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平台—群柱基础对浪流冲击的动力反应分析

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摘　　要

本文研究平台—群柱基础系统对浪流冲击的动力反应分析· 文中考虑了群柱基础与水流和地基土的动力相互作用, 由解析法给出了系统对浪流力激励的动力响应分析, 给出了算例的位移响应结果, 并讨论了系统参数对动力特性的影响·

关键词　平台—群柱基础系统　相互作用　浪流激励　动力特性

中图分类号　O347, TU435

§ 1. 引　　言

由于海洋资源开发利用的需要, 近年来建造了许多大型的海洋结构· 这些结构在极其恶劣的水流环境之中, 要经受严酷的荷载环境, 结构耗资巨大, 其安全可靠性及抗灾害能力问题尤为突出, 结构动力特性及抗振研究引人注目, 促进了大型结构动力学问题的研究[1]· 固定式海上采油平台的基础通常为圆柱(大型桩)结构, 为支承平台的巨大重量并维持其稳定和正常工作, 柱体结构的尺寸比较大, 振动时结构与水流之间的相互作用比较明显· 海上采油平台大多建造在深水区, 柱腿穿过海水进入海底地层, 结构振动与水介质运动相互耦合, 浪流压力通过柱体传入地层, 使基础与土体产生相互作用, 因此, 柱体与水流和土体之间的相互作用是系统动力分析的关键· Williams 和 Liaw 等人[2~3]分析了竖直圆柱体的水平地震反应, 给出了动力压力和位移响应结果· 文献[4]考虑水的压缩性及基础与水介质和地基土之间的相互作用, 分析了单桩平台的地震反应, 给出了平台位移幅值响应结果· 文献[5]则给出了单层半空间地基上海洋平台沉箱基础动力反应的解析解, 导出了地基的阻抗函数, 并由线性波理论模拟海浪, 给出了海浪对基础的水平合力和力矩的解析表达式· 本文讨论平台—群柱基础对浪流冲击力的动力反应分析·

海洋波浪典型波长为数十米至数百米, 波幅为数米[6], 在工程意义上, 波幅与波长相比可为小量· 因此, 文中对深水平台情况由线性波理论近似模拟波浪, 假定水介质为可压缩的理想流体, 土体为 Winkler 地基土, 平台为矩形刚块, 考虑水流、土体与群柱基础的相互作用, 由解析法给出系统对浪流激励响应结果, 并讨论了系统参数对其动力特性的影响·

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§2. 系统分析模型及基本方程

平台—群柱基础系统见图 1 所示• 刚性平台具有质量 M 和绕其质心轴的转动惯量J_0, 由群柱基础支承, 柱腿穿过海水和土层嵌入基岩或相对硬层; 水介质和土层分别视为可压缩的理想流体和 Winkler 弹性地基土• 结构系统受 x 方向的浪流冲击• 取基岩上指向平台质心的坐标为整体坐标($Oxyz$), 而通过各圆柱轴线的坐标为局部坐标($O_jx_jy_jz$)•

2.1 圆柱的振动方程

假定圆柱外径与其长度相比为很小, 把它们视为具有均匀抗弯刚度 EI_j, 抗压刚度 EA_j 和单位长度质量 m_{sj} 的梁构件, 入土部分受到的土压力与水平位移成正比, 忽略轴向的土体阻力; 各柱为图中的对称分布• 其横向和轴向振动方程可表示为

$$\left.\begin{aligned}
&EI_j\frac{\partial^4 u_{1j}}{\partial z^4}+m_{sj}\frac{\partial^2 u_{1j}}{\partial t^2}+k_{sj}u_{1j}=0 \quad (0\leqslant z\leqslant h_1)\\
&EI_j\frac{\partial^4 u_{2j}}{\partial z^4}+m_{sj}\frac{\partial^2 u_{2j}}{\partial t^2}=F_{wj}(z,t) \quad (h_1< z\leqslant h_2)\\
&EI_j\frac{\partial^4 u_{3j}}{\partial z^4}+m_{sj}\frac{\partial^2 u_{3j}}{\partial t^2}=0 \quad (h_2< z\leqslant h_0)\\
&EA_j\frac{\partial^2 v_j}{\partial z^2}-m_{sj}\frac{\partial^2 v_j}{\partial t^2}=0 \quad (0\leqslant z\leqslant h_0)
\end{aligned}\right\}\quad (j=1,2,\cdots,N) \tag{2.1}$$

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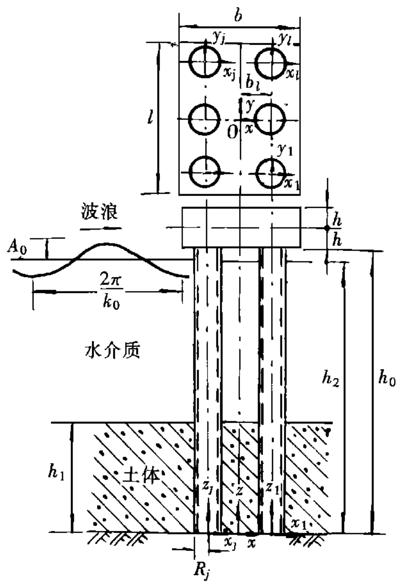

图 1 平台—群柱—水流—土体系统

式中, u_{1j}, u_{2j}, u_{3j} 是第j 根柱在土中、水中和水面之上的横向位移, v_j 是其轴向位移, k_{sj} 是各柱对应的 Winkler 水平地基系数, F_{wj} 是各柱单位长度上的动水压力•

$$F_{wj}(z,t)=-R_j\rho_w\int_0^{2\pi}\frac{\partial\Phi}{\partial t}\bigg|_{r_j=R_j}\cos(\pi-\theta_j)\,d\theta_j \tag{2.2}$$

其中, $\Phi(r,\theta,z,t)$ 是水的速度势, ρ_w 是水的质量密度, R_j 为第j 根圆柱的外半径• 边界条件和连续条件则表示为

$$\left.\begin{aligned}
&u_{1j}=0,\quad \frac{\partial u_{1j}}{\partial z}=0,\quad v_j=0,\quad z=0\\
&\frac{\partial^\alpha u_{1j}}{\partial z^\alpha}-\frac{\partial^\alpha u_{2j}}{\partial z^\alpha}=0\ (\alpha=0,1,2,3),\quad z=h_1\\
&\frac{\partial^\alpha u_{2j}}{\partial z^\alpha}-\frac{\partial^\alpha u_{3j}}{\partial z^\alpha}=0\ (\alpha=0,1,2,3),\quad z=h_2\\
&u_{3j}-(u_M-h\theta_M)=0,\quad v_j-(u_M+b_j\theta_M)=0,\quad \frac{\partial u_{3j}}{\partial z}-\theta_M=0,\quad z=h_0
\end{aligned}\right\}\quad (j=1,2,\cdots,N) \tag{2.3}$$

式中, u_M, v_M, θ_M 为平台的水平和竖向位移及绕质心轴的转角, b_j 为第j 根柱圆心的x 坐轴值•

2.2　平台的振动方程

平台为矩形刚块, 振动时其底面受各柱端剪力, 轴力和弯矩的作用• 其振动方程表示为

$$\left.\begin{aligned} & M\frac{\partial^2 u_M}{\partial t^2}-\sum_{j=1}^{N}EI_j\frac{\partial^3 u_{3j}(h_0,t)}{\partial z^3}=0 \\ & J_0\frac{\partial^2\theta_M}{\partial t^2}-\sum_{j=1}^{N}\left\{EI_j\left[\frac{\partial^2 u_{3j}(h_0,t)}{\partial z^2}+h\frac{\partial^3 u_{3j}(h_0,t)}{\partial z^3}\right]+EA_jb_j\frac{\partial v_j(h_0,t)}{\partial z}\right\}=0 \\ & M\frac{\partial^2 v_M}{\partial t^2}+\sum_{j=1}^{N}EA_j\frac{\partial v_j(h_0,t)}{\partial z}=0 \end{aligned}\right\} \tag{2.4}$$

2.3　水介质的运动方程

本文假定浪流的波幅与其波长相比为小量, 水流较深, 水介质视为可压缩的理想流体, 由线性波理论近似模拟波浪, 以简化分析• 波浪对应的自由面位移由入射波 η^I和散射波η^S对应的两部分自由面位移组成, 表示为

$$\eta(x,y,t)=\eta^I(x,t)+\eta^S(x,y,t) \tag{2.5}$$

其中, $\eta^I=A_0\exp[i(k_0x-\omega t)]$ 是波浪在x 方向入射波的自由面位移, A_0 为波幅; 波数k_0和频率 ω 之间满足一定的色散关系[6]• 水介质的速度势方程在极坐标中表示为[2]

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} \tag{2.6}$$

式中, 速度势 $\Phi=\Phi^I+\Phi^S$; Φ^I, Φ^S分别是入射波和散射波的速度势, c 是水的声波速度, 边界条件表示为

$$\frac{\partial\Phi}{\partial z}=0,\qquad z=h_1 \tag{2.7a}$$

$$\frac{\partial\Phi}{\partial z}-\frac{\partial\eta}{\partial t}=0,\qquad \frac{\partial\Phi}{\partial t}+g\eta=0,\qquad z=h_2 \tag{2.7b}$$

$$\frac{\partial\Phi}{\partial r_j}-\frac{\partial u_{2j}}{\partial t}\cos\theta_j=0,\qquad r_j=R_j,\qquad h_1\leqslant z\leqslant h_2\quad(j=1,2,\cdots,N) \tag{2.7c}$$

式中, g 为重力加速度• 另外当频率 $\omega\geqslant\omega$ 时, Φ^S 应满足辐射条件, 当 $\omega<\omega$ 时, 应满足 $\lim\limits_{r\to\infty}\Phi^S=0$, 其中 ω 是第一截止频率[2]•

§3. 系统方程的求解

为简明起见, 以下分析中省略各式中的时间因子 $\exp[-i\omega t]$•

3.1　水介质的运动

对应于波浪入射波 η^I的速度势表示为

$$\Phi^I=-\frac{i\omega A_0}{\beta_0}\frac{\operatorname{ch}\beta_0(z-h_1)}{\operatorname{sh}\beta_0(h_2-h_1)}\exp[ik_0r\cos\theta]\qquad\left(\beta_0=\sqrt{k_0^2-(\omega/c)^2}\right) \tag{3.1a}$$

利用关系式

$$\left.\begin{aligned} & r\cos\theta=r_l\cos\theta_l+b_l\qquad(l=1,2,3,\cdots,N) \\ & \exp[ik_0r_l\cos\theta_l]=\sum_{m=0}^{\infty}(i)^m\mathscr{E}_mJ_m(k_0r_l)\cos m\theta_l,\qquad \mathscr{E}_m=\begin{cases}1 & (m=0)\\ 2 & (m\geqslant1)\end{cases} \end{aligned}\right\} \tag{3.1b}$$

式(3.1a)可在局部坐标(r_l, θ_l, z)中表示为

$$\Phi^I = -\frac{\omega A_0 \exp[ik_0 b_l]}{\beta_0 \mathrm{sh}\,\beta_0(h_2 - h_1)} \sum_{m=0}^{\infty} (i)^{m+1} \mathscr{E}_m J_m(k_0 r_l) \cos m\theta_l \,\mathrm{ch}\,\beta_0(z - h_1) \tag{3.2}$$

入射波频率 ω 与波数 k_0 之间满足如下的色散关系

$$\beta_0 \mathrm{th}\,\beta_0(h_2 - h_1) - \omega^2/g = 0 \tag{3.3}$$

由式(2.6)把对应于散射波的速度势表示为各局部坐标(r_j, θ_j, z)中的解之和, 考虑辐射条件后, 散射波的速度势表示为

$$\Phi^S = \sum_{j=1}^{N} \Phi_j^S(r_j, \theta_j, z) = \sum_{j=1}^{N}\sum_{m=0}^{\infty}[(A_{mj}^{(1)}\cos\beta z + B_{mj}^{(1)}\sin\beta z)\cos m\theta_j$$
$$+ (A_{mj}^{(2)}\cos\beta z + B_{mj}^{(2)}\sin\beta z)\sin m\theta_j] R_{mj}(r_j) \tag{3.4}$$

式中

$$R_{mj}(r_j) = \begin{cases} H_m^{(1)}(\lambda^* r_j) & ((\omega/c)^2 - \beta^2 \geqslant 0) \\ K_m(\lambda' r_j) & ((\omega/c)^2 - \beta^2 < 0) \end{cases} \tag{3.5}$$

其中, $\lambda^* = \sqrt{(\omega/c)^2 - \beta^2}$, $\lambda' = -i\lambda^*$. 把式(3.2)、(3.4)代入边界条件(2.7a)、(2.7b)可得

$$A_{mj}^{(1)}/B_{mj}^{(1)} = A_{mj}^{(2)}/B_{mj}^{(2)} = \mathrm{ctg}\,\beta h_1 \tag{3.6a}$$

$$\beta \,\mathrm{ctg}\,\beta(h_2 - h_1) + g/\omega^2 = 0 \tag{3.6b}$$

式(3.6b)是确定常数 β 的特征方程, 有无限多个根 $\beta_n(n = 1, 2, 3, \ldots)$, 由叠加法并利用式(3.6a), Φ^S 可进一步表示为

$$\Phi^S = \sum_{j=1}^{N}\sum_{m=0}^{\infty}\Big[\sum_{n=1}^{N_0}(A_{mn}^j \cos m\theta_j + B_{mn}^j \sin m\theta_j) H_m^{(1)}(\lambda_n r_j)\cos\beta_n(z - h_1)$$
$$+ \sum_{n=N_0+1}^{\infty}(A_{mn}^j \cos m\theta_j + B_{mn}^j \sin m\theta_j) K_m(\lambda_n' r_j)\cos\beta_n(z - h_1)\Big] \tag{3.7}$$

式中, 常数 A_{mn}^j, B_{mn}^j 可由边界条件(2.7c)确定, $H_m^{(1)}$, K_m 为第一类和虚宗量 Hankel 函数; $\lambda_n = \sqrt{(\omega/c)^2 - \beta_n}$, $\lambda_n' = -i\lambda_n$, N_0 是满足$(\omega/c)^2 - \beta_n^2 \geqslant 0$的 n 的最大值, 第一截止频率是满足$(\omega/c)^2 - \beta_1^2 = 0$的频率, 即 $\omega = \beta_1 c$. 为了利用边界条件(2.7c)确定常数 A_{mn}^j, B_{mn}^j, 利用如下的 Graf 加法公式(参考图2)[7,8]

当 $d_{jl} > r_l$ 时

$$H_m^{(1)}(\lambda_n r_j)\begin{Bmatrix}\cos m\theta_j \\ \sin m\theta_j\end{Bmatrix} = \sum_{k=0}^{\infty} \underset{\text{群 柱}}{J_k(\lambda_n r_l)} \begin{Bmatrix} P_{mk}^{jl}\cos k\theta_l + Q_{mk}^{jl}\sin k\theta_l \\ R_{mk}^{jl}\cos k\theta_l + S_{mk}^{jl}\sin k\theta_l \end{Bmatrix} \tag{3.8a}$$

$$K_m(\lambda_n' r_j)\begin{Bmatrix}\cos m\theta_j \\ \sin m\theta_j\end{Bmatrix} = \sum_{k=0}^{\infty} I_k(\lambda_n' r_l) \begin{Bmatrix} P'^{jl}_{mk}\cos k\theta_l + Q'^{jl}_{mk}\sin k\theta_l \\ R'^{jl}_{mk}\cos k\theta_l + S'^{jl}_{mk}\sin k\theta_l \end{Bmatrix} \tag{3.8b}$$

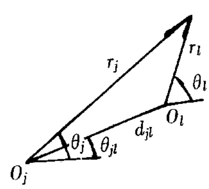

图2 坐标转换参考图

把式(3.2)、(3.7)表示的速度势变换为局部坐标(r_l, θ_l, z)中的表达式后, 得

$$\Phi \ddagger \sum_{m=0}^{\infty}\Bigg\{-\frac{\omega A_0 (i)^{m+1}\mathscr{E}_m \exp[ik_0 b_l]}{\beta_0 \mathrm{sh}\,\beta_0(h_2 - h_1)} J_m(k_0 r_l)\cos m\theta_l \,\mathrm{ch}\,\beta_0(z - h_1) + \sum_{n=1}^{\infty}\Big\{(A_{mn}^l \cos m\theta_l$$

$$=$$

$$+ B^l_{mn}\sin m\theta_l) R^{(1)}_{mn}(r_l) + \sum_{j=1, j\neq l}^{N}\sum_{k=0}^{\infty}[(T^{jl}_{mk}\cos k\theta_l + U^{jl}_{mk}\sin k\theta_l) A^j_{mn} + (V^{jl}_{mk}\cos k\theta_l$$

$$\left. + W^{jl}_{mk}\sin k\theta_l) B^j_{mn}] R^{(2)}_{kn}(r_l)\right\} \cdot \cos\beta_n(z - h_1)\Big] \qquad (3.9)$$

式中

$$(R^{(1)}_{mn}, R^{(2)}_{kn}, T^{jl}_{mk}, U^{jl}_{mk}, V^{jl}_{mk}, W^{jl}_{mk})^T = \begin{cases} (H^{(1)}_m(\lambda_n r_l), J_k(\lambda_n r_l), P^{jl}_{mk}, Q^{jl}_{mk}, R^{jl}_{mk}, S^{jl}_{mk})^T & (n \leqslant N_0) \\ (K_m(\lambda'_n r_l), I_k(\lambda'_n r_l), P'^{jl}_{mk}, Q'^{jl}_{mk}, R'^{jl}_{mk}, S'^{jl}_{mk})^T & (n \geqslant N_0 + 1) \end{cases} \qquad (3.10)$$

其中，J_k，I_k 是 Bessel 函数和虚宗量 Bessel 函数，且

$$\left.\begin{aligned} (P^{jl}_{mk}, Q^{jl}_{mk}, R^{jl}_{mk}, S^{jl}_{mk})^T &= (-1)^k \frac{\mathscr{E}_k}{2}[H^{(1)}_{k+m}(\lambda_n d_{jl})\Phi_1(\theta_{jl}) + (-1)^m H^{(1)}_{k-m}(\lambda_n d_{jl})\Phi_2(\theta_{jl})] \\ (P'^{jl}_{mk}, Q'^{jl}_{mk}, R'^{jl}_{mk}, S'^{jl}_{mk})^T &= (-1)^k \frac{\mathscr{E}_k}{2}[K_{k+m}(\lambda'_n d_{jl})\Phi_1(\theta_{jl}) + K_{k-m}(\lambda'_n d_{jl})\Phi_2(\theta_{jl})] \end{aligned}\right\} \qquad (3.11a)$$

$$\left.\begin{aligned} \Phi_1(\theta_{jl}) &= (\cos(k+m)\theta_{jl}, \sin(k+m)\theta_{jl}, \sin(k+m)\theta_{jl}, -\cos(k+m)\theta_{jl})^T \\ \Phi_2(\theta_{jl}) &= (\cos(k-m)\theta_{jl}, \sin(k-m)\theta_{jl}, -\sin(k-m)\theta_{jl}, \cos(k-m)\theta_{jl})^T \end{aligned}\right\} \qquad (3.11b)$$

3.2 圆柱及平台的振动

由式(2.2)、(3.9)可得作用于第 l 根柱上单位长度的动水压力

$$F_{wl} = -i\omega\pi R_l\rho_w\left\{\frac{2\omega A_0\exp[ik_0 b_l]}{\beta_0\operatorname{sh}\beta_0(h_2 - h_1)} J_1(k_0R_l)\operatorname{ch}\beta_0(z - h_1) + \sum_{n=1}^{\infty}\Big[A^l_{1m}R^{(1)}_{1n}(R_l)\right.$$

$$\left. + \sum_{m=0}^{\infty}\sum_{j=1, j\neq l}^{N}(A^j_{mn}T^{jl}_{m1} + B^j_{mn}V^{jl}_{m1})R^{(2)}_{1n}(R_l)\Big]\cos\beta_n(z - h_1)\right\} \qquad (3.12)$$

把式(3.12)代入式(2.1)后求得圆柱的振动位移

$$u_{1l} = a^l_1\cos\lambda_{1l}z + b^l_1\sin\lambda_{1l}z + c^l_1\operatorname{ch}\lambda_{1l}z + d^l_1\operatorname{sh}\lambda_{1l}z \qquad (3.13a)$$

$$u_{2l} = a^l_2\cos\lambda_{2l}z + b^l_2\sin\lambda_{2l}z + c^l_2\operatorname{ch}\lambda_{2l}z + d^l_2\operatorname{sh}\lambda_{2l}z - i\pi\omega R_l\rho_w$$

$$\cdot\left\{\frac{2\omega A_0\exp[ik_0b_l]J_1(k_0R_l)}{(EI_l\beta_0^4 - m_{sl}\omega^2)\beta_0\operatorname{sh}\beta_0(h_2 - h_1)}\operatorname{ch}\beta_0(z - h_1) + \sum_{n=1}^{\infty}\Big[A^l_{1n}R^{(1)}_{1n}(R_l)\right.$$

$$\left. + \sum_{j=1, j\neq l}^{N}\sum_{m=0}^{\infty}(A^j_{mn}T^{jl}_{m1} + B^j_{mn}V^{jl}_{m1})R^{(2)}_{1n}(R_l)\Big]\frac{\cos\beta_n(z - h_1)}{EI_l\beta_n^4 - m_{sl}\omega^2}\right\} \qquad (3.13b)$$

$$u_{3l} = a^l_3\cos\lambda_{2l}z + b^l_3\sin\lambda_{2l}z + c^l_3\operatorname{ch}\lambda_{2l}z + d^l_3\operatorname{sh}\lambda_{2l}z \qquad (3.13c)$$

$$v_l = a^l_4\cos\lambda_{3l}z + b^l_4\sin\lambda_{3l}z \qquad (3.13d)$$

式中，$\lambda_{1l} = [(\omega^2 m_{sl} - k_l)/EI_l]^{1/4}$，$\lambda_{2l} = (\omega^2 m_{sl}/EI_l)^{1/4}$，$\lambda_{3l} = \omega\sqrt{m_{sl}/EA_l}$

类似地由式(2.4)、(3.13c)、(3.13d)得平台的振动位移

$$u_M = -\sum_{j=1}^{N}\frac{\lambda_{2j}^3 EI_j}{\omega^2 M}(a^j_3\sin\lambda_{2j}h_0 - b^j_3\cos\lambda_{2j}h_0 + c^j_3\operatorname{sh}\lambda_{2j}h_0 + d^j_3\operatorname{ch}\lambda_{2j}h_0) \qquad (3.14a)$$

$$\theta_M = -\sum_{j=1}^{N}\frac{1}{\omega^2 J_0}\{\lambda_{2j}^2 EI_j[d_3^j(\lambda_{2j}h\sin\lambda_{2j}h_0 - \cos\lambda_{2j}h_0) - b_3^j(\lambda_{2j}h\sin\lambda_{2j}h_0 + \sin\lambda_{2j}h_0) + c_3^j(\lambda_{2j}h\,\mathrm{sh}\,\lambda_{2j}h_0 + \mathrm{ch}\,\lambda_{2j}h_0) + d_3^j(\lambda_{2j}h\,\mathrm{ch}\,\lambda_{2j}h_0 + \mathrm{sh}\,\lambda_{2j}h_0)] + EA_j b_j\lambda_{3j}(-a_4^j\sin\lambda_{3j}h_0 + b_4^j\cos\lambda_{3j}h_0)\} \tag{3.14b}$$

$$v_M = \sum_{j=1}^{N}\frac{\lambda_{3j}EA_j}{\omega^2 M}[-a_4^j\sin\lambda_{3j}h_0 + b_4^j\cos\lambda_{3j}h_0] \tag{3.14c}$$

3.3 积分常数的确定

式(3.9)、(3.13)中的常数 $A_{mn}^j, B_{mn}^j, a_j^l, b_j^l, c_j^l, d_j^l$ 可由边界条件和连续条件(2.3)、(2.7c)确定. 当 $r_l = R_l$ 时满足 $d_{jl} > r_l$, 把式(3.9)、(3.13b)代入式(2.7c)可得

$$\left.\begin{aligned}&\sum_{n=1}^{\infty}[A_{kn}^l I_{kn}^l + \sum_{j=1,j\neq l}^{N}\sum_{m=0}^{\infty}J_{kn}^l(A_{mn}^j T_{mk}^{jl} + B_{mn}^j V_{mk}^{jl})]\cos\beta_n(z-h_1) + i\omega\psi_{kl}(z) = 0\\&\sum_{n=1}^{\infty}[B_{kn}^l K_{kn}^l + \sum_{j=1,j\neq l}^{N}\sum_{m=0}^{\infty}L_{kn}^l(A_{mn}^j U_{mk}^{jl} + B_{mn}^j W_{mk}^{jl})]\cos\beta_n(z-h_1) = 0\end{aligned}\right\} \tag{3.15}$$

式中

$$\psi_{kl}(z) = \delta_{1k}[a_2^l\cos\lambda_{2l}(z-h_1) + b_2^l\sin\lambda_{2l}(z-h_1) + c_2^l\,\mathrm{ch}\,\lambda_{2j}(z-h_1) + d_2^l\,\mathrm{sh}\,\lambda_{2j}(z-h_1)] - (\delta_{1k}\alpha_0 + \alpha_{kl})A_0\,\mathrm{ch}\,\beta_0(z-h_1) \tag{3.16}$$

$$(I_{kn}^l, J_{kn}^l, K_{kn}^l, L_{kn}^l)^T = \left.\left(\frac{\partial R_{kn}^{(1)}}{\partial r_l} + \frac{\delta_{1k}R_{kn}^{(1)}}{EI_l\beta_n^4 - m_{sn}\omega^2}, \frac{\partial R_{kn}^{(2)}}{\partial r_l} + \frac{\delta_{1k}R_{kn}^{(2)}}{EI_l\beta_n^4 - m_{sn}\omega^2}, \frac{\partial R_{kn}^{(1)}}{\partial r_l}, \frac{\partial R_{kn}^{(2)}}{\partial r_l}\right)^T\right|_{r_l=R_l} \tag{3.17}$$

$$\alpha_0 = \frac{2i\pi\omega^2 R_l\rho_w J_1(k_0R_l)\exp[ik_0b_l]}{(EI_l\beta_0^4 - m_{sl}\omega^2)\beta_0\,\mathrm{sh}\,\beta_0(h_2-h_1)},\quad \alpha_{kl} = \frac{(i)^k\mathscr{E}_k\exp[ik_0b_l]}{\beta_0\,\mathrm{sh}\,\beta_0(h_2-h_1)}\frac{\partial J_k(k_0R_l)}{\partial r_l},\quad \delta_{1k} = \begin{cases}1, & k=1\\0, & k\neq 1\end{cases} \tag{3.18}$$

根据 Sturm_Liouville 本征值理论[9], 把 $\psi_{kl}(z)$ 在区间$[h_1, h_2]$展开为 $\cos\beta_n(z-h_1)$ 的级数

$$\left.\begin{aligned}&\psi_{kl}(z) = \sum_{n=1}^{\infty}C_{kl}^n\cos\beta_n(z-h_1),\quad C_{kl}^n = (a_{nl}^*a_2^l + b_{nl}^*b_2^l + c_{nl}^*c_2^l + d_{nl}^*d_2^l) - a_{kl}^nA_0\\&(a_{nl}^*, b_{nl}^*, c_{nl}^*, d_{nl}^*, a_{kl}^n)^T = \frac{4\beta_n}{2(h_2-h_1)\beta_n + \sin 2\beta_n(h_2-h_1)}\int_{h_1}^{h_2}(\cos\lambda_{2l}(z-h_1),\\&\sin\lambda_{2l}(z-h_1), \mathrm{ch}\,\lambda_{2l}(z-h_1), \mathrm{sh}\,\lambda_{2l}(z-h_1), (\delta_{1k}\alpha_0 + \alpha_{kl})\mathrm{ch}\,\beta_0(z-h_1))^T\cos\beta_n(z-h_1)dz\end{aligned}\right\} \tag{3.19}$$

由式(3.15)、(3.19)可得确定常数 A_{mn}^j, B_{mn}^j 的代数方程

$$\left.\begin{aligned}&I_{kn}^lA_{kn}^l + \sum_{j=1,j\neq l}^{N}\sum_{m=0}^{\infty}J_{kn}^l(A_{mn}^jT_{mk}^{jl} + B_{mn}^jV_{mk}^{jl}) + i\omega C_{kl}^n = 0\\&K_{kn}^lB_{kn}^l + \sum_{j=1,j\neq l}^{N}\sum_{m=0}^{\infty}L_{kn}^l(A_{mn}^jU_{mk}^{jl} + B_{mn}^jW_{mk}^{jl}) = 0\end{aligned}\right\} \tag{3.20}$$

式中, $k = 0, 1, 2, 3, \cdots; n = 1, 2, 3, \cdots, l = 1, 2, \cdots, N$.

由边界条件和连续条件(2.3)并利用柱和平台振动位移(3.13)、(3.14)可得

$$\left.\begin{aligned}&(a_1^l, b_1^l, c_1^l, d_1^l)^T = [B_1]^l(a_2^l, b_2^l, c_2^l, d_2^l)^T + \sum_{n=1}^{\infty}\{D_{nl}^1\}A_{1n}^l + \{D_l^1\}\\&(a_3^l, b_3^l, c_3^l, d_3^l)^T = [B_2]^l(a_2^l, b_2^l, c_2^l, d_2^l)^T + \sum_{n=1}^{\infty}\{D_{nl}^2\}A_{1n}^l + \{D_l^2\},\ a_4^l = 0\end{aligned}\right\} \tag{3.21}$$

$$[B]^l\{A_2^l\} - \sum_{j=1}^{N}[C]^j\{A_2^j + \sum_{n=1}^{\infty}(\{D_{nlj}A_{1n}^l - \sum_{j=1}^{N}\{D_{nj}A_{1n}^j) = \{D_l^*\} \qquad (3.22)$$

式中 $\{A_2^l\} = (a_2^l, b_2^l, c_2^l, d_2^l, b_4^l)^T$, $\{A_2^j\} = (a_2^j, b_2^j, c_2^j, d_2^j, b_4^j)^T$, $[B]^l$, $[C]^j$ 为 5 × 5 阶方阵, $\{D_l^*\}$, $\{D_{nlj}\}$, $\{D_{nj}\}$ 为 5 维列向量; $[B_1]^l$, $[B_2]^l$ 为 4 × 4 阶方阵, $\{D_{nl}^1\}$, $\{D_{nl}^2\}$, $\{D_l^1\}$, $\{D_l^2\}$ 为 4 维列向量• 以上矩阵和列向量的具体表达式由附录给出•

由式(3.20)、(3.22) 求得常数 A_{mn}^j, B_{mn}^j, $\{A_2^j\}$ $(j = 1, 2, \cdots, N)$ 后, 再由式(3.21) 求得常数 $a_1^j, \cdots, a_3^j, \cdots$, 便可确定满足系统所有基本方程, 边界条件和连续条件的精确解•

§4. 数值结果与讨论

本文对混凝土两等径圆柱—平台结构系统在浪流冲击下的动力响应作了数值计算, 给出了海洋波浪典型周期范围内平台水平位移幅值响应的结果(见图 3~6)• 两柱圆心连线与 y 轴重合, 计算参数如下, 混凝土: $E = 35$GPa, $\rho_{sl} = 2300$kg/m; 圆柱: $h_0 = 100$m, $h_2/h_0 = 0.9$, $R_{内} = 0.7R_a$(R_a 为外径); 土层: $k_{s1}/R_a = k_{s2}/R_a = 20$MPa; 水介质: $c = 1430$m/s, $\rho_w = 1000$kg/m^3; 平台: $h = h_0/35$, $l = b = 20h$• 其余参数由图中标明, 图中 U_c 为平台质心水平位移振幅, T 是两柱心距离, $\lambda_R = 2R_a/h_0$, $\lambda_M = M/\pi R_a^2 h_0 \rho_w$, $\lambda_h = h_1/h_2$•

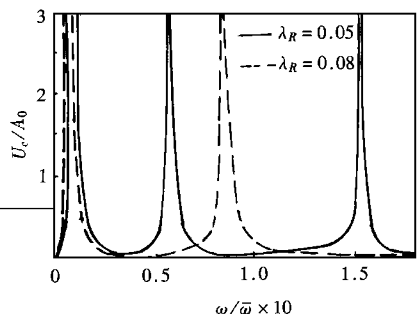

图 3 平台水平位移幅值响应
($T = 4R_a$, $\lambda_M = 0.6$, $\lambda_h = 0.30$)

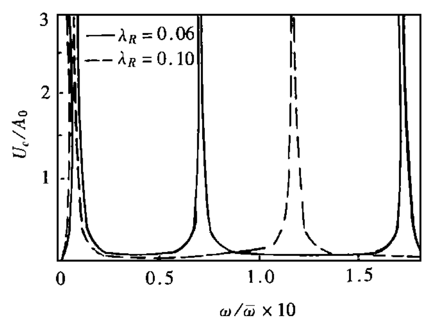

图 4 平台水平位移幅值响应
($T = 6R_a$, $\lambda_M = 1.0$, $\lambda_h = 0.40$)

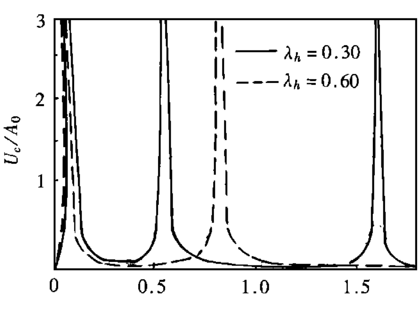

图 5 平台水平位移幅值响应
($T = 6R_a$, $\lambda_M = 0.6$, $\lambda_R = 0.05$)

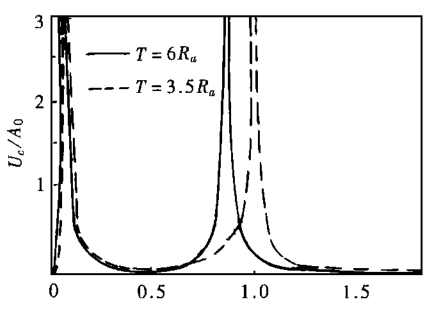

图 6 平台水平位移幅值响应
($\lambda_R = 0.08$, $\lambda_M = 0.6$, $\lambda_h = 0.30$)

从以上给出的计算结果可知, 平台水平位移幅值响应谱有多个峰值并存• 由于波浪力的激励频率小于第一截止频率, 水介质的辐射阻尼为系统的阻尼, 其值很小, 因而响应的峰值很大• 当土层厚度增加 (λ_n 增大) 或圆柱直径增大时, 峰值响应频率提高, 这是由于土层厚度增加减小了水深, 系统中水介质的附加质量减小, 而圆柱半径增大时, 抗弯刚度增加, 这两个因素均使系统固有频率提高而使峰值响应频率提高• 此外, 当两柱距离减小时, 峰值响应频率有所提高, 这反映了两柱对水波相互反射产生的相互作用•

附 录

$$[B]^l = \begin{bmatrix} B_l^1 & B_l^2 \\ B_l^3 & B_l^4 \end{bmatrix}, \quad [C]^j = \begin{bmatrix} C_j^1 & C_j^2 \\ C_j^3 & C_j^4 \end{bmatrix}, \quad \{D_{nl}\} = \begin{Bmatrix} D'_{nl} \\ D''_{nl} \end{Bmatrix}, \quad \{D_{nj}\} = \begin{Bmatrix} D'_{nj} \\ D''_{nj} \end{Bmatrix}$$

$$\{D_l^*\} = -\begin{Bmatrix} [D_l]\{D_l^2\} \\ [I^*]\{D_l^1\} \end{Bmatrix} + \sum_{j=1}^{N} \begin{Bmatrix} [E_j]\{D_j^2\} \\ 0 \end{Bmatrix}, \quad [B_1]^l = [G_1^l(h_1)]^{-1}[G_2^l(0)],$$

$$[B_2]^l = [G_3^l(h_2)]^{-1}[G_2^l(0)]$$

$$\{D_{nl}^1\} = [G_1^l(h_1)]^{-1}\{R_{nl}^2(0)\}, \{D_{nl}^2\} = [G_3^l(h_2)]^{-1}\{R_{nl}^2(h_2 - h_1)\}, \{D_n^1\} = [G_1^l(h_1)]^{-1}\{R_l^2(0)\}$$

$$\{D_n^2\} = [G_3^l(h_2)]^{-1}\{R_l^2(h_2 - h_1)\}, [B_n^1] = [D_l][B_2]^l, [B_l^2] = (0, 0, \sin\lambda_{3l}h_0)^T, [B_l^3] = [I^*][B_1]^l$$

$$[B_l^4] = (0, 0)^T, \quad [C_j^1] = [E_j][B_2]^j, \quad [C_j^2] = \alpha_j^2\left(-h\cos\lambda_{3j}h_0, \cos\lambda_{3j}h_0, \left(\frac{J_0}{Mb_j} - b_l\right)\cos\lambda_{3j}h_0\right)^T$$

$$[C_j^3] = (0)_{2\times 4}, \quad [C_j^4] = (0, 0)^T, \quad \{D'_{nl}\} = [D_l]\{D_{nl}^2\}, \quad \{D''_{nl}\} = [I^*]\{D_{nl}^1\}, \quad \{D'_{nj}\} = [E_j]\{D_{nj}^2\}$$

$$\{D''_{nj}\} = (0, 0)^T, \; [I^*] = \begin{bmatrix} 1, 0, 1, 0 \\ 0, 1, 0, 1 \end{bmatrix},$$

$$\{R_{nl}^2(z_2)\} = -\frac{i\pi\omega R_l\rho_w[J_{1n}^l R_{1n}^{(1)}(R_l) - I_{1n}^l R_{1n}^{(2)}(R_l)]}{J_{1n}^l(EI_l\beta_n^4 - m_{sl}\omega^2)}(\cos\beta_n z_2, -\beta_n\sin\beta_n z_2, -\beta_n^2\cos\beta_n z_2, \beta_n^3\sin\beta_n z_2)^T$$

$$\{R_l^2(z_2)\} = -\alpha_0 A_0(\mathrm{ch}\beta_0 z_2, \beta_0\mathrm{sh}\beta_0 z_2, \beta_0^2\mathrm{ch}\beta_0 z_2, \beta_0^3\mathrm{sh}\beta_0 z_2)^T + \sum_{n=0}^{\infty}\frac{\pi\omega^2 R_l\rho_w R_{1n}^{(2)}(R_l)a_{1l}^n A_0}{J_{1nl}^l(EI_l\beta_n^4 - m_{sl}\omega^2)}\cdot$$

$$(\cos\beta_n z_2, -\beta_n\sin\beta_n z_2, -\beta_n^2\cos\beta_n z_2, \beta_n^3\sin\beta_n z_2)^T,$$

$$\{R_{nl}^1(z_1)\} = \{R_{nl}^3(z_3)\} = \{R_l^1(z_1)\} = \{R_l^3(z_3)\} = \{0 \quad 0\}$$

$$[E_j] = \alpha_j^1\left[\begin{matrix} \dfrac{-\lambda_{2j}J_0}{M}\sin\lambda_{2j}h_0 + (\lambda_{2j}h\sin\lambda_{2j}h_0 - \cos\lambda_{2j}h_0), & \dfrac{\lambda_{2j}J_0}{M}\cos\lambda_{2j}h_0 - h(\lambda_{2j}h\cos\lambda_{2j}h_0 + \sin\lambda_{2j}h_0), \\ -(\lambda_{2j}h\sin\lambda_{2j}h_0 - \cos\lambda_{2j}h_0), & (\lambda_{2j}h\cos\lambda_{2j}h_0 + \sin\lambda_{2j}h_0), \\ -b_l(\lambda_{2j}h\sin\lambda_{2j}h_0 - \cos\lambda_{2j}h_0), & b_l(\lambda_{2j}h\cos\lambda_{2j}h_0 + \sin\lambda_{2j}h_0), \end{matrix}\right.$$

$$\left.\begin{matrix} \dfrac{-\lambda_{2j}J_0}{M}\mathrm{sh}\lambda_{2j}h_0 + h(\lambda_{2j}h\mathrm{sh}\lambda_{2j}h_0 + \mathrm{ch}\lambda_{2j}h_0), & \dfrac{-\lambda_{2j}J_0}{M}\mathrm{ch}\lambda_{2j}h_0 + h(\lambda_{2j}h\mathrm{ch}\lambda_{2j}h_0 + \mathrm{sh}\lambda_{2j}h_0) \\ -(\lambda_{2j}h\mathrm{sh}\lambda_{2j}h_0 + \mathrm{ch}\lambda_{2j}h_0), & -(\lambda_{2j}h\mathrm{ch}\lambda_{2j}h_0 + \mathrm{sh}\lambda_{2j}h_0) \\ -b_l(\lambda_{2j}h\mathrm{sh}\lambda_{2j}h_0 + \mathrm{ch}\lambda_{2j}h_0), & -b_l(\lambda_{2j}h\mathrm{ch}\lambda_{2j}h_0 + \mathrm{sh}\lambda_{2j}h_0) \end{matrix}\right]$$

$$[D_l] = \begin{bmatrix} \cos\lambda_{2l}h_0, & \sin\lambda_{2l}h_0, & \mathrm{ch}\lambda_{2l}h_0, & \mathrm{sh}\lambda_{2l}h_0, & 0 \\ -\lambda_{2l}\sin\lambda_{2l}h_0, & \lambda_{2l}\cos\lambda_{2l}h_0, & \lambda_{2l}\mathrm{sh}\lambda_{2l}h_0, & \lambda_{2l}\mathrm{ch}\lambda_{2l}h_0, & 0 \\ 0, & 0, & 0, & 0, & \sin\lambda_{3l}h_0 \end{bmatrix}$$

$$[G_j^l(z_j)] = \begin{bmatrix} \cos\lambda_{jl}z_j, & \sin\lambda_{jl}z_j, & \mathrm{ch}\,\lambda_{jl}z_j, & \mathrm{sh}\,\lambda_{jl}z \\ -\lambda_{jl}\sin\lambda_{jl}z_j, & \lambda_{jl}\cos\lambda_{jl}z_j, & \lambda_{jl}\mathrm{sh}\,\lambda_{jl}z_j, & \lambda_{jl}\mathrm{ch}\,\lambda_{jl}z_j \\ -\lambda_{jl}^2\mathrm{cosn}\,\lambda_{jl}z_j, & -\lambda_{jl}^2\sin\lambda_{jl}z_j, & \lambda_{jl}^2\mathrm{ch}\,\lambda_{jl}z_j, & \lambda_{jl}^2\mathrm{sh}\,\lambda_{jl}z_j \\ \lambda_{jl}^3\sin\lambda_{jl}z_j, & -\lambda_{jl}^3\cos\lambda_{jl}z_j, & \lambda_{jl}^3\mathrm{sh}\,\lambda_{jl}z_j, & \lambda_{jl}^3\mathrm{ch}\,\lambda_{jl}z_j \end{bmatrix} + \sum_{n=1}^{\infty} \frac{i\pi\omega\rho_w R_l R_{1n}^{(2)}(R_l)}{J_{1n}^l(EI_l\rho_n^4 - m_{sl}\omega^2)}$$

$$\cdot \begin{Bmatrix} \cos\beta_n z_j \\ -\beta_n\sin\beta_n z_j \\ -\beta_n^2\cos\beta_n z_j \\ \beta_n^3\sin\beta_n z_j \end{Bmatrix} \quad (j = 1, 2, 3, \lambda_{3l} = \lambda_{2l}), \quad \alpha_j^1 = \frac{EI_j\lambda_{2j}^2}{\omega^2 J_0}, \alpha_j^2 = \frac{\lambda_{3j} b_j EA_j}{\omega^2 J_0}$$

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Dynamic Response Analysis of Platform_Cylinder Group Foundation due to Impact by Water Wave Flow

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Abstract

This paper deals with the problem of dynamic response of platform_cylinder group foundation. Dynamic interaction of cylinder group foundation_water_soil is taken into account and the analysis of dynamic response to excitation of water wave force is given by analytic method. The numerical examples are presented and the influence of system's parameters on the dynamic behaviour is discussed.

Key words platform_cylinder group foundation system, interaction, excitation of water wave, dynamic behaviour