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各向异性复合材料的平面周期焊接问题*

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(周焕文推荐)

摘要: 利用平面弹性复变方法和解析函数边值问题的基本理论, 讨论不同材料的各向异性弹性半平面和弹性长条的周期焊接问题, 并给出应力分布封闭形式的解•

关 键 词: 各向异性; 平面弹性复变方法; 焊接; 边值问题
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引 言

对于各向同性材料的平面焊接问题, 目前已有了许多的研究成果, 如[1], [2]等, 而对各向异性材料的平面焊接问题, 仅有个别情况进行了讨论[3], [4]• 有许多问题尚有待于进一步地进行探讨•

本文研究不同材料的各向异性弹性半平面与弹性长条的周期焊接问题, 利用 Hilbert 积分公式和分析函数等方法, 得到了应力分布封闭形式的解•

1 应力与位移的复变函数表示

对于各向异性材料的平面问题, 应力与位移可用两个函数 $\Phi(z)$ 与 $\Psi(z)$ 表示为[5]:

$$\left.\begin{aligned} \sigma_x &= 2\mathrm{Re}[\lambda_1^2\Phi'(z_1)+\lambda_2^2\Psi'(z_2)], \\ \sigma_y &= 2\mathrm{Re}[\Phi'(z_1)+\Psi'(z_2)], \\ \tau_{xy} &= -2\mathrm{Re}[\lambda_1\Phi'(z_1)+\lambda_2\Psi'(z_2)], \end{aligned}\right\} \tag{1}$$

以及

$$\left.\begin{aligned} u_x &= 2\mathrm{Re}[p_1\Phi(z_1)+p_2\Psi(z_2)], \\ u_y &= 2\mathrm{Re}[q_1\Phi(z_1)+q_2\Psi(z_2)], \end{aligned}\right\} \tag{2}$$

其中

$$z_j = x+\lambda_j y,$$
$$p_j = a_{11}\lambda_j^2+2a_{12}-a_{16}\lambda_j \qquad (j=1,2),$$
$$q_j = a_{12}\lambda_j+\frac{a_{22}}{\lambda_j}-a_{26},$$

$a_{ij}\,(i,j=1,2,6)$ 为各向异性材料的弹性系数, $\lambda_j, \bar{\lambda}_j\,(j=1,2)$ 为特征方程

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$$a_{11}\lambda^4-2a_{16}\lambda^3+2(a_{12}+a_{66})\lambda^2-2a_{16}\lambda+a_{22}=0, \tag{3}$$

的根, 且设 $\operatorname{Im}\lambda_j=0(j=1,2)$, 这样假设是合理的, 因为特征方程无实根[5] •

在边界 L 给定外力 X_n, Y_n 的情况下, 边界条件为

$$\left.\begin{aligned}&2\operatorname{Re}[\Phi(z_1)+\Psi(z_2)]=-\int Y_n\mathrm{d}s+C_1,\\&2\operatorname{Re}[\lambda_1\Phi(z_1)+\lambda_2\Psi(z_2)]=\int X_n\mathrm{d}s+C_2\end{aligned}\quad(z\in L),\right\} \tag{4}$$

在边界 L 给定位移 $u(z)+\mathrm{i}v(z)$ 的情况下, 边界条件为

$$\left.\begin{aligned}&2\operatorname{Re}[p_1\Phi(z_1)+p_2\Psi(z_2)]=u(z),\\&2\operatorname{Re}[q_1\Phi(z_1)+q_2\Psi(z_2)]=v(z)\end{aligned}\quad z\in L\right\} \tag{5}$$

2 问题的提法和解法

设有不同材料的各向异性弹性半平面和弹性长条, 它们分别占有下半平面 S^- 和带形区域 S^+ ($|x|<+\infty, 0<y<b$), 其弹性系数分别为 a_{jk}^-, $a_{jk}^+(j,k=1,2,6)$, 相应地有 S_j^-, S_j^+; p_j^-, p_j^+; q_j^-, q_j^+ ($j=1,2$), 设在 X 轴上两种材料表面不能密切粘合, 在 $z=x$ 处, 上、下岸纵坐标有位移差 $h(x)$, 以 $a\pi$ 为周期, 且 $h(x)\in\hat{H}$, $h(\pm\infty)=0$, 把这两种材料沿整个 X 轴焊接起来, 且使有相同横坐标的两点焊接在一起, 所以上、下岸横坐标位移为零; 长条的另一边作用着以 $a\pi$ 为周期的外力 $X(t)+\mathrm{i}Y(t)$, 记 L 为其一周期段($|x|<\frac{1}{2}a\pi$, $z=x+b\mathrm{i}$), 假定无穷远点无应力, 无转动, 求弹性平衡•

仿射变换 $z_j=x+\lambda_j y(j=1,2)$ 将 Z 平面的下半平面 S^- 和长条 S^+ 分别映射为 $Z_j(j=1,2)$ 平面的下半平面 S_j^- ($j=1,2$) 和长条 S_j^+ ($j=1,2$), 把 L 映成 $L_j(j=1,2)$, Z 平面的实轴 X 映成 Z_j 平面的实轴 $X_j(j=1,2)$, 且有

$$Z|_{y=0}=Z_j|_{y=0}=x\quad(j=1,2),$$

由以上条件, 问题可转化为如下的周期边值问题

$$A\Phi_1(t_1)+B\overline{\Phi_1(t_1)}+\Psi_1(t_2)=F(t)+C_*\qquad(t\in L, t_1\in L_1, t_2\in L_2), \tag{6}$$

$$\begin{aligned}&(1+\mathrm{i}\lambda_1^+)\Phi_1(x)+(1+\mathrm{i}\lambda_1^+)\overline{\Phi_1(x)}+(1+\mathrm{i}\lambda_2^+)\Psi_1(x)+(1+\mathrm{i}\lambda_2^+)\overline{\Psi_1(x)}=\\&\quad(1+\mathrm{i}\lambda_1^-)\Phi_2(x)+(1+\mathrm{i}\lambda_1^-)\overline{\Phi_2(x)}+(1+\mathrm{i}\lambda_2^-)\Psi_2(x)+\\&\quad(1+\mathrm{i}\lambda_2^-)\overline{\Psi_2(x)})\qquad(x\in X_1),\end{aligned} \tag{7}$$

$$\begin{aligned}&(p_1^++\mathrm{i}q_1^+)\Phi_1(x)+(p_1^++\mathrm{i}q_1^+)\overline{\Phi_1(x)}+(p_2^++\mathrm{i}q_2^+)\Psi_1(x)+(p_2^++\mathrm{i}q_2^+)\overline{\Psi_1(x)}=\\&\quad(p_1^-+\mathrm{i}q_1^-)\Phi_2(x)+(p_1^-+\mathrm{i}q_1^-)\overline{\Phi_2(x)}+(p_2^-+\mathrm{i}q_2^-)\Psi_2(x)+\\&\quad(p_2^-+\mathrm{i}q_2^-)\overline{\Psi_2(x)}+\mathrm{i}h(x)\ (x\in X_1),\end{aligned} \tag{8}$$

其中

$$\Phi(z)=\begin{cases}\Phi_1(z)&(z\in S_1^+),\\\Phi_2(z)&(z\in S_1^-),\end{cases}\qquad \Psi(z)=\begin{cases}\Psi_1(z)&(z\in S_2^+),\\\Psi_2(z)&(z\in S_2^-),\end{cases}$$

$$X_1:\ |x|\leqslant\frac{1}{2}a\pi,\quad y=0,$$

$$A=\frac{\lambda_1^+-\lambda_2^+}{\bar\lambda_2^+-\lambda_2^+},\quad B=\frac{\bar\lambda_1^+-\lambda_2^+}{\bar\lambda_2^+-\lambda_2^+}$$

$$C_*=\frac{(1-\mathrm{i}\bar\lambda_2^+)C-(1+\mathrm{i}\lambda_2^+)\bar C}{2\mathrm{i}(\bar\lambda_2^+-\lambda_2^+)},\quad(C\text{ 为待定复常数})•$$

$$F(t)=\frac{(1-\mathrm{i}\lambda_2^+)f(t)-(1+\lambda_2^+)\overline{f(t)}}{2\mathrm{i}(\lambda_2^+-\lambda_2^+)},$$

$$f(t)=\mathrm{i}\int_{t_0}^{t}[X(t)+\mathrm{i}Y(t)]\,\mathrm{d}t.$$

下面我们来求解边值问题(6)~(8)

令 $$\omega(\tau)=\Phi_1(\tau_1),(\tau\in L,\ \tau_1\in L_1),\tag{9}$$

由(6),有

$$\Psi_1(\tau_2)=F(\tau)-A\omega(\tau)-B\,\overline{\omega(\tau)},\quad(\tau\in L,\ \tau_2\in L_2),\tag{10}$$

构造函数

$$\Phi_{10}(z)=\frac{1}{2a\pi\mathrm{i}}\int_{L_1}\omega(\tau)\cot\frac{\tau_1-z}{a}\mathrm{d}\tau_1,\ (z\notin L_1),\tag{11}$$

$$\Psi_{10}(z)=\frac{1}{2a\pi\mathrm{i}}\int_{L_2}[F(\tau)-A\omega(\tau)-B\,\overline{\omega(\tau)}]\cot\frac{\tau_2-z}{a}\mathrm{d}\tau_2,\ (z\notin L_2)\tag{12}$$

且 $\Phi_{10}(\pm\infty\mathrm{i})=\Psi_{10}(\pm\infty\mathrm{i})=0$.

显然, $\Phi_{10}(z)$, $\Psi_{10}(z)$ 分别以 L_1, L_2 为跳跃的分区全纯函数, 且以 $a\pi$ 为周期, 由此我们可以作上半平面全纯且能连续延拓到 X 轴上的周期函数 $\Phi_{11}(z)$, $\Psi_{11}(z)$ 为

$$\Phi_{11}(z)=\begin{cases}\Phi_{10}(z)+\Phi_1(z) & (z\in S_1^+),\\ \Phi_{10}(z) & (\mathrm{Im}z>\mathrm{Im}(\lambda_1 b)),\end{cases}\tag{13}$$

$$\Psi_{11}(z)=\begin{cases}\Psi_{10}(z)+\Psi_1(z) & (z\in S_2^+),\\ \Psi_{10}(z) & (\mathrm{Im}z>\mathrm{Im}(\lambda_2 b)).\end{cases}\tag{14}$$

将(13),(14)分别代入(6)~(8)得

$$A[\Phi_{11}(t_1)-\Phi_{10}(t_1)]+B[\overline{\Phi_{11}(t_1)}-\overline{\Phi_{10}(t_1)}]+\Psi_{11}(t_2)-\Psi_{10}(t_2)=$$
$$F(t)+C_*\qquad(t\in L,t_1\in L_1,t_2\in L_2),\tag{15}$$

$$(1+\mathrm{i}\lambda_1^+)[\Phi_{11}(x)-\Phi_{10}(x)]+(1+\mathrm{i}\lambda_2^+)[\overline{\Phi_{11}(x)}-\overline{\Phi_{10}(x)}]+(1+\mathrm{i}\lambda_2^+)$$
$$[\Psi_{11}(x)-\Psi_{10}(x)]+(1+\mathrm{i}\lambda_1^+)[\Psi_{11}(x)-\Psi_{10}(x)]=(1+\mathrm{i}\lambda_1^-)\Phi_2(x)+$$
$$(1+\mathrm{i}\lambda_1^-)\overline{\Phi_2(x)}+(1+\mathrm{i}\lambda_2^-)\Psi_2(x)+(1+\mathrm{i}\lambda_2^-)\overline{\Psi_2(x)}\qquad(x\in X_1),\tag{16}$$

$$(p_1^++\mathrm{i}q_1^+)[\Phi_{11}(x)-\Phi_{10}(x)]+(p_1^++\mathrm{i}q_1^+)[\overline{\Phi_{11}(x)}-\overline{\Phi_{10}(x)}]+$$
$$(p_2^++\mathrm{i}q_2^+)[\Psi_{11}(x)-\Psi_{10}(x)]+(p_2^++\mathrm{i}q_2^+)[\overline{\Psi_{11}(x)}-\overline{\Psi_{10}(x)}]=$$
$$(p_1^-+\mathrm{i}q_1^-)\Phi_2(x)+(p_1^-+\mathrm{i}q_1^-)\overline{\Phi_2(x)}+(p_2^-+\mathrm{i}q_2^-)\Psi_2(x)+$$
$$(p_2^-+\mathrm{i}q_2^-)\overline{\Psi_2(x)}+\mathrm{i}h(x)\qquad(x\in X_1).\tag{17}$$

将等式(16)两边同乘以 $\frac{1}{2a\pi\mathrm{i}}\cot\frac{x-z}{a}\mathrm{d}x\,(\mathrm{Im}z<0)$, 且沿 X_1 积分, 利用 Hilbert 积分公式[2], 并考虑已给的条件, 得

$$(1+\mathrm{i}\lambda_1^+)\Phi_{10}(z)-(1+\mathrm{i}\lambda_1^+)\overline{\Phi_{11}(z)}+(1+\mathrm{i}\lambda_2^+)\Psi_{10}(z)-(1+\mathrm{i}\lambda_2^+)\overline{\Psi_{11}(z)}=$$
$$-(1+\mathrm{i}\lambda_1^-)\Phi_2(z)-(1+\mathrm{i}\lambda_2^-)\Psi_2(z)\qquad(\mathrm{Im}z<0).\tag{18}$$

将(16)两边取共轭, 类似上面积分, 得

$$-(1-\mathrm{i}\lambda_1^+)\overline{\Phi_{11}(z)}+(1-\mathrm{i}\lambda_1^+)\Phi_{10}(z)-(1-\mathrm{i}\lambda_2^+)\overline{\Psi_{11}(z)}+(1-\mathrm{i}\lambda_2^+)\Psi_{10}(z)=$$
$$-(1-\mathrm{i}\lambda_1^-)\Phi_2(z)-(1-\mathrm{i}\lambda_2^-)\Psi_2(z)\qquad(\mathrm{Im}z<0).\tag{19}$$

联立(18),(19)并解之, 得

$$\Phi_2(z)=\frac{\lambda_2^- -\lambda_1^+}{\lambda_2^- -\lambda_1^-}\overline{\Phi_{11}(z)}+\frac{\lambda_2^- -\lambda_2^+}{\lambda_2^- -\lambda_1^-}\overline{\Psi_{11}(z)}+\frac{\lambda_1^+ -\lambda_2^-}{\lambda_2^- -\lambda_1^-}\Phi_{10}(z)+\frac{\lambda_2^+ -\lambda_2^-}{\lambda_2^- -\lambda_1^-}\Psi_{10}(z)\quad(\mathrm{Im}z<0),\tag{20}$$

$$\Psi_2(z)=\frac{\lambda_1^+ -\lambda_1^-}{\lambda_2^- -\lambda_1^-}\overline{\Phi_{11}(z)}+\frac{\lambda_2^+ -\lambda_1^-}{\lambda_2^- -\lambda_1^-}\overline{\Psi_{11}(z)}+\frac{\lambda_1^- -\lambda_1^+}{\lambda_2^- -\lambda_1^-}\Phi_{10}(z)+\frac{\lambda_1^- -\lambda_2^+}{\lambda_2^- -\lambda_1^-}\Psi_{10}(z)\quad(\mathrm{Im}z<0),\tag{21}$$

将(20),(21)代入(17),整理得

$$\alpha_1\Phi_{11}(x)+\alpha_2\overline{\Phi_{11}(x)}+\alpha_3\Phi_{10}(x)+\alpha_4\overline{\Phi_{10}(x)}+\alpha_5\Psi_{11}(x)+\alpha_6\overline{\Psi_{11}(x)}+\alpha_7\Psi_{10}(x)+\alpha_8\overline{\Psi_{10}(x)}=\mathrm{i}h(x)\quad(x\in X_1),\tag{22}$$

其中 $\alpha_j(j=1,2,\cdots,8)$ 为易确定的常数•

类似上面的做法,由(22)式,得

$$\Phi_{11}(z)=\frac{\alpha_5\alpha_6-\alpha_3\alpha_4}{\alpha_1\alpha_3-\alpha_2\alpha_5}\overline{\Phi_{10}(z)}+\frac{\alpha_5\alpha_7-\alpha_3\alpha_8}{\alpha_1\alpha_3-\alpha_2\alpha_5}\overline{\Psi_{10}(z)}+\frac{1}{2a\pi(\alpha_1\alpha_3-\alpha_2\alpha_5)}\times\int_{X_1}(\alpha_3h(x)+\alpha_5\overline{h(x)})\cot\frac{x-z}{a}\mathrm{d}x\quad(\mathrm{Im}z>0),\tag{23}$$

$$\Psi_{11}(z)=\frac{\alpha_2\alpha_4-\alpha_1\alpha_6}{\alpha_1\alpha_3-\alpha_2\alpha_5}\overline{\Phi_{10}(z)}+\frac{\alpha_2\alpha_8-\alpha_1\alpha_7}{\alpha_1\alpha_3-\alpha_2\alpha_5}\overline{\Psi_{10}(z)}-\frac{1}{2a\pi(\alpha_1\alpha_3-\alpha_2\alpha_5)}\times\int_{X_1}(\alpha_2h(x)+\alpha_1\overline{h(x)})\cot\frac{x-z}{a}\mathrm{d}x\quad(\mathrm{Im}z>0),\tag{24}$$

将(23),(24)代入(15),整理得

$$\beta_1\overline{\Phi_{10}(t_1)}+\beta_2\overline{\Psi_{10}(t_1)}-A\Phi_{10}(t_1)+\beta_3\Phi_{10}(t_1)+\beta_4\Psi_{10}(t_1)-B\overline{\Phi_{10}(t_1)}+\beta_5\overline{\Phi_{10}(t_2)}+\beta_6\overline{\Psi_{10}(t_2)}-\Psi_{10}(t_2)=F_*(t)+C_*\quad(t\in L,t_1\in L_1,t_2\in L_2),\tag{25}$$

其中,$\beta_j(j=1,2,\cdots,6)$ 为易确定的常数•

$$F_*(t)=F(t)-\frac{A}{2a\pi(\alpha_1\alpha_3-\alpha_2\alpha_5)}\int_{X_1}[\alpha_3h(x)+\alpha_5\overline{h(x)}]\cot\frac{x-t_1}{a}\mathrm{d}x-\frac{B}{2a\pi(\alpha_1\alpha_3-\alpha_2\alpha_5)}\int_{X_1}[\alpha_3\overline{h(x)}+\alpha_5h(x)]\cot\frac{x-t_1}{a}\mathrm{d}x+\frac{1}{2a\pi(\alpha_1\alpha_5-\alpha_2\alpha_5)}\int_{X_1}[\alpha_2h(x)+\alpha_1\overline{h(x)}]\cot\frac{x-t}{a}\mathrm{d}x•$$

将(11),(12)代入(25),考虑到,当 $t=x+b\mathrm{i}\in L$ 时,有 $t_1=x+\lambda_1^+b\in L_1$,$t_2=x+\lambda_2^+b\in L_2$,且当 $z\to t$ 时,有 $z_1\to t_1$,$z_2\to t_2$•

根据 Plemelj 公式,整理得

$$\frac{1}{2a\pi\mathrm{i}}\int_{-\frac{1}{2}a\pi}^{\frac{1}{2}a\pi}\overline{\omega(x+b\mathrm{i})}k_1(x_0,x)\mathrm{d}x+\frac{1}{2a\pi\mathrm{i}}\int_{-\frac{1}{2}a\pi}^{\frac{1}{2}a\pi}\omega(x+b\mathrm{i})k_2(x_0,x)\mathrm{d}x=G(x_0)+C_*\quad\left(|x_0|\leqslant\frac{1}{2}a\pi\right),\tag{26}$$

其中

$$k_1(x_0,x) = -\beta_1\cot\frac{x-x_0+(\lambda_1^+-\lambda_1^+)b}{a} + \beta_2 A\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a} - \beta_4 B\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a} - \beta_5\cot\frac{x-x_0+(\lambda_1^+-\lambda_2^+)b}{a} + \beta_6 A\cot\frac{x-x_0+(\lambda_2^+-\lambda_2)b}{a} + 2B\cot\frac{x-x_0}{a},$$

$$k_2(x_0,x) = \beta_2 B\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a} + \beta_3\cot\frac{x-x_0+(\lambda_1^+-\lambda_1^+)b}{a} - \beta_4 A\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a} + \beta_6 B\cot\frac{x-x_0+(\lambda_2^+-\lambda_2^+)b}{a},$$

$$G(x_0) = F_*(x_0+bi) + \frac{1}{2}F(x_0+bi) + \frac{1}{2a\pi i}\int_{-\frac{1}{2}a\pi}^{\frac{1}{2}a\pi}\overline{F(x+bi)}\left[\beta_2\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a} + \beta_6\cot\frac{x-x_0+(\lambda_2^+-\lambda_2^+)b}{a}\right]\mathrm{d}x + \frac{1}{2a\pi i}\int_{-\frac{1}{2}a\pi}^{\frac{1}{2}a\pi}F(x+bi)\left[\cot\frac{x-x_0}{a} - \beta_4\cot\frac{x-x_0+(\lambda_2^+-\lambda_1^+)b}{a}\right]\mathrm{d}x, \qquad (*)$$

再令 $\tau = \mathrm{e}^{2ix/a}$, $\tau_0 = \mathrm{e}^{2ix_0/a}$, 并把圆周 $|\tau| = 1$ 记为 C, 取逆时针方向为 C 的正向, 这时, (26) 式变为

$$\frac{1}{\pi i}\int_C\frac{\omega_1(\tau)}{\tau-\tau_0}\mathrm{d}\tau = \frac{1}{\pi i}\int_C k_3(\tau_0,\tau)\,\omega_1(\tau)\,\mathrm{d}\tau + \frac{1}{\pi i}\int_C k_4(\tau_0,\tau)\,\overline{\omega_1(\tau)}\,\mathrm{d}\tau + G\left(\frac{a}{2i}\ln\tau_0\right) + C_* \quad (\tau_0\in C), \qquad (27)$$

其中 $\omega_1(\tau) = \overline{\omega\left(\frac{a}{2i}\ln\tau + bi\right)}$,

$k_3(\tau_0,\tau)$, $k_4(\tau_0,\tau)$ 由 $k_1(x_0,x)$, $k_2(x_0,x)$ 确定•

由Cauchy 型积分反演公式和 Poincare_Bertrand 置换公式, 得

$$\omega_1(\tau_0) = \frac{1}{\pi i}\int_C k_3(\tau_0,\tau)\,\omega_1(\tau)\,\mathrm{d}\tau + \frac{1}{\pi i}\int_C k_4(\tau_0,\tau)\,\overline{\omega_1(\tau)}\,\mathrm{d}\tau + G_*(\tau_0) + C_* \quad (\tau_0\in C), \qquad (28)$$

这里, $G_*(\tau_0) = \frac{1}{\pi i}\int_C G\left(\frac{a}{2i}\ln\tau\right)\frac{\mathrm{d}\tau}{\tau-\tau_0}$•

易知, $k_3(\tau_0,\tau)$, $k_4(\tau_0,\tau)$ 可展成如下级数

$$k_3(\tau_0,\tau) = \frac{1}{\tau}\sum_{-\infty}^{+\infty}\delta_1^{(n)}\frac{\tau_n}{\tau_0^n}, \quad k_4(\tau_0,\tau) = \frac{1}{\tau}\sum_{-\infty}^{+\infty}\delta_2^{(n)}\frac{\tau_n}{\tau_0^n}, \qquad (29)$$

其中 $\delta_1^{(n)}$, $\delta_2^{(n)}$ 为易确定的常数•

将(29) 代入(28), 得

$$\omega(\tau_0) = \sum_{-\infty}^{+\infty}(d_{n-1}\delta_1^{(n)} + d_{-n-1}\delta_2^{(n)})\,\tau_0^{\,n} + G_*(\tau_0) + C_* \quad (\tau_0\in C), \qquad (30)$$

其中 $d_n = \frac{1}{2\pi i}\int_C \omega_1(\tau)\tau^n d\tau \quad (n = 0, \pm 1, \pm 2, \cdots)$•

类似[6]的方法可证方程(30)的解存在,且任何两解之间相差一常数,由于常数不影响物体的应力状况,我们可取 $d_{-1} = 0$,在方程(30)两边都乘以$\frac{1}{2\pi i}\tau_0^j d\tau$,($j = 0, 1, 2, \cdots$),再沿 C 积分,得

$$(\delta_1^{(j+1)} - 1) d_j + \delta_2^{(j+1)} d_{j-2} = G_j, \tag{31}$$

这里 $G_j = -\frac{1}{2\pi i}\int_C G_*(\tau)\tau_j d\tau$•

方程(30)两边取共轭,类似上面积分,得

$$\delta_2^{(j-1)} d_j + (\delta^{(-j-1)} - 1) d_{-j-2} = G_{-j-2}, \tag{32}$$

将(31),(32)联立,就可以求出 d_j($j = 0, \pm 1, \pm 2, \cdots$)代入(33)得到方程(28)的解为

$$\omega_1(\tau) = \sum_{n=1}^{\infty} E_n \tau^n + \sum_{n=1}^{\infty} E_{-n}\tau^{-n} + G_*(\tau) \quad (\tau \in C), \tag{33}$$

这里 $E_n = d_{-n-1}\delta_1^{(-n)} + d_{n-1}\delta_2^{(-n)}$, $E_{-n} = d_{n-1}\delta_1^{(n)} + d_{-n-1}\delta_2^{(n)}$•

从而得出方程(26)的解为

$$\omega(x + bi) = \sum_{n=1}^{\infty} E_n e^{-2nix/a} + \sum_{n=1}^{\infty} E_{-n} e^{2nix/a} + \overline{G(e^{2ix/a})} \quad \left(|x| \leqslant \frac{1}{2}a\pi\right)• \tag{34}$$

将(34)分别代入(11),(12),求出 $\Phi_{10}(z)$, $\Psi_{10}(z)$,再由(23),(24)求得 $\Phi_{11}(z)$, $\Psi_{11}(z)$,进一步由(20),(21)求得 $\Phi_2(z)$、$\Psi_2(z)$的值,至此,问题得已解决•

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Problem of the Periodic Welding of Anisotropic Elastic Plane with Different Materials

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Abstract: In this paper, the problem of the periodic welding of an anisotropic elastic half_plane and a strip with different materials is discussed. By means of plane elastic complex variable method and theory of boundary value problems for analytic function, the stress distribution is given in closed forms.

Key words: anisotropic; plane elastic complex variable method; welding; boundary valve problem