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大挠度板混沌运动的双模态分析*

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摘要: 研究了大挠度矩形薄板受迫振动时的混沌运动, 导出了矩形薄板的非线性控制方程; 利用 Galerkin 原理, 将其化为二自由度的常微分方程组, 从理论上证明了在讨论其混沌运动时可以归结为一个单模态问题; 利用 Melnikov 函数法给出了发生混沌运动的临界条件, 揭示出在此类新的非线性动力系统中, 同样存在着发生混沌的可能•

关 键 词: 屈曲板; 混沌; Poincar 映射
中图分类号: O34 文献标识码: A

引 言

近年来, 弹塑性系统混沌运动的研究引起了人们的极大关注• 然而对这一问题的研究仍然显得比较简单, 绝大部分对于混沌运动的研究是在低维系统中进行的• 不仅如此, 即使是对于梁这样的简单系统, 人们也只是采用单自由度模型来描述梁这一无穷维系统的混沌运动, 高阶模态对无穷维系统混沌运动会产生什么影响尚不得而知•

本文研究了大挠度矩形薄板受迫振动时的混沌运动, 导出了矩形薄板的非线性控制方程; 利用 Galerkin 原理, 将其化为二自由度的常微分方程组, 从理论上证明了在讨论其混沌运动时可以归结为一个单模态问题; 利用 Melnikov 函数法给出了发生混沌运动的临界条件, 揭示出在此类新的非线性动力系统中, 同样存在着发生混沌的可能• 并进行了相应的数值计算, 理论分析表明高阶模态对混沌运动没有影响•

1 控制方程

考虑图 1 所示的矩形薄板, 板厚为 h, 周边简支, 且在其边界上受有均匀压力 N_1 和 N_2, 我们的讨论基于这样的基本假设:

(1) 变形前垂直于中面的直线变形后仍为一直线, 并保持与中面垂直;

(2) 忽略沿中面垂直方向的法向应力;

(3) 只计及横向惯性效应•

$$p = q\cos\omega t, \tag{1}$$

板单元的动力方程为:

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$$\left.\begin{aligned}&D\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}+\delta_0\frac{\partial w}{\partial t}-hL(\varphi,w)+N_1\frac{\partial^2 w}{\partial x^2}+N_2\frac{\partial^2 w}{\partial y^2}=q\cos\omega t,\\&\nabla^4\varphi+\frac{E}{2}L(w,w)=0,\end{aligned}\right\}\tag{2}$$

其中 $\nabla^4=\frac{\partial^4}{\partial x^4}+2\frac{\partial^4}{\partial x^2\partial y^2}+\frac{\partial^4}{\partial x^4}$, $L(\varphi,w)=\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2\varphi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\varphi}{\partial x\partial y}$,

$D=Eh^3/12(1-\mu^2)$, E 为弹性模量, h 为板厚, μ 为Poisson 比, w 为横向挠度, δ_0 为阻尼系数, φ 为应力函数•

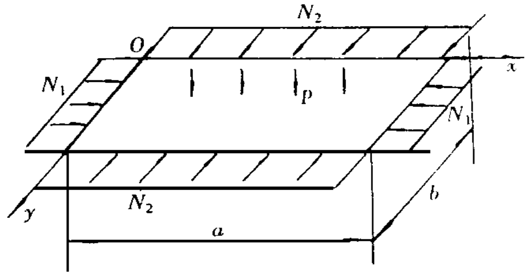

图 1　周边简支的矩形薄板

四边简支板的边界条件为:

$$\left.\begin{aligned}&x=0,\ a:\quad W=\frac{\partial^2 W}{\partial x^2}=0,\\&y=0,\ b:\quad W=\frac{\partial^2 W}{\partial y^2}=0,\end{aligned}\right\}\tag{3}$$

取位移模式形如:

$$w(x,t)=T_1(t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}+T_2(t)\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}•\tag{4}$$

由此可见:

$$\begin{aligned}\varphi=&ET_1^2\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\frac{1}{2}\left[\left(\frac{a}{2\pi}\right)^4\cos\frac{2\pi x}{a}+\left(\frac{b}{2\pi}\right)^4\cos\frac{2\pi y}{b}\right]+\\&ET_2^2\left(\frac{2\pi}{a}\right)^2\left(\frac{2\pi}{b}\right)^2\frac{1}{2}\left[\left(\frac{a}{4\pi}\right)^4\cos\frac{4\pi x}{a}+\left(\frac{b}{4\pi}\right)^4\cos\frac{4\pi y}{b}\right]+\\&ET_1T_2\left(\frac{\pi}{a}\right)^2\left(\frac{2\pi}{b}\right)^2\frac{1}{2}\left[A\cos\frac{3\pi x}{a}\cos\frac{\pi y}{b}+B\cos\frac{3\pi y}{b}\cos\frac{\pi x}{a}\right],\end{aligned}\tag{5}$$

其中 $A=\frac{2}{\left[\left(\frac{3\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2}$, $B=\frac{2}{\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{3\pi}{b}\right)^2\right]^2}$,

显然, 给定的位移模式满足四边简支的边界条件• 同时, 由伽辽金原理应有:

$$\left.\begin{aligned}&\iint_S\left[D\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}+\delta_0\frac{\partial w}{\partial t}-hL(\varphi,w)+N_1\frac{\partial^2 w}{\partial x^2}+N_2\frac{\partial^2 w}{\partial y^2}-q\cos\omega t\right]w_1^*\,\mathrm{d}s=0,\\&\iint_S\left[D\nabla^4 w+\rho h\frac{\partial^2 w}{\partial t^2}+\delta_0\frac{\partial w}{\partial t}-hL(\varphi,w)+N_1\frac{\partial^2 w}{\partial x^2}+N_2\frac{\partial^2 w}{\partial y^2}-q\cos\omega t\right]w_2^*\,\mathrm{d}s=0,\end{aligned}\right\}\tag{6}$$

其中 $w_1^*=\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$, $w_2^*=\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}$,

积分可求得

$$\left.\begin{aligned}&\ddot{T}_1-\alpha_1T_1+\beta_1T_1^3+\gamma_1T_1T_2^2=\varepsilon(f\cos\omega t-\varepsilon'\dot{T}_1),\\&\ddot{T}_2-\alpha_2T_2+\beta_2T_2^3+\gamma_2T_2T_1^2=-\frac{\delta_0}{\rho h}\dot{T}_2,\end{aligned}\right\}\tag{7}$$

其中

$$
\left.\begin{aligned}
&\alpha_1 = \frac{N_1\left(\frac{\pi}{a}\right)^2 + N_2\left(\frac{\pi}{b}\right)^2 - D\left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]^2}{\rho h},\\
&\alpha_2 = \frac{N_1\left(\frac{2\pi}{a}\right)^2 + N_2\left(\frac{2\pi}{b}\right)^2 - D\left[\left(\frac{2\pi}{a}\right)^2 + \left(\frac{2\pi}{b}\right)^2\right]^2}{\rho h},\\
&\beta_1 = \frac{E}{16\rho}\left[\left(\frac{\pi}{a}\right)^4 + \left(\frac{\pi}{b}\right)^4\right],\quad \beta_2 = \frac{E}{16\rho}\left[\left(\frac{2\pi}{a}\right)^4 + \left(\frac{2\pi}{b}\right)^4\right],\\
&\gamma_1 = \frac{4E}{\rho}(A + B)\left(\frac{\pi}{a}\right)^4\left(\frac{\pi}{b}\right)^4,\quad \gamma_2 = \frac{8E}{\rho}(A + B)\left(\frac{\pi}{a}\right)^4\left(\frac{\pi}{b}\right)^4,\\
&f = \frac{16g}{\rho h \pi^2 \varepsilon},\quad \bar{\mu} = \frac{\delta_0}{\rho h \varepsilon}.
\end{aligned}\right\}
\tag{8}
$$

2 混沌运动分析

不难发现如果取 $T_1 = T(t)$, $T_2 = 0$, 则相互耦合的常微分方程组可以解耦为:

$$
\ddot{T} - \alpha_1 T + \beta_1 T^3 = \varepsilon(f\cos\omega t - \bar{\mu}\dot{T}),
\tag{9}
$$

这是一个 Duffing 方程, 这个动力系统对应的无扰动系统是

$$
\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} T \\ \dot{T} \end{pmatrix} = \begin{pmatrix} \dot{T} \\ \alpha_1 T - \beta_1 T^3 \end{pmatrix}
\tag{10}
$$

这是一个 Hamilton 系统• 其首次积分为

$$
H = \frac{1}{2}\dot{T}^2 - \frac{1}{2}\alpha_1 T^2 + \frac{1}{4}\beta_1 T^4;
\tag{11}
$$

相应的同宿轨道是

$$
(T_0, \dot{T}_0) = \left(\pm\sqrt{\frac{2\alpha_1}{\beta_1}}\operatorname{sech}(\sqrt{\alpha_1}t),\ \pm\alpha_1\sqrt{\frac{2\alpha_1}{\beta_1}}\operatorname{sech}(\sqrt{\alpha_1}t)\tanh(\sqrt{\alpha_1}t)\right),
\tag{12}
$$

相应的 Melnikov 函数为

$$
M(\tau_0) = -\bar{\mu}\frac{4\sqrt{\alpha_1^3}}{3\beta_1} + \pi f\omega\sqrt{\frac{2}{\beta_1}}\operatorname{sech}\left(\frac{\omega\pi}{2\sqrt{\alpha_1}}\right)\sin\omega\tau_0 .
\tag{13}
$$

故此, 发生混沌的临界条件可表为:

$$
\frac{f}{\bar{\mu}} \geqslant \frac{4\alpha_1^{3/2}}{3\pi\omega\sqrt{2\beta_1}}\cosh\left(\frac{\pi\omega}{2\sqrt{\alpha_1}}\right).
\tag{14}
$$

3 数 值 分 析

由于混沌运动与其它定常运动相比, 有其独特的数字特征, 这些数字特征对于了解混沌运动的规律具有十分重要的意义, 它们可以作为一个定常运动是否混沌的判断标准• 因此, 为了证明混沌的存在, 采用 Melnikov 函数法并通过 (1) Poincar 截面; (2) 相平面图; (3) 时程运动轨迹的直接观察来做出相应的判断• 图 3 和图 2 曲线均计算了 3000 个点, 初始条件为 $T(0) = 0.05$, $\dot{T}(0) = 0$, 数值分析表明, 具有同宿轨道的非线性动力系统的数值分析较具有异宿轨道的非线性动力系统的数值分析容易得多, 并且定常运动较混沌运动的数值分析也容易一些• 从图 2 我们不难发现, 在此组参数下, 系统的运动是一个定常运动而非混沌解, 特别是其时程曲线具有明显的规律; 图 3 有很大的不同, 其相平面图和 Poincar 映射具有类似的特点, 它们的形状恰好似一副倾斜放置的眼镜, 其时程曲线毫无规律可言; 再结合这组参数落在 Melnikov 函数法确定的混沌区内, 由此可以判定这是一个混沌运动•

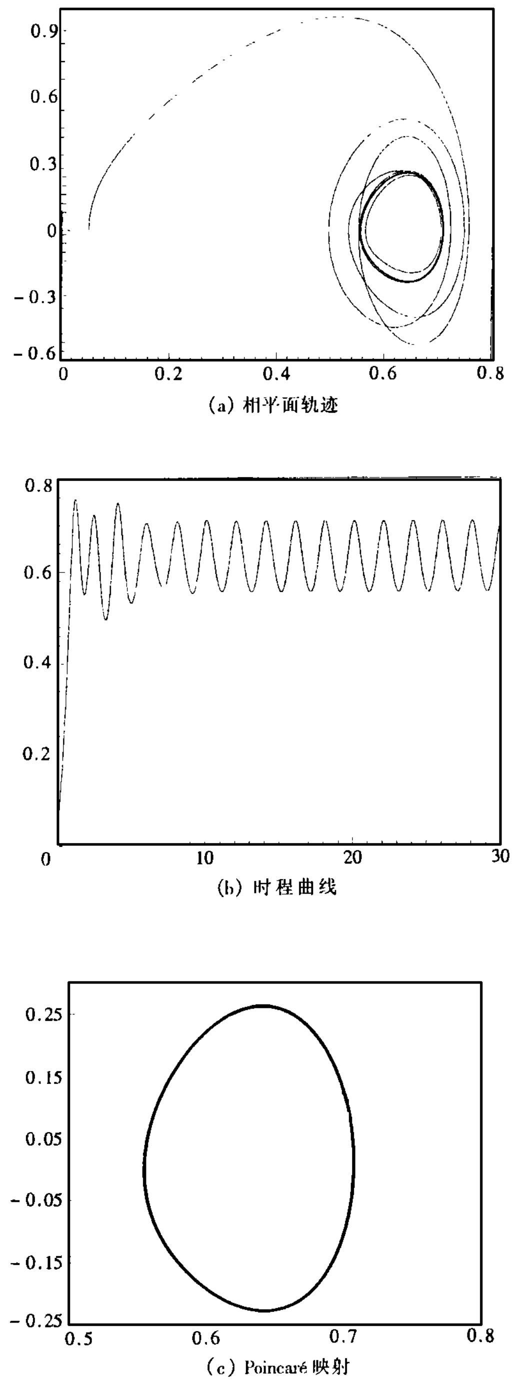

图 2　$\alpha = 10, \beta = 25, \omega = \pi, g = 0.8, \varepsilon = 1$

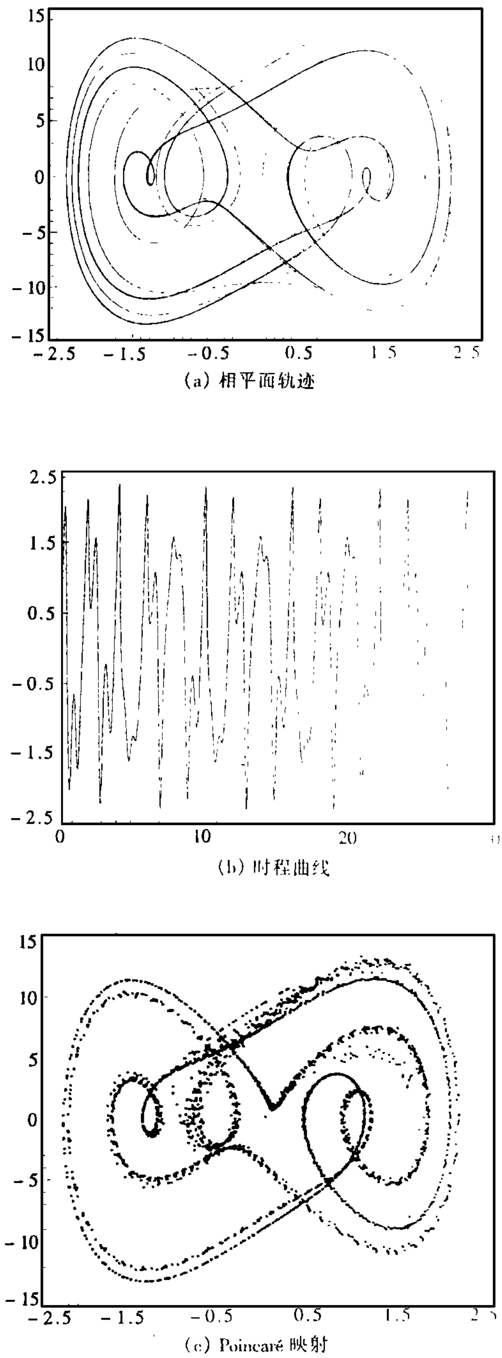

图 3　$\alpha = 10, \beta = 25, \omega = \pi, g = 59.1, \varepsilon = 1$

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The Double Mode Model of the Chaotic Motion for a Large Deflection Plate

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Abstract: The primary aim of this paper is to study the chaotic motion of a large deflection plate. Considered here is a buckled plate, which is simply supported and subjected to a lateral harmonic excitation. At first, the partial differential equation governing the transverse vibration of the plate is derived. Then, by means of the Galerkin approach, the partial differential equation is simplified into a set of two ordinary differential equations. It is proved that the double mode model is identical with the single mode model. The Melnikov method is used to give the approximate excitation thresholds for the occurrence of the chaotic vibration. Finally numerical computation is carried out.

Key words: buckled plate; chaos; Poincar section