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三维热传导方程的一族两层显式格式*

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摘要: 提出了一族三维热传导方程的两层显式差分格式, 当截断误差阶为 $O(\Delta t+(\Delta x)^2)$ 时, 稳定性条件为网格比 $r=\frac{\Delta t}{(\Delta x)^2}=\frac{\Delta t}{(\Delta y)^2}=\frac{\Delta t}{(\Delta z)^2}\leqslant\frac{1}{2}$, 优于其他显式差分格式· 而当截断误差阶为 $O((\Delta t)^2+(\Delta x)^4)$ 时, 稳定性条件为 $r\leqslant 1/6$, 包含了已有的结果·

关 键 词: 三维热传导方程; 显式差分格式; 截断误差; 稳定性
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引 言

求解区域 $D:\{0\leqslant x, y, z\leqslant L, 0\leqslant t\leqslant T\}$ 上的三维热传导方程初边值问题

$$\begin{cases}\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}, & (1)\\ u|_{x=0}=f_1(y,z,t),\quad u|_{x=L}=f_2(y,z,t), & (2)\\ u|_{y=0}=g_1(z,x,t),\quad u|_{y=L}=g_2(z,x,t), & (3)\\ u|_{z=0}=h_1(x,y,t),\quad u|_{z=L}=h_2(x,y,t), & (4)\\ u|_{t=0}=\varphi(x,y,z)\cdot & (5)\end{cases}$$

解上述方程的古典显式格式的精度不高, 截断误差阶仅为 $O(\Delta t+(\Delta x)^2)$, 且稳定性条件 $r=\frac{\Delta t}{(\Delta x)^2}=\frac{\Delta t}{(\Delta y)^2}=\frac{\Delta t}{(\Delta z)^2}\leqslant\frac{1}{6}$, 也较为苛刻[1]· 在文[2] 中构造了精度较高且绝对稳定的差分格式, 其截断误差阶达到 $O((\Delta t)^2+(\Delta x)^4)$, 但却是三层隐式格式, 徒增了很多计算量和存储量· 文[3~4] 给出了三层高精度显式格式, 其稳定性条件为 $r\leqslant 1/6$, 但它们也不能计算第一层上的网格函数值, 需用其他方法先启动·

本文构造了一族两层显式格式, 不仅能计算第一层上的网格函数数值, 而且稳定性也优于(或不逊于) 其他同精度显式格式· 当截断误差阶为 $O(\Delta t+(\Delta x)^2)$ 时, 稳定性比其他显式格式好, 可放宽到 $r\leqslant\frac{1}{2}$; 而当截断误差阶为 $O((\Delta t)^2+(\Delta x)^4)$ 时, 稳定性条件为 $r\leqslant\frac{1}{6}$, 包含了文[4] 的一族两层高精度显式格式· 数值例子表明理论分析是正确的·

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1　差分格式的建立

设 Δt 为时间步长, Δx, Δy, Δz 分别为x, y, z 方向的空间步长, 且为简便计, 设 $\Delta x = \Delta y = \Delta z = \dfrac{L}{M}$($M$ 为正整数)• 用如下含参数 a、b、c、d 的两层显式格式逼近微分方程(1):

$$w^{n+1}_{i,j,k} = a\Diamond_2 w^n_{i,j,k} + b\Diamond_1 w^n_{i,j,k} + c\Box w^n_{i,j,k} + d w^n_{i,j,k}, \tag{6}$$

其中 $w^n_{i,j,k}$ 表示在节点($i\Delta x$, $j\Delta y$, $k\Delta z$, $n\Delta t$) 处的网格函数值, 且记

$$\left.\begin{aligned}
&\Box w^n_{i,j,k} = ({}_z\Box + {}_y\Box + {}_x\Box) w^n_{i,j,k},\\
&\Diamond_l w^n_{i,j,k} = ({}_z\Diamond_l + {}_y\Diamond_l + {}_x\Diamond_l) w^n_{i,j,k},\\
&{}_z\Box w^n_{i,j,k} = w^n_{i+1,j+1,k} + w^n_{i-1,j+1,k} + w^n_{i+1,j-1,k} + w^n_{i-1,j-1,k} - 4w^n_{i,j,k},\\
&{}_z\Diamond_l w^n_{i,j,k} = w^n_{i+l,j,k} + w^n_{i,j+l,k} + w^n_{i-l,j,k} + w^n_{i,j-l,k} - 4w^n_{i,j,k}\\
&\qquad\qquad\qquad\qquad (l = 1, 2)•
\end{aligned}\right\} \tag{7}$$

其余类推• 适当选取待定参数 a、b、c、d 可以使差分格式(6) 逼近微分方程(1) 时, 不仅具有尽可能高阶的截断误差, 而且有较好的稳定性•

当微分方程(1) 的解充分光滑时, 有如下关系式成立:

$$\frac{\partial^n}{\partial t^n}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^m w = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)^{m+2n} w, \tag{8}$$

其中 m, n 为非负整数• 在节点($i\Delta x$, $j\Delta y$, $k\Delta z$, $n\Delta t$) 处进行Taylor 展开(今后为简便计, 略去 $w^n_{i,j,k}$ 的下标而简记为w^n, 等等), 并利用关系式(8) 得

$$\frac{1}{(\Delta x)^2}\,{}_z\Diamond_2 w^n = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w^n + \frac{(2\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4}\right) w^n + O((\Delta x)^4),$$

$$\frac{1}{(\Delta x)^2}\,{}_z\Diamond_1 w^n = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w^n + \frac{(\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4}\right) w^n + O((\Delta x)^4),$$

$$\frac{1}{2(\Delta x)^2}\,{}_z\Box w^n = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w^n + \frac{(\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^4} + 6\frac{\partial^4}{\partial x^2 y^2} + \frac{\partial^4}{\partial y^4}\right) w^n + O((\Delta x)^4),$$

等等, 及

$$\begin{aligned}\frac{1}{2(\Delta x)^2}\Diamond_2 w^n = {}& \frac{\partial w^n}{\partial t} + \frac{(2\Delta x)^2}{12}\frac{\partial^2 w^n}{\partial t^2} - \frac{2(2\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^2\partial y^2} + \right.\\ &\left.\frac{\partial^4}{\partial y^2\partial z^2} + \frac{\partial^4}{\partial z^2\partial x^2}\right) w^n + O((\Delta x)^4),\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{1}{2(\Delta x)^2}\Diamond_1 w^n = {}& \frac{\partial w^n}{\partial t} + \frac{(\Delta x)^2}{12}\frac{\partial^2 w^n}{\partial t^2} - \frac{2(\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^2\partial y^2} + \right.\\ &\left.\frac{\partial^4}{\partial y^2\partial z^2} + \frac{\partial^4}{\partial z^2\partial x^2}\right) w^n + O((\Delta x)^4),\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{1}{4(\Delta x)^2}\Box w^n = {}& \frac{\partial w^n}{\partial t} + \frac{(\Delta x)^2}{12}\frac{\partial^2 w^n}{\partial t^2} + \frac{(\Delta x)^2}{12}\left(\frac{\partial^4}{\partial x^2\partial y^2} + \right.\\ &\left.\frac{\partial^4}{\partial y^2\partial z^2} + \frac{\partial^4}{\partial z^2\partial x^2}\right) w^n + O((\Delta x)^4),\end{aligned} \tag{11}$$

$$w^{n+1} = w^n + \Delta t\frac{\partial w^n}{\partial t} + \frac{(\Delta t)^2}{2}\frac{\partial^2 w^n}{\partial t^2} + O((\Delta t)^3)• \tag{12}$$

将它们代入差分格式(6) 可得:

$$w^n+\Delta t\frac{\partial w^n}{\partial t}+\frac{(\Delta t)^2}{2}\frac{\partial^2 w^n}{\partial t^2}+O((\Delta t)^3)=$$
$$dw^n+(2a+2b+4c)(\Delta x)^2\frac{\partial w^n}{\partial t}+\frac{1}{12}(8a+2b+4c)(\Delta x)^4\frac{\partial^2 w^n}{\partial t^2}+$$
$$\frac{1}{12}(-16a-4b+4c)(\Delta x)^4\left(\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial y^2\partial z^2}+\frac{\partial^4}{\partial z^2\partial x^2}\right)w^n+$$
$$O((\Delta x)^4). \tag{13}$$

于是立得 $\left(记\ r=\frac{\Delta t}{(\Delta x)^2}=\frac{\Delta t}{(\Delta y)^2}=\frac{\Delta t}{(\Delta z)^2}\right)$:

1. 一般两层显式格式

为了使格式(6)逼近方程(1)的截断误差阶为 $O(\Delta t+(\Delta x)^2)$, 参数必须满足下列条件:

$$\begin{cases}d=1,\\ 2a+2b+4c=r.\end{cases}\quad 或\quad \begin{cases}d=1,\\ b=\frac{r}{2}-a-2c.\end{cases}$$

将上式代入(6)式, 即得一族含参数 a、c, 截断误差阶为 $O(\Delta t+(\Delta x)^2)$ 的一般两层显式格式

$$w^{n+1}=a\Diamond_2 w^n+\left(\frac{r}{2}-a-2c\right)\Diamond_1 w^n+c\,\Box w^n+w^n. \tag{14}$$

2. 高精度两层显格式

为了使格式(6)逼近方程(1)的截断误差阶为 $O((\Delta t)^2+(\Delta x)^4)$, 参数必须同时满足

$$\begin{cases}d=1,\quad 2a+2b+4c=r.\\ 8a+2b+4c=6r^2,\quad 4a+b=c.\end{cases}$$

解得 $a=r^2-\frac{1}{6}r,\quad b=\frac{2}{3}r-3r^2,\quad c=r^2,\quad d=1.$

于是得截断误差阶为 $O((\Delta t)^2+(\Delta x)^4)$ 的高精度两层显式差分格式如下:

$$w^{n+1}=\left(r^2-\frac{1}{6}r\right)\Diamond_2 w^n+\left(\frac{2}{3}r-3r^2\right)\Diamond_1 w^n+r^2\Box w^n+w^n. \tag{15}$$

特别地, 当 $r=1/6$ 时有两层高精度显式格式

$$w^{n+1}=\left(1+\frac{1}{36}\Diamond_1+\frac{1}{36}\Box\right)w^n. \tag{16}$$

即

$$w^{n+1}=\left\{1+\frac{1}{6}(\delta_x^2+\delta_y^2+\delta_z^2)+\frac{1}{36}(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)\right\}w^n. \tag{16$'$}$$

它与文[4]中格式(25)相同, 是一个简洁而实用的两层显式格式. 其中 δ_x^2, δ_y^2, δ_z^2 分别为 w^n 关于 x, y, z 方向的二阶中心差分.

2 稳定性分析

利用 Fowror 稳定性分析法, 令

$$w_{i,j,k}^n=\lambda^n e^{i^*(p\pi x_i+q\pi y_j+s\pi z_k)}\qquad (i^*=\sqrt{-1}),$$

且记

$$\Diamond_1 w_{i,j,k}^n=-8(s_1^2+s_2^2+s_3^2)w_{i,j,k}^n,$$
$$\Diamond_2 w_{i,j,k}^n=-32\left\{(s_1^2+s_2^2+s_3^2)-(s_1^4+s_2^4+s_3^4)\right\}w_{i,j,k}^n,$$
$$\Box w_{i,j,k}^n=-16\left\{(s_1^2+s_2^2+s_3^2)-(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2)\right\}w_{i,j,k}^n,$$

式中 $s_1 = \sin\dfrac{p\pi\Delta x}{2},\ s_2 = \sin\dfrac{q\pi\Delta y}{2},\ s_3 = \sin\dfrac{s\pi\Delta z}{2}$•

将上述各式代入两层显格式(14),经整理得传播因子为

$$\begin{aligned}\lambda = &\ 1 + a\left\{-32(s_1^2+s_2^2+s_3^2)+32(s_1^4+s_2^4+s_4^4)\right\}+\\&\frac{r-2a-4c}{2}\left\{-8(s_1^2+s_2^2+s_3^2)\right\}+\\&c\left\{-16(s_1^2+s_2^2+s_3^2)+16(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2)\right\}=\\&1-4(r+6a)(s_1^2+s_2^2+s_3^2)+32a(s_1^4+s_2^4+s_3^4)+\\&16c(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2)\bullet\end{aligned}$$

因此,由 Von Neumann 条件 $|\lambda| \leqslant 1$ 知,差分格式(14)的稳定性条件为

$$\begin{aligned}-2 \leqslant &\ 32a(s_1^4+s_2^4+s_3^4)-4(r+6a)(s_1^2+s_2^2+s_3^2)+\\&16c(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2) \leqslant 0\bullet\end{aligned}\tag{17}$$

相应地,我们分别讨论两类格式的稳定性•

1. 一般两层显式格式(14)

A) 若取 $a = c = 0$,则格式(14)退化为熟知的古典显式格式

$$w^{n+1} = \frac{r}{2}\Diamond_1 w^n + w^n\bullet\tag{18}$$

即

$$\frac{w^{n+1}-w^n}{\Delta t} = \frac{\delta_x^2+\delta_y^2+\delta_z^2}{(\Delta x)^2}w^n\bullet\tag{18$'$}$$

其稳定性条件为 $r \leqslant 1/6$•

B) 若取 $a = 0$,则稳定性条件(17)退化为

$$-2 \leqslant -4r(s_1^2+s_2^2+s_3^2)+16c(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_2^2) \leqslant 0,$$

它等价于

$$\begin{cases}r(s_1^2+s_2^2+s_3^2) \geqslant 4c(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2),\\2r(s_1^2+s_2^2+s_3^2) \leqslant 1+8c(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2)\bullet\end{cases}$$

由此可见,当 $c = r/4$ 时稳定性最好,其稳定性条件为

$$2r(s_1^2+s_2^2+s_3^2) \leqslant 1+2r(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2)\bullet$$

或

$$2r(s_1^2+s_2^2+s_3^2-s_1^2s_2^2-s_2^2s_3^2-s_3^2s_1^2) \leqslant 1\bullet$$

令 $s_i^* = s_i^2 (i = 1, 2, 3)$,则 $0 \leqslant s_i^* \leqslant 1$• 令 $F(s_1^*, s_2^*, s_3^*) = s_1^*+s_2^*+s_3^*-s_1^*s_2^*-s_2^*s_3^*-s_3^*s_1^*$,则 $\max F(s_1^*, s_2^*, s_3^*) = F(1,1,0) = F(1,0,0) = 1$• 故得当 $a = 0$、$c = r/4$ 时格式(14)的稳定性条件为 $r \leqslant 1/2$,此时格式(14)成为

$$w^{n+1} = \frac{r}{4}\Box w^n + w^n\bullet\tag{19}$$

即

$$w^{n+1} = r(\delta_x^2+\delta_y^2+\delta_z^2)w^n+\frac{r}{4}(\delta_x^2\delta_y^2+\delta_y^2\delta_z^2+\delta_z^2\delta_x^2)w^n+w^n\bullet\tag{19$'$}$$

今后将称格式(19)或(19)$'$为改进格式,

其稳定性条件优于古典显式格式及其他显格式•

C) 若取 $c = 0$,则稳定性条件(17)成为

$$-2 \leqslant 32a(s_1^4+s_2^4+s_3^4)-4(r+6a)(s_1^2+s_2^2+s_3^2) \leqslant 0.$$

它等价于

$$8a(s_1^4+s_2^4+s_3^4) \leqslant (r+6a)(s_1^2+s_2^2+s_3^2),$$

$$2(r+6a)(s_1^2+s_2^2+s_3^2) \leqslant 1+16a(s_1^4+s_2^4+s_3^4).$$

易知, 当 $r=2a$ 时稳定性最好, 其稳定性条件为

$$8r(s_1^2+s_2^2+s_3^2-s_1^4-s_2^4-s_3^4) \leqslant 1.$$

令 $G(s_1^*, s_2^*, s_3^*) = s_1^*+s_2^*+s_3^*-s_1^{*2}-s_2^{*2}-s_3^{*2}$, $\dfrac{\partial G}{\partial s_i^*} = 1-2s_i^* = 0(i=1,2,3)$, 解得 $s_i^* = 1/2(i=1,2,3)$, 故

$$\max G(s_1^*, s_2^*, s_3^*) = G\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right) = \frac{3}{4}.$$

故当 $a=r/2$, $c=0$ 时格式(14) 的稳定性条件为 $8r\cdot\dfrac{3}{4} \leqslant 1$, 即 $r \leqslant 1/6$, 此时格式(14) 成为

$$w^{n+1} = \frac{r}{2}\Diamond_2 w^n + w^n. \tag{20}$$

其稳定性条件与古典显式格式(18) 相当, 但所用点数多, 边界处理麻烦, 故不宜采用此格式.

2. 两层高精度显式格式(15)

此时稳定性条件(17) 成为

$$-2 \leqslant 32\left(r^2-\frac{1}{6}r\right)(s_1^4+s_2^4+s_3^4)-24r^2(s_1^2+s_2^2+s_3^2)+$$
$$16r^2(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2) \leqslant 0. \tag{21}$$

它等价于

$$\left.\begin{aligned}&\frac{2}{3}(s_1^4+s_2^4+s_3^4)+3r(s_1^2+s_2^2+s_3^2)-4r(s_1^4+s_2^4+s_3^4)-\\&\quad 2r(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2) \geqslant 0,\\&1+16r^2(s_1^4+s_2^4+s_3^4)-12r^2(s_1^2+s_2^2+s_3^2)-\\&\quad \frac{8}{3}r(s_1^4+s_2^4+s_3^4)+8r^2(s_1^2s_2^2+s_2^2s_3^2+s_3^2s_1^2) \geqslant 0.\end{aligned}\right\} \tag{22}$$

事实上, 当 $r \leqslant 2/9$ 时, $\dfrac{2}{3}-3r \geqslant 0$, 所以,

(22) 式的第 1 式

$$\begin{aligned}\text{左边} &= \frac{2}{3}(s_1^4+s_2^4+s_3^4)+3r\{(s_1^2+s_2^2+s_3^2)-(s_1^4+s_2^4+s_3^4)\}-\\&\quad r(s_1^2+s_2^2+s_3^2) = \left(\frac{2}{3}-3r\right)(s_1^4+s_2^4+s_3^4)+\\&\quad r\{3-(s_1^2+s_2^2+s_3^2)\}(s_1^2+s_2^2+s_3^2) \geqslant 0,\end{aligned}$$

而当 $r \leqslant \dfrac{1}{6}$ 时, $16r^2-\dfrac{8}{3}r = 8r\left(2r-\dfrac{1}{3}\right) \leqslant 0$,

所以(22) 式第二式

$$\begin{aligned}\text{左边} &\geqslant 1+\left(16r^2-\frac{8}{3}r\right)(s_1^4+s_2^4+s_3^4)-12r^2(s_1^2+s_2^2+s_3^2) \geqslant\\&\quad 1+3\cdot\left(16r^2-\frac{8}{3}-12\cdot 3\cdot r^2\right) =\end{aligned}$$

$$12r^2-8r+1=12\left(r-\frac{1}{2}\right)\left(r-\frac{1}{6}\right)\geqslant 0.$$

综上所述可得: 当 $r\leqslant\min\left(\frac{1}{6},\frac{2}{9}\right)=\frac{1}{6}$ 时, 稳定性条件(22)(等价地(17))成立, 高精度格式(15) 稳定． 特别地, 两层高精度显格式(16) 是稳定的．

3　数值试验

考虑三维热传导方程混合问题

$$\begin{cases}\dfrac{\partial u}{\partial t}=\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2} & (0\leqslant x,y,z\leqslant 1,\ t>0),\\ u(x,y,z,0)=\sin(x+y+z) & (0\leqslant x,y,z\leqslant 1),\\ u|_{\text{边界}}=\text{精确解在边界上的值},\end{cases}\tag{23}$$

其精确解为

$$u(x,y,z,t)=e^{-3t}\sin(x+y+z).\tag{24}$$

取 $\Delta x=\Delta y=\Delta z=0.1$, $\Delta t=0.0016$ 及 0.05(此时相应地 $r=1/6$ 及 $1/2$) 按古典显式格式(18)、改进格式(19) 及高精度格式(16) 分别计算到 $n=200$ 列表如表 1 所示． 当 $r=1/6$ 时, 三种格式均稳定, 其平均平方误差分别为 5.4555897×10^{-5}、2.8109402×10^{-5} 及 3.7037035×10^{-8}, 与各种格式精度分析相一致． 且由表可看出, 当 $r=1/2$ 时古典显式格式已溢出, 高精度格式也偏离较大, 这是由于它们不满足稳定性条件所致． 而改进格式仍相当精确, 稳定性较好． 此与格式的稳定性分析结果相一致．

表 1　各种格式计算结果比较表 ($\Delta x=\Delta y=\Delta z=0.1, n=200$)

Δt	(x,y,z)	精确解 (24)	古典显式格式 (18)	改进格式 (19)	高精度格式 (16)
0.0016	(0.2, 0.7, 0.8)	0.36481310	0.36476455	0.36483827	0.36481313
	(0.4, 0.3, 0.6)	0.35447325	0.35438527	0.35451852	0.35447331
	(0.6, 0.5, 0.7)	0.35825852	0.35816703	0.35830558	0.35825858
	(0.8, 0.2, 0.5)	0.36695790	0.36690365	0.36698592	0.36695794
0.005	(0.2, 0.7, 0.8)	0.04937208	溢出	0.04935518	0.05929834
	(0.4, 0.3, 0.6)	0.04797274		0.04794235	0.06696802
	(0.6, 0.5, 0.7)	0.04848502		0.04845343	0.06828631
	(0.8, 0.2, 0.5)	0.04966235		0.04964354	0.06089017

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A Class of Two_Level Explicit Difference Schemes for Solving Three Dimensional Heat Conduction Equation

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Abstract: A class of two_level explicit difference schemes are presented for solving three_dimensional heat conduction equation. When the order of truncation error is $O(\Delta t+(\Delta x)^2)$, the stability condition is mesh ratio $r=\frac{\Delta t}{(\Delta x)^2}=\frac{\Delta t}{(\Delta y)^2}=\frac{\Delta t}{(\Delta z)^2}\leqslant\frac{1}{2}$, which is better than that of a all the other explicit difference schemes. And when the order of truncation error is $O((\Delta t)^2+(\Delta x)^4)$, the stability condition is $r\leqslant 1/6$, which contains the known results.

Key words: three_dimensional heat conduction equation; explicit difference scheme; truncation error; stability condition