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非完善加筋圆柱壳在外压和热荷载共同作用下的后屈曲

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摘 要

基于壳体屈曲的边界层理论, 本文给出有限长加筋圆柱壳在侧向外压和均布热荷载共同作用下的后屈曲分析• 分析中同时考虑壳体非线性前屈曲变形, 大挠度和初始几何缺陷的影响• 肋条的处理采用“平均刚度”法• 采用奇异摄动方法导得壳体屈曲载荷关系曲线和后屈曲平衡路径, 并给出完善和非完善, 纵向加筋或环向加筋圆柱壳数值算例•

关键词 后屈曲 热后屈曲 加筋圆柱壳 复合加载 壳体屈曲边界层理论 奇异摄动法
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§1. 引 言

加筋圆柱壳广泛用于各种结构, 如宇航、石化及核能工程• 实际中壳体结构经常受到热荷载和机械荷载的共同作用, 且此类壳体结构不可避免地存在不同程度的初始几何缺陷, 因此, 有必要弄清非完善加筋圆柱壳在外压和热荷载共同作用下的后屈曲性态•

对于加筋圆柱壳在单纯轴压、外压及其共同作用下的屈曲和后屈曲已作过诸多研究, 但是对于加筋圆柱壳的热屈曲问题的研究却相对较少, 如 Chang 和 Card[1] 以及 Bushnell[2] 等• 据作者所知, 尚无公开发表的文献讨论非完善加筋圆柱壳在外压和热荷载共同作用下的后屈曲行为•

正如 Bushnell 和 Smith[3] 所指出, 壳体热屈曲问题(如同压缩问题一样) 存在边界层现象• 在壳体边界层中前屈曲和屈曲时的位移变化非常剧烈• 沈惠申和陈铁云[4, 5] 曾建议一个壳体屈曲的边界层理论, 这一理论可同时计及壳体非线性前屈曲变形, 大挠度和初始几何缺陷的影响, 发展了非线性大挠度理论• 基于这一理论沈惠申和陈铁云[4~6] 及沈惠申等[7, 8] 给出了完善和非完善, 加筋和未加筋圆柱壳在各类机械荷载作用下的后屈曲分析• 最近, 沈惠申[9] 又给出了完善和非完善, 加筋复合材料层合圆柱壳在均布和非均布热荷载作用下的后屈曲分析• 本文将这一工作进一步推广到非完善加筋圆柱壳在外压和均布热荷载共同作用时的情况• 肋条的处理采用“平均刚度”法• 壳体的材料性质假定与温度变化无关• 采用奇异摄动法导出壳体屈曲载荷关系曲线和后屈曲平衡路径• 分析中同时计及非线性前屈曲变形和初始几何

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缺陷的影响,初始几何缺陷的形式取作和圆柱壳初始屈曲模态一致•

§2. 基本方程

考虑半径为 R, 长度为 L, 壳板厚度为 t 的加筋圆柱壳承受侧向外压 q 和均布热荷载 T_0 的共同作用• 壳体几何参数与坐标系如图 1 所示• 设纵向肋条数为 n_s, 环向肋条数为 n_r• 取纵向肋条和环向肋条的截面面积分别为 A_1 和 A_2, 惯性矩分别为 I_1 和 I_2, 扭矩分别为 J_1 和 J_2, 偏心距分别为 e_1 和 e_2, 肋间间距分别为 d_1 和 d_2• 纵向肋条、环向肋条和壳板的弹性模数分别为 E_1, E_2 和 E• 通常纵肋、环肋和壳板的几何参数与材料常数可以不同• 取 U, V 和 W 为对应右手坐标系(X, Y, Z)的位移分量, 并以 $W^*(X, Y)$和 $W(X, Y)$分别表示初始的和附加的挠度, 以 $F(X, Y)$表示应力函数即使 $N_X = F_{,YY}$, $N_Y = F_{,XX}$和$N_{XY} = -F_{,XY}$•

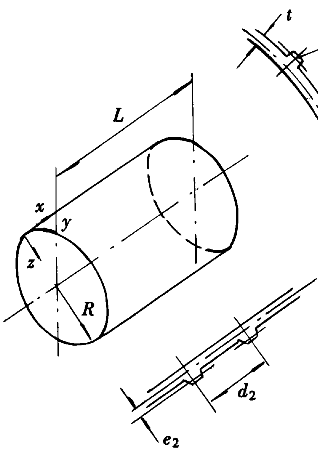

图 1 加筋圆柱壳几何参数及坐标系

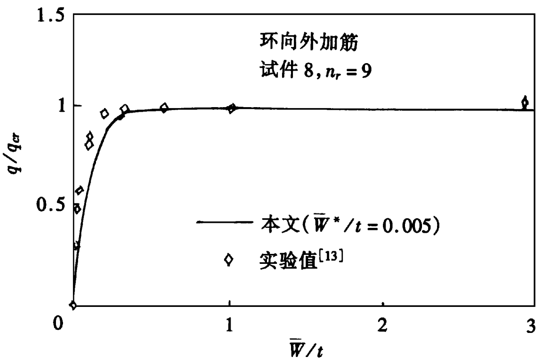

图 2 环向加筋圆柱壳在外压作用下,
后屈曲载荷—挠度曲线比较

根据经典壳体理论, 并计及热效应, 那么加筋圆柱壳大挠度方程可表为如下形式

$$L_1(W) + L_3(F) - L_4(N^T) - L_6(M^T) - \frac{1}{R}F_{,XX} = L(W + W^*, F) + q \qquad (2.1)$$

$$L_2(F) - L_3(W) - L_5(N^T) + \frac{1}{R}W_{,XX} = -\frac{1}{2}L(W + 2W^*, W) \qquad (2.2)$$

其中算子

$$L_1(\ ) = D_{11}\frac{\partial^4}{\partial X^4} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} + D_{22}\frac{\partial^4}{\partial Y^4}$$

$$L_2(\ ) = A_{22}\frac{\partial^4}{\partial X^4} + (2A_{12} + A_{66})\frac{\partial^4}{\partial X^2 \partial Y^2} + A_{11}\frac{\partial^4}{\partial Y^4}$$

$$L_3(\ ) = B_{21}\frac{\partial^4}{\partial X^4} + (B_{11} + B_{22})\frac{\partial^4}{\partial X^2 \partial Y^2} + B_{12}\frac{\partial^4}{\partial Y^4}$$

$$L_4(\ ) = (B_{11} + B_{21})\frac{\partial^2}{\partial X^2} + (B_{12} + B_{22})\frac{\partial^2}{\partial Y^2}$$

$$L_5(\ ) = (A_{12} + A_{22})\frac{\partial^2}{\partial X^2} + (A_{11} + A_{12})\frac{\partial^2}{\partial Y^2}$$

$$L_6(\ ) = \frac{\partial^2}{\partial X^2} + 2\frac{\partial^2}{\partial X\partial Y} + \frac{\partial^2}{\partial Y^2}$$

$$L(\ ) = \frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial X^2} - 2\frac{\partial^2}{\partial X\partial Y}\frac{\partial^2}{\partial X\partial Y} + \frac{\partial^2}{\partial Y^2}\frac{\partial^2}{\partial X^2}$$

式中抗弯、抗拉及耦合刚度在附录Ⅰ中给出•

由温度变化 T_0 引起的热力和热弯矩定义为

$$(N^T, M^T) = \frac{E\alpha}{1-\nu}\int_{-t/2}^{t/2}(1, Z)T_0 dz \tag{2.3}$$

式中, α 为壳体热膨胀系数•

注意, 方程(2.1)和(2.2)不仅包括拉伸和弯曲的耦合效应, 同时包括热引起的耦合效应•根据式(2.3), 我们有 $M^T = 0$ 和 $L_4(N^T) = L_5(N^T) = 0$•

单位端部缩短为

$$\frac{\Delta_X}{L} = -\frac{1}{2\pi RL}\int_0^{2\pi R}\int_0^L \frac{\partial U}{\partial X}dXdY = -\frac{1}{2\pi RL}\int_0^{2\pi R}\int_0^L\left\{\left[A_{11}\frac{\partial^2 F}{\partial Y^2} + A_{12}\frac{\partial^2 F}{\partial X^2}\right.\right.$$
$$\left.\left. - \left[B_{11}\frac{\partial^2 W}{\partial X^2} + B_{12}\frac{\partial^2 W}{\partial Y^2}\right] - \frac{1}{2}\left[\frac{\partial W}{\partial X}\right]^2 - \frac{\partial W}{\partial X}\frac{\partial W^*}{\partial X} - (A_{11} + A_{12})N^T\right\}dXdY \tag{2.4}$$

且我们有闭合条件(或称周期性条件)

$$\int_0^{2\pi R}\frac{\partial V}{\partial Y}dY = 0 \tag{2.5a}$$

即

$$\int_0^{2\pi R}\left\{\left[A_{22}\frac{\partial^2 F}{\partial X^2} + A_{12}\frac{\partial^2 F}{\partial Y^2}\right] - \left[B_{21}\frac{\partial^2 W}{\partial X^2} + B_{22}\frac{\partial^2 W}{\partial Y^2}\right]\right.$$
$$\left. + \frac{W}{R} - \frac{1}{2}\left[\frac{\partial W}{\partial Y}\right]^2 - \frac{\partial W}{\partial Y}\frac{\partial W^*}{\partial Y} - (A_{12} + A_{22})N^T\right\}dY = 0 \tag{2.5b}$$

壳体的两端边界假定受到约束阻止纵向热膨胀, 那么边界条件为

$X = 0, L;$

$$W = M_X = 0 \quad (\text{简支}) \tag{2.6a}$$

$$W = W_{,X} = 0 \quad (\text{固支}) \tag{2.6b}$$

$$U = 0 \tag{2.6c}$$

式(2.1)~(2.6)即为所讨论加筋圆柱壳后屈曲大挠度问题的控制方程•

§ 3. 分析方法与渐近解

引进无量纲参数(其中 λ_T^* 和 λ_q^* 仅用于数值分析)

$$x = \pi X/L,\ y = Y/R,\ \beta = L/\pi R,\ z = L^2/Rt,\ \varepsilon = (\pi^2 R/L^2)[D_{11}D_{22}A_{11}A_{22}]^{1/4},$$

$$(W, W^*) = \varepsilon(W, W^*)/[D_{11}D_{22}A_{11}A_{22}]^{1/4},\ F = \varepsilon^2 F/D_{11}D_{22}^{1/2},$$

$$\gamma_T = \frac{EtR}{1-\nu}[A_{11}A_{22}/D_{11}D_{22}]^{1/4},\ \gamma_{12} = (D_{12} + 2D_{66})/D_{11},\ \gamma_{14} = [D_{22}/D_{11}]^{1/2},$$

$$\gamma_{22} = (A_{12} + A_{66}/2)/A_{22},\ \gamma_{24} = [A_{11}/A_{22}]^{1/2},\ \gamma_5 = -A_{12}/A_{22},$$

$$(\gamma_{30}, \gamma_{32}, \gamma_{34}, \gamma_{311}, \gamma_{322}) = (B_{21}, B_{11} + B_{22}, B_{12}, B_{11}, B_{22})/[D_{11}D_{22}A_{11}A_{22}]^{1/4}$$

$$M_X = \varepsilon^2 M_X L^2/\pi^2 D_{11}[D_{11}D_{22}A_{11}A_{22}]^{1/4},\ \lambda_T = \alpha T_0,\ \lambda_T^* = \alpha T_0\times 10^3,$$
$$\lambda_q = q(3)^{3/4}LR^{3/2}[A_{11}A_{22}]^{1/8}/4\pi[D_{11}D_{22}]^{3/8},$$
$$\lambda_q^* = q(3)^{3/2}LR^{3/2}[1-\nu^2]^{3/4}/\pi E t^{5/2}(2)^{1/2},$$
$$\delta_T = (\Delta_X/L)/(2/R)[D_{11}D_{22}A_{11}A_{22}]^{1/4},$$
$$\delta_q = (\Delta_X/L)(3)^{3/4}LR^{1/2}/4\pi[D_{11}D_{22}A_{11}A_{22}]^{3/8} \tag{3.1}$$

那么,方程(2.1)和(2.2)可化为如下无量纲形式

$$\varepsilon^2 L_1(W) + \varepsilon\gamma_{14}L_3(F) - \gamma_{14}F_{,XX} = \gamma_{14}\beta^2 L(W+W^*, F) + \gamma_{14}\frac{4}{3}(3)^{1/4}\lambda_q\varepsilon^{3/2} \tag{3.2}$$

$$L_2(F) - \varepsilon\gamma_{24}L_3(W) + \gamma_{24}W_{,XX} = -\frac{1}{2}\gamma_{24}\beta^2 L(W+2W^*, W) \tag{3.3}$$

其中

$$L_1(\ ) = \frac{\partial^4}{\partial x^4} + 2\gamma_{12}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} + \gamma_{14}^2\beta^4\frac{\partial^4}{\partial y^4}$$
$$L_2(\ ) = \frac{\partial^4}{\partial x^4} + 2\gamma_{22}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} + \gamma_{24}^2\beta^4\frac{\partial^4}{\partial y^4}$$
$$L_3(\ ) = \gamma_{30}\frac{\partial^4}{\partial x^4} + \gamma_{32}\beta^2\frac{\partial^4}{\partial x^2\partial y^2} + \gamma_{34}\beta^4\frac{\partial^4}{\partial y^4}$$
$$L(\ ) = \frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} - 2\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y} + \frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}$$

单位端部缩短化为

$$\delta_q = -\frac{(3)^{3/4}}{8\pi^2\gamma_{24}}\varepsilon^{-3/2}\int_0^{2\pi}\int_0^{\pi}\left\{\left[\gamma_{24}^2\beta^2\frac{\partial^2 F}{\partial y^2} - \gamma_5\frac{\partial^2 F}{\partial x^2}\right] - \varepsilon\gamma_{24}\left[\gamma_{311}\frac{\partial^2 W}{\partial x^2}\quad A \right.\right.$$
$$\left.\left. + \gamma_{34}\beta^2\frac{\partial^2 W}{\partial y^2}\right] - \frac{1}{2}\gamma_{24}\left[\frac{\partial W}{\partial x}\right]^2 - \gamma_{24}\frac{\partial W}{\partial x},\frac{\partial W^*}{\partial x} + (\gamma_{24}^2 - \gamma_5)\gamma_T\lambda_T\varepsilon\right\}dxdy \tag{3.4a}$$

或

$$\delta_T = -\frac{1}{4\pi^2\gamma_{24}}\varepsilon^{-1}\int_0^{2\pi}\int_0^{\pi}\left\{\left[\gamma_{24}^2\beta^2\frac{\partial^2 F}{\partial y^2} - A\gamma_5\frac{\partial^2 F}{\partial x^2}\right. - \varepsilon\gamma_{24}\left[\gamma_{311}\frac{\partial^2 W}{\partial x^2}\right.\right.$$
$$\left.\left. + \gamma_{34}\beta^2\frac{\partial^2 W}{\partial y^2}\right] - \frac{1}{2}\gamma_{24}\left[\frac{\partial W}{\partial x}\right]^2 - \gamma_{24}\frac{\partial W}{\partial x}\frac{\partial W^*}{\partial x} + (\gamma_{24}^2 - \gamma_5)\gamma_T\lambda_T\varepsilon\right\}dxdy \tag{3.4b}$$

闭合条件化为

$$\int_0^{2\pi}\left\{\left[\frac{\partial^2 F}{\partial x^2} - \gamma_5\beta^2\frac{\partial^2 F}{\partial y^2}\right] - \varepsilon\gamma_{24}\left[\gamma_{30}\frac{\partial^2 W}{\partial x^2} + \gamma_{322}\beta^2\frac{\partial^2 W}{\partial y^2}\right]\right.$$
$$\left. + \gamma_{24}W - \frac{1}{2}\gamma_{24}\beta^2\left[\frac{\partial W}{\partial y}\right]^2 - \gamma_{24}\beta^2\frac{\partial W}{\partial y}\frac{\partial W^*}{\partial y} + (1-\gamma_5)\gamma_T\lambda_T\varepsilon\right\}dy = 0 \tag{3.5}$$

边界条件化为

$x = 0,\ \pi$;

$$W = M_x = 0\ (\text{简支}) \tag{3.6a}$$
$$W = W_{,x} = 0\ (\text{固支}) \tag{3.6b}$$

$$\delta_q(\text{或}\ \delta_T)=0 \tag{3.6c}$$

对于未加筋圆柱壳, 根据式(3.1)我们有 $\varepsilon=\pi^2/z_B\sqrt{12}$, 其中 $z_B=(L^2/Rt)[1-\nu^2]^{1/2}$ 为Batdorf 壳体参数, 在经典壳体屈曲分析中 z_B 必须大于 2.85(参见 Batdorf[10]• 由 $z_B>2.85$, 导出 $\varepsilon<1$, 因此方程(3.2)和(3.3)即为边界层型方程• 采用奇异摄动方法可构造其大挠度渐近解•

设方程(3.2)和(3.3)的解可表为

$$\begin{aligned} W &= w(x,y,\varepsilon)+\overline{W}(x,\xi,y,\varepsilon)+\widetilde{W}(x,\zeta,y,\varepsilon)\\ F &= f(x,y,\varepsilon)+\overline{F}(x,\xi,y,\varepsilon)+\widetilde{F}(x,\zeta,y,\varepsilon) \end{aligned} \tag{3.7}$$

其中, ε 为摄动小参数, $w(x,y,\varepsilon)$, $f(x,y,\varepsilon)$ 称为壳体"外解"或正则解, $\overline{W}(x,\xi,y,\varepsilon)$, $\overline{F}(x,\xi,y,\varepsilon)$ 和 $\widetilde{W}(x,\zeta,y,\varepsilon)$, $\widetilde{F}(x,\zeta,y,\varepsilon)$ 分别为壳体在 $x=0$ 和 $x=\pi$ 端的边界层解, 且边界层变量

$$\xi=x/\sqrt{\varepsilon},\quad \zeta=(\pi-x)/\sqrt{\varepsilon} \tag{3.8}$$

(这意味着对于未加筋圆柱壳, 边界层宽度为 $\sqrt{Rt}$ 量级)• 在式(3.7)中设正则解和边界层解为如下渐近展开

$$w(x,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2}w_{j/2}(x,y),\quad f(x,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2}f_{j/2}(x,y) \tag{3.9a}$$

$$\overline{W}(x,\xi,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2+1}\overline{W}_{j/2+1}(x,\xi,y),\quad \overline{F}(x,\xi,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2+2}\overline{F}_{j/2+2}(x,\xi,y) \tag{3.9b}$$

$$\widetilde{W}(x,\zeta,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2+1}\widetilde{W}_{j/2+1}(x,\zeta,y),\quad \widetilde{F}(x,\zeta,y,\varepsilon)=\sum_{j=0}\varepsilon^{j/2+2}\widetilde{F}_{j/2+2}(x,\zeta,y) \tag{3.9c}$$

取壳体小挠度经典解

$$w_2(x,y)=A_{11}^{(2)}\sin mx\sin ny \tag{3.10}$$

并假定壳体初始几何缺陷与屈曲模态一致, 即

$$w^*(x,y,\varepsilon)=\varepsilon^2A_{11}^*\sin mx\sin ny=\varepsilon^2\mu A_{11}^{(2)}\sin mx\sin ny \tag{3.11}$$

其中, $\mu=A_{11}^*/A_{11}^{(2)}$ 为缺陷参数•

将式(3.7)~(3.9)代入方程(3.2)和(3.3)导出一系列摄动方程组, 并可逐级求解•

正如文[9, 4]所指出, 对于受热屈曲圆柱壳边界层对解的影响为 ε^1 级, 对于受外压屈曲圆柱壳边界层对解的影响为 $\varepsilon^{3/2}$ 级• 因此, 在外压和热荷载复合加载时必须考虑两种加载条件•

第(1)种加载情况, 外压较大而温度变化相对较小•

令

$$\frac{2\lambda_c\varepsilon}{\frac{4}{3}(3)^{1/4}\lambda_q\varepsilon^{3/2}}=\frac{b_1}{2} \tag{3.12}$$

其中, $\lambda_c=\sigma_xRt[A_{11}A_{22}/D_{11}D_{22}]^{1/4}/2$, σ_x 为由温度变化 T_0 引起的平均轴向压应力, 且我们有 $\lambda_c=(\gamma_{24}^2-\gamma_5)\gamma_T\lambda_T/2\gamma_{24}^2$(这意味着对于未加筋圆柱壳 $\sigma_x=E\alpha T_0$)•

利用式(3.10)和(3.11)求解各级摄动方程, 并在壳体两端匹配正则解和边界层解, 我们可以导得满足固支边界条件的大挠度渐近解

$$W = \varepsilon^{3/2}\left[A_{00}^{(3/2)} - A_{00}^{(3/2)}\left(\cos\phi\frac{x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right)\right. \qquad 1$$

$$- A_{00}^{(3/2)}\left(\cos\phi\frac{\pi - x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{\pi - x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi - x}{\sqrt{\varepsilon}}\right)\Bigg]$$

$$+ \varepsilon^2\left[A_{11}^{(2)}\sin mx\sin ny\right] + \varepsilon^3\left[A_{11}^{(3)}\sin mx\sin ny\right]$$

$$+ \varepsilon^4\left[A_{11}^{(4)} + A_{20}^{(4)}\cos 2mx + A_{02}^{(4)}\cos 2ny\right] + O(\varepsilon^5) \tag{3.13}$$

$$F = -\frac{1}{2}B_{00}^{(0)}\left(\beta^2 x^2 + b_1\frac{y^2}{2}\right) + \varepsilon^2\left[-\frac{1}{2}B_{00}^{(2)}\left(\beta^2 x^2 + b_1\frac{y^2}{2}\right) + B_{11}^{(2)}\sin mx\sin ny\right]$$

$$+ \varepsilon^{5/2}\left[A_{00}^{3/2}\left(\gamma_{24}\left(\frac{1}{b} + \gamma_{30}\right)\cos\phi\frac{x}{\sqrt{\varepsilon}} - \gamma_{24}\left(\frac{1}{b} - \gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right)\right.$$

$$+ A_{00}^{3/2}\left(\gamma_{24}\left(\frac{1}{b} + \gamma_{30}\right)\cos\phi\frac{\pi - x}{\sqrt{\varepsilon}} - \gamma_{24}\left(\frac{1}{b} - \gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{\pi - x}{\sqrt{\varepsilon}}\right)$$

$$\cdot \exp\left(-\alpha\frac{\pi - x}{\sqrt{\varepsilon}}\right)\Bigg] + \varepsilon^3\left[-\frac{1}{2}B_{00}^{(3)}\left(\beta^2 x^2 + b_1\frac{y^2}{2}\right)\right]$$

$$+ \varepsilon^4\left[-\frac{1}{2}B_{00}^{(4)}\left(\beta^2 x^2 + b_1\frac{y^2}{2}\right) + B_{11}^{(4)}\sin mx\sin ny\right.$$

$$\left. + B_{20}^{(4)}\cos 2mx + B_{02}^{(4)}\cos 2ny\right] + O(\varepsilon^5) \tag{3.14}$$

将式(3.13)和(3.14)代入边界条件(3.6c)并利用式(3.4a)进而可导得后屈曲平衡路径

$$\lambda_q = \frac{1}{4}(3)^{3/4}\varepsilon^{-3/2}\left[\lambda_q^{(0)} + \lambda_q^{(2)}(A_{11}^{(2)}\varepsilon^2)^2 + \cdots\right] \tag{3.15}$$

注意，在式(3.13)和(3.14)中所有系数皆相互关联并可表为 $A_{11}^{(2)}$ 的函数· 在式(3.15)中可将 $(A_{11}^{(2)}\varepsilon^2)$ 取作二次摄动参数· 如若最大挠度设在 $(x, y) = (\pi/2m, \pi/2n)$，那么

$$A_{11}^{(2)}\varepsilon^2 = w_m - \Theta_1 w_m^2 + \cdots \tag{3.16a}$$

其中，w_m 为壳体最大无量纲挠度，并可表为

$$w_m = \left[\varepsilon\frac{t}{\sqrt[4]{D_{11}D_{22}A_{11}A_{22}}}\frac{w}{t} - \Theta_3\lambda_q^{(\Delta)}\right]\left[1 - \frac{g_3}{m^2}\varepsilon\right] \tag{3.16b}$$

第(2)种加载情况，温度变化较大而外压相对较小·

令

$$\frac{\frac{4}{3}(3)^{1/4}\lambda_q\varepsilon^{3/2}}{2\lambda_c\varepsilon} = 2b_2 \tag{3.17}$$

将式(3.17)代入方程(3.2)，并取 $a_2 = 2b_2$，采用奇异摄动方法，类似地可导得满足固支边界条件的大挠度渐近解·

$$W = \varepsilon\left[A_{00}^{(1)} - A_{00}^{(1)}\left(\cos\phi\frac{x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right)\right.$$

$$\left. - A_{00}^{(1)}\left(\cos\phi\frac{\pi - x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{\pi - x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi - x}{\sqrt{\varepsilon}}\right)\right]$$

$$+ \varepsilon^2\left[A_{11}^{(2)}\sin mx\sin ny + A_{02}^{(2)}\cos 2ny - (A_{02}^{(2)}\cos 2ny)\right.$$

$$\cdot\left(\cos\phi\frac{x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right) - (A_{02}^{(2)}\cos 2ny)$$

$$\cdot\left(\cos\phi\frac{\pi - x}{\sqrt{\varepsilon}} + \frac{\alpha}{\phi}\sin\phi\frac{\pi - x}{\sqrt{\varepsilon}}\right)\exp\left(-\alpha\frac{\pi - x}{\sqrt{\varepsilon}}\right)\Bigg]$$

$$+\ \varepsilon^3[A_{11}^{(3)}\sin mx\sin ny+A_{02}^{(3)}\cos 2ny]+\varepsilon^4[A_{00}^{(4)}+A_{20}^{(4)}\cos 2mx$$
$$+A_{02}^{(4)}\cos 2ny+A_{13}^{(4)}\sin mx\sin 3ny+A_{04}^{(4)}\cos 4ny]+O(\varepsilon^5) \qquad (3.18)$$

$$F=-\frac{1}{2}B_{00}^{(0)}(y^2+a_2\beta^2x^2)+\varepsilon\left[-\frac{1}{2}B_{00}^{(1)}(y^2+a_2\beta^2x^2)\right]$$
$$+\ \varepsilon^2\left[-\frac{1}{2}B_{00}^{(2)}(y^2+a_2\beta^2x^2)+B_{11}^{(2)}\sin mx\sin ny\right.$$
$$+A_{00}^{(1)}\left\{\gamma_{24}\left(\frac{1}{b}+\gamma_{30}\right)\cos\phi\frac{x}{\sqrt{\varepsilon}}-\gamma_{24}\left(\frac{1}{b}-\gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right\}\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right)$$
$$+A_{00}^{(1)}\left\{\gamma_{24}\left(\frac{1}{b}+\gamma_{30}\right)\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}-\gamma_{24}\left(\frac{1}{b}-\gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right\}$$
$$\left.\cdot\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\right]+\varepsilon^3\left[-\frac{1}{2}B_{00}^{(3)}(y^2+a_2\beta^2x^2)+B_{02}^{(3)}\cos 2ny\right.$$
$$+(A_{02}^{(2)}\cos 2ny)\left\{\gamma_{24}\left(\frac{1}{b}+\gamma_{30}\right)\cos\phi\frac{x}{\sqrt{\varepsilon}}-\gamma_{24}\left(\frac{1}{b}-\gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{x}{\sqrt{\varepsilon}}\right\}$$
$$\cdot\exp\left(-\alpha\frac{x}{\sqrt{\varepsilon}}\right)+(A_{02}^{(2)}\cos 2ny)\left\{\gamma_{24}\left(\frac{1}{b}+\gamma_{30}\right)\cos\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right.$$
$$\left.\left.-\gamma_{24}\left(\frac{1}{b}-\gamma_{30}\right)\frac{\alpha}{\phi}\sin\phi\frac{\pi-x}{\sqrt{\varepsilon}}\right\}\exp\left(-\alpha\frac{\pi-x}{\sqrt{\varepsilon}}\right)\right]$$
$$+\ \varepsilon^4\left[-\frac{1}{2}B_{00}^{(4)}(y^2+a_2\beta^2x^2)+B_{11}^{(4)}\sin mx\sin ny+B_{20}^{(4)}\cos 2mx\right.$$
$$\left.+B_{02}^{(4)}\cos 2ny+B_{13}^{(4)}\sin mx\sin 3ny\right]+O(\varepsilon^5) \qquad (3.19)$$

将式(3.18)和(3.19)代入边界条件(3.6c)并利用(3.4b),我们进而可以导得热后屈曲平衡路径

$$\lambda_T=C_{11}[\lambda_T^{(0)}-\lambda_T^{(2)}(A_{11}^{(2)}\varepsilon)^2+\lambda_T^{(4)}(A_{11}^{(2)}\varepsilon)^4\cdots] \qquad (3.20)$$

类似地,在式(3.20)中$(A_{11}^{(2)}\varepsilon)$可取作二次摄动参数,且可表为

$$A_{11}^{(2)}\varepsilon=w_m-\Theta_2w_m^2+\cdots \qquad (3.21a)$$

相应的最大无量纲挠度可表为

$$w_m=\left[\frac{t}{\sqrt[4]{D_{11}D_{22}A_{11}A_{22}}}\frac{w}{t}+\Theta_4\lambda_T^{(0)}\right]\Big/\left[1-\frac{g_3}{m^2}\varepsilon\right] \qquad (3.21b)$$

所有在式(3.15)~(3.16)和式(3.20)~(3.21)中所用的符号在附录Ⅱ中给出•

式(3.15)~(3.16)和(3.20)~(3.21)可用于加筋圆柱壳在外压和均布热荷载共同作用下的后屈曲载荷—挠度曲线计算•加筋圆柱壳在外压单独作用下的后屈曲以及在均布热荷载单独作用下的后屈曲可视作本文的两种特例•加筋圆柱壳的屈曲载荷关系曲线可通过分别增加荷载比参数b_1和b_2得到两条曲线来构成•注意到$b_2=1/b_1$,因此仅有一个荷载比参数需要由实验确定•在式(3.15)或(3.20)中,取$\mu=0$(或$w^*/t=0$),并取$w_m=0$(或$w/t=0$)容易得到完善壳体的屈曲载荷,即对应不同屈曲模态(m,n)时的最小屈曲载荷,其中(m,n)分别为x方向的半波数和y方向的全波数•由式(3.13)和(3.18)可以看出,由于边界层的贡献,壳体前屈曲变形是非线性的,因此本文得到的屈曲载荷与小挠度经典解是有差别的•

§4. 数值结果与讨论

本文给出了加筋圆柱壳在外压和均布热荷载共同作用下的后屈曲分析•数值算例讨论完

善和非完善, 纵向加筋或环向加筋圆柱壳在多种荷载组合时的情况• 主要结果以无量纲图示形式给出• 在所示算例中(除图2外) Poisson 比取 $\nu=0.3$, 热膨胀系数取 $\alpha=1.0\times10^{-6}/℃$•

表 1 未加筋圆柱壳热屈曲载荷比较

($R/t=289.7765$, $z=24814.9381$, $\nu=0.3$, $\alpha=8.4\times10^{-6}/°F$)

壳体	实验值 Ross et al.[11]	本文
1006 冷轧钢	227	221

表 2 加筋圆柱壳在静水外压作用下屈曲载荷比较

($R/t=82.1693$, $z=1692.9697$, $A_1/d_1t=A_2/d_2t=0.1471$,

$I_1/d_1t^3=I_2/d_2t^3=0.0652$, $e_1/t=e_2/t=\pm1.653$, $\nu=0.3$)

壳 体	Baruch and Singer[12]	本 文
未加筋	102	100.7(1, 4) [a]
环向外加筋	326	325.7(1, 3)
环向内加筋	370	368.3(1, 3)
纵向外加筋	106	106.2(1, 4)
纵向内加筋	103	102.2(1, 4)
正交外加筋	346	343.(1, 3)
正交内加筋	377	374.1(1, 3)

a 括号里的值表示屈曲模态(m, n)•

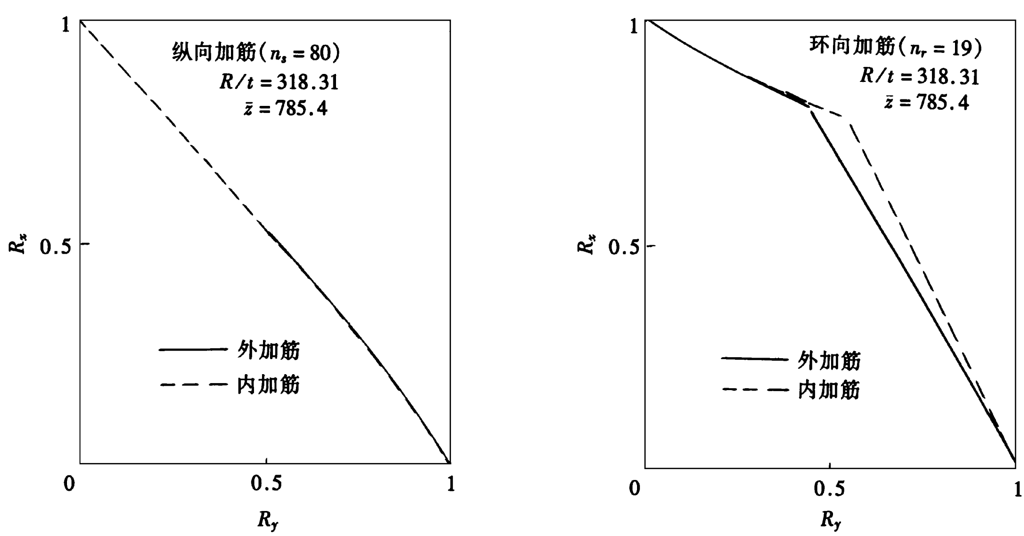

图 3 加筋圆柱壳在外压和均布热荷载共同作用下屈曲载荷关系曲线

作为对本文分析方法的部分验证, 表 1 给出未加筋圆柱壳在单纯均匀温度变化作用下的热屈曲载荷, 并与 Ross 等[11] 实验结果有很好的符合• 表 2 给出加筋和未加筋圆柱壳在单纯静水外压作用下的屈曲载荷, 并与 Baruch 和 Singer[12] 的计算结果有很好的符合• 此外, 图 2

给出环向外加筋圆柱壳在单纯侧向外压作用下的后屈曲载荷—挠度曲线· 可以看出, 当考虑不大的初始几何缺陷时, 本文结果与 Seleim 和 Roorda[13] 实验结果有相当合理的符合·

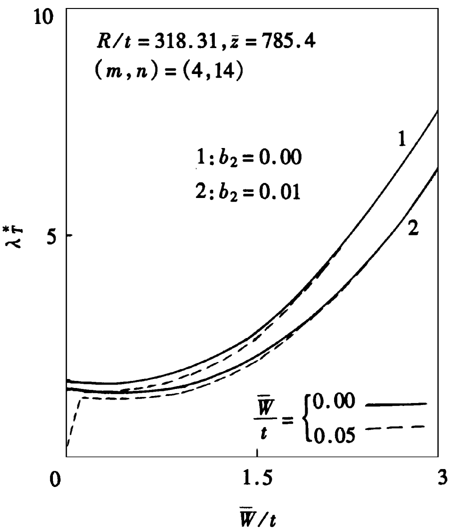

图 4　未加筋圆柱壳在复合加载下, 热后屈曲载荷—挠度曲线

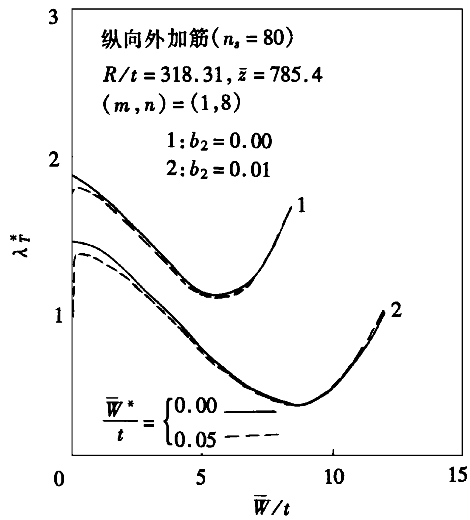

图 5　纵向加筋圆柱壳在复合加载下, 热后屈曲载荷—挠度曲线

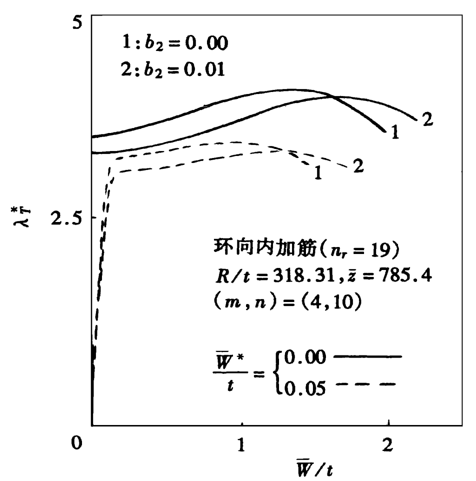

图 6　环向加筋圆柱壳在复合加载下, 热后屈曲载荷—挠度曲线

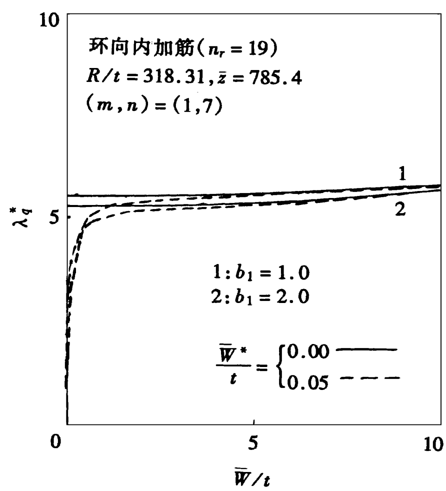

图 7　环向加筋圆柱壳在复合加载下, 后屈曲载荷—挠度曲线

图 3 给出外加筋或内加筋, 纵向加筋或环向加筋圆柱壳($R/t = 318.31$, $z = 785.4$) 在外压和均布热荷载共同作用下的屈曲载荷关系曲线· 肋条的几何和材料参数为: $E_1/E = E_2/E = 1.0$, $A_1/d_1t = A_2/d_2t = 0.048$, $I_1/d_1t^3 = I_2/d_2t^3 = 0.288$, $e_1/t = e_2/t = \pm 3.5$ 和 $G_1J_1/d_1D = G_2J_2/d_2D = 0.000672$· 计算表明, 对于纵向加筋圆柱壳($n_s = 80$), 侧向外压单独作用时对应屈曲模态$(m, n) = (1, 9)$, 均布热荷载单独作用时对应屈曲模态$(m, n) =$

(1, 8); 对于环向加筋圆柱壳(n_r = 19), 侧向外压单独作用时对应屈曲模态(m, n) = (1, 7), 均布热荷载单独作用时对应屈曲模态(m, n) = (4, 11) 或(4, 10)• 可以看出, 随着荷载比参数 b_2(或 b_1) 的增加, 屈曲模态将发生改变• 同时可以看出, 纵向加筋圆柱壳与环向加筋圆柱壳屈曲载荷关系曲线有很大差别•

图 4 给出完善和非完善, 未加筋圆柱壳在第(2) 种加载情况下对应 b_2 = 0.0, 0.01 时的热后屈曲载荷—挠度曲线• 图5和图6分别给出纵向外加筋或环向内加筋圆柱壳在第(2) 种加载情况下的热后屈曲载荷—挠度曲线• 可以看出, 随着荷载比参数 b_2 的增加, 热屈曲载荷减小, 且对于纵向筋圆柱壳后屈曲平衡路径变得相当低, 而对于未加筋圆柱壳后屈曲平衡路径仅有不大的下降• 计算结果表明, 当温度变化超过临界值时未加筋圆柱壳的热后屈曲载荷—挠度曲线显著向上翘, 因而壳结构变得对初始几何缺陷表现不敏感• 相反, 对于纵向加筋圆柱壳仍具有常见的"跳跃"式后屈曲响应, 因而可计算相应的缺陷敏感度•

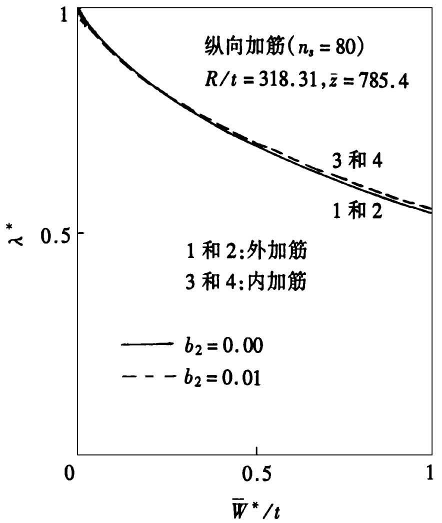

图 8 纵向加筋圆柱壳在复合加载下, 缺陷敏感度曲线

图 7 给出环向内加筋圆柱壳在第(1) 种加载情况下对应 b_1 = 1.0, 2.0 时后屈曲载荷—挠度曲线• 可以看出, 在第(1) 种加载情况下, 环向内加筋圆柱壳的后屈曲平衡路径是稳定的, 而在第(2) 种加载情况下, 后屈曲平衡路径从稳定转向不稳定(见图 6)•

图8 给出纵向加筋圆柱壳在第(2) 种加载情况下的缺陷敏感度曲线• λ^* = (非完善壳的 λ_T)/(完善壳的 λ_T)• 图示表明, 纵向内加筋比纵向外加筋圆柱壳的缺陷敏感度略小, 但两者差别不大• 在不同加载此时壳体的缺陷敏感度几乎相同•

§5. 结 论

本文将壳体屈曲的边界层理论发展到用于非完善加筋圆柱壳在外压和均布热荷载共同作用下后屈曲分析• 算例给出完善和非完善, 纵向加筋或环向加筋圆柱壳的数值结果• 计算结果表明, 在外压和均布热荷载共同作用下, 壳体后屈曲行为取决于加筋方式, 壳体初始几何缺陷和荷载比参数• 在第(2) 种加载条件的许多情况下, 未加筋圆柱壳的热后屈曲平衡路径是稳定的, 此时壳结构对初始几何缺陷表现不敏感• 这一结果或许可以用来解释为什么在实验研究中我们经常获得比较高的临界温度值• 相反, 对于纵向加筋圆柱壳, 当热荷载为主要作用时具有常见的'跳跃'式后屈曲性态, 此时壳结构对初始几何缺陷是敏感的• 一般说来, 环向加筋圆柱壳后屈曲性态是稳定的, 在第(2) 种加载情况下, 有可能发生从稳定到不稳定的转变•

附　录 I

式(2.1)和(2.2)中各刚度系数定义为

$$D_{11}=Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{E_1}{E}\frac{I_1}{d_1t^3}+\frac{E_1}{E}\frac{A_1}{d_1t}\left(\frac{e_1}{t}\right)^2-\frac{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)^2(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_{22}=Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{E_2}{E}\frac{I_2}{d_2t^3}+\frac{E_2}{E}\frac{A_2}{d_2t}\left(\frac{e_2}{t}\right)^2-\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)^2(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_{12}=\nu Et^3\left[\frac{1}{12(1-\nu^2)}+\frac{\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}\right]$$

$$D_{66}=\frac{D}{2}\left[(1-\nu)+\frac{1}{2}\left(\frac{G_1J_1}{d_1D}+\frac{G_2J_2}{d_2D}\right)\right]$$

$$A_{11}=\frac{1}{Et}\frac{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right](1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$A_{22}=\frac{1}{Et}\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right](1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$A_{12}=-\frac{\nu}{Et}\frac{(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$A_{66}=\frac{1}{Gt}$$

$$B_{11}=-t\frac{\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]\left(\frac{E_1}{E}\frac{A_1}{d_1t}\frac{e_1}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$B_{22}=-t\frac{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$B_{12}=\nu t\frac{\left(\frac{E_2}{E}\frac{A_2}{d_2t}\frac{e_2}{t}\right)(1-\nu^2)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2t}(1-\nu^2)\right]-\nu^2}$$

$$B_{21} = \nu t \frac{\left(\frac{E_1}{E}\frac{A_1}{d_1 t}E_1\frac{e_1}{t}(1-\nu^2)\right)}{\left[1+\frac{E_1}{E}\frac{A_1}{d_1 t}(1-\nu^2)\right]\left[1+\frac{E_2}{E}\frac{A_2}{d_2 t}(1-\nu^2)\right]-\nu^2}$$

在式(3. 15)~(3. 16)中

$$g_T = \left[(\gamma_{24}^2-\gamma_5)+\frac{4}{\pi}\frac{\alpha}{b}\gamma_5(1-\gamma_5)\varepsilon^{1/2}\right]\gamma_T$$

$$\Theta_1 = \left\{\Theta_3\lambda_q^{(2)}-\frac{1}{8}\frac{(1-\gamma_5)\gamma_T}{g_T}\left[m^2\left(1+2\mu+\frac{1}{\pi\alpha}\varepsilon^{1/3}-2g_3\varepsilon+\frac{g_3^2}{m^2}\varepsilon^2\right)\right]\right\}\Big\backslash\left[1-\frac{g_3}{m^2}\varepsilon\right]$$

$$\Theta_3 = \frac{\gamma_{24}^2-\gamma_5^2}{\gamma_{24}}\frac{\left(1-\frac{1}{2}b_1\right)\gamma_T}{g_T}$$

$$\lambda_q^{(0)} = \frac{\gamma_{24}m^4}{(1+\mu)\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]g_2}+\frac{(2+\mu)}{(1+\mu)^2}\frac{\gamma_{24}m^2}{\left[n^2\beta^2+\frac{1}{2}b_2m^2\right]}\frac{g_3}{g_2}\varepsilon$$
$$+\frac{1}{(1+\mu)\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]}\left[\frac{g_1}{\gamma_{14}}+\frac{(1-\mu-\mu^2)}{(1+\mu)^2}\frac{\gamma_{24}g_3^2}{g_2}\right]\varepsilon^2$$
$$-\frac{\mu}{(1+\mu)^2}\frac{g_3}{m^2\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]}\left[1+\varepsilon\frac{g_3}{(1+\mu)m^2}\right]\left[\frac{g_1}{\gamma_{14}}+\frac{(4+4\mu+\mu^2)}{(1+\mu)^2}\frac{\gamma_{24}g_3^2}{g_2}\right]\varepsilon^2$$

$$\lambda_q^{(2)} = \frac{1}{4}\frac{m^4n^2\beta^2}{g_2}\left\{2\gamma_{24}(1+\mu)(2+\mu)+\frac{1}{4}\frac{(1+2)g_2}{\gamma_{24}n^2\beta^2\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]}\right.$$
$$-\frac{\gamma_{24}n^2\beta^2g_2}{(1+\mu)\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]g_2-2b_1m^6}\left[2(1+\mu)^2+\frac{\frac{1}{2}b_1m^2(1+2\mu)}{\left[n^2\beta^2+\frac{1}{2}b_1m^2\right]}\right.$$
$$\left.\left.+\frac{(1+2\mu)g_2+8m^4(1+\mu)}{g_2}(2+\mu)\right]\right\}$$

在式(3. 20)和(3. 21)中

$$C_{11} = \left[\gamma_{24}^2-a_2\gamma_5-\frac{4}{\pi}\frac{\alpha}{b}\gamma_5(\gamma_5-a_2)\varepsilon^{1/2}\right]g_T$$

$$\Theta_2 = \left\{\left[\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right]\frac{m^4(1+\mu)}{16n^2\beta^2g_2}\varepsilon^{-1}+\frac{1}{32}\left[\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\frac{m^2}{\gamma_{24}n^2\beta^2}\left[\frac{\gamma_{34}}{\gamma_{24}}(1+2\mu)\right.\right.\right.$$
$$\left.-\frac{2\gamma_{24}g_3}{g_2}\right]+\frac{1}{8}\frac{\gamma_5-a_2}{\gamma_{24}^2-a_2\gamma_5-\frac{4}{\pi}\frac{\alpha}{b}\gamma_5(\gamma_5-a_2)\varepsilon^{1/2}}\left[m^2\left(1+2\mu+\frac{1}{\alpha\pi}\varepsilon^{1/3}\right)\varepsilon\right.$$
$$\left.\left.-2g_3\varepsilon^2+\frac{g_3^2}{m^2}\varepsilon^3\right]+\Theta_4\lambda_T^{(2)}\right\}\Big\backslash\left[1-\frac{g_3}{m^2}\varepsilon\right]$$

$$\Theta_4 = \frac{\gamma_{24}^2-\gamma_5^2}{\gamma_{24}}\frac{(1-a_2)\gamma_T}{g_T}$$

$$\lambda_T^{(0)} = \left[\frac{m^2}{m^2+a_2n^2\beta^2}\right]\left\{\frac{\gamma_{24}m^2}{(1+\mu)g_2}\varepsilon^{-1}+\frac{(2+\mu)}{(1+\mu)^2}\frac{\gamma_{24}g_3}{g_2}\right.$$
$$+\frac{1}{(1+\mu)m^2}\left[\frac{g_1}{\gamma_{14}}+\frac{(1-\mu-\mu^2)}{(1+\mu)^2}\frac{\gamma_{24}g_3^2}{g_2}\right]\varepsilon$$
$$\left.-\frac{\mu}{(1+\mu)^2}\frac{g_3}{m^4}\left[1+\frac{g_3}{(1+\mu)m^2}\varepsilon\right]\left[\frac{g_1}{\gamma_{14}}+\frac{(4+4\mu+\mu^2)}{(1+\mu)^2}\frac{\gamma_{24}g_3^2}{g_2}\right]\varepsilon^2\right\}$$

$$\lambda_T^{(2)} = \frac{1}{4}\left[\frac{m^2}{m^2+a_2n^2\beta^2}\right]\left\{\left[\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right]\frac{\gamma_{24}m^6(2+\mu)}{2g_2^2}\varepsilon^{-1}\right.$$

$$+\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)\frac{m^4}{2g_2}\left[\frac{\gamma_{34}}{\gamma_{24}}\frac{(1+\mu)^2+(1+2\mu)}{(1+\mu)}+\frac{\mu(3+\mu)}{(1+\mu)}\frac{\gamma_{24}g_3}{g_2}\right.$$

$$-\frac{1}{4}\left(2\frac{\gamma_{14}}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)m^2(1+2\mu)\varepsilon+\varepsilon\frac{\gamma_{24}m^2n^4\beta^4}{g_2}\left[g_2\left[(4+9\mu+4\mu^2)\right.\right.$$

$$+\left(\frac{m^2}{m^2+a_2n^2\beta^2}\right)(1+2\mu)\Big]+8m^4(1+\mu)(1+2\mu)\Big]\Big/\left[g_2(1+\mu)-4m^4\left(\frac{m^2}{m^2+a_2n^2\beta^2}\right)\right]$$

$$-\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{24}^2}\right)\frac{m^2g_3}{4g_2}\left[\frac{\gamma_{34}}{\gamma_{24}}\frac{(4+12\mu+15\mu^2+4\mu^3)}{(1+\mu)^2}\frac{}{2}+\frac{2(4+\mu+2\mu^2+\mu^3)}{(1+\mu)^2}\frac{\gamma_{24}g_3}{g_2}\right]\varepsilon$$

$$-\frac{1}{2}\frac{\gamma_{24}}{\gamma_{24}^2-a_2\gamma_5-\frac{4}{\pi}\frac{\alpha}{b}\gamma_5(\gamma_5-a_2)\varepsilon^{1/3}}\left[m^2\left(1+2\mu+\frac{1}{\alpha\pi}\varepsilon^{1/2}\right)\varepsilon-2g_3\varepsilon^2+\frac{g_3^2}{m^2}\varepsilon^3\right\}$$

$$\lambda_T^{(4)}=\frac{1}{64}\left(\frac{m^2}{m^2+a_2n^2\beta^2}\right)^4\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)^2\frac{\gamma_{24}m^{10}(1+\mu)}{g_2^3}\varepsilon^{-1}\left\{g_{13}\left[\left(\frac{m^2+9a_2n^2\beta^2}{m^2+a_2n^2\beta^2}\right)(1+3\mu+\mu^2)\right.\right.$$

$$+\left(\frac{m^2+5a_2n^2\beta^2}{m^2+a_2n^2\beta^2}\right)(4+2\mu)+(1+\mu)\Big]+\gamma g_2\left[\left(\frac{m^2+5a_2n^2\beta^2}{m^2+a_2n^2\beta^2}\right)(6+8\mu+2\mu^2)\right.$$

$$-(2\mu+3\mu^2+\mu^3)\Big]\Big\}\Big/\left[g_{13}\left(\frac{m^2+9a_2n^2\beta^2}{m^2+a_2n^2\beta^2}\right)-g_2(1+\mu)\right]$$

$$+\frac{1}{64}\frac{\gamma_{24}}{\gamma_{24}^2-a_2\gamma_5-\frac{4}{\pi}\frac{\alpha}{b}\gamma_5(\gamma_5-a_2)\varepsilon^{1/2}}\left(\frac{m^2}{m^2+a_2n^2\beta^2}\right)$$

$$\times\left\{\frac{1}{32\pi\alpha}\left(\frac{\gamma_{24}^2}{\gamma_{14}\gamma_{24}+\gamma_{34}^2}\right)^2\frac{m^8(1+\mu)^2}{n^4\beta^4g_2^2}\varepsilon^{-3/2}\right.$$

$$+m^2n^4\beta^4(1+\mu)^2\varepsilon^3\left[g_2(1+2\mu)+8m^4(1+\mu)\right]^2\Big/\left[g_2(1+\mu)\right.$$

$$\left.-4m^4\left(\frac{m^2}{m^2+a_2n^2\beta^2}\right)\right]^2\Big\}$$

在以上公式中

$$g_1=m^4+2\gamma_{12}m^2n^2\beta^2+\gamma_{14}^2n^4\beta^4,\quad g_2=m^4+2\gamma_{22}m^2n^2\beta^2+\gamma_{24}^2n^4\beta^4$$

$$g_3=\gamma_{30}m^4+\gamma_{32}m^2n^2\beta^2+\gamma_{34}n^4\beta^4,\quad g_{13}=m^4+18\gamma_{22}m^2n^2\beta^2+81\gamma_{24}^2n^4\beta^4$$

$$\alpha=\left[\frac{1}{2}(b-c)\right]^{1/2},\quad \phi=\left[\frac{1}{2}(b+c)\right]^{1/2}$$

$$b=\left[\frac{\gamma_{14}\gamma_{24}}{1+\gamma_{14}\gamma_{24}\gamma_{30}^2}\right]^{1/2},\quad c=-\frac{\gamma_{14}\gamma_{24}\gamma_{30}}{1+\gamma_{14}\gamma_{24}\gamma_{30}^2}$$

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Postbuckling of Imperfect Stiffened Cylindrical Shells under Combined External Pressure and Heating

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Abstract

A postbuckling analysis is presented for a stiffened cylindrical shell of finite length subjected to combined loading of external pressure and a uniform temperature rise. The formulations are based on a boundary layer theory of shell buckling, which includes the effects of nonlinear prebuckling deformations, nonlinear large deflections in the postbuckling range and initial geometrical imperfections of the shell. The "smeared stiffener" approach is adopted for the stiffeners. The analysis uses a singular perturbation technique to determine the interactive buckling loads and the postbuckling equilibrium paths. Numerical examples cover the performances of perfect and imperfect, stringer and ring stiffened cylindrical shells. Typical results are presented in dimensionless graphical form.

Key words postbuckling, thermal postbuckling, stiffened cylindrical shell, combined loading, a boundary layer theory of shell buckling, singular perturbation technique