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线布载荷作用下碟形扁壳的非线性稳定*

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摘 要

本文对轴对称线布载荷作用下碟形扁壳的非线性稳定问题进行了分析, 得到了各种常见边界条件下碟形扁壳特征关系的二次近似表达式• 讨论了几何参数 β, γ 和 k 对非线性特性的影响•

关键词 碟形扁壳 非线性稳定 修正迭代法
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§1. 引 言

在仪表自控系统中, 常利用碟形扁壳失稳时的跳跃现象作为一种控制信号• 由于碟形扁壳非线性特性分析的复杂性, 尚未有研究成果见诸报道• 本文作者在文献[1]中已处理了均布荷载下该类壳体的非线性稳定问题• 目前的设计主要是在经验——试验的基础上完成的[2], 缺乏足够的理论依据• 理论研究的缺乏使碟形扁壳的工程设计带有一定的盲目性, 同时也限制了该类扁壳在各工程领域的进一步应用• 因此, 碟形扁壳非线性问题的研究工作是有意义的•

本文对多种边界条件下, 碟形扁壳在轴对称线布载荷作用下的非线性稳定问题进行了研究• 首先将问题的控制微分方程组转化为等价的积分方程组, 然后用修正迭代法进行求解[3,4], 得到了碟形扁壳非线性稳定的二次近似特征关系式• 文中给出了一组算例, 讨论了边界约束和几何参数 β, γ, k 对稳定特性的影响•

§2. 基 本 方 程

考虑图 1 所示碟形扁壳• 中心圆板的半径为 b, 锥壳底面半径为 a, 圆板和锥壳的厚度均为 h, 轴对称线布载荷的作用半径为 c•

选取如下的无量纲量

$$x = \frac{r}{a}, \quad W = \sqrt{12(1-\mu^2)}\,\frac{w}{h}$$

$$\theta = \frac{dW}{dx}, \quad \beta = \frac{b}{a}, \quad \gamma = \frac{c}{a}$$

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$$k = \sqrt{12(1-\mu^2)}\,\frac{f}{h},\quad S = -\frac{12(1-\mu^2)}{Eh^2}a^2 x N_r$$

$$P^* = \frac{\sqrt{12(1-\mu^2)}}{Dh}a^2 cP \tag{2.1}$$

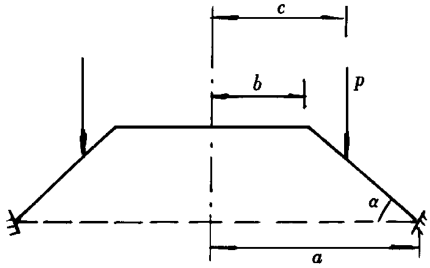

图 1 碟形扁壳的几何尺寸及受力

式中, W 为挠度, μ 为泊松比, D 为抗弯刚度, E 为弹性模量, P 为载荷参数, f 为扁锥壳矢高($f = a\mathrm{tg}\alpha$)•

由板壳非线性理论[5~9], 可得平板和锥壳的无量纲控制微分方程分别为

(1) 平板部份

$$\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x\theta) = -\frac{S}{x}\theta$$

$$\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xS) = \frac{1}{2x}\theta^2 \qquad (0 \leqslant x < \beta) \tag{2.2}$$

(2) 锥壳部份

$$\left.\begin{aligned}&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x\theta) = -\frac{S}{x}(\theta + k)\\&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xS) = \frac{\theta^2}{2x} + \frac{k\theta}{x} \qquad (\beta \leqslant x < \gamma)\end{aligned}\right\} \tag{2.3}$$

$$\left.\begin{aligned}&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x\theta) = \frac{P^*}{x} - \frac{S}{x}(\theta + k)\\&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xS) = \frac{\theta^2}{2x} + \frac{k\theta}{x} \qquad (\gamma \leqslant x \leqslant 1)\end{aligned}\right\} \tag{2.4}$$

引入 Heaviside 阶梯函数

$$\mathscr{U}(x-\xi) = \begin{cases}1 & (x \geqslant \xi)\\ 0 \text{ 小} & (x < \xi)\end{cases} \tag{2.5}$$

由(2.2)~(2.5), 可得碟形扁壳在轴对称线布载荷作用下的无量纲控制微分方程为

$$\left.\begin{aligned}&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(x\theta) = \frac{P^*}{x}\mathscr{U}(x-\gamma) - S(\theta + \mathscr{U}(x-\beta)k)/x\\&\frac{d}{dx}\frac{1}{x}\frac{d}{dx}(xS) = \frac{\theta^2}{2x} + \frac{k\theta}{x}\mathscr{U}(x-\beta)\end{aligned}\right\} \tag{2.6a, b}$$

无量纲边界条件的一般提法为

$$\left.\begin{aligned}&x = 0,\quad \theta = 0,\quad S = 0\\&x = 1,\quad x\frac{d\theta}{dx} + \nu_1\theta = 0,\quad x\frac{dS}{dx} - \nu_2 S = 0\end{aligned}\right\} \tag{2.7}$$

ν_1 和 ν_2 与边界约束有关, 对常见边界情形, 取

固定夹紧 $\nu_1 = \infty,\quad \nu_2 = \mu$

可移夹紧 $\nu_1 = \infty,\quad \nu_2 = \infty$

固定铰支 $\nu_1 = \mu,\quad \nu_2 = \mu$

可移铰支 $\nu_1 = \mu,\quad \nu_2 = \infty$ (2.8)

边值问题(2.6a、b),(2.7)即构成了本文所讨论的碟形扁壳非线性稳定问题的求解方程组•
令

$$\left.\begin{aligned} F(x) &= -S[\theta + k\mathscr{U}(x-\beta)] \\ f(x) &= \theta\left[\frac{\theta}{2} + k\mathscr{U}(x-\beta)\right] \end{aligned}\right\} \quad x \tag{2.9a, b}$$

将(2.9a、b)代入(2.6),并直接积分,同时引入边界条件(2.7),可得与边值问题(2.6a、b),(2.7)等价的积分方程组

$$\left.\begin{aligned} \text{板壳}\quad \theta &= \frac{xP^*}{2}\left[\left(\ln\frac{x}{\gamma} - \frac{1-(\gamma/x)^2}{2}\right)\mathscr{U}(x-\gamma) + \ln\gamma - \frac{G}{2}(1-\gamma^2)\right. \\ &\quad + \int_0^1 K_1(x,\xi)F(\xi)\,d\xi \\ S &= \int_0^1 K_2(x,\xi)f(\xi)\,d\xi \end{aligned}\right\} \tag{2.10a, b}$$

式中,$K_1(x,\xi)$,$K_2(x,\xi)$为积分核

$$\left.\begin{aligned} K_1(x,\xi) &= \begin{cases} -\dfrac{1}{2}\left(\dfrac{\xi}{x} + Gx\xi\right) & (0 \leqslant \xi < x) \\ -\dfrac{1}{2}\left(\dfrac{x}{\xi} + G\xi x\right) & (x \leqslant \xi \leqslant 1) \end{cases} \\ K_2(x,\xi) &= \begin{cases} -\dfrac{1}{2}\left(\dfrac{\xi}{x} + gx\xi\right) & (0 \leqslant \xi < x) \\ -\dfrac{1}{2}\left(\dfrac{x}{\xi} + g\xi x\right) & (x \leqslant \xi \leqslant 1) \end{cases} \end{aligned}\right\} \tag{2.11}$$

G、g 是与边界约束有关的常数

$$G = \frac{1-\nu_1}{1+\nu_1}, \quad g = \frac{1+\nu_1}{1-\nu_2} \tag{2.12}$$

至此,本文问题归结为求解积分方程组(2.10)•

§3. 迭 代 求 解

记无量纲中心挠度为 W_0($W_0 = W|_{x=0}$),则

$$W_0 = \int_1^0 \theta\,dx \tag{3.1}$$

将(2.10a)代入(3.1),并注意到有下式成立

$$\int_1^0 x\,dx\int_0^1 xF(x)\,dx + \int_1^0 x\,dx\int_1^x \frac{F(x)}{x}dx = 0 \tag{3.2}$$

可以得到下述关系

$$P^* = \alpha\left[W_0 - \frac{(1+G)}{4}\int_0^1 xF(x)\,dx - \frac{1}{2}\int_0^1\frac{dx}{x}\int_0^x xF(x)\,dx\right] \tag{3.3}$$

式中

$$\alpha = \left[\frac{1-\gamma^2}{8}(2+G) + \frac{\gamma^2}{4}\ln\gamma\right]^{-1} \tag{3.4}$$

利用(3.3)式,可使碟形扁壳弹性特征的计算过程大为简化•

本文用修正迭代法求解• 在一次近似中,忽略 S 的影响,即取

$$F_1(x) = 0 \tag{3.5}$$

由(2.10a)得一次近似为

$$\theta_1 = P^* x\left[\frac{1}{4}\left(2\ln\frac{x}{\gamma} - 1 + \frac{\gamma^2}{x^2}\right)\mathscr{U}(x-\gamma) + \frac{\ln\gamma}{2} - \frac{G}{4}(1-\gamma^2)\right] \tag{3.6}$$

(3.6)代入(3.1)式或直接由(3.3)式,得一次近似的特征关系为

$$P^* = \alpha W_0 \tag{3.7}$$

(3.7)式即圆板小挠度解[5]。将(3.7)代入(3.6)有

$$\theta_1 = \frac{\alpha W_0}{2}x\left[-\left(\ln\frac{\gamma}{x} + \frac{1-(\gamma/x)^2}{2}\right)\mathscr{U}(x-\gamma) + \ln\gamma - \frac{G}{2}(1-\gamma^2)\right] \tag{3.8}$$

二次近似的迭代格式为

$$\left.\begin{aligned} S_2 &= \int_0^1 K_2(x,\xi) f_1(\xi)\, d\xi \\ \theta_2 &= \frac{x}{2}P^*\left[\left(\ln\frac{x}{\gamma} - \frac{1-(\gamma/x)^2}{2}\right)\mathscr{U}(x-\gamma) + \ln\gamma - \frac{G}{2}(1-\gamma^2)\right] \\ &\quad + \int_0^1 K_1(x,\xi) F_2(\xi)\, d\xi \end{aligned}\right\} \tag{3.9a, b}$$

式中

$$\left.\begin{aligned} f_1(\xi) &= \theta_1\left[\frac{\theta_1}{2} + k\mathscr{U}(\xi-\beta)\right] \\ F_2(\xi) &= -S_2[\theta_1 + k\mathscr{U}(\xi-\beta)] \end{aligned}\right\} \tag{3.10a, b}$$

将(3.8)代入(3.10a),得 $f_1(\xi)$ 的表达式,再将该式代入(3.9a),积分后,得

$$\begin{aligned} S_2 = &-\frac{\alpha k W_0}{2}\left\{\left[-\frac{1}{3}x^2\ln x + \frac{1}{3}x^2\left(\frac{10}{3} + \ln\gamma - \frac{\gamma}{4}x(5\gamma+1)\right)\right.\right. \\ &+ \left.\frac{\gamma^3}{x}\left(\frac{1}{3} - \ln\gamma\right) + \frac{\gamma^2}{2}\right]\mathscr{U}(x-\gamma) + a_1\left(\beta x - \frac{\beta^3}{3x} - \frac{2}{3}x^2\right) \\ &\left.\cdot\,\mathscr{U}(x-\beta) + (a_4 + g a_8)x\right\} - \frac{\alpha^2 W_0^2}{2}\left\{\left(\frac{a_1^2}{4} + a_6 + g a_9\right)x\right. \\ &- \frac{a_1^2}{8}x^3 + \mathscr{U}(x-\gamma)\left[a_5 x + \frac{\gamma^2}{32}(4a_2 - 1)x\right. \\ &- \frac{1}{32}x(x^2 + 2\gamma^2)\ln^2 x + \frac{\gamma^2}{16}(1-4a_2)x\ln x + \frac{\gamma^4}{32}\frac{\ln x}{x} \\ &\left.\left.+\left(\frac{\gamma^4}{64} + a_7\right)\frac{1}{x} + \left(\frac{7}{256} + \frac{3a_2}{8} - \frac{a_3}{8}\right)x^3\right]\right\} \end{aligned} \tag{3.11}$$

式中

$$a_1 = \frac{1}{2}\ln\gamma - \frac{1}{4}G(1-\gamma^2)$$

$$a_2 = -\frac{1}{4}[1 + G(1-\gamma^2)]$$

$$a_3 = \frac{1}{16} - \frac{1}{4}\ln^2\gamma + \frac{1}{8}G(1-\gamma^2)(1+2\ln\gamma)$$

$$a_4 = \gamma - \frac{\gamma^2}{4} - \frac{\ln\gamma}{2} - \frac{3}{4} + a_1(1-\beta)$$

$$a_5 = \frac{\gamma^2}{8}\left[\ln\gamma\left(\ln\gamma - \frac{1}{2}\right) + a_2(4\ln\gamma - 1) + \frac{1}{8} + 2a_3\right]$$

$$
\begin{aligned}
a_6 &= \frac{1}{32} - \frac{\gamma^4}{64} - \frac{a_2}{8} + \frac{a_3}{4} - a_5 \\
a_7 &= -\frac{\gamma^4}{8}\left[\frac{1}{4}\ln^2\gamma + \left(\frac{5}{8} + a_2\right)\ln\gamma + \frac{3}{4}a_2 + a_3 + \frac{9}{32}\right] \\
a_8 &= \frac{a_1}{3}(1 - \beta^3) - \frac{1}{6}\ln\gamma + \frac{\gamma^2}{4} + \frac{\gamma^3}{9} - \frac{5}{36} \\
a_9 &= \frac{1}{256} + \frac{a_1^2}{8} + \frac{a_3^2}{8} + \frac{a_2}{32} - \frac{(4a_2 - 1)}{32}\gamma^2 + a_7
\end{aligned}
\tag{3.12}
$$

我们只计算到二次近似• 因此, 直接由(3.3)式计算弹性特征是方便的, 这样可避免 θ_2 的繁杂计算• 此时, (3.3)式可写为

$$
P^* = \alpha\left[W_0 - \frac{1+G}{4^2}\int_0^1 xF_2(x)\,dx - \frac{1}{2}\int_0^1\frac{dx}{x}\int_0^x xF_2(x)\,dx\right.
\tag{3.13}
$$

将(3.6)、(3.11)代入(3.10b)可得

$$
xF_2(x) = \frac{\alpha k^2 W_0}{2}\varphi_1 + \frac{\alpha^3 k W_0^2}{2}\varphi_2 + \frac{\alpha^3 W_0^3}{2}\varphi_3
\tag{3.14}
$$

式中, φ_1, φ_2, φ_3 均是 x 的函数, 其表达式见附录• 将(3.14)代入(3.13), 并记

$$
\left.
\begin{aligned}
\delta_0 &= \int_0^1\varphi_1\,dx, & \delta_3 &= \int_0^1\frac{dx}{x}\int_0^x\varphi_1\,dx \\
\xi_0 &= \int_0^1\varphi_2\,dx, & \xi_3 &= \int_0^1\frac{dx}{x}\int_0^x\varphi_2\,dx \\
\eta_0 &= \int_0^1\varphi_3\,dx, & \eta_3 &= \int_0^1\frac{dx}{x}\int_0^x\varphi_3\,dx
\end{aligned}
\right\}
\tag{3.15}
$$

则得

$$
P^* = (\alpha + Ak^2)W_0 + BkW_0^2 + CW_0^3
\tag{3.16}
$$

常数 A、B、C 为

$$
\left.
\begin{aligned}
A &= -\frac{\alpha^2}{4}\left(\frac{1+G}{2}\delta_0 + \delta_3\right) \\
B &= -\frac{\alpha^3}{4}\left(\frac{1+G}{2}\xi_0 + \xi_3\right) \\
C &= -\frac{\alpha^4}{4}\left(\frac{1+G}{2}\eta_0 + \eta_3\right)
\end{aligned}
\right\}
\tag{3.17}
$$

(3.16)式即为本文所求碟形扁壳在线布载荷作用下的二次近似特征关系式•

壳体的上、下临界载荷值是反映稳定特性的重要参数值, 它们可由(3.16)式方便地求出•

由 $\dfrac{dP^*}{dW_0} = 0$, 得

$$
\alpha + Ak^2 + 2BkW_0 + 3CW_0^2 = 0
$$

其解为

$$
W_{0_{1,2}} = \frac{1}{3C}\left[-Bk \mp \sqrt{B^2k^2 - 3(\alpha + Ak^2)C}\right]
\tag{3.18}
$$

此解只有当下式满足时才有实根

$$
k \geqslant k_{cr} = \sqrt{\frac{3\alpha C}{B^2 - 3AC}}
\tag{3.19}
$$

(3.19)式给出了碟形扁壳失稳的几何必要条件, 即只有 $k > k_{cr}$ 的壳体才会产生跳跃现象•

将(3.18)代入(3.16), 即得上、下临界载荷值 P_1^* 和 P_2^*

$$P_i^* = (\alpha + Ak^2)W_{0i} + BkW_{0i}^2 + CW_{0i}^2 \quad (i = 1, 2) \tag{3.20}$$

§4. 算例与讨论

在下面的各算例中均取泊松比 $\mu = 0.3$•

算例 1 边缘固定铰支碟形扁壳, 取 $\beta = 0.3$, $\gamma = 0.3$• 由(3.14)～(3.17)诸式可算得碟形扁壳的特征关系为

$$P^* = (3.8217 + 0.6758k^2)W_0 - 1.3491kW_0^2 + 0.5965W_0^3 \tag{4.1}$$

壳体的临界几何参数为

$$k_{cr} = 3.3464$$

图 2 给出了在 $k = 4$、5、6 时碟形扁壳的特征曲线• 图形表明, 随着 k 的增大, 壳体的上临界载荷不断增大($k = 6$ 时, $P_1^* = 29.1946$), 即壳体抵抗失稳的能力随 k 的增大而增强•

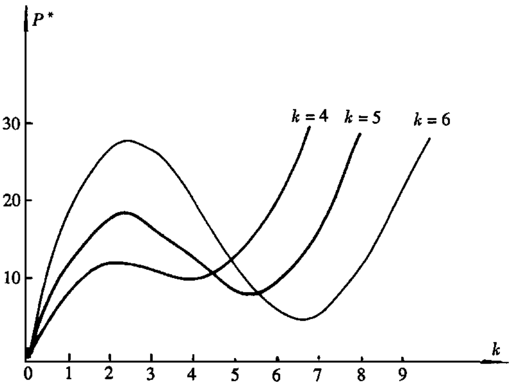

图 2 边缘固定铰支时的特征曲线
($\beta = 0.3$, $\gamma = 0.3$)

算例 2 边缘固定铰支碟形扁壳, 取 $\beta = 0.3$•

图 3 给出了碟形扁壳上的线布载荷作用在不同位置时的稳定曲线• 可以看到, 在相同 k 值下, γ 越小, 壳体的上临界载荷越小, 上、下临界载荷差也越小• 即, 线布载荷的半径越小, 碟形扁壳越易失稳•

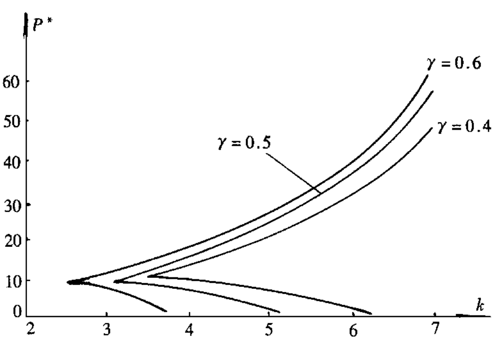

图 3 边缘固定铰支时的稳定曲线
($\beta = 0.3$)

算例 3 考虑边缘支承条件对碟形扁壳稳定特性的影响•

表 1 临界参数($\beta = 0.3$, $\gamma = 0.3$)

边界条件	k_{cr}	$k = 8$	
		P_1^*	P_2^*
铰 支	3.2234	67.9251	−19.3499
简 支	7.1899	25.8115	23.7360

表 1 给出了碟形扁壳($\beta = 0.2$, $\gamma = 0.3$)在边缘固定铰支和边缘可移铰支情形下各稳定参数的计算结果• 可以看到, 边界支承条件越强, 壳体的 k_{cr} 越小, 亦即对屈曲反应越灵敏• 同时, 边缘支承条件越弱, 壳体的上临界载荷越小, 壳体越易失稳•

§5. 结 束 语

本文联合应用积分方程法和修正迭代法分析了碟形扁壳在轴对称均布线载荷作用下的非

线性稳定问题• 得到问题的二次近似特征关系式,通过算例分析了几何参数 β, γ, k 及边缘支承情况对碟形扁壳非线性稳定特性的影响• 所得结果可供工程设计参考•

附　　录

(3.14) 中 φ_1, φ_2, φ_3 的表达式为

$$\left.\begin{aligned}
\varphi_1 &= \left(-\frac{1}{3}x^3\ln x + a_1x^3 \; 6-7\,\gamma x^2 + \frac{\gamma^2}{2}x - \frac{\gamma^3}{9}\right)\mu(x-\gamma) \\
&\quad + \left[-\frac{2}{3}a_2x^3 + (\beta a_2 + a_3)x^2 - \frac{1}{3}\beta^3a_2\right]\mu(x-\beta) \\
\varphi_2 &= a_2a_3x^3 + \bullet\left(-\frac{19}{24}a_2^2x^4 + \beta a_2^2x^3 + a_9x^2 - \frac{1}{3}\beta^3a_2^2x\right)\bullet\mu(x-\beta) \\
&\quad + \left[\left(a_7 + \frac{\gamma^4}{8}\right. + b_1\# + b_2x^2 + b_3x^3 + b_4x^4 + \frac{b_5}{x} + b_6x^4\ln^2x + b_7x^4\ln x \right. \\
&\quad \left. + b_8x^3\ln x + b_9x^2\ln^2x + b_{10}x^2\ln x + b_{11}x\ln x + \frac{\gamma^4}{32}\ln x\right]\mu(x-\gamma) \\
\varphi_3 &= a_2x^3\left(a_9 - \frac{1}{8}a_2^2x^2\right) + \left[C_1x + C_2x^3 + C_3x^5 + \frac{a_7\gamma^2}{4x} - \frac{1}{64}x^5\ln^3x + C_4x^5\ln^2x\right. \\
&\quad \left. + C_5x^5\ln x + C_6x^3\ln^2x - \frac{\gamma^2}{32}x^3\ln^3x + C_7x^3\ln x + \frac{\gamma^6}{128x}\ln x + C_8x\ln x\right]\mu(x-\gamma)
\end{aligned}\right\} \quad \text{(a)}$$

其中

$$b_1 = -\frac{\gamma^3}{9}d_1 + \frac{\gamma^2}{4}(a_3 - \gamma) + \beta a_2\left[\frac{\gamma^2}{4} + \frac{\beta^2}{3}\left(\frac{1}{2}\ln\gamma + \frac{1}{4}\right)\right]$$

$$b_2 = a_4 + \frac{\gamma^2}{2}d_1 + \frac{\gamma^2}{2}\left(\frac{a_1}{2} - \frac{a_2}{3}\right.$$

$$b_3 = -d_1(\gamma + a_3) - \left(\frac{1}{2}\ln\gamma + \frac{1}{4}\right.(\beta a_2 - \gamma)$$

$$b_4 = d_1a_1 + a_8 + \frac{2}{3}a_2\left(\frac{1}{2}\ln\gamma + \frac{1}{4}\right.$$

$$b_5 = -\frac{\gamma^2}{12}\left(\frac{\gamma^3}{3} + \beta^3a_2\right)$$

$$b_6 = -\frac{19}{96}$$

$$b_7 = a_5 - \frac{2}{3}a_2 + \frac{1}{2}a_1 + \frac{1}{6}\left(\ln\gamma + \frac{1}{2}\right.$$

$$b_8 = \frac{1}{2}\beta a_2 + \frac{1}{2}a_3 - \frac{\gamma}{2}$$

$$b_9 = -\frac{\gamma^2}{16}$$

$$b_{10} = a_6 + \frac{\gamma^2}{6}$$

$$b_{11} = -\frac{\gamma^3}{18} - \frac{\beta^3}{6}a_2$$

$$C_1 = d_1a_7 + \frac{\gamma^2}{4}(a_4 + a_9)$$

$$C_2 = d_1a_4 - \frac{1}{2}a_9\left(\ln\gamma + \frac{1}{2}\right. - \frac{\gamma^2}{4}\left(\frac{1}{8}a_2^2 + a_8\right)$$

$$C_3 = d_1a_8 + \frac{1}{16}a_2^2\left(\ln\gamma + \frac{1}{2}\right.$$

$$C_4 = -\frac{1}{32}d_1 + \frac{1}{2}a_5$$

$$C_5 = d_1a_5 - \frac{1}{16}a_2^2 + \frac{1}{2}a_8$$

$$C_6 = -\frac{\gamma^2}{16}d_1 - \frac{\gamma^2}{128} + \frac{1}{2}a_6$$

$$C_7 = d_1 a_6 + \frac{1}{2}(a_4 + a_9) + \frac{\gamma^2}{4}a_5$$

$$C_8 = \frac{\gamma^4}{32}d_1 + \frac{1}{2}a_7 + \frac{\gamma^2}{4}a_6$$

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Snap_Buckling of Dished Shallow Shells under Line Loads

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Abstract

A theoretical analysis is presented for the snap_buckling behavior of dished shallow shells under axisymmetric distributed line loads. The second approximate formulae of elastic behavior of dished shallow shells with various boundary conditions are qiven, and effects of geometrical parameters γ, β and k on non_linear behavior are discussed.

Key words dished shallow shells, snap_buckling, modified iteration method