我们可知，存在两个非负的光滑函数 $s_{k+1}(\cdot)$ 和 $\psi_{k+1}(\cdot)$ 使得：

$$\begin{cases}|F_{k+1}(z,x,t)| \leqslant \\ \quad (\|z\|^{p_{k+1}} + |\xi_1|^{p_{k+1}} + \cdots + |\xi_{k+1}|^{p_{k+1}})s_{k+1}(z,\xi_1,\cdots,\xi_{k+1}),\\ |G_{k+1}(z,x,t)| \leqslant \psi_{k+1}(z,\xi_1,\cdots,\xi_{k+1}).\end{cases} \tag{28}$$

令 $$V_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) = V_k(z,\xi_1,\cdots,\xi_k) + \frac{\xi_{k+1}^{2mp_0-p_{k+1}+1}}{2mp_0 - p_{k+1} + 1}, \tag{29}$$

由(26)、(27)，得到

$$\dot V_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) + y^{2mp_0} - (k+2)\beta\|w\|^{2m} = \dot V_k(z,\xi_1,\cdots,\xi_k) + y^{2mp_0} - (k+1)\beta\|w\|^{2m} + \xi_{k+1}^{2mp_0-p_{k+1}}\dot\xi_{k+1} - \beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^k} + (r-k+1)(\xi_1^{2mp_0}+\cdots+\xi_k^{2mp_0})\right]W(z) + \xi_k^{2mp_0-p_k}(x_{k+1}^{p_k} - x_{k+1}^{*\,p_k}) + \xi_{k+1}^{2mp_0-p_{k+1}}(x_{k+2}^{p_{k+1}} + F_{k+1}(\cdot) + G_{k+1}(\cdot)w) - \beta\|w\|^{2m}. \tag{30}$$

由(28)、(30)和引理,存在非负光滑函数 $\tilde s_{k+1}(\cdot)$ 和 $\hat s_{k+1}(\cdot)$ 使得:

$$\dot V_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) + y^{2mp_0} - (k+2)\beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^{k+1}} + (r-k)(\xi_1^{2mp_0}+\cdots+\xi_k^{2mp_0})\right]W(z) + \xi_{k+1}^{2mp_0-p_{k+1}}(x_{k+2}^{p_{k+1}} - x_{k+2}^{*\,p_{k+1}}) + \xi_{k+1}^{2mp_0-p_{k+1}}\left[x_{k+2}^{*\,p_{k+1}} + \xi_{k+1}^{p_{k+1}}\left(\tilde s_{k+1}(\cdot) + \hat s_{k+1}(\cdot) + \left(1-\frac{1}{2m}\right)\left(\frac{\xi_{k+1}^{2m(p_0-p_{k+1})}\psi_{k+1}^{2m}(\cdot)}{2m\beta}\right)^{1/(2m-1)}\right)\right]. \tag{31}$$

令

$$x_{k+2}^*(z,x_1,\cdots,x_{k+1}) = -\xi_k\alpha_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) = -\xi_{k+1}\left[(r-k)W(z) + \tilde s_{k+1}(\cdot) + \hat s_{k+1}(\cdot) + \left(1-\frac{1}{2m}\right)\left(\frac{\xi_{k+1}^{2m(p_0-p_{k+1})}\psi_{k+1}^{2m}(\cdot)}{2m\beta}\right)^{1/(2m-1)}\right]^{1/p_{k+1}},$$

由 $\alpha_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) > 0$

和(31)得到:

$$\dot V_{k+1}(z,\xi_1,\cdots,\xi_{k+1}) + y^{2mp_0} - (k+2)\beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^{k+1}} + (r-k)(\xi_1^{2mp_0}+\cdots+\xi_{k+1}^{2mp_0})\right]W(z) + \xi_{k+1}^{2mp_0-p_{k+1}}(x_{k+2}^{p_{k+1}} - x_{k+2}^{*\,p_{k+1}}).$$

仿以上步骤,第 r 步,我们可以显式构造出整体的坐标变换

$$\xi_1 = x_1 - x_1^*,\cdots,\ \xi_r = x_r - x_r^*$$

和光滑的状态反馈:

$$u^* = x_{r+1}^* = -\xi_r\alpha_r(z,\xi_1,\cdots,\xi_r),\ \alpha_r(\cdot) > 0, \tag{32}$$

并得到类似于(29)的 Liapunov 函数 $V_r(z,\xi_1,\cdots,\xi_r)$ 使得:

$$\dot V_r(z,\xi_1,\cdots,\xi_r) + y^{2mp_0} - (r+1)\beta\|w\|^{2m} \leqslant -\left[\frac{\|z\|^{2mp_0}}{2^r} + \xi_1^{2mp_0} + \cdots + \xi_r^{2mp_0}\right]W(z) + \xi_r^{2mp_0-p_r}(u^{p_r} - u^{*\,p_r}). \tag{33}$$

令 $u_\lambda(\cdot) = u^*(z,\xi_1,\cdots,\xi_r)$ 及 $(r+1)\beta = \lambda^2$,

由此

$$\dot V_r(z,\xi_1,\cdots,\xi_r) + y^{2mp_0} - \lambda^2\|w\|^{2m} \leqslant -\left(\frac{\|z\|^{2mp_0}}{2^r} + \xi_1^{2mp_0} + \cdots + \xi_r^{2mp_0}\right)W(z). \tag{34}$$

由(34)知道,当 $w = 0$ 时,闭环系统(1)、(32)在 $x = 0$ 是一致渐近稳定的. 由 V_r 的正定性及 $V_r(0,\cdots,0) = 0$,我们得到:

$$\int_0^t |y(s)|^{2mp_0}ds \leqslant \lambda^2\int_0^t \|w(s)\|^{2m}ds, \qquad \forall t \geqslant 0, 当 x(0) = 0.$$

证毕.

3 结 论

本文讨论了一类最小相高阶串联系统的几乎干扰解耦问题的反馈设计,对一般的由 L_{2m}-L_{2mp} 所定义的非线性增益指标,在一定的增长性条件下,我们构造性的给出了一种光滑控制律的设计方法,保证闭环系统在内稳定的基础上,使系统达到干扰衰减.

[参 考 文 献]

[1] Willems J C. Almost invariant subspaces: an approach to high-gain feedback design-Part Ⅰ: Almost controlled invariant subspaces[J]. *IEEE Trans Automat Control*, 1981, AC-**26**(2): 235—252.

[2] Willems J C. Almost invariant subspaces: an approach to high-gain feedback design-Part Ⅱ: Almost conditionally invariant subspaces[J]. *IEEE Trans Automat Control*, 1982, AC-**27**(5): 1071—1085.

[3] Marino R, Respondek W, Van der Schaft A J. Almost disturbance decoupling for single-input single-output nonlinear systems[J]. *IEEE Trans Automat Control*, 1989, **34**(5): 1013—1017.

[4] Marino R, Respondek W, Van der Schaft A J, *et al*. Nonlinear H_∞ almost disturbance decoupling[J]. *Systems Control Lett*, 1994, **23**(2): 159—168.

[5] Isidori A. A note on almost disturbance decoupling for nonlinear minimum-phase systems[J]. *Systems Control Lett*, 1996, **27**(2): 191—194.

[6] Qian C J, Lin W. Almost disturbance decoupling for a class of high-order nonlinear systems[J]. *IEEE Trans Automat Control*, 2000, **45**(6): 1208—1214.

[7] Lin W, Qian C J. Adding one power integrator: A tool for global stabilization of high-order lower-triangular systems[J]. *Systems Control Lett*, 2000, **39**(3): 339—351.

[8] Lin W, Qian C J. Adaptive regulation of high-order lower-triangular systems: an adding power integrator technique[J]. *Systems Control Lett*, 2000, **39**(3): 353—364.

[9] Isidori A. Global almost disturbance decoupling with stability for nonminimum-phase single-input single-output nonlinear systems[J]. *Systems Control Lett*, 1996, **28**(2): 115—122.

[10] Lin Z. Almost disturbance decoupling with global stability for nonlinear systems with disturbance affected unstable zero dynamics[J]. *Systems Control Lett*, 1998, **33**(2): 163—169.

Almost Disturbance Decoupling for a Class of Minimum-Phase Higher-Order Cascade Nonlinear Systems

BI Wei-ping, MU Xiao-wu, LIN Lan

(*Department of Mathematics, Zhengzhou University, Zhengzhou* 450052, *P. R. China*)

Abstract: The problem of almost disturbance decoupling (ADD) with internal stability is discussed, for a class of high-order cascade nonlinear systems having zero dynamics. Using adding power integrator techniques, the ADD problems via a smooth static state feedback is solved.

Key words: almost disturbance decoupling; internal stability; nonlinear gain

.net