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复合材料旋转壳非线性稳定性分析计算*

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(周承倜推荐)

摘要: 利用前屈曲一致理论和能量变分法分析计算了复合材料旋转壳非线性稳定性· 前屈曲应变_位移关系采用非线性的卡门方程, 能量积分采用数值积分, 用势能最小原理求解前屈曲位移和内力, 提出了求解临界载荷的实用计算方法, 用 FORTRAN 语言编制了相应的计算机程序, 并给出了算例·

关 键 词: 复合材料; 旋转壳; 非线性; 稳定性
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前 言

对于本征值 ——求自振频率也好, 求屈曲载荷也好, 最小势能原理仍不失为一个强有力的工具, 尤其是对一些几何形状复杂、铺层顺序复杂, 或需要考虑湿热效应与横向剪切变形等情形· 本文利用前屈曲一致理论和能量变分法分析复合材料旋转壳非线性稳定性· 前屈曲应变_位移关系采用非线性的卡门方程, 能量积分采用数值积分, 用势能最小原理求解前屈曲位移和内力, 然后求解线性的临界载荷 P^* (外压)、F^* (轴压)、T^* (轴 / 外压), 在 $[0, 2P^*]$、$[0, 2F^*]$、$[0, 2T^*]$ 内给定步长求解前屈曲方程, 得前屈曲内力及变形· 再代入屈曲方程, 求得若干组满足某一给定精度的载荷值及相应的屈曲波数· 然后减小步长, 求得每组屈曲波数中精度最高的载荷值, 在这些载荷值中取最小的便为所求临界载荷值·

1 复合材料层合板本构关系[1]

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ & & A_{66} & B_{12} & B_{16} & B_{66} \\ & & & D_{11} & D_{12} & D_{16} \\ & & & & D_{22} & D_{26} \\ & & & & & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

简记为:

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$$\begin{Bmatrix} \boldsymbol{N} \\ \boldsymbol{M} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{D} \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix},$$

其中, **[A]:** 拉伸刚度矩阵**, [D]:** 弯曲刚度矩阵**, [B] :** 拉伸 _ 弯曲耦合刚度矩阵•

各矩阵系数计算如下:

$$[A_{ij}] = \sum_{k=1}^{n} (Q_{ij})_k [h_k - h_{k-1}],$$

$$[B_{ij}] = \frac{1}{2} \sum_{k=1}^{n} (Q_{ij})_k [h_k^2 - h_{k-1}^2],$$

$$[D_{ij}] = \frac{1}{3} \sum_{k=1}^{n} (Q_{ij})_k [h_k^3 - h_{k-1}^3],$$

其中, k 为层板序号: h_k、h_{k-1} 为第 k 层板上下板面垂向坐标; $(Q_{ij})_k$ 为第 k 层板在参考坐标系下的弹性矩阵系数•

2　壳体非线性应变位移关系

前屈曲应变_位移关系采用非线性的卡门方程[2]:

$$\left.\begin{aligned}
\varepsilon_{10} &= \frac{1}{A_1}\frac{\partial u}{\partial \alpha_1} + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}v - \frac{w}{R_1} + \frac{1}{2}\left(\frac{1}{A_1}\frac{\partial w}{\partial \alpha_1}\right)^2, \\
\varepsilon_{20} &= \frac{1}{A_2}\frac{\partial v}{\partial \alpha_2} + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}u - \frac{w}{R_2} + \frac{1}{2}\left(\frac{1}{A_2}\frac{\partial w}{\partial \alpha_2}\right)^2, \\
\gamma_{120} &= \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{u}{A_1}\right) + \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{v}{A_2}\right) + \frac{1}{A_1A_2}\frac{\partial w}{\partial \alpha_1}\frac{\partial w}{\partial \alpha_2}, \\
\chi_1 &= \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}\left(\frac{1}{A_1}\frac{\partial w}{\partial \alpha_1}\right) + \frac{1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2}\left(\frac{1}{A_2}\frac{\partial w}{\partial \alpha_2}\right), \\
\chi_2 &= \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}\left(\frac{1}{A_2}\frac{\partial w}{\partial \alpha_2}\right) + \frac{1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1}\left(\frac{1}{A_1}\frac{\partial w}{\partial \alpha_1}\right), \\
\chi_{12} &= \frac{2}{A_1A_2}\left(\frac{\partial^2 w}{\partial \alpha_1 \partial \alpha_2} - \frac{1}{A_1}\frac{\partial A_1}{\partial \alpha_2}\frac{\partial w}{\partial \alpha_1} - \frac{1}{A_2}\frac{\partial A_2}{\partial \alpha_1}\frac{\partial w}{\partial \alpha_2}\right) \bullet
\end{aligned}\right\} \tag{2}$$

3　能 量 变 分 法

系统总势能为[3]

$$\begin{aligned}
\Pi = {} & \frac{1}{2}\iint (N_1\varepsilon_{10} + N_2\varepsilon_{20} + N_{12}\gamma_{120} + M_1\chi_1 + M_2\chi_2 + M_{12}\chi_{12}) A_1A_2 \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 - \\
& \Big\{ \iint qw A_1A_2 \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 + \int \Big[N_1^0 u + N_{12}^0 v - M_1^0 \frac{\partial w}{\partial x} + V_1^0 w \Big]_0^{2\pi} A_2 \mathrm{d}\alpha_2 + \\
& \int \Big[N_2^0 v + N_{12}^0 u - M_2^0 \frac{\partial w}{\partial y} + V_2^0 w \Big]_{\varphi_1}^{\varphi_2} A_1 \mathrm{d}\alpha_1 + S^0 w \,|_s \Big\} \bullet
\end{aligned} \tag{3}$$

又 M_1^0, M_2^0, V_1^0, V_2^0, S^0, $w\,|_s$ 依次代表边界上已给的弯矩, 等值剪力, 角点力和角点挠度•

各应变分量的一次变分为:

$$\left.\begin{aligned}
\delta\varepsilon_{10} &= \frac{1}{A_1}\frac{\partial(\delta u)}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\delta v-\frac{\delta w}{R_1}+\frac{1}{A_1^2}\frac{\partial w}{\partial\alpha_1}\cdot\frac{\partial(\delta w)}{\partial\alpha_1},\\
\delta\varepsilon_{20} &= \frac{1}{A_2}\frac{\partial(\delta v)}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\delta u-\frac{\delta w}{R_2}+\frac{1}{A_2^2}\frac{\partial w}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_2},\\
\delta\gamma_{120} &= \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{\delta u}{A_1}\right)+\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{\delta v}{A_2}\right)+\frac{1}{A_1A_2}\frac{\partial w}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial w}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_1},\\
\delta\chi_1 &= \frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(\delta w)}{\partial\alpha_1}\right)+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(\delta w)}{\partial\alpha_2}\right),\\
\delta\chi_2 &= \frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(\delta w)}{\partial\alpha_2}\right)+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(\delta w)}{\partial\alpha_1}\right),\\
\delta\chi_{12} &= \frac{2}{A_1A_2}\left(\frac{\partial^2(\delta w)}{\partial\alpha_1\partial\alpha_2}-\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_1}-\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}\right)\cdot
\end{aligned}\right\}\tag{4}$$

各应变分量的二次变分为

$$\left.\begin{aligned}
\delta^2\varepsilon_{10} &= \frac{1}{A_1^2}\left(\frac{\partial(\delta w)}{\partial\alpha_1}\right)^2,\\
\delta^2\varepsilon_{20} &= \frac{1}{A_2^2}\left(\frac{\partial(\delta w)}{\partial\alpha_2}\right)^2,\\
\delta^2\varepsilon_{120} &= \frac{2}{A_1A_2}\frac{\partial(\delta w)}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}\cdot
\end{aligned}\right\}\tag{5}$$

对于线弹性应力_应变关系有:

$$\left.\begin{aligned}
\delta N_1\varepsilon_{10}+\delta N_2\varepsilon_{20}+\delta N_{12}\gamma_{120} &= N_1\delta\varepsilon_{10}+N_2\delta\varepsilon_{20}+N_{12}\delta\gamma_{120},\\
\delta M_1\chi_1+\delta M_2\chi_2+\delta M_{12}\chi_{12} &= M_1\delta\chi_1+M_2\delta\chi_2+M_{12}\delta\chi_{12}\cdot
\end{aligned}\right\}\tag{6}$$

将式(6)代入总势能的一次变分式(3),可得:

$$\begin{aligned}
\delta\Pi = &\iint(N_1\delta\varepsilon_{10}+N_2\delta\varepsilon_{20}+N_{12}\delta\gamma_{120}+M_1\delta\chi_1+M_2\delta\chi_2+M_{12}\delta\chi_{12})A_1A_2\mathrm{d}\alpha_1\mathrm{d}\alpha_2-\\
&\left\{\iint q\delta wA_1A_2\mathrm{d}\alpha_1\mathrm{d}\alpha_2+\int\left[N_1^0\delta u+N_{12}^0\delta v-M_1^0\frac{\partial(\delta w)}{\partial x}+V_1^0\delta w\right]_0^{2\pi}A_2\mathrm{d}\alpha_2+\right.\\
&\left.\int\left[N_2^0\delta v+N_{12}^0\delta u-M_2^0\frac{\partial(\delta w)}{\partial y}+V_2^0\delta w\right]_{\varphi_1}^{\varphi_2}A_1\mathrm{d}\alpha_1+S^0\delta w\mid_s\right\}\cdot
\end{aligned}\tag{7}$$

将应变分量的一次变分式(4)代入式(7),并使 $\delta\Pi=0$,即可求得屈曲前的平衡方程、边界条件等· 这就是熟知的由最小势能原理所应求得的结果·

由变分原理可知,要判断所得到的平衡的稳定性,则必须研究二次变分 $\delta^2\Pi$ 的性质· 在任何一可能的变形体系下,当 $\delta^2\Pi>0$ 时(正定)势能为最小,平衡是稳定的;当 $\delta^2\Pi<0$ 时(负定),则转为不稳定的,不能再保持在原有的平衡位置上· 为此,在临界点上 $\delta^2\Pi=0$,平衡体系开始转入一个新的变形状态,称为屈曲·

从式(6)可以得到二次变分

$$\begin{aligned}
\delta^2\Pi = \iint\Big\{&[N_1\delta^2\varepsilon_{10}+N_2\delta^2\varepsilon_{20}+N_{12}\delta^2\gamma_{120}]+[\delta N_1\delta\varepsilon_{10}+\delta N_2\delta\varepsilon_{20}+\delta N_{12}\delta\gamma_{120}+\\
&\delta M_1\delta\chi_1+\delta M_2\delta\chi_2+\delta M_{12}\delta\chi_{12}]\Big\}A_1A_2\mathrm{d}\alpha_1\mathrm{d}\alpha_2\cdot
\end{aligned}\tag{8}$$

为导出屈曲方程,Trefftz 指出,屈曲前 $\delta^2\Pi$ 总是正定二次型,所以达到临界点 $\delta^2\Pi=0$,$\delta^2\Pi$ 又是最小的· 从这个极值性质出发,对于所有可能变化的新的虚位移体系 $\delta^*(\delta u)$,$\delta^*(\delta v)$,

$\delta^{*}(\delta w)$, 必有

$$\delta^{*}(\delta^{2}\Pi)=0\cdot$$

4 前屈曲分析

前屈曲位移模式

$$\begin{cases} u=A_{0}x, \\ w=B_{0}x^{2}+C_{0}\sin^{2}\pi x\,e^{\beta x}, \end{cases} \tag{9}$$

其中 $x=\dfrac{\varphi-\varphi_{1}}{\varphi_{2}-\varphi_{1}}$, $\beta=-\sqrt[4]{\dfrac{3(1-\mu^{2})R_{2}^{4}}{R_{1}^{2}h^{2}}}$

为考虑曲率半径 R_2 的变化引起的刚度变化而引入的, 若不考虑此项影响, 可令其为零, A_0、B_0、C_0 为待定常数·

边界条件, 当 $\varphi=\varphi_1$(顶部) 时, $x=1$, $u\neq0$, $w\neq0$; 当 $\varphi=\varphi_2$(底部) 时, $x=0$, $u=0$, $w=0$, $\partial w/\partial\varphi=0$·

将式(9) 代入式(4) 有

$$\left.\begin{aligned}
\varepsilon_{10}={}&\frac{1}{A_1}\frac{\partial u}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}v-\frac{w}{R_1}+\frac{1}{2}\left(\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}\right)^2= \\
&-\frac{1}{R_1(\varphi_2-\varphi_1)}\cdot A_0-\frac{x^2}{R_1}\cdot B_0-\frac{1}{R_1}\sin^2\pi x\,e^{\beta x}\cdot C_0+ \\
&\frac{2x^2}{R_1^2(\varphi_2-\varphi_1)^2}\cdot B_0^2+\frac{2x}{R_1^2(\varphi_2-\varphi_1)^2}(2\pi\sin\pi x\cos\pi x\,e^{\beta x}+\beta e^{\beta x}\sin^2\pi x)\cdot B_0C_0+ \\
&\frac{1}{2R_1^2(\varphi_2-\varphi_1)^2}(2\pi\sin\pi x\cos\pi x\,e^{\beta x}+\beta e^{\beta x}\sin^2\pi x)^2\cdot C_0^2, \\
\varepsilon_{20}={}&\frac{1}{A_2}\frac{\partial v}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}u-\frac{w}{R_2}+\frac{1}{2}\left(\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}\right)^2= \\
&\frac{\cot\varphi}{R_1}x\cdot A_0-\frac{x^2}{R_2}\cdot B_0-\frac{\sin^2\pi x\,e^{\beta x}}{R_2}\cdot C_0, \\
\gamma_{120}={}&\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{u}{A_1}\right)+\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{v}{A_2}\right)+\frac{1}{A_1A_2}\frac{\partial w}{\partial\alpha_1}\frac{\partial w}{\partial\alpha_2}=0, \\
\chi_1={}&\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}\right)+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}\right)= \\
&\frac{2}{R_1^2(\varphi_2-\varphi_1)^2}\cdot B_0+\frac{e^{\beta x}}{R_1^2(\varphi_2-\varphi_1)^2}[2\pi^2(\cos^2\pi x-\sin^2\pi x)+ \\
&4\pi\beta\sin\pi x\cos\pi x\,\beta+\beta^2\sin^2\pi x]\cdot C_0, \\
\chi_2={}&\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2}\right)+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1}\right)= \\
&-\frac{2x\cot\varphi}{R_1^2(\varphi_2-\varphi_1)}\cdot B_0-\frac{\cot\varphi}{R_1^2(\varphi_2-\varphi_1)}(2\pi\sin\pi x\cos\pi x\,e^{\beta x}+\beta\sin^2\pi x\,e^{\beta x})\cdot C_0, \\
\chi_{12}={}&\frac{2}{A_1A_2}\left(\frac{\partial^2 w}{\partial\alpha_1\partial\alpha_2}-\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial w}{\partial\alpha_1}-\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial w}{\partial\alpha_2}\right)=0\cdot
\end{aligned}\right\} \tag{10}$$

将式(10) 及复合材料层合板的本构关系式(1) 代入系统总势能的表达式(7), 由最小势能

原理有:

$$\frac{\partial \Pi}{\partial A_0}=0,\ \frac{\partial \Pi}{\partial B_0}=0,\ \frac{\partial \Pi}{\partial C_0}=0, \tag{11}$$

可得关于 A_0、B_0、C_0 的非线性方程组, 求解可得前屈曲内力及位移· 式中能量积分采用数值积分·

5 屈 曲 分 析

屈曲位移模式

$$\begin{cases}\delta u = A\cos m\pi x\sin n\theta,\\ \delta v = B\sin m\pi x\cos n\theta,\\ \delta w = C\sin m\pi x\sin n\theta,\end{cases} \tag{12}$$

其中, A、B、C 为待定常数·

边界条件, 当 $\varphi=\varphi_1$ 时, $x=1$, $v=0$; 当 $\varphi=\varphi_2$ 时, $x=0$, $v=0$·

将式(12) 代入应变的一次变分式(4) 有

$$\begin{aligned}\delta\varepsilon_{10} &= \frac{1}{A_1}\frac{\partial(\delta u)}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\delta v-\frac{\delta w}{R_1}+\frac{1}{A_1^2}\frac{\partial w}{\partial\alpha_1}\cdot\frac{\partial(\delta w)}{\partial\alpha_1}=\\ &\frac{m\pi}{R_1(\varphi_2-\varphi_1)}\sin m\pi x\sin n\theta\cdot A-\frac{1}{R_1}\sin m\pi x\sin n\theta\cdot C+\\ &\frac{2m\pi x}{R_1^2(\varphi_2-\varphi_1)^2}\cos m\pi x\sin n\theta\cdot B_0C+\\ &\frac{m\pi}{R_1^2(\varphi_2-\varphi_1)^2}(2\pi\sin\pi x\cos\pi x\,e^{\beta x}+\beta\sin^2\pi x\,e^{\beta x})\cos m\pi x\sin n\theta\cdot C_0C,\end{aligned} \tag{13a}$$

$$\begin{aligned}\delta\varepsilon_{20} &= \frac{1}{A_2}\frac{\partial(\delta v)}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\delta u-\frac{\delta w}{R_2}+\frac{1}{A_2^2}\frac{\partial w}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_2}=\\ &\frac{\cot\varphi}{R_1}\cos m\pi x\sin n\theta\cdot A-\frac{n}{R_2\sin\varphi}\sin m\pi x\sin n\theta\cdot B-\\ &\frac{1}{R_2}\sin m\pi x\sin n\theta\cdot C,\end{aligned} \tag{13b}$$

$$\begin{aligned}\delta\gamma_{120} &= \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{\delta u}{A_1}\right)+\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{\delta v}{A_2}\right)+\frac{1}{A_1A_2}\frac{\partial w}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial w}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_1}=\\ &\frac{n}{R_2\sin\varphi}\cos m\pi x\cos n\theta\cdot A+\left(\frac{\cot\varphi}{R_1}\sin m\pi x\cos n\theta+\frac{m\pi}{R_1(\varphi_2-\varphi_1)}\cos m\pi x\cos n\theta\right)\cdot B-\\ &\frac{2nx}{R_1R_2(\varphi_2-\varphi_1)\sin\varphi}\sin m\pi x\cos n\theta\cdot B_0C-\\ &\frac{2\pi\sin\pi x\cos\pi x\,e^{\beta x}+\beta\sin^2\pi x\,e^{\beta x}}{R_1R_2(\varphi_2-\varphi_1)\sin\varphi}\sin m\pi x\cos n\theta\cdot C_0C,\end{aligned} \tag{13c}$$

$$\begin{aligned}\delta\chi_1 &= \frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(\delta w)}{\partial\alpha_1}\right)+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(\delta w)}{\partial\alpha_2}\right)=\\ &-\frac{m^2\pi^2}{R_1^2(\varphi_2-\varphi_1)^2}\sin m\pi x\sin n\theta\cdot C,\end{aligned} \tag{13d}$$

$$\delta\chi_2=\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\frac{\partial(\delta w)}{\partial\alpha_2}\right)+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\left(\frac{1}{A_1}\frac{\partial(\delta w)}{\partial\alpha_1}\right)=$$

$$-\left[\frac{m\pi\cot\varphi}{R_1^2(\varphi_2-\varphi_1)}\cos m\pi x\sin n\theta+\frac{n^2}{R_2^2\sin^2\varphi}\sin m\pi x\sin n\theta\right]\cdot C, \tag{13e}$$

$$\delta\chi_{12}=\frac{2}{A_1A_2}\left(\frac{\partial^2(\delta w)}{\partial\alpha_1\partial\alpha_2}-\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_2}\frac{\partial(\delta w)}{\partial\alpha_1}-\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}\right)=$$

$$-\frac{2}{R_1R_2\sin\varphi}\left(\frac{\pi mn}{\varphi_2-\varphi_1}\cos m\pi x\cos n\theta+n\cot\varphi\sin m\pi x\cos n\theta\right)\cdot C\cdot \tag{13f}$$

将式(12)代入应变的二次变分式(5)有

$$\left.\begin{aligned}
\delta^2\varepsilon_{10}&=\frac{1}{A_1^2}\left(\frac{\partial(\delta w)}{\partial\alpha_1}\right)^2=\left(\frac{m\pi}{R_1(\varphi_2-\varphi_1)}\cos m\pi x\sin n\theta\right)^2C^2,\\
\delta^2\varepsilon_{20}&=\frac{1}{A_2^2}\left(\frac{\partial(\delta w)}{\partial\alpha_2}\right)^2=\left(\frac{n\sin m\pi x\cos n\theta}{R_2\sin\varphi}\right)^2\cdot C^2,\\
\delta^2\varepsilon_{120}&=\frac{2}{A_1A_2}\frac{\partial(\delta w)}{\partial\alpha_1}\frac{\partial(\delta w)}{\partial\alpha_2}=\\
&\frac{2mn\pi}{R_1R_2(\varphi_2-\varphi_1)\sin\varphi}\sin m\pi x\cos m\pi x\sin n\theta\cos n\theta\cdot C^2\cdot
\end{aligned}\right\} \tag{14}$$

将式(13)和式(14)代入总势能的二次变分式(8),并作 δ^* 变分,可得屈曲方程•

6 求解方法

首先求解线性的临界载荷 P^*(外压)、F^*(轴压)、T^*(轴/外压),在$[0, 2P^*]$、$[0, 2F^*]$、$[0, 2T^*]$内给定步长求解前屈曲方程,得前屈曲内力及变形;再代入屈曲方程,求得若干组满足某一给定精度的载荷值及相应的屈曲波数;然后减小步长,求得每组屈曲波数中精度最高的载荷值,在这些载荷值中取最小的便为所求临界载荷值•

利用本文所推导的方法,编制了相应的计算机程序,程序采用 FORTRAN 语言,其中数值积分采用蒙特卡罗方法• 采用梯度法和修正的牛顿迭代法相结合的方法求解非线性方程组• 梯度法精度较差,但对任意的初值均可收敛,修正的牛顿迭代法有较高的精度,但不是对任意的初值都收敛,程序中用梯度法给出初值,然后用修正的牛顿迭代法提高解的精度•

为证明本文方法的正确性,程序中加入无矩理论的计算模块,该模块利用无矩理论计算壳体屈曲前的内力,而屈曲位移模式和临界载荷的求解方法与主模块公用相同的计算程序•

本方法适用于正高斯曲率的旋转壳的稳定性计算,可计算轴压、外压(分顶部封闭和不封闭两种情况)、轴/外压、给定轴压下外压和给定外压下轴压六种形式的临界载荷•

7 算例

算例取自文献[1],柱壳长 500 mm,半径 500 mm,单层厚为 0.15 mm,材料参数弹性模量为 117.6 GPa、5.88 GPa,剪切模量为 2.94 GPa,泊松比为 0.3,计算的结构分 5 种铺层情况:1) 正交对称铺层,共 40 层• 2) ±45°角对称铺层,共 40 层• 3) ±45°角对称铺层,共 20 层• 4) ±22.5°角对称铺层,共 40 层• 5) ±22.5°角对称铺层,共 20 层• 计算结果见表 1,由于本文所编制的程序中,前屈曲位移模式不适用于圆柱壳在轴压下的变形形态,结果中该项未列出•

表 1 柱壳轴压和外压下临界载荷计算对比 F^*/kN, P^*/MPa

序号	工况	文[1]剪切理论	文[1]经典理论	无矩理论	本文方法
1	轴压 F^*	2 758.0(2,7)	2 804.8(2,6)	2 577.953(2,6)	
1	外压 P^*	0.590(1,7)	0.605(1,7)	0.586 377(1,7)	0.589 323(1,7)
2	轴压 F^*	2 954.8(4,2)	2 988.9(4,2)	2 551.853(4,1)	
2	外压 P^*	0.627(1,8)	0.645(1,8)	0.573 606(1,7)	0.560 094(1,7)
3	轴压 F^*	711.2(6,2)	717.4(6,2)	638.097(6,1)	
3	外压 P^*	0.097 6(1,9)	0.099 0(1,9)	0.088 278(1,8)	0.086 167(1,8)
4	轴压 F^*	3 155.7(3,2)	3 235.8(3,2)	3 109.077(3,1)	
4	外压 P^*	0.336(1,11)	0.342(1,11)	0.312 382(1,10)	0.322 148(1,10)
5	轴压 F^*	784.8(4,2)	792.2(4,2)	756.287(4,1)	
5	外压 P^*	0.052 7(1,13)	0.053 1(1,13)	0.047 807(1,12)	0.047 492(1,12)

对比算例可以看出本文采用的方法是比较满意的•

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Analysis and Calculation of the Nonlinear Stability of the Rotational Composite Shell

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Abstract: By adopting the energy method, a new method to calculate the stability of the composite shell of revolution is presented. This method takes the influence of nonlinear prebuckling deformations and stresses on the buckling of the shell into account. The relationships between the prebuckling deformations and strains are calculated by nonlinear Kûrmûn equations. The numerical method is used to calculate the energy of the total system. The nonlinear equations are solved by combining gradient method and amendatory Newton iterative method. The computer program is also developed. An example is given to demonstrate the accuracy of the method presented.

Key words: composite material; rotational shell; stability; nonlinear