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一类非线性时滞差分方程的全局吸引性

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摘要: 研究非线性时滞差分方程 $x_n + a_n x_n + f(n, \sum_{s=-k}^{0} q_{s,n} x_{s+n}) = 0$, 给出保证方程每一解 $\{x_n\}$ 趋于 0 的一族充分条件, 推广并改进了已有的结果

关 键 词: 全局吸引性; 非线性; 非自治; 时滞差分方程
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引 言

研究非线性时滞差分方程

$$x_n + a_n x_n + f(n, \sum_{s=-k}^{0} q_{s,n} x_{s+n}) = 0 \quad (n = 0, 1, 2, \quad), \tag{1}$$

(1) 的初始条件

$$x_s = \quad _s \quad (s = -k, -k+1, \quad , -1, 0) \tag{2}$$

其中基本假设

D1) $\{a_n\}$ 为非负实数列($0 \quad a_n < 1$, k 为自然数);

D2) $xf(n, x) > 0$ $x \quad 0$, $f(n, x)$ 关于 x 是 Lipschitz 连续的;

D3) 对任意 n, $f(n, x)$ 关于 x 单调增加;

D4) $q_{s,n} \quad 0$ 且 $\sum_{s=-k}^{0} q_{s,n} = 1$;

D5) 存在非负数列 $\{p_n\}$, 对任意的 $x \quad 0$ 有

$$\left|\frac{f(n, x)}{x}\right| \quad p_n$$

按通常规定 $x_n = x_{n+1} - x_n$ 方程(1) 包含许多具有生态学意义的时滞差分方程为特殊情况(见参见[1]), 本文试图将文[1] 的结果推广到差分方程(1) 中, 事实上, 这种将微分方程有关结论推广或离散到差分方程的工作是十分有意义的(见[2, 3]) 本文结果也可视为文[4] 结果的离散化, 同时又是文[2, 3] 中结果的推广

第 1 节先给出引理, 结束时给出本文主要结果, 同时给出主要结果的应用

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文中规定 $a < b$ 时 $\sum_{b}^{a} x_i = 0,\ \prod_{b}^{a} x_i = 1$

1　引理和主要结果

设条件 $D_1 \sim D_4$ 成立, 定义数列 $\{q_n\}$

$$q_1 = 1,\quad q_{n+1} = \frac{1}{1-a_n} q_n = \prod_{s=1}^{n} \frac{1}{1-a_s},\quad n = 1, 2, \quad , \tag{3}$$

由于 $a_n \in [0, 1)$, 故 $q_n > 0$

引理 1　若存在非负数列 $\{l_n\}$ 使得

$$\left|\frac{f(n, x)}{x}\right| \le l_n \text{ 且}$$

$$\liminf_{n} \sum_{s=N_1}^{n} \frac{q_{s+1}}{q_{N_1}} l_s = \quad > 1 \quad (\quad N_1 \quad N), \tag{4}$$

则方程(1)的任意最终正解(或最终负解) $\{x_n\}$ 均满足 $\lim_{n} x_n = 0$

证明　不妨设 x_n 是(1)的最终正解, 当 x_n 为最终负解时类似证明　于是存在 N_0, 当 $n \quad N_0$ 时,

$$x_n > 0,\quad n \quad N_0 \text{ 时},$$

由(1)立知 $\Delta x_n < 0$, x_n 最终单调减小, $\lim x_n = a \quad 0$ 存在, 反设 $a > 0$　于是 $n \quad N_0$ 时 $x_n \quad a$

另一方面　由(1)

$$q_{n+1} \Delta x_n + q_{n+1} a_n x_n = - q_{n+1} f(n, \sum_{s=-k}^{0} q_{s,n} x_{n+s})$$

即

$$\Delta(q_n x_n) = - q_{n+1} f(n, \sum_{s=-k}^{0} q_{s,n} x_{n+s}) \quad - q_{n+1} f(n, \sum_{s=-k}^{0} q_{s,n} a)$$

$$- q_{n+1} f(n, a) \quad - a q_{n+1} l_n$$

取 $1 < \quad_1 < \quad$, $0 < \quad < a(\quad_1 - 1)$, 由于 $\{x_n\}$ 单减趋于 a, 故存在 N_1, 使 $x_{N_1} < a + \quad$, 又由于 $\liminf \sum_{s=N_1}^{n} \frac{q_{s+1}}{q_{N_1}} l_s = \quad > \quad_1 > 1$, 故当 n 充分大时 $\sum_{s=N_1}^{n} \frac{q_{s+1}}{q_{N_1}} l_s > \quad_1$, 对上式从 N_1 到 n(充分大) 求和并化简

$$x_{n+1} \quad \frac{q_{N_1}}{q_{n+1}} (x_{N_1} - a \sum_{s=N_1}^{n} \frac{q_{s+1}}{q_{N_1}} l_s) \quad \frac{q_{N_1}}{q_{n+1}} (a + \quad - a \quad_1) < 0,$$

导出 $x_{n+1} < 0$ 与 x_n 最终正矛盾　故 $a \quad 0$ 错误, 即 $\lim x_n = a = 0$, 证完

引理 2　设条件$(D_1) \sim (D_5)$成立且

$$\limsup_{n} \sum_{t=n-k}^{n} \frac{q_{t+1}}{q_{n-k}} p_t = \quad < \frac{3}{2} + \frac{1}{2(k+1)},$$

则方程(1)的任意振动解 $\{x_n\}$ 满足 $\lim_{n} x_n = 0$

证明　设 $\{x_n\}$ 为(1)的振动解, 我们将证 $\lim_{n} x_n = 0$　先证 x_n 有界　反设 x_n 无界, 取 $\quad > 0$, N_1 使

$$\frac{1}{1+k} + 1 < \quad + \quad < \frac{3}{2} + \frac{1}{2(k+1)} \quad (\text{注意 } k \quad 1), \tag{5}$$

$$\sum_{t=n-k}^{n} \frac{q_{t+1}}{q_{n-k}} p_t \qquad + \qquad (n \quad N_1), \tag{6}$$

因 x_n 无界, 存在 n^* $(> N_1 + 2k)$ 使得

$$| x_n | < | x_{n^*} |, N_1 < n < n^*$$

不失一般性, 设 $x_{n^*} > 0$, 则 $x_{n^*} - x_{n^*-1} \quad 0$, 由

$$0 \qquad x_{n^*-1} = - a_{n^*-1} x_{n^*-1} - f(n^* - 1, \sum_{s=-k}^{0} q_{sn} x_{n^*-1+s})$$

知存在 $0 \quad l \quad k$, 使得

$$x_{n^*-l} \quad 0, \quad x_{n^*-l-1} < 0, \quad x_n \quad 0 \, (n \quad (n^* - l + 1, n^*)$$

于是存在 $\quad (n^* - l - 1, n^* - l]$, 使得

$$q_{n^*-l-1} x_{n^*-l-1} + (q_{n^*-l} x_{n^*-l} - q_{n^*-l-1} x_{n^*-l-1})(\quad - n^* + l + 1) = 0$$

当 $n \quad n^*$ 时

$$(q_n x_n) = - q_{n+1} f(n, \sum_{s=-k}^{0} q_{sn} x_{n+s})$$

$$q_{n+1} p_n \left| \sum_{s=-k}^{0} q_{s,n} x_{n+s} \right|$$

$$q_{n+1} p_n \sum_{s=-k}^{0} q_{s,n} | x_{n+s} |$$

$$q_{n+1} p_n \sum_{s=-k}^{0} q_{s,n} x_{n^*} = q_{n+1} p_n \sum_{s=-k}^{0} q_{s,n} x_{n^*},$$

即 $\qquad (q_n x_n) \qquad q_{n+1} p_n x_{n^*}$ (7)

当 $N_1 \quad n \quad n^* - l - 1$ 时, 对上式从 n 到 $n^* - l - 2$ 求和得

$$- q_n x_n \quad - q_{n^*-l-1} x_{n^*-l-1} + x_{n^*} \sum_{s=n}^{n^*-l-2} q_{s+1} p_s =$$

$$(q_{n^*-l} x_{n^*-l} - q_{n^*-l-1} x_{n^*-l-1})(\quad - n^* + l + 1) + x_{n^*} \sum_{s=n}^{n^*-l-2} q_{s+1} p_s$$

$$x_{n^*} \left\{ q_{n^*-l} p_{n^*-l-1}(\quad - n^* + l + 1) + \sum_{s=n}^{n^*-l-2} q_{s+1} p_s \right\} =$$

$$x_{n^*} \left(\sum_{s=n}^{n^*-l-1} q_{s+1} p_s - q_{n^*-l} p_{n^*-l-1}(n^* - \quad - l) \right),$$

$$- x_n \qquad x_{n^*} \frac{1}{q_n} \left(\sum_{s=n}^{n^*-l-1} q_{s+1} p_s - q_{n^*-l} p_{n^*-l-1}(n^* - \quad - l) \right),$$

而当 $n^* - l \quad n \quad n^* - 1$ 时上式显然成立 从而当 $n^* - l \quad n \quad n^*$ 时, 注意到 q_n 数列单调增且 $q_n \quad 1$

$$(q_n x_n) \quad - q_{n+1} f \left(n, - \sum_{s=-k}^{0} q_s \left(x_{n^*} \frac{1}{q_{n+s}} \left(\sum_{t=n+s}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1}(n^* - l - \quad) \right) \right) \right)$$

$$- q_{n+1} f \left(n, - \sum_{s=-k}^{0} q_s x_{n^*} \left(\frac{1}{q_{n-k}} \left(\sum_{t=n-k}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1}(n^* - l - \quad) \right) \right) \right)$$

$$q_{n+1} p_n \frac{1}{q_{n-k}} x_{n^*} \left(\sum_{t=n-k}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1}(n^* - l - \quad) \right),$$

即 $(q_n x_n) \quad x_{n^*} \dfrac{q_{n+1}}{q_{n-k}} p_n \left(\sum_{t=n-k}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1} (n^* - l - \quad) \right)$ (8)

下面分两种情形讨论:

情形 1 $\quad d = \sum_{n=n^*-l}^{n^*-1} \dfrac{q_{n+1}}{q_{n^*}} p_n + (n^* - l - \quad) \dfrac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} \quad 1$

$$q_{n^*} x_{n^*} = q_{n^*-l} x_{n^*-l} + \sum_{n=n^*-l}^{n^*-1} (q_{n+1} x_{n+1} - q_n x_n) =$$

$$(q_{n^*-l} x_{n^*-l} - q_{n^*-l-1} x_{n^*-l-1})(n^* - \quad - l) + \sum_{n=n^*-l}^{n^*-1} (q_{n+1} x_{n+1} - q_n x_n)$$

$$x_{n^*} (n^* - l - \quad) \frac{q_{n^*-l}}{q_{n^*-l-1-k}} p_{n^*-l-1} \left(\sum_{t=n^*-l-1-k}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1} (n^* - l - \quad) \right) +$$

$$\sum_{n=n^*-l}^{n^*-1} x_{n^*} \frac{q_{n+1}}{q_{n-k}} p_n \left(\sum_{t=n-k}^{n^*-l-1} q_{t+1} p_t - q_{n^*-l} p_{n^*-l-1} (n^* - l - \quad) \right),$$

$$x_{n^*} \quad x_{n^*} (n^* - l - \quad) \frac{q_{n^*-l}}{q_{n^*-l-1-k}} p_{n^*-l-1} \left(\sum_{t=n^*-l-1-k}^{n^*-l-1} \frac{q_{t+1}}{q_{n^*}} p_t - \frac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} (n^* - l - \quad) \right) +$$

$$\sum_{n=n^*-l}^{n^*-1} x_{n^*} \frac{q_{n+1}}{q_{n-k}} p_n \left(\sum_{t=n-k}^{n^*-l-1} \frac{q_{t+1}}{q_{n^*}} p_t - \frac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} (n^* - l - \quad) \right) =$$

$$x_{n^*} \left\{ (n^* - l - \quad) \frac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} \left(\sum_{t=n^*-l-1-k}^{n^*-l-1} \frac{q_{t+1}}{q_{n^*-l-1-k}} p_t - \frac{q_{n^*-l}}{q_{n^*-l-1}} p_{n^*-l-1} (n^* - l - \quad) \right) + \right.$$

$$\left. \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \left(\sum_{t=n-k}^{n} \frac{q_{t+1}}{q_{n-k}} p_t - \sum_{t=n^*-l}^{n} \frac{q_{t+1}}{q_{n-k}} p_t - \frac{q_{n^*-l}}{q_{n-k}} p_{n^*-l-1} (n^* - l - \quad) \right) \right\}$$

$$x_{n^*} \left\{ (\quad + \quad) d - (n^* - l - \quad)^2 p^2_{n^*-l-1} \frac{q^2_{n^*-l}}{q_{n^*} q_{n^*-l-1}} - \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \sum_{t=n^*-l}^{n} \frac{q_{t+1}}{q_{n-k}} p_t - \right.$$

$$\left. \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \frac{q_{n^*-l}}{q_{n-k}} p_{n^*-l-1} (n^* - l - \quad) \right\} =$$

$$x_{n^*} \left\{ (\quad + \quad) d - (n^* - l - \quad)^2 p^2_{n^*-l-1} \frac{q^2_{n^*-l}}{q_{n^*} q_{n^*-l-1}} - \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \sum_{t=n^*-l}^{n} \frac{q_{t+1}}{q_{n-k}} p_t - \right.$$

$$\left. \frac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} (n^* - l - \quad) \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n-k}} p_n \right\},$$

注意到

$$\sum_{n=1}^{m} y_n \sum_{i=1}^{n} y_i = \frac{1}{2} \left(\sum_{n=1}^{m} y_n \right)^2 + \frac{1}{2} \sum_{n=1}^{m} y_n^2, \ q_{n-k} \quad q_{n^*},$$

$$x_{n^*} \quad x_{n^*} \left\{ (\quad + \quad) d - (n^* - l - \quad)^2 p^2_{n^*-l-1} \frac{q^2_{n^*-l}}{q_{n^*} q_{n^*-l-1}} - \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \sum_{t=n^*-l}^{n} \frac{q_{t+1}}{q_{n^*}} p_t - \right.$$

$$\left. \frac{q_{n^*-l}}{q_{n^*}} p_{n^*-l-1} (n^* - l - \quad) \sum_{n=n^*-l}^{n^*-1} \frac{q_{n+1}}{q_{n^*}} p_n \right\}$$

$$x_{n^*}\left\{(\quad+\quad)d-(n^*-l-\quad)^2p^2_{n^*-l-1}\frac{q^2_{n^*-l}}{q^2_{n^*}}-\frac{1}{2}\left(\sum_{n=n^*-l}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\right)^2-\right.$$

$$\left.\frac{1}{2}\sum_{n=n^*-l}^{n^*-1}\left(\frac{q_{t+1}}{q_{n^*}}p_n\right)^2-\frac{q_{n^*-l}}{q_{n^*}}p_{n^*-l-1}(n^*-l-\quad)\sum_{n=n^*-l}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\right\}=$$

$$x_{n^*}\left\{(\quad+\quad)d-\frac{1}{2}\left(\sum_{n=n^*-l}^{n^*-1}\frac{q_{n+l}}{q_{n^*}}p_n+(n^*-l-\quad)\frac{q_{n^*-l}}{q_{n^*}}p_{n^*-l-1}\right)^2-\right.$$

$$\left.\frac{1}{2}(n^*-l-\quad)^2p^2_{n^*-l-1}\frac{q^2_{n^*-l}}{q^2_{n^*}}-\frac{1}{2}\sum_{n=n^*-l}^{n^*-1}\left(\frac{q_{n+1}}{q_{n^*}}p_n\right)^2\right\},$$

又注意到

$$\left(\sum_{n=1}^{m}y_n\right)^2\quad m\sum_{n=1}^{m}y_n^2$$ 得

$$x_{n^*}\quad x_{n^*}=\left[(\quad+\quad)d-\frac{1}{2}d^2-\frac{1}{2}\frac{1}{l+1}\left(\sum_{n=n^*-l}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+\right.\right.$$

$$\left.\left.(n^*-l-\quad)\frac{q_{n^*-l}}{q_{n^*}}p_{n^*-l-1}\right)^2\right]=$$

$$x_{n^*}\left((\quad+\quad)d-\left(\frac{1}{2}+\frac{1}{2(l+1)}\right)d^2\right)$$

$$x_{n^*}\left((\quad+\quad)d-\frac{k+2}{2(k+1)}d^2\right),$$

由于$(\quad+\quad)d-\frac{k+2}{2(k+1)}d^2$, 在 $d=\frac{k+1}{k+2}(\quad+\quad)$ 取最大值而(5) 知$\frac{k+1}{k+2}(\quad+\quad)>1$, 故(5) 又导出

$$x_{n^*}\quad x_{n^*}\left(\quad+\quad-\frac{k+2}{2(k+1)}\right)<x_{n^*}$$ 这不可能

情形 2 $\quad d=\sum_{n=n^*-l}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(n^*-l-\quad)\frac{q_{n^*-l}}{q_{n^*}}p_{n^*-l-1}>1$

这时存在 $n_1\quad[n^*-l,n^*]$ 使得

$$\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\quad 1,\quad\sum_{n=n_1-1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n>1$$

从而存在 $\quad(n_1-1,n_1)$ 使得

$$\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}=1\tag{9}$$

于是由(7), (8) 得:

$$q_{n^*}x_{n^*}=q_{n_1}x_{n_1}+\sum_{n=n_1}^{n^*-1}(q_{n+1}x_{n+1}-q_nx_n)=$$

$$q_{n_1-1}x_{n_1-1}+(\quad-n_1+1)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+(n_1-\quad)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+$$

$$\sum_{n=n_1}^{n^*-1}(q_{n+1}x_{n+1}-q_nx_n)=$$

$$q_{n^*-l}x_{n^*-l}+\sum_{n=n^*-l}^{n_1-2}(q_{n+1}x_{n+1}-q_nx_n)+(\quad-n_1+1)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+$$

$$(n_1-\quad)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+\sum_{n=n_1}^{n^*-1}(q_{n+1}x_{n+1}-q_nx_n)=$$

$$(q_{n^*-l}x_{n^*-l}-q_{n^*-l-1}x_{n^*-l-1})(n^*-\quad-l)+\sum_{n=n^*-l}^{n_1-2}(q_{n+1}x_{n+1}-q_nx_n)+$$

$$(\quad-n_1+1)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+(n_1-\quad)(q_{n_1}x_{n_1}-q_{n_1-1}x_{n_1-1})+$$

$$\sum_{n=n_1}^{n^*-1}(q_{n+1}x_{n+1}-q_nx_n)$$

$$x_{n^*}\Big[(n^*-\quad-l)q_{n^*-l}p_{n^*-l-1}+\sum_{n=n^*-l}^{n_1-2}q_{n+1}p_n+(\quad-n_1+1)q_{n_1}p_{n_1-1}\Big]$$

$$x_{n^*}\Bigg[(n_1-\quad)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}\Big(\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*-l-\quad)\Big)+$$

$$\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\Big(\sum_{t=n-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*-l-\quad)\Big)\Bigg]=$$

$$x_{n^*}\Bigg\{\Big[(n^*-\quad-l)q_{n^*-l}p_{n^*-l-1}+\sum_{n=n^*-l}^{n_1-2}q_{n+1}p_n+(\quad+1-n_1)q_{n_1}p_{n_1-1}\Big]$$

$$\Big[\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\Big]+\Big[(n_1-\quad)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}\Big(\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t-$$

$$q_{n^*-l}p_{n^*-l-1}(n^*-l-\quad)\Big)+\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\Big(\sum_{t=n-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*-l-\quad)\Big)\Big]\Bigg\}=$$

$$x_{n^*}\Bigg\{(n^*-\quad-l)q_{n^*-l}p_{n^*-l-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(n^*-\quad-l)q_{n^*-l}p_{n^*-l-1}(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$\sum_{n=n^*-l}^{n_1-2}q_{n+1}p_n\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+\sum_{n=n^*-l}^{n_1-2}p_nq_{n+1}(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$(\quad+1-n_1)q_{n_1}p_{n_1-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(\quad+1-n_1)q_{n_1}p_{n_1-1}(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$(n_1-\quad)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*-l-\quad)(n_1-\quad)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}+$$

$$\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\sum_{t=n-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*-\quad-l)\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\Bigg\}$$

注意到$\{q_n\}$为单调增数列, $q_n\quad 1$, 从而通过抵消得

$$q_{n^*}x_{n^*}\qquad x_{n^*}\Bigg\{(n^*-l-\quad)q_{n^*-l}p_{n^*-l-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+$$

$$(n^*-\quad-l)q_{n^*-l}p_{n^*-l-1}(n_1-\quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$\sum_{n=n^*-l}^{n_1-2} q_{n+1}p_n \sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+\sum_{n=n^*-l}^{n_1-2} p_n q_{n+1}(n_1- \quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$(\quad +1-n_1)q_{n_1}p_{n_1-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(\quad +1-n_1)q_{n_1}p_{n_1-1}(n_1- \quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$(n_1- \quad)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*- \quad - l)(n_1- \quad)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$\left.\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\sum_{t=n-k}^{n^*-l-1}q_{t+1}p_t-q_{n^*-l}p_{n^*-l-1}(n^*- \quad - l)\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\right\}$$

$$x_{n^*}\left\{\sum_{n=n^*-l}^{n_1-1}q_{n+1}p_n\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+\sum_{n=n^*-l}^{n_1-1}p_nq_{n+1}(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+\right.$$

$$(G-n_1)q_{n_1}p_{n_1-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n+(G-n_1)q_{n_1}p_{n_1-1}(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$\left.(n_1-G)\frac{q_{n_1}}{q_{n_1-k-1}}p_{n_1-1}\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t+\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n-k}}p_n\sum_{t=n-k}^{n^*-l-1}q_{t+1}p_t\right\},$$

从而

$$x_{n^*} \quad [\quad x_{n^*}\left\{\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\left(\sum_{n=n^*-l}^{n_1-1}\frac{q_{n+1}}{q_{n^*}}p_n+\sum_{t=n-k}^{n^*-l-1}\frac{q_{t+1}}{q_{n-k}}p_t\right)+\right.$$

$$(G-n_1)\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}+$$

$$(n_1-G)\left[\frac{q_{n_1}}{q_{n^*}\ q_{n_1-k-1}}p_{n_1-1}\sum_{t=n_1-1-k}^{n^*-l-1}q_{t+1}p_t+\sum_{n=n^*-l}^{n_1-1}q_{n+1}p_n\frac{q_{n_1}}{q^2_{n^*}}p_{n_1-1}\right]+$$

$$\left.(G-n_1)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right\} \quad [$$

$$x_{n^*}\left\{\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\left(\sum_{t=n-k}^{n_1-1}\frac{q_{t+1}}{q_{n-k}}p_t-(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)+\right.$$

$$\left.(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\left(\sum_{t=n_1-1-k}^{n_1-1}\frac{q_{t+1}}{q_{n_1-k-1}}p_t-(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)\right\} \quad [$$

$$x_{n^*}\left\{\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\left(L+D-\sum_{t=n_1}^{n}\frac{q_{t+1}}{q_{n-k}}p_t-(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)+\right.$$

$$\left.(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\left(L+D-(n_1-G)\frac{q_{n}}{q_{n^*}}p_{n_1-1}\right)\right\}=$$

$$x_{n^*}\left\{L+D-\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\sum_{t=n_1}^{n}\frac{q_{t+1}}{q_{n-k}}p_t-(n_1-G)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n-\right.$$

$$\left.(n_1-G)^2\left(\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2\right\}\#$$

注意到

$$2\sum_{n=1}^{m} y_n \sum_{i=1}^{m} y_i = \left(\sum_{n=1}^{m} y_n\right)^2 + \sum_{n=1}^{m} y_n^2,$$

$$\left(\sum_{n=1}^{m} y_n\right)^2 \le m\sum_{n=1}^{m} y_n^2 \text{ 及 } q_{n-k} \le q_{n^*},$$

我们可得

$$x_{n^*} \le x_{n^*}\left\{L + D - \frac{1}{2}\left(\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n\right)^2 - \frac{1}{2}\sum_{n=n_1}^{n^*-1}\left(\frac{q_{n+1}}{q_{n^*}}p_n\right)^2 - \right.$$

$$\left.(n_1-\varepsilon)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n - (n_1-\varepsilon)^2\left(\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2\right\} =$$

$$x_{n^*}\left\{L + D - \frac{1}{2}\left(\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n + (n_1-\varepsilon)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2 - \frac{1}{2}\sum_{n=n_1}^{n^*-1}\left(\frac{q_{n+1}}{q_{n^*}}p_n\right)^2 - \right.$$

$$\left.\frac{1}{2}(n_1-\varepsilon)^2\left(\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2\right\} =$$

$$x_{n^*}\left\{L + D - \frac{1}{2} - \frac{1}{2}\left[\sum_{n=n_1}^{n^*-1}\left(\frac{q_{n+1}}{q_{n^*}}p_n\right)^2 + (n_1-\varepsilon)^2\left(\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2\right]\right\} \le$$

$$x_{n^*}\left\{L + D - \frac{1}{2} - \frac{1}{2}\,\frac{1}{n^*-n_1+1}\left(\sum_{n=n_1}^{n^*-1}\frac{q_{n+1}}{q_{n^*}}p_n + (n_1-\varepsilon)\frac{q_{n_1}}{q_{n^*}}p_{n_1-1}\right)^2\right\} =$$

$$x_{n^*}\left\{L + D - \frac{1}{2} - \frac{1}{2}\,\frac{1}{n^*-n_1+1}\right\} \le$$

$$x_{n^*}\left(L + D - \frac{1}{2} - \frac{1}{2(k+1)}\right) < x_{n^*},$$

这也不可能,故 x_n 有界证完。　下证 $\lim x_n = 0$,设 x_n 的上下极限如下

$$A = \limsup_{n\to\infty} x_n, \quad B = \liminf_{n\to\infty} x_n$$

则 $-\infty < B \le 0 \le A < +\infty$,只证 $A = B = 0$ 即可。　任给 $\varepsilon > 0$,必存在 $N_2 > N_1 + k$ 使得

$$-B_1 = B - \varepsilon < x_n < A + \varepsilon = A_1, \quad n \ge N_2 + k,$$

于是由于 $\Delta(q_n x_n) = -q_{n+1} f\left(n, \sum_{s=-k}^{0} q_{s,n} x_{n+s}\right) \le$

$$-q_{n+1} f\left(n, -\sum_{s=-k}^{0} q_{s,n} B_1\right) =$$

$$-q_{n+1} f(n, -B_1) \le B_1 q_{n+1} p_n,$$

即　$\Delta(q_n x_n) \le B_1 q_{n+1} p_n, \quad n \ge N_2 + k,$ (10)

同理　$\Delta(q_n x_n) \ge -A_1 q_{n+1} p_n, \quad n \ge N_2 + k$。 (11)

由于 x_n 是振动解。　于是存在子列 x_{n_i},使得 $x_{n_i} \ge x_{n_i-1}, x_{n_i} > 0, \lim_{i\to\infty} x_{n_i} = A$,由方程(1),用 B_1 代替(7)式中的 x_{n^*},运用证 x_n 有界的方法可得

$$x_{n_i} \le B_1\left(L + D - \frac{1}{2} - \frac{1}{2(k+1)}\right),$$

令 $i \to +\infty$,$\varepsilon \to 0$ 立得

$$A \le B\left(L + D - \frac{1}{2} - \frac{1}{2(k+1)}\right)。 \quad (12)$$

又可取子列 x_{n_j}，使得 $x_{n_j} \le x_{n_j-1}, x_{n_j} < 0, \lim_{j\to\infty} x_{n_j} = B$，于是 $-x_{n_j} \ge -x_{n_j-1}, -x_{n_j} > 0$，(11) 式化为

$$\Delta(q_n(-x_n)) \le A_1 q_{n+1} p_n. \tag{13}$$

用 A_1 代替(7)式中的 x_n^*，用证明 x_n 有界的相同方法可得

$$-x_{n_j} \le A_1\left(L + D - \frac{1}{2} - \frac{1}{2(k+1)}\right).$$

令 $j\to+\infty$，$G\to 0$ 立得

$$-B \le A\left(L + D - \frac{1}{2} - \frac{1}{2(k+1)}\right). \tag{14}$$

由于 $0 < L + D - \frac{1}{2} - \frac{1}{2(k+1)} < 1$. 故(12)、(14) 导出 $A = B = 0$. 证毕.

由引理 1, 2 立得

定理 1 设(D_1) ~ (D_5) 成立，且引理 1 与 2 的条件满足，则方程(1)的任一解 $\{x_n\}$ 满足 $\lim_{n\to\infty} x_n = 0$.

注 1 考查方程

$$\Delta x_n + a_n x_n + Q_n x_{n-k} = 0, \tag{15}$$

其中 $a_n \in [0, 1), p_n > 0, k$ 为自然数.

定理 2 设

$$\liminf_{n\to\infty} \sum_{s=N_1}^{n} \frac{q_{s+1}}{q_{N_1}} p_s = K > 1 \quad (PN_1 \in N), \tag{16}$$

$$\limsup_{n\to\infty} \sum_{t=n-k}^{n} \prod_{s=n-k}^{t} (1 - a_s)^{-1} p_t = L < \frac{3}{2} + \frac{1}{2(k+1)}. \tag{17}$$

则(15)的每一解满足 $\lim x_n = 0$.

定理 2 推广了文献[5]的结果. (当 $a_n \equiv 0$ 时，定理 2 即为文[5]的主要结果).

注 2 当 $a_n \equiv K \in (0, 1)$ 时，定理 2 为文[3]的主要结果，因此定理 2 推广了文[5]的部分定理.

注 3 考虑多时滞线性时滞差分方程

$$\Delta x_n + a_n x_n + \sum_{s=1}^{m} p_{s,n} x_{n-k_s} = 0, \tag{18}$$

其中 $a_n \in [0, 1), p_{s,n} > 0 (s = 1, 2, \cdots, m, n = 1, 2, \cdots), k_s$ 为自然数，$\max\{k_1, k_2, \cdots, k_m\} = k$. 则令

$$q_s = \frac{p_{s,n}}{\sum_{s=1}^{m} p_{s,n}}, \quad p_n = \sum_{s=1}^{n} p_{s,n},$$

从而(18)化为方程(3)的形式，有如下定理

定理 3 设如下条件成立

$$\liminf_{n\to\infty} \sum_{s=N_1}^{n} \prod_{t=N_1}^{s} (1 - a_t)^{-1} p_s = K > 1 \quad (PN_1 \in N), \tag{19}$$

$$\limsup_{n\to\infty} \sum_{t=n-k}^{n} \prod_{s=n-k}^{t} (1 - a_s)^{-1} p_t = L < \frac{3}{2} + \frac{1}{2(k+1)}, \tag{20}$$

则方程(18)的每一解$\{x_n\}$满足$\lim x_n = 0$#

方程(18)的全局吸引性至今未见有较好结果发表[6]# 定理3也是新的# 当 $a_n S K I [0, 1)$ 时,条件(20)化为

$$\limsup_{n y]} \sum_{t=n-k}^{n} (1-K)^{-t+n+k} p_t = L < \frac{3}{2} + \frac{1}{2(k+1)}\#$$

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Global Attractivity for a Class of Nonlinear Delay Difference Equations

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Abstract: The global attractivity of the delay difference equation $\$ x_n + a_n x_n + f\left(n, \sum_{s=-k}^{0} q_{s,n} x_{s+n}\right) = 0$, *which includes the discrete type of many mathematical ecologic equations, was dicussed. The sufficient conditions that guarantee every solution to converge to zero were obtained. Many known results are improved and generated.*

Key words: global attractivity; nonlinear; nonautonomous; delay difference equation