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均布载荷作用下的两端简支压电梁的解析解*

林启荣, 刘正兴, 金占礼

(上海交海大学 工程力学系, 上海 200030)

(沈惠申推荐)

摘要: 按平面应力问题推导出两端简支压电梁在均布载荷作用下的位移、电势、应力分布的解析表达式, 并与压电有限元的计算结果进行了比较, 为探索压电层的分布感测机理以及验证有限元等数值方法提供了参考依据•

关 键 词: 压电梁; 分布感测; 平面应力
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引 言

由于压电材料在智能结构中的广泛应用前景, 所以对其感测与作用机理的研究显得格外重要[1]• 由于力_电耦合效应导致控制方程的解析十分困难, 因此有限元、边界元等的数值计算方法得到了极大的重视[2~6]• 显然这些方法及其计算结果的正确性和精度的验证成了关键• 诚然, 实验是较有效的, 但由于其代价昂贵和条件所限, 因此解析解仍是最常用的依据• 解析解的主要作用并不在于实际应用, 而主要作为数值解的验证标准以及揭示物理本质, 因此大可不必对很复杂的结构进行研究• 但对于几种基本的受力状态很有研究的必要, 单轴拉伸和纯弯曲状态下剪应力为零, 解析相对比较简单• 梁的平面弯曲状态下由于剪应力的存在解析困难, 特别是考虑层合梁时由于层间应力的存在将使解析解变得更加复杂, 很难得到满足各种边界条件特别是层间连续性条件的解析解•

我们从压电弹性介质三维本构方程出发, 导出了平面应力、平面应变问题的物理方程, 在此基础上本文将讨论压电梁在均布载荷作用下的位移、电势、应力分布状态, 给出了简洁明了的解析表达式, 并且与有限元的计算结果进行了比较•

1 基 本 方 程

图 1 为沿 z 轴极化的压电梁, 在 x_y 平面内呈各向同性, 在 x_z 平面内是正交异性的• 假定厚度 t 远小于长度 l 及高度 H, 那么可以作为 x_z 平面内的平面应力问题进行讨论•

1) 平衡方程(无体力作用) [7]

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{zx}}{\partial z}=0,\quad \frac{\partial \sigma_z}{\partial z}+\frac{\partial \tau_{zx}}{\partial x}=0; \tag{1}$$

2) 协调方程[7]

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作者简介: 林启荣(1973~) , 男, 福建人, 博士.

$$\frac{\partial^2 \varepsilon_x}{\partial z^2}+\frac{\partial^2 \varepsilon_z}{\partial x^2}-\frac{\partial^2 \gamma_{zx}}{\partial x \partial z}=0; \tag{2}$$

3) 物理方程[8]

$$\begin{Bmatrix} \varepsilon_x^p \\ \varepsilon_z^p \\ \gamma_{zx}^p \end{Bmatrix}=\begin{bmatrix} s_{11}^p & s_{13}^p & 0 \\ s_{13}^p & s_{33}^p & 0 \\ 0 & 0 & s_{44}^p \end{bmatrix}\begin{Bmatrix} \sigma_x^p \\ \sigma_z^p \\ \tau_{zx}^p \end{Bmatrix}-\begin{bmatrix} 0 & d_{31} \\ 0 & d_{33} \\ d_{15} & 0 \end{bmatrix}\begin{Bmatrix} E_x \\ E_z \end{Bmatrix}, \tag{3}$$

$$\begin{Bmatrix} D_x \\ D_z \end{Bmatrix}=\begin{bmatrix} 0 & 0 & d_{15} \\ d_{31} & d_{33} & 0 \end{bmatrix}\begin{Bmatrix} \sigma_x^p \\ \sigma_z^p \\ \tau_{zx}^p \end{Bmatrix}+\begin{bmatrix} g_{11} & 0 \\ 0 & g_{33} \end{bmatrix}\begin{Bmatrix} E_x \\ E_z \end{Bmatrix}, \tag{4}$$

$$\begin{Bmatrix} E_x \\ E_z \end{Bmatrix}=\begin{Bmatrix} -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial z} \end{Bmatrix}\varphi; \tag{5}$$

4) 几何方程[7]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{zx} \end{Bmatrix}=\begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & \dfrac{\partial}{\partial z} \\ \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial x} \end{bmatrix}\begin{Bmatrix} u \\ w \end{Bmatrix}; \tag{6}$$

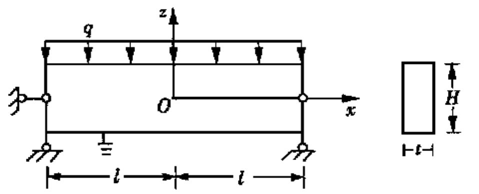

图 1 均布载荷作用下的两端简支压电梁

5) 无表面自由电荷的电学方程[9]

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_z}{\partial z}=0; \tag{7}$$

其中

$$s_{11}^p=\frac{c_{33}c_{11}-c_{13}^2}{c_{33}c_{11}^2-c_{33}c_{12}^2-2c_{13}^2c_{11}+2c_{13}^2c_{12}},\quad s_{13}^p=\frac{-c_{13}}{c_{33}c_{12}-2c_{13}^2+c_{33}c_{11}},$$

$$s_{33}^p=\frac{c_{11}+c_{12}}{c_{33}c_{12}-2c_{13}^2+c_{33}c_{11}},\quad s_{44}^p=\frac{1}{c_{44}},\quad d_{31}=\frac{-e_{33}c_{13}+e_{31}c_{33}}{c_{33}c_{12}-2c_{13}^2+c_{33}c_{11}},$$

$$d_{33}=\frac{e_{33}c_{12}-2e_{31}c_{13}+e_{33}c_{11}}{c_{33}c_{12}-2c_{13}^2+c_{33}c_{11}},\quad d_{15}=\frac{e_{15}}{c_{44}},\quad g_{11}=\frac{e_{15}^2}{c_{44}}+g_{11}^p,$$

$$g_{33}=\frac{c_{12}g_{33}^pc_{33}+c_{12}e_{33}^2-2c_{13}^2g_{33}^p+2c_{33}e_{31}^2-4c_{13}e_{33}e_{31}+c_{11}g_{33}^pc_{33}+c_{11}e_{33}^2}{c_{33}c_{12}-2c_{13}^2+c_{33}c_{11}},$$

上面式中$\{\sigma\}$、$\{\varepsilon\}$、$\{\boldsymbol{D}\}$、$\{\boldsymbol{E}\}$分别表示应力、应变、电位移、电场强度, u、w、φ表示位移和电势, s_{ij}^p、d_{ij}、g_{ij} 分别表示压电材料的短路柔度系数、压电应变常数与自由介电常数矩阵· c_{ij}、e_{ij}、g_{ij}^p 分别为压电体三维状态下的短路刚度系数、压电应力常数与夹持介电常数·

2 均布载荷作用下的两端简支压电梁解析解

如图 1, 两端简支的压电梁受均布载荷 q 作用, 下表面接地·

由平衡方程可知, 平面应力问题存在应力函数 ϕ, 使

$$\sigma_x=\frac{\partial^2\phi}{\partial z^2},\quad \sigma_z=\frac{\partial^2\phi}{\partial x^2},\quad \tau_{zx}=-\frac{\partial^2\phi}{\partial z\partial x}. \tag{8}$$

把(8)、(5)代入(4)再代入(7)得

$$(d_{33}-d_{15})\frac{\partial^3\phi}{\partial z\partial x^2}-g_{11}\frac{\partial^2\varphi}{\partial x^2}+d_{31}\frac{\partial^3\phi}{\partial z^3}-g_{33}\frac{\partial^2\varphi}{\partial z^2}=0. \tag{9}$$

把式(7)、(5)代入(3)再代入(2), 可得

$$s_{11}\frac{\partial^4\phi}{\partial z^4}+(2s_{13}+s_{44})\frac{\partial^4\phi}{\partial z^2\partial x^2}+s_{33}\frac{\partial^4\phi}{\partial x^4}-d_{31}\frac{\partial^3\varphi}{\partial z^3}-(d_{33}-d_{15})\frac{\partial^3\varphi}{\partial z\partial x^2}=0. \tag{10}$$

对于均布载荷作用下的两端简支压电梁, 可假设

$$\sigma_z=\frac{\mathrm{d}}{\mathrm{d}z}f2(z). \tag{11}$$

把(11)代入(8), 并积分得应力函数为

$$\phi=\frac{1}{2}\left(\frac{\mathrm{d}}{\mathrm{d}z}f2(z)\right)x^2+f0(z)+f1(z)x. \tag{12}$$

把上式代入(8), 得

$$\left.\begin{aligned}\sigma_x&=\frac{1}{2}\left(\frac{\mathrm{d}^3}{\mathrm{d}z^3}f2(z)\right)x^2+\frac{\mathrm{d}^2}{\mathrm{d}z^2}f0(z)+\frac{\mathrm{d}^2}{\mathrm{d}z^2}f1(z)x,\\ \tau_{zx}&=-\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)x-\frac{\mathrm{d}}{\mathrm{d}z}f1(z).\end{aligned}\right\} \tag{13}$$

由对称性 $\sigma_x(x)=\sigma_x(-x)$, $\tau_{zx}(x)=-\tau_{zx}(-x)$ 可令

$$f1(z)=0. \tag{14}$$

把(12)代入(9)、(10)可得

$$-d_{15}\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)-g_{11}\frac{\partial^2}{\partial x^2}\varphi+d_{31}\left(\frac{1}{2}\frac{\mathrm{d}^4}{\mathrm{d}z^4}f2(z)x^2+\frac{\mathrm{d}^3}{\mathrm{d}z^3}f0(z)\right)+d_{33}\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)-g_{33}\frac{\partial^2}{\partial z^2}\varphi=0, \tag{15}$$

$$s^p_{11}\left(\frac{1}{2}\frac{\mathrm{d}^5}{\mathrm{d}z^5}f2(z)x^2+\frac{\mathrm{d}^4}{\mathrm{d}z^4}f0(z)\right)+2s^p_{13}\frac{\mathrm{d}^3}{\mathrm{d}z^3}f2(z)-d_{31}\frac{\partial^3}{\partial z^3}\varphi-d_{33}\frac{\partial^3}{\partial z\partial x^2}\varphi+s^p_{44}\frac{\mathrm{d}^3}{\mathrm{d}z^3}f2(z)+d_{15}\frac{\partial^3}{\partial z\partial x^2}\varphi=0. \tag{16}$$

由(15)式解得 $\partial^2\varphi/\partial x^2$, 再代入(16)式并积分可得

$$\varphi=\frac{1}{2}\left[(s^p_{11}g_{11}-d_{33}d_{31}+d_{31}d_{15})\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)x^2+2\frac{\mathrm{d}}{\mathrm{d}z}f0(z)\right)+(4s^p_{13}g_{11}+4d_{33}d_{15}-2d^2_{33}+2s^p_{44}g_{11}-2d^2_{15})f2(z)\right]\Big/(d_{31}g_{11}-d_{33}g_{33}+d_{15}g_{33})+\frac{1}{2}f3(x)z^2+f4(x)z+f5(x). \tag{17}$$

上面式子中, $f2(z)$、$f0(z)$、$f3(x)$、$f4(x)$、$f5(x)$ 均为待定函数.

把上式代入(15)式, 可得

$$-d_{15}\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)-g_{11}\left[\frac{1}{2}\frac{s^p_{11}g_{33}-d^2_{31}}{d_{31}g_{11}-d_{33}g_{33}+d_{15}g_{33}}\frac{\mathrm{d}^4}{\mathrm{d}z^4}f2(z)x^2+\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2}f3(x)z^2+\frac{\mathrm{d}^2}{\mathrm{d}x^2}f4(x)z+\frac{\mathrm{d}^2}{\mathrm{d}x^2}f5(x)+\frac{s^p_{11}g_{11}-d_{33}d_{31}+d_{15}d_{31}}{d_{31}g_{11}-d_{33}g_{33}+d_{15}g_{33}}\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)\right]+d_{31}\frac{\mathrm{d}^3}{\mathrm{d}z^3}f0(z)+d_{33}\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)-\frac{g_{33}}{d_{31}g_{11}-d_{33}g_{33}+d_{15}g_{33}}\left[(s^p_{11}g_{11}-d_{33}d_{11}+d_{15}d_{31})\frac{\mathrm{d}^3}{\mathrm{d}z^3}f0(z)+(2s^p_{13}g_{11}+2d_{33}d_{15}-d^2_{33}+s^p_{44}g_{11}-d^2_{15})\frac{\mathrm{d}^2}{\mathrm{d}z^2}f2(z)\right]-g_{33}f3(x)=0. \tag{18}$$

上式对于任意的 x、z 都成立, 由此可得到 $f2(z)$、$f0(z)$、$f3(x)$、$f4(x)$、$f5(x)$ 函数的表达式.

至此已得到压电体的应力函数 ϕ、电势 φ 的一般表达式, 把它们分别代入(5)、(8) 得到应力、电场强度的分布, 再代入(6) 并积分可得压电体的位移表达式• 其中待定常数可由下列边界条件确定:

主要应力边界条件:

$$\sigma_z \mid_{z=\pm H/2} = 0, \quad \tau_{zx} \mid_{z=\pm H/2} = 0; \tag{19}$$

放松应力边界条件:

$$x = \pm l: \int_{-H/2}^{H/2} \sigma_x \mathrm{d}z = 0, \tag{20}$$

$$\int_{-H/2}^{H/2} \sigma_x z \mathrm{d}z = 0, \tag{21}$$

$$\int_{-H/2}^{H/2} \tau_{zx} \mathrm{d}z = ql; \tag{22}$$

位移边界条件:

$$x = -l, \; z = 0: \; u = 0, \; w = 0, \tag{23}$$

$$x = l, \; z = 0: \; w = 0; \tag{24}$$

电学边界条件:

$$D_z \mid_{z=H/2} = 0, \quad \varphi \mid_{z=-H/2} = 0 \tag{25}$$

把应力、位移、电位移及电势的表达式代入上述边界条件, 最终可得到位移和电势:

$$\begin{aligned} u = \; & \frac{q}{10H^3 g_{33}^2}(5d_{31}^2 g_{11} x H^3 + 5d_{31}^2 g_{11} l H^3 - 12d_{31}^2 g_{11} z x H^2 + 60d_{31}^2 z x g_{33} l^2 - 20d_{31}^2 z x^3 g_{33} + \\ & 5d_{31} d_{15} x g_{33} H^3 + 20d_{31} z^3 d_{33} x g_{33} - 9d_{31} z d_{15} x g_{33} H^2 + 5d_{31} l H^3 g_{33} d_{15} - \\ & 20d_{31} z^3 d_{15} x g_{33} + 9d_{31} z d_{33} x g_{33} H^2 + 5d_{31} x H^3 d_{33} g_{33} + 5d_{31} l H^3 d_{33} g_{33} + \\ & 20zx^3 g_{33}^2 s_{11}^p - 5xH^3 g_{33}^2 s_{13}^p - 20z^3 s_{13}^p x g_{33}^2 - 5lH^3 s_{13}^p g_{33}^2 - 20z^3 s_{44}^p x g_{33}^2 - \\ & 9zx g_{33}^2 H^2 s_{13}^p + 3z s_{44}^p x g_{33}^2 H^2 - 60 s_{11}^p z x g_{33}^2 l^2), \end{aligned} \tag{26}$$

$$\begin{aligned} w = \; & \frac{q}{20(-g_{33} s_{11}^p + d_{31}^2) g_{33}^2 H^3}(12d_{31}^2 g_{11} s_{11}^p g_{33} l^2 H^2 + 50 (s_{11}^{p})^2 g_{33}^3 l^4 + 10 s_{11}^p z^4 d_{33}^2 g_{33}^2 + \\ & 15 s_{11}^p g_{33}^3 s_{33}^p z^2 H^2 - 10 s_{11}^p g_{33}^3 s_{33}^p z^4 - 60 s_{13}^p x^2 z^2 g_{33}^3 s_{11}^p - 3 s_{13}^p z^2 g_{33}^3 H^2 s_{44}^p + 10 (s_{11}^{p})^2 g_{33}^3 x^4 + \\ & 15 s_{11}^p d_{33} z^2 g_{33}^2 H^2 d_{15} - 10 s_{11}^p d_{33}^2 z g_{33}^2 H^3 - 10 s_{11}^p d_{33} z g_{33}^2 H^3 d_{15} + 10 g_{33}^3 s_{13}^p z^4 s_{44}^p - \\ & 15 s_{11}^p d_{33}^2 z^2 g_{33}^2 H^2 + 60 s_{11}^p s_{13}^p z^2 g_{33}^3 l^2 - 6 (s_{13}^{p})^2 z^2 g_{33}^3 H^2 - 10 d_{33} z^4 d_{15} g_{33}^2 s_{11}^p - \\ & 9 s_{11}^p g_{33}^3 s_{13}^p x^2 H^2 + 9 s_{11}^p g_{33}^3 l^2 H^2 s_{13}^p + 10 s_{11}^p g_{33}^3 s_{33}^p z H^3 - 12 s_{11}^p g_{33}^3 x^2 H^2 s_{44}^p - 60 (s_{11}^{p})^2 g_{33}^3 x^2 l^2 + \\ & 12 s_{11}^p g_{33}^3 l^2 H^2 s_{44}^p + 20 g_{33}^3 z^4 (s_{13}^{p})^2 - 10 d_{31}^2 s_{33}^p z g_{33}^2 H^3 - 12 d_{31}^2 l^2 g_{33}^2 H^2 s_{44}^p + 60 d_{31}^2 s_{13}^p x^2 z^2 g_{33}^2 + \\ & 12 d_{31}^2 x^2 g_{33}^2 H^2 s_{44}^p - 60 d_{31}^2 s_{13}^p z^2 g_{33}^2 l^2 - 20 d_{31}^2 x^4 g_{33}^2 s_{11}^p - 9 d_{31}^2 d_{33} z^2 H^2 d_{15} g_{33} - \\ & 9 d_{31}^2 l^2 g_{33}^2 H^2 s_{13}^p + 9 d_{31}^2 s_{13}^p x^2 g_{33}^2 H^2 - 100 d_{31}^2 l^4 g_{33}^2 s_{11}^p + 10 d_{31}^2 s_{33}^p z^4 g_{33}^2 + 10 d_{31}^2 d_{33}^2 z H^3 g_{33} - \\ & 15 d_{31}^2 s_{33}^p z^2 g_{33}^2 H^2 + 15 d_{31} g_{11} s_{11}^p d_{33} z^2 g_{33} H^2 - 10 d_{31} g_{11} s_{11}^p d_{33} z g_{33} H^3 - \\ & 10 d_{31} g_{11} s_{11}^p d_{33} z^4 g_{33} + 9 d_{31} s_{11}^p g_{33}^2 d_{33} x^2 H^2 - 40 d_{31} d_{33} z^4 s_{13}^p g_{33}^2 + 3 d_{31} d_{33} z^2 g_{33}^2 H^2 s_{44}^p + \\ & 12 d_{31} d_{33} z^2 g_{33}^2 H^2 s_{13}^p - 24 d_{31} s_{11}^p g_{33}^2 d_{15} x^2 H^2 - 10 d_{31} d_{33} z^4 s_{44}^p g_{33}^2 + 24 d_{31} s_{11}^p g_{33}^2 d_{15} l^2 H^2 - \\ & 60 d_{31} d_{33} z^2 g_{33}^2 l^2 s_{11}^p - 9 d_{31} s_{11}^p g_{33}^2 d_{33} l^2 H^2 + 60 d_{31} s_{11}^p d_{33} z^2 x^2 g_{33}^2 - 6 d_{31} s_{13}^p z^2 d_{15} g_{33}^2 H^2 + \\ & 20 d_{31} s_{13}^p z^4 d_{15} g_{33}^2 - 12 d_{31}^4 g_{11} l^2 H^2 + 12 d_{31}^4 g_{11} x^2 H^2 - 60 d_{31}^4 x^2 g_{33} l^2 + 10 d_{31}^3 g_{11} d_{33} z H^3 - \\ & 12 d_{31}^3 g_{11} d_{33} z^2 H^2 - 9 d_{31}^3 g_{33} d_{33} x^2 H^2 - 24 d_{31}^3 l^2 H^2 d_{15} g_{33} + 9 d_{31}^3 g_{33} d_{33} l^2 H^2 - \end{aligned}$$

$$60d_{31}^3d_{33}z^2x^2g_{33}+24d_{31}^3x^2d_{15}g_{33}H^2+60d_{31}^3d_{33}z^2g_{33}l^2-3d_{31}^2g_{11}s_{13}^pz^2g_{33}H^2-$$
$$12d_{31}^2g_{11}s_{11}^pg_{33}x^2H^2+10d_{31}^2g_{11}s_{13}^pz^4g_{33}+10d_{31}^2z^4d_{33}^2g_{33}+120d_{31}^2x^2g_{33}^2l^2s_{11}^p-$$
$$10d_{31}^2d_{33}z^4d_{15}g_{33}+9d_{31}^2d_{33}^2z^2H^2g_{33}+10d_{31}^4x^4g_{33}+50d_{31}^4l^4g_{33}+10d_{31}^2d_{33}d_{15}zg_{33}H^3); \tag{27}$$

$$\varphi=\frac{q}{160(g_{33}s_{11}^p+d_{31}^2)g_{33}^2H^3}(-80d_{31}^3g_{11}H^3z+65g_{33}^2d_{15}s_{11}^pH^4-120g_{33}^2d_{15}s_{11}^pH^2z^2+$$
$$15g_{33}^2d_{33}s_{11}^pH^4+80g_{33}^2d_{15}s_{11}^pH^3z+80d_{33}^2d_{15}s_{11}^pz^4-80g_{33}^2d_{33}s_{11}^pz^4+80g_{33}^2d_{33}s_{11}^pH^3z+$$
$$120g_{33}^2d_{33}s_{11}^pH^2z^2-80d_{31}^2d_{33}g_{33}z^4-80d_{31}^2d_{15}g_{33}H^3z-72d_{31}^2d_{33}g_{33}H^2z^2-$$
$$17d_{31}^2d_{33}g_{33}H^4-63d_{31}^2d_{15}g_{33}H^4+96d_{31}^3g_{15}H^2z^2-480d_{31}^3g_{33}l^2z^2-64d_{31}^3g_{11}H^4-$$
$$24g_{33}^2d_{31}s_{44}^pH^2z^2-480d_{31}g_{33}^2s_{11}^px^2z^2+120d_{31}g_{33}^2s_{11}^pH^2x^2+g_{33}^2d_{31}s_{44}^pH^4-$$
$$48g_{33}^2d_{31}s_{11}^pH^2z^2+80d_{31}g_{33}^2s_{44}^pz^4-120d_{31}g_{33}^2s_{11}^pH^2l^2+480g_{33}^2d_{31}s_{11}^pl^2z^2+$$
$$80g_{11}g_{33}s_{11}^pH^3z+80d_{31}g_{11}g_{33}s_{11}^pz^4+65d_{31}g_{11}g_{33}s_{11}^pH^4-120d_{31}g_{11}g_{33}s_{11}^pH^2z^2+$$
$$160g_{33}^2d_{31}s_{13}^pz^4+2d_{31}g_{33}^2s_{13}^pH^4+80d_{31}^2d_{15}g_{33}z^4-80d_{31}^2d_{33}g_{33}H^3z+$$
$$72d_{31}^2d_{15}g_{33}H^2z^2+120d_{31}^3g_{33}l^2H^2-120d_{31}^3g_{33}H^2x^2+480d_{31}^3g_{33}x^2z^2)\cdot \tag{28}$$

3 算　　例

作为应用，图 2~ 12，给出位移、应力和电势的计算结果，并与文[5] 有限元结果进行比较• 压电梁的材料为 PZT_4 压电陶瓷，其几何参数为 $l=0.3$ m，$H=0.02$ m，$t=0.002$ m• 作用在其上表面的均布载荷 $q=10\ \mathrm{N/m^2}$，其力学与电学性能参数分别列于表 1•

表 1　　PZT_4 的物理参数[6]

物理参数	c_{11}	c_{12}	c_{13}	c_{33}	c_{44}	e_{31}	e_{33}	e_{15}	g_{11}	g_{33}
量纲	$\times 10^{10}$Pa					C/m^2			$\times 10^{-9}$C/V•m	
数值	13.9	7.78	7.43	11.3	2.56	−6.98	13.84	13.44	6.00	5.47

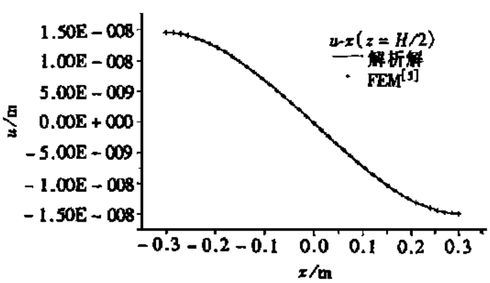

图 2　位移 u 沿 x 轴的变化 ($z=H/2$)

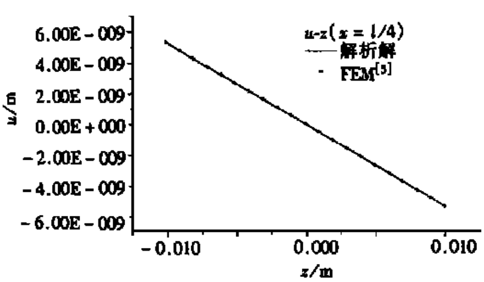

图 3　位移 u 沿 z 轴的变化 ($x=l/4$)

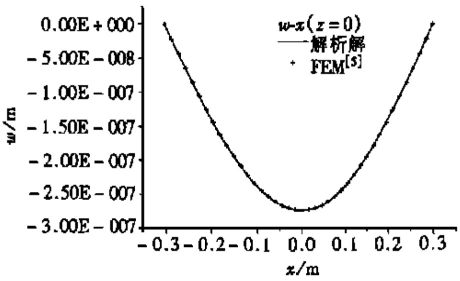

图 4　位移 w 沿 x 轴的变化 $(z=0)$

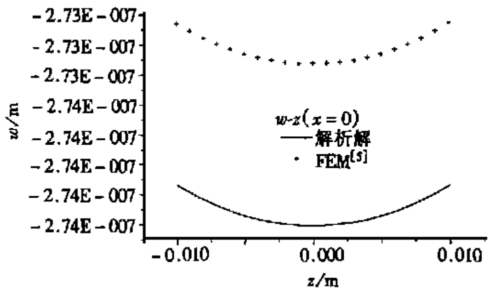

图 5　位移 w 沿 z 轴的变化 $(x=0)$

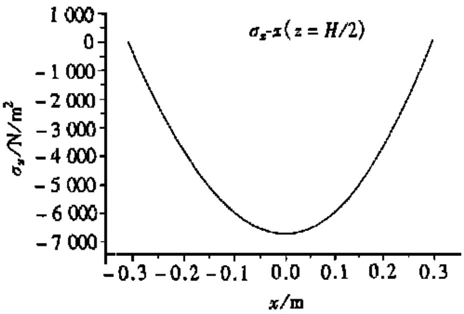

图 6　应力 σ_x 沿 x 轴的变化 $(z=H/2)$

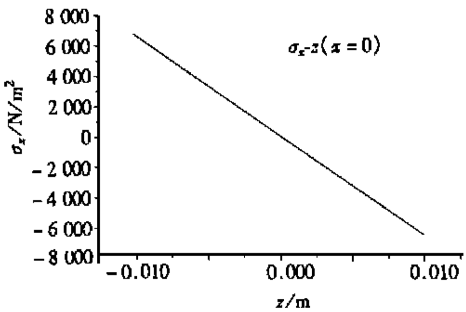

图 7　应力 σ_x 沿 z 轴的变化 $(x=0)$

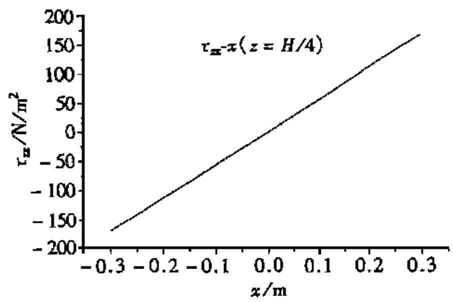

图 8　应力 τ_{zx} 沿 x 轴的变化 $(z=H/4)$

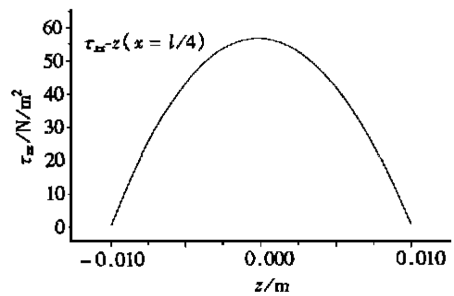

图 9　应力 τ_{zx} 沿 z 轴的变化 $(x=l/4)$

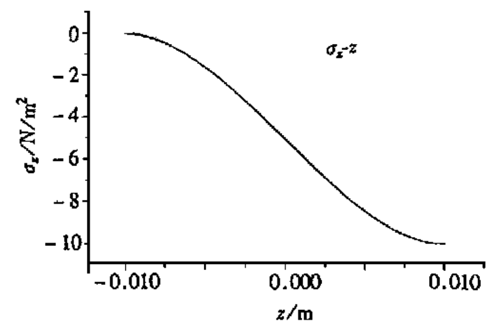

图 10　应力 σ_z 沿 z 轴的变化

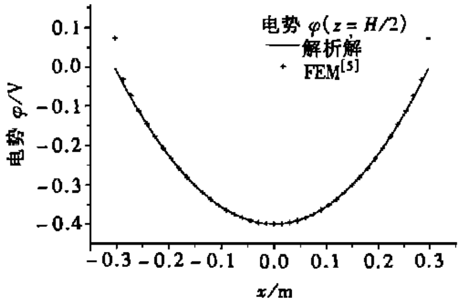

图 11 电势沿 x 轴的变化 $(z = H/2)$

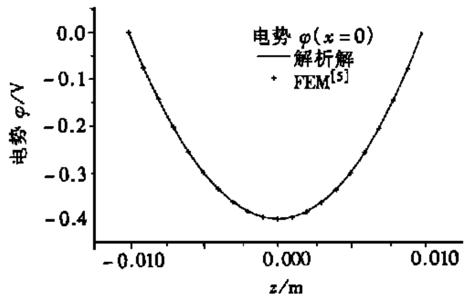

图 12 电势沿 z 轴的变化 $(x = 0)$

4 结束语

本文利用压电介质的二维本构关系，推导出压电梁在均布载荷作用下的位移、电势分布的解析表达式，可为有限元等数值解提供检验依据，例如文献[5]的有限元方法采用八节点非协调块元，其计算结果与本文的解析解吻合很相当好• 当考虑层合梁时，由于层间剪切的作用，解析解变得十分困难• 作为基础，本文研究了纯压电梁平面弯曲的情况，其结论对深入了解压电层的力_电耦合特性是有价值的• 对于层合梁平面弯曲的解析解的研究，尚待进一步探讨•

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A Close_Form Solution to Simply Supported Piezoelectric Beams Under Uniform Exterior Pressure

Lin Qirong, Liu Zhengxing, Jin Zhanli
(Department of Engineering Mechanics, Shanghai Jiaotong University, Shanghai 200030, P R China)

Abstract: On the basis of two_dimensional constitutive relationships of piezoelectric materials, the analytic solution for simply supported piezoelectric beams under uniform exterior pressure was derived. Furthermore the results were also compared with the ones of FEM for piezoelectric materials. Thus the foundation for further research of piezoelectric materials' distribution sensing mechanism and the validation of numerical methods such as FEM is provided.

Key words: piezoelectric beams; distribution sensing; plane state of stress