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双参数地基上 Reissner 板弯曲问题的边界积分方程*

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摘　要

本文应用广义函数的 Fourier 积分变换, 导出了双参数地基上 Reissner 板弯曲问题的两个基本解· 在此基础上, 从虚功原理出发, 依据胡海昌导出的 Reissner 板弯曲理论, 推导出适用于任意形状、任意荷载、任意边界条件情形的三个边界积分方程, 为边界元法在这一问题中的应用提供了理论基础· 文中给出了固支、简支、自由三类边界的算例, 并与解析解比较, 均得到满意的结果·

关键词　Reissner 板　双参数地基　基本解　边界积分方程
中图分类号　TU471

§1. 引　言

弹性地基厚板在工程上有着广泛的应用, 如水工建筑物的底板、机场面板、高层建筑基础等· 对于具有复杂边界条件或不规则形状的弹性地基厚板, 用解析法求解是非常困难的· 近年来, 边界元法已成功地用于各类弹性地基薄板弯曲问题的分析[1],[2],[3], 但是在厚板问题分析的应用中, 尚只有极少数文献论及[4],[5]· 本文作者在文献[4]中利用广义函数的 Fourier 积分变换, 导出了 Winkler 地基上厚板弯曲问题的基本解· 本文用同样的方法求得了双参数地基上厚板弯曲问题的基本解, 并将它展成一致收敛的级数形式, 从虚功原理出发, 利用胡海昌的 Reissner 板弯曲问题理论, 推导出适用于任意形状、任意荷载、任意边界条件的三个边界积分方程· 数值计算的算例表明, 本文方法的精度是满意的·

§2. 控制微分方程及基本解

1. 控制微分方程

根据胡海昌关于 Reissner 板弯曲问题的理论[6], 经适当推广, 双参数地基上 Reissner 板的控制微分方程可表为:

$$\nabla^4 F - 2\alpha_0^2 \nabla^2 F + \beta_0^4 F = q/D_0 \tag{2.1}$$

$$\frac{1}{2}(1-\mu) D \nabla^2 f - Cf = 0 \tag{2.2}$$

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式中，∇^4 为双重 Laplace 算子，∇^2 为 Laplace 算子，

$$\alpha_0^2 = \frac{K/C + G_p/D}{2(1 + G_p/C)},\quad \beta_0^4 = \frac{K/D}{(1 + G_p/C)},\quad D_0 = D\left(1 + \frac{G_p}{C}\right)$$

$D = Eh^3/12(1-\mu^2)$ 为板的弯曲刚度，$C = 5Eh/12(1+\mu)$ 为板的剪切刚度，K，G_p 则为表征土模型的两个弹性常数，对于不同的模型，K，G_p 取值亦不同，工程中一般采用的有 Filomenko_Borodich 模型、Pasternak 模型、Vlazov 模型以及 Reissner 模型•

F，f 为胡海昌引入的两个位移函数，它们与挠度 w，转角 ϕ_x，ϕ_y 的关系是：

$$w = F - \frac{D}{C}\nabla^2 F \tag{2.3}$$

$$\phi_x = \frac{\partial F}{\partial x} + \frac{\partial f}{\partial y} \tag{2.4}$$

$$\phi_y = \frac{\partial F}{\partial y} - \frac{\partial f}{\partial x} \tag{2.5}$$

内力与位移关系：

$$M_x = -D\left(\frac{\partial \phi_x}{\partial x} + \mu\frac{\partial \phi_y}{\partial y}\right) \tag{2.6}$$

$$M_y = -D\left(\frac{\partial \phi_y}{\partial y} + \mu\frac{\partial \phi_x}{\partial x}\right) \tag{2.7}$$

$$M_{xy} = -\frac{1}{2}(1-\mu)D\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \tag{2.8}$$

$$V_x = C\left(\frac{\partial w}{\partial x} - \phi_x\right)$$

$$L_1 = \nabla^4 - 2\alpha_0^2\nabla^2 + \beta_0^4$$

$$L_2 = \frac{1}{2}(1-\mu)D\nabla^2 - C$$

对方程(2.14)、(2.15)施行广义函数的 Fourier 积分变换[4], 可得:

$$F^* = \frac{1}{8D_0\sqrt{\beta_0^4-\alpha_0^4}}\left[H_0^{(1)}(a_1 r) - H_0^{(1)}(a_2 r)\right] \tag{2.16}$$

$$f^* = \frac{\lambda^2}{2\pi C}K_0(\lambda r) \tag{2.17}$$

其中,

$$\begin{matrix} a_1^2 \\ a_2^2 \end{matrix} = -\alpha_0^2 \pm i\sqrt{\beta_0^4-\alpha_0^4}, \quad \lambda = \frac{\sqrt{10}}{h}$$

$H_0^{(1)}(x)$ 为第一种零阶 Hankle 函数, $K_0(x)$ 为零阶 Kelvin 函数。

将基本解 F^*, f^* 展成下列级数形式[4]:

$$\begin{aligned} F^* = \frac{1}{8D_0\sqrt{\beta_0^4-\alpha_0^4}}\Bigg[& \frac{2}{\pi}(\theta_2-\theta_1)\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{\beta_0 r}{2}\right)^{2k}\cos 2k\theta_1 \\ & - \frac{4}{\pi}\ln\frac{\beta_0 r}{2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{\beta_0 r}{2}\right)^{2k}\sin 2k\theta_1 \\ & + \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\psi(k+1)\left(\frac{\beta_0 r}{2}\right)^{2k}\sin 2k\theta_1\Bigg] \end{aligned} \tag{2.18}$$

$$f^* = -\frac{\lambda}{2\pi C}\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left[\ln\frac{\lambda r}{2} - \psi(k+1)\right]\left(\frac{\lambda r}{2}\right)^{2k} \tag{2.19}$$

式中, $\psi(k+1) = 1 + 1/2 + \cdots + 1/k - C_E$, $C_E = 0.57721$, 为 Euler 常数, β_0 亦即 a_1, a_2 的模, θ_1, θ_2 分别是 a_1, a_2 之幅角, $\theta_1 + \theta_2 = \pi$, 式(2.18)、(2.19)对 $0 < r < +\infty$ 是一致收敛的。

§3. 积分方程解答

根据文献[6]在板三类变量广义变分原理中导出的结果, 其虚功原理可表述为:

$$\begin{aligned} & -\iint_\Omega\left[M_x\frac{\partial\psi_x^*}{\partial x} + M_{xy}\left(\frac{\partial\psi_x^*}{\partial y} + \frac{\partial\psi_y^*}{\partial x}\right) + M_y\frac{\partial\psi_y^*}{\partial y}\right]dxdy \\ & + \iint_\Omega\left[V_x\left(\frac{\partial w^*}{\partial x} - \psi_x^*\right) + V_y\left(\frac{\partial w^*}{\partial y} - \psi_y^*\right)\right]dxdy \\ & = \iint_\Omega\left[\left(\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_x}{\partial x} - V_x\right)\psi_x^* + \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - V_y\right)\psi_y^*\right. \\ & \left. - \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y}\right)w^*\right]dxdy + \int_\Gamma\left(V_n w^* - M_n\psi_n^* - M_{ns}\psi_s^*\right)ds \end{aligned} \tag{3.1}$$

式中, Ω 是板所占区域, Γ 是 Ω 之边界, M_x, M_y, M_{xy}, V_x, V_y, w^*, ψ_x^*, ψ_y^* 是 8 个独立无关的函数。边界弯矩 M_n, 扭矩 M_{ns} 及剪力 V_n 的正方向如图 1 所示, 转角 ψ_n, ψ_s 方向如图 2 所示。

不妨在式(3.1)中交换真实解与基本解的位置, 使独立变量为 M_x^*, M_y^*, M_{xy}^*, V_x^*, V_y^*, w, ψ_x, ψ_y, 即得:

$$-\iint_\Omega\left[M_x^*\frac{\partial\psi_x}{\partial x} + M_{xy}^*\left(\frac{\partial\psi_x}{\partial y} + \frac{\partial\psi_y}{\partial x}\right) + M_y^*\frac{\partial\psi_y}{\partial y}\right]dxdy$$

$$+\iint_{\Omega}\left[V_x^*\left(\frac{\partial w}{\partial x}-\phi_x\right)+V_y^*\left(\frac{\partial w}{\partial y}-\phi_y\right)\right]dxdy$$

$$=\iint_{\Omega}\left[\left(\frac{\partial M_{xy}^*}{\partial y}+\frac{\partial M_x^*}{\partial x}-V_x^*\right)\phi_x+\left(\frac{\partial M_{xy}^*}{\partial x}+\frac{\partial M_y^*}{\partial y}-V_y^*\right)\phi_y$$

$$-\left(\frac{\partial V_x^*}{\partial x}+\frac{\partial V_y^*}{\partial y}\right)w\right]dxdy+\int_{\Gamma}(V_n^*w-M_n^*\phi_n-M_{ns}^*\phi_s)ds \qquad (3.2)$$

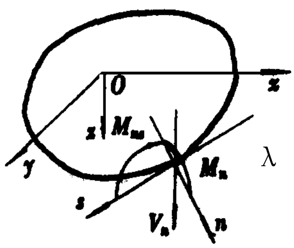

图　1

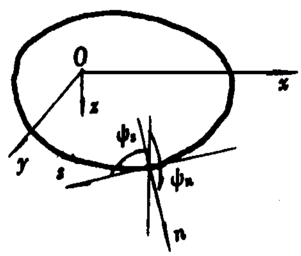

图　2

在(3.1)、(3.2)式中出现的上标带“＊”的量均与基本解 F^*, f^* 有关, 它们可由方程(2.3)～(2.10)分别求得•

利用方程(2.3)～(2.13), 可将上式化为:

$$\nabla^2 f(p)-w(p)=-\iint_{\Omega}qw^*\,d\Omega+\iint_{\Omega}G_p(\nabla^2 w^*\cdot w-\nabla^2 w\cdot w^*)\,d\Omega$$

$$+\int_{\Gamma}\left[\left(V_n^*w-M_n^*\phi_n-M_{ns}^*\phi_s\right)-\left(V_nw^*-M_n\phi_n^*-M_{ns}\phi_s^*\right)\right]d\Gamma \qquad (3.3)$$

在式(3.3)的左端, 可依次令 $F^*=0$, $f^*=0$(两种虚位移状态), 则:

当 $f^*=0$ 时, 显然 $f(p)$ 不出现, 则,

$$w(p)=\iint_{\Omega}qw_1^*\,d\Omega+\iint_{\Omega}G_p(\nabla^2 w\cdot w_1^*-\nabla^2 w_1^*\cdot w)\,d\Omega$$

$$+\int_{\Gamma}[(V_nw_1^*-M_n\phi_{n1}^*-M_{ns}\phi_{s1}^*)-(V_{n1}^*w-M_{n1}^*\phi_n-M_{ns1}^*\phi_s)]\,d\Gamma \qquad (3.4)$$

根据 Green 公式

$$\iint_{\Omega}(\nabla^2 w\cdot w_1^*-\nabla^2 w_1^*\cdot w)\,d\Omega=\oint_{\Gamma}\left(w_1^*\frac{\partial w}{\partial n}-w\frac{\partial w_1^*}{\partial n}\right)d\Gamma$$

式(3.4)可化为:

$$w(p)=\iint_{\Omega}qw_1^*\,d\Omega+\int_{\Gamma}\left\{\left[\left(V_n+G_p\frac{\partial w}{\partial n}\right)w_1^*-M_n\phi_{n1}^*-M_{ns}\phi_{s1}^*\right]\right.$$

$$\left.-\left[\left(V_{n1}^*+G_p\frac{\partial w}{\partial n}\right)w-M_{n1}^*\phi_n-M_{ns1}^*\phi_{s1}\right]\right\}d\Gamma \qquad (3.5)$$

当 $F^*=0$ 时, 显然 $F(p)$ 不出现, 且 $w^*=0$, 则由式(3.3)可得

$$\nabla^2 f(p)=\int_{\Gamma}[(V_{n2}^*w-M_{n2}^*\phi_n-M_{ns2}^*\phi_s)+(M_n\phi_{n2}^*+M_{ns}\phi_{s2}^*)]\,d\Gamma \qquad (3.6)$$

再由式(2.3)可得:

$$\frac{\partial F}{\partial n}=\frac{\partial w}{\partial n}+\frac{D}{C}\frac{\partial}{\partial n}(\nabla^2 F) \qquad (3.7)$$

$$\frac{\partial F}{\partial s}=\frac{\partial w}{\partial s}+\frac{D}{C}\frac{\partial}{\partial s}(\nabla^2 F) \qquad (3.8)$$

由方程(2.2)可得

$$\frac{\partial f}{\partial n} = \frac{1}{2}(1-\mu)\frac{D}{C}\frac{\partial}{\partial n}(\nabla^2 f) \tag{3.9}$$

$$\frac{\partial f}{\partial s} = \frac{1}{2}(1-\mu)\frac{D}{C}\frac{\partial}{\partial s}(\nabla^2 f) \tag{3.10}$$

则:

$$\phi_n = \frac{\partial F}{\partial n} + \frac{\partial f}{\partial s} = \frac{\partial w}{\partial n} + \frac{D}{C}\frac{\partial}{\partial n}(\nabla^2 F) + \frac{1}{\lambda^2}\frac{\partial}{\partial s}(\nabla^2 f) \tag{3.11}$$

$$\phi_s = \frac{\partial F}{\partial s} - \frac{\partial f}{\partial n} = \frac{\partial w}{\partial s} + \frac{D}{C}\frac{\partial}{\partial s}(\nabla^2 F) - \frac{1}{\lambda^2}\frac{\partial}{\partial n}(\nabla^2 f) \tag{3.12}$$

再一次利用式(2.3),将 $\nabla^2 F = C(F-w)/D$ 代入控制方程 $\nabla^4 F - 2\alpha_0^2\nabla^2 F + \beta_0^4 F = q/D_0$ 后得到:

$$\nabla^2 F = H\left(\frac{C}{D}\nabla^2 w - \beta_0^4 w + \frac{9}{D_0}\right) \tag{3.13}$$

式中, $$H = \frac{CD}{C^2 - 2CD\alpha_0^2 + D^2\beta_0^4}$$

将式(3.13)代入式(3.11)、(3.12)即得到:

$$\phi_n = \left(1 - \frac{D\beta_0^4}{C}H\right)\frac{\partial w}{\partial n} + H\frac{\partial w}{\partial n}(\nabla^2 w) + \frac{1}{\lambda^2}\frac{\partial}{\partial s}(\nabla^2 f) \tag{3.14}$$

$$\phi_s = \left(1 - \frac{D\beta_0^4}{C}H\right)\frac{\partial w}{\partial s} + H\frac{\partial w}{\partial s}(\nabla^2 w) - \frac{1}{\lambda^2}\frac{\partial}{\partial n}(\nabla^2 f) \tag{3.15}$$

将式(3.6)代入以上两式,即可得到两个积分方程表达式,它们与积分方程(3.5)共同构成了求解双参数地基上 Reissner 板弯曲问题的积分方程表述形式• 在以上的积分方程组中涉及的上标带"*"、下标为"1"的基本解是由 F^* 引出的量,上标带"*"、下标为"2"的基本解是由 f^* 引出的量•

§4. 边界积分方程

可以证明,在积分方程(3.5)和(3.6)中,若令 $p \to P \in \Gamma$,即可得到下列的边界积分方程:

$$C(P)w(P) = \iint_\Omega q w_1^* \, d\Omega + G_p\int_\Gamma\left(w_1^*\frac{\partial w}{\partial n} - w\frac{\partial w_1^*}{\partial n}\right)d\Gamma + \int_\Gamma[(V_n w_1^* - M_n\phi_{n1}^* - M_{ns}\phi_{s1}^*) - (V_{n1}^* w - M_{n1}^*\phi_n - M_{ns1}^*\phi_s)]\,d\Gamma \tag{4.1}$$

$$C(P)\nabla^2 f(P) = \int_\Gamma[(V_{n2}^* w - M_{n2}^*\phi_n - M_{ns2}^*\phi_s) + (M_n\phi_{n2}^* + M_{ns}\phi_{s2}^*)]\,d\Gamma \tag{4.2}$$

其中, $C(P) = \alpha/2\pi$,对于光滑边界, $C(P) = 1/2$• α 见图 3• 将式(4.1)、(4.2)代入式(3.14)、(3.15),即得到另两个以广义位移表示的边界积分方程:

再由

$$\begin{aligned} C(P)\phi_n(P) = {} & \iint_\Omega\left[H_0\frac{\partial w_1^*}{\partial n_p} + H\frac{\partial}{\partial n_p}(\nabla^2 w_1^*)\right]q\,d\Omega + \int_\Gamma\Bigg[-H_0\frac{\partial}{\partial n_p} \\ & \cdot\left(V_{n1}^* + G_p\frac{\partial w_1^*}{\partial n}\right) - H\frac{\partial}{\partial n_p}\nabla^2\left(V_{n1}^* + G_p\frac{\partial w_1^*}{\partial n}\right) + \frac{1}{\lambda^2}\frac{\partial V_{n2}^*}{\partial s_p}\Bigg]w\,d\Gamma \\ & + \int_\Gamma\left[H_0\frac{\partial}{\partial n_p}\left(M_{n1}^* + G_p w_1^*\right) + H\frac{\partial}{\partial n_p}\nabla^2\left(M_{n1}^* + G_p w_1^*\right) - \frac{1}{\lambda^2}\frac{\partial M_{n2}^*}{\partial s_p}\right]\phi_n d\Gamma \\ & + \int_\Gamma\left[H_0\frac{\partial}{\partial n_p}M_{ns1}^* + H\frac{\partial}{\partial n_p}\nabla^2 M_{ns1}^* - \frac{1}{\lambda^2}\frac{\partial M_{ns2}^*}{\partial s_p}\right]\phi_s d\Gamma \end{aligned}$$

$$+\int_\Gamma\left[H_0\left(1+\frac{G_p}{C}\right)\frac{\partial w_1^*}{\partial n_p}+H\left(1+\frac{G_p}{C}\right)\frac{\partial}{\partial n_p}\nabla^2 w_1^*\right]V_n d\Gamma$$

$$-\int_\Gamma\left[H_0\frac{\partial \phi_{n1}^*}{\partial n_p}+H\frac{\partial}{\partial n_p}\nabla^2\phi_{n1}^*-\frac{1}{\lambda^2}\frac{\partial\phi_{n2}^*}{\partial s_p}\right]M_n d\Gamma$$

$$-\int_\Gamma\left[H_0\frac{\partial \phi_{s1}^*}{\partial n_p}+H\frac{\partial}{\partial n_p}\nabla^2\phi_{s1}^*-\frac{1}{\lambda^2}\frac{\partial\phi_{s2}^*}{\partial s_p}\right]M_{ns} d\Gamma \tag{4.3}$$

$$C(P)\phi_s(P)=\iint_\Omega\left[H_0\frac{\partial w_1^*}{\partial s_p}+H\frac{\partial}{\partial s_p}(\nabla^2 w_1^*)\right]q d\Omega+\int_\Gamma\left[-H_0\cdot\frac{\partial}{\partial s_p}\left(V_{n1}^*+G_p\frac{\partial w_1^*}{\partial n}\right)-H\frac{\partial}{\partial s_p}\nabla^2\left(V_{n1}^*+G_p\frac{\partial w_1^*}{\partial n}\right)-\frac{1}{\lambda^2}\frac{\partial V_{n2}^*}{\partial n_p}\right]w d\Gamma$$

$$+\int_\Gamma\left[H_0\frac{\partial}{\partial s_p}\left(M_{n1}^*+G_p w_1^*\right)+H\frac{\partial}{\partial s_p}\nabla^2\left(M_{n1}^*+G_p w_1^*\right)+\frac{1}{\lambda^2}\frac{\partial M_{n2}^*}{\partial n_p}\right]\phi_n d\Gamma$$

$$+\int_\Gamma\left[H_0\frac{\partial M_{ns1}^*}{\partial s_p}+H\frac{\partial}{\partial s_p}\nabla^2 M_{ns1}^*+\frac{1}{\lambda^2}\frac{\partial M_{ns2}^*}{\partial n_p}\right]\phi_s d\Gamma$$

$$+\int_\Gamma\left[H_0\left(1+\frac{G_p}{C}\right)\frac{\partial w_1^*}{\partial s_p}+H\left(1+\frac{G_p}{C}\right)\frac{\partial}{\partial s_p}(\nabla^2 w_1^*)\right]V_n d\Gamma$$

$$-\int_\Gamma\left[H_0\frac{\partial \phi_{n1}^*}{\partial s_p}+H\frac{\partial}{\partial s_p}\nabla^2\phi_{n1}^*+\frac{1}{\lambda^2}\frac{\partial\phi_{n2}^*}{\partial n_p}\right]M_n d\Gamma$$

$$-\int_\Gamma\left[H_0\frac{\partial \phi_{s1}^*}{\partial s_p}+H\frac{\partial}{\partial s_p}\nabla^2\phi_{s1}^*+\frac{1}{\lambda^2}\frac{\partial\phi_{s2}^*}{\partial n_p}\right]M_{ns} d\Gamma \tag{4.4}$$

式中 $H_0=1-D\beta_0^4H/C$, 各积分项中的核函数见附录•

式(4.1)、(4.3)、(4.4)即为用三个广义位移 w, ϕ_n, ϕ_s 表示的三个边界积分方程, 经适当的数值离散, 引入边界条件即可求得数值解•

§5. 算　　例

根据上述的三个以广义位移表示边界积分方程, 本文给出下列的数值解算例, 以验证其正确性, 本例中采用的边界单元为常数单元•

Winkler 地基上圆板作用一轴对称圆形均布荷载(图3), 地基弹性常数 $K=5\times10^4\mathrm{kN/m^3}$, $G_p=0$, 板弹性模量 $E=2.45\times10^7\mathrm{kN/m^2}$, 泊松比 $\mu=0.167$, 圆板半径为1.35m, 板厚 $H=0.25$m, 荷载作用半径为0.18m, 均布荷载 $q=1000\mathrm{kN/m^2}$, 按周边固支、简支、自由三种边界条件进行计算, 表1中列出了板中心挠度、应力的计算结果, 并与精确解[7]与有限层分析数值解[8]进行比较, 误差均很小•

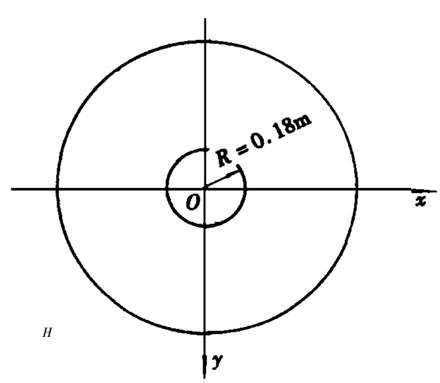

图　3

致谢　感谢陈山林教授的大力帮助及提出的宝贵意见•

表 1 **Winkler 地基圆板中心挠度及径向应力**

计算方法	自由边		简支边		固支边	
	w (10^{-4}m)	σ_r(MPa)	w (10^{-4}m)	σ_r(MPa)	w (10^{-4}m)	σ_r(MPa)
解析解	4. 685	1. 924	2. 574	2. 332	1. 127	1. 787
有限层法	4. 653	1. 930	2. 557	2. 330	1. 143	1. 789
本文方法	4. 681	1. 879	2. 486	2. 280	1. 087	1. 764

附　　录

由 F^* 引出的各基本解:

$$w_1^* = A\left[u_1(r) - \frac{D}{C}\left(u_1''(r) + \frac{u_1'(r)}{r}\right)\right]$$

$$\phi_{n1}^* = A u_1'(r)\cos\beta$$

$$\phi_{s1}^* = A u_1'(r)\sin\beta$$

$$V_{n1}^* + G_p\frac{\partial w_1^*}{\partial n} = A\left[G_p u_1'(r) - D\left(1+\frac{G_p}{C}\right)\left(u_1'''(r) + \frac{u_1''(r)}{r} - \frac{u_1'(r)}{r^2}\right)\right]\cos\beta$$

$$M_{n1}^* + G_p w_1^* = -DA\left[(\cos^2\beta + \mu\sin^2\beta)u_1''(r) + (\sin^2\beta + \mu\cos^2\beta)\frac{u_1'(r)}{r}\right.$$

$$\left. + G_p A\left[u_1(r) - \frac{D}{C}\left(u_1''(r) + \frac{u_1'(r)}{r}\right)\right]\right]$$

$$M_{ns1}^* = -\frac{1}{2}(1-\mu)DA\left(u_1''(r) - \frac{u_1'(r)}{r}\right)\sin 2\beta$$

由 f^* 引出的各基本解:

$$\phi_{n2}^* = -Bu_2'(r)\sin\beta$$

$$\phi_{s2}^* = -Bu_2'(r)\cos\beta$$

$$V_{n2}^* = BCu_2'(r)\sin\beta$$

$$M_{n2}^* = \frac{1}{2}DB(1-\mu)\left[u_2''(r) + \frac{u_1'(r)}{r}\right]\sin 2\beta$$

$$M_{ns2}^* = \frac{1}{2}DB(1-\mu)\left[u_2''(r) + \frac{u_1'(r)}{r}\right]\cos 2\beta$$

式中,

$$A = \frac{1}{8D_0\sqrt{\beta_0^4 - \alpha_0^4}}, \quad B = -\frac{\lambda^2}{2\pi C}$$

$$u_1(r) = \frac{F^*(r)}{A}, \quad u_2(r) = \frac{f^*(r)}{B}$$

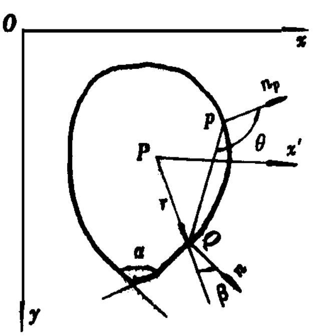

图 4

式(4. 3)、(4. 4)中各积分项中的导数项可依据下列两式依次求得, 表达式较繁冗, 不再列出•

$$\begin{cases}\dfrac{\partial}{\partial n_p} = \cos\beta\dfrac{\partial}{\partial r} + \dfrac{\sin\beta}{r}\dfrac{\partial}{\partial\theta}\\[2ex] \dfrac{\partial}{\partial s_p} = -H\sin\beta\dfrac{\partial}{\partial r} + \dfrac{\cos\beta}{r}\dfrac{\partial}{\partial\theta}\end{cases}$$

上述公式中出现的 $\beta = \langle \boldsymbol{r}, \boldsymbol{n}\rangle$, $\theta = \langle \boldsymbol{n}_p, \boldsymbol{r}\rangle$, 见图 4•

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Boundary Integral Equations for Bending Problem of Reissner's Plates on Two_Parameter Foundation

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Abstract

Two fundamental solutions for bending problem of Reissner's plates on two_parameter foundation are derived by means of Fourier integral transformation of generalized function in this paper. On the basis of virtual work principles, three boundary integral equations which fit for arbitrary shapes, loads and boundary conditions of thick plates are presented according to Hu Haichang's theory about Reissner's plates. It provides the fundamental theories for the application of BEM. A numerical example is given for clamped, simply supported and free boundary conditions. The results obtained are satisfactory as compared with the analytical methods.

Key words　Reissner's plate, two_parameter foundation, fundamental solution, boundary integral equation