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轴压加筋圆柱壳Koiter-边界层奇异摄动法

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摘　要

将Koiter理论和奇异摄动理论中的边界层法相结合，处理加筋圆柱壳无因次化非线性边界层型Karman-Donnell方程由分支点和边界层导致的双重奇异性，提出轴压加筋圆柱壳Koiter-边界层奇异摄动法。对AS-2壳分析表明，本方法具有很好的计算效率和计算精度，与数值解相比更能揭示其内在的影响规律。

关键词　奇异摄动理论　加筋圆柱壳　屈曲和后屈曲

一、引　言

轴压加筋和不加筋圆柱壳屈曲和后屈曲的研究，经历了近一个世纪的发展历史。经典线性理论的预测值与试验值的差异主要归结为前屈曲变形、边界条件和初始几何缺陷的影响[1]。壳体屈曲和初始后屈曲的一般理论首先由Koiter[2]创立，实用标准化方法由Budiansky[3]给出。为了计及前屈曲变形、边界条件和初始几何缺陷的影响，Yamaki[4]，Arbocz和Hol[5]以及Flores和Godey[6]分别采用Galerkin法、微分方程"打靶法"和有限元法等数值方法对Koiter理论的各级线性化摄动方程进行数值解。沈惠申、陈铁云等[7],[8]将Karman-Donnell方程化为边界层型方程，采用奇异摄动理论[9]中的边界层法进行求解，提出了"边界层理论"。

加筋圆柱壳无因次化非线性边界层型Karman-Donnell方程，具有由分支点和边界层导致的双重奇异性。本文将Koiter理论和奇异摄动理论中的边界层法相结合，对此双重奇异性分别处理，提出轴压加筋圆柱壳Koiter-边界层奇异摄动法。首先采用Koiter理论将纵向加筋圆柱壳无因次化非线性边界层型Karman-Donnell方程逐级化为线性的边界层型方程，然后将每级线性化的边界层型方程采用边界层法求解。文中为了计及前屈曲变形和边界条件的影响，采用Galerkin法对正则分支屈曲载荷进行修正。对AS-2壳[10]的详细计算和分析表明，本文提出的奇异摄动法具有很好的计算效率和计算精度。从理论分析中给出的若干结论，具有较大的理论意义和实用价值。

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二、基本表达式及无因次化

考虑半径为R、壳厚为t、长为L的加筋圆柱壳．壳体中面上的一点由轴向坐标X和径向坐标Y表示，壳体中面上法向位移W向内为正，且初始几何缺陷W^*满足

$$|W^*|\ll R,\quad |W^*_{,X}|\ll 1,\quad |W^*_{,Y}|\ll 1$$

由扁壳理论，可以得到如下以法向位移W和应力函数F表达的加筋圆柱壳非线性 Kaman-Donnell型控制方程[11]

$$\left.\begin{aligned}&L_H[F]+L_Q[W]+\frac{1}{R}\frac{\partial^2 W}{\partial X^2}=-\frac{1}{2}L_{NL}(W,\ W+2W^*)\\&L_D[W]-L_Q[F]-\frac{1}{R}\frac{\partial^2 F}{\partial X^2}=L_{NL}(F,\ W+W^*)\end{aligned}\right\}\tag{2.1}$$

式中

$$\left.\begin{aligned}&L_D[\quad]=D_{XX}\frac{\partial^4}{\partial X^4}+D_{XY}\frac{\partial^4}{\partial X^2\partial Y^2}+D_{YY}\frac{\partial^4}{\partial Y^4}\\&L_H[\quad]=H_{XX}\frac{\partial^4}{\partial X^4}+H_{XY}\frac{\partial^4}{\partial X^2\partial Y^2}+H_{YY}\frac{\partial^4}{\partial Y^4}\\&L_Q[\quad]=Q_{XX}\frac{\partial^4}{\partial X^4}+Q_{XY}\frac{\partial^4}{\partial X^2\partial Y^2}+Q_{YY}\frac{\partial^4}{\partial Y^4}\\&L_{NL}(S,T)=\frac{\partial^2 S}{\partial X^2}\frac{\partial^2 T}{\partial Y^2}-2\frac{\partial^2}{\partial X}\frac{S}{\partial Y}\frac{\partial^2 T}{\partial X\partial Y}+\frac{\partial^2 S}{\partial Y^2}\frac{\partial^2 T}{\partial X^2}\\&N_{XX}=\frac{\partial^2 F}{\partial Y^2},\quad N_{XY}=-\frac{\partial^2 F}{\partial X\partial Y},\quad N_{YY}=\frac{\partial^2 F}{\partial X^2}\end{aligned}\right\}\tag{2.2}$$

固支边界条件、闭合条件和单位端部缩短为

$$\left.\begin{aligned}&X=0,L,\quad W=W_{,X}=0;\quad \int_0^{2\pi R}\frac{\partial^2 F}{\partial Y^2}dY+P=0\\&\int_0^{2\pi R}\frac{\partial V}{\partial Y}dY=0,\quad \frac{\Delta}{L}=-\frac{1}{3\pi RL}\int_0^{2\pi R}\int_0^L\frac{\partial U}{\partial X}dXdY\end{aligned}\right\}\tag{2.3}$$

引入如下无量纲化因子[8]：

$$\left.\begin{aligned}&x=\frac{\pi}{L}X,\quad y=\frac{Y}{R},\quad \beta=\frac{L}{\pi R},\quad Z=\frac{L^2}{Rt}\sqrt{1-\nu^2}\\&C=\sqrt{3(1-\nu^2)},\quad \varepsilon=\frac{\pi^2}{\sqrt{12}Z},\quad (w,\ w^*)=\frac{(W,W^*)}{t}2C\varepsilon\\&f=\frac{F}{D}\varepsilon^2,\quad \sigma_{plc}=\frac{Et}{CR},\quad \lambda_p=\frac{\sigma}{\sigma_{plc}},\quad \delta_p=\frac{\Delta}{L}\frac{E}{\sigma_{pcl}}\end{aligned}\right\}\tag{2.4}$$

得加筋圆柱壳无因次化非线性边界层型Karman-Donnell方程

$$\left.\begin{aligned}&L_h[f]+\varepsilon L_q[w]+\frac{\partial^2 w}{\partial x^2}=-\frac{1}{2}\beta^2(w;w+2w^*)\\&\varepsilon^2L_d[w]-\varepsilon L_p[f]-\frac{\partial^2 f}{\partial x^2}=\beta^2(f;w+w^*)\end{aligned}\right\}\tag{2.5}$$

式中

$$
\left.\begin{aligned}
L_d[\quad] &= h_1\frac{\partial^4}{\partial x^4}+2h_2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+h_4\beta^4\frac{\partial^4}{\partial y^4}\\
L_h[\quad] &= s_1\frac{\partial^4}{\partial x^4}+2s_2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+s_4\beta^4\frac{\partial^4}{\partial y^4}\\
L_q[\quad] &= e_1\frac{\partial^4}{\partial x^4}+2e_2\beta^2\frac{\partial^4}{\partial x^2\partial y^2}+e_4\beta^4\frac{\partial^4}{\partial y^4}\\
(s;t) &= \frac{\partial^2 s}{\partial x^2}\frac{\partial^2 t}{\partial y^2}-2\frac{\partial^2 s}{\partial x\partial y}\frac{\partial^2 t}{\partial x\partial y}+\frac{\partial^2 s}{\partial y^2}\frac{\partial^2 t}{\partial x^2}
\end{aligned}\right\}\tag{2.6}
$$

固支边界条件、闭合条件和单位端部缩短无因次化为

$$
\left.\begin{aligned}
&x=0,\pi,\ w=w_{,x}=0;\ \frac{1}{2\pi}\int_0^{2\pi}\beta^2\frac{\partial^2 f}{\partial y^2}dy+2\lambda_p\varepsilon=0\\
&\int_0^{2\pi}\left[\left(s_1\frac{\partial^2 f}{\partial x^2}-v_{xy}\beta^2\frac{\partial^2 f}{\partial y^2}\right)+\varepsilon\left(e_1\frac{\partial^2 w}{\partial x^2}+e_y\beta^2\frac{\partial^2 w}{\partial y^2}\right)+w\right.\\
&\qquad\left.-\frac{1}{2}\beta^2\left(\frac{\partial w}{\partial y}\right)^2-\beta^2\frac{\partial w}{\partial y}\frac{\partial w^*}{\partial y}\right]dy=0\\
&\delta_p=-\frac{1}{4\pi^2\varepsilon}\int_0^{2\pi}\int_0^{\pi}\left[\left(s_4\beta^2\frac{\partial^2 f}{\partial y^2}-v_{xy}\frac{\partial^2 f}{\partial x^2}\right)+\varepsilon\left(e_4\beta^2\frac{\partial^2 w}{\partial y^2}+e_x\frac{\partial^2 w}{\partial x^2}\right)\right.\\
&\qquad\left.-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-\frac{\partial w}{\partial x}\frac{\partial w^*}{\partial x}\right]dxdy=0
\end{aligned}\right\}\tag{2.7}
$$

方程(2.5)为无因次化非线性边界层型Karman-Donnell方程. 当应用方程(2.5)进行加筋圆柱壳屈曲和后屈曲分析时，由于具有由分支点和边界层导致的双重奇异性，数值解对此的处理存在着较大的困难，本文采用Koiter理论和边界层法相结合的奇异摄动法进行求解.

三、完善壳体的Koiter-边界层奇异摄动解

在分支点附近，假定解可作如下渐近展开

$$
\left.\begin{aligned}
w&=w_0(\lambda_p,\ x,\ y,\ \varepsilon)+\eta w_1(x,\ y,\ \varepsilon)+\eta^2 w_2(x,\ y,\ \varepsilon)+\cdots\\
f&=f_0(\lambda_p,\ x,\ y,\ \varepsilon)+\eta f_1(x,\ y,\ \varepsilon)+\eta^2 f_2(x,\ y,\ \varepsilon)+\cdots\\
\frac{\lambda_p}{\lambda_{pc}}&=1+\eta b_1+\eta^2 b_2+\cdots
\end{aligned}\right\}\tag{3.1}
$$

式中 w_0, f_0为前屈曲法向位移和应力函数，w_1, f_1为分支屈曲法向位移和应力函数小增量，w_k, $f_k(k\geqslant 2)$为初始后屈曲法向位移和应力函数；λ_{pc}为分支屈曲载荷，$b_k(k\geqslant 1)$为初始后屈曲载荷系数，η为分支屈曲模态幅值. 根据奇异摄动理论中的边界层 法，w_k, $f_k(k\geqslant 0)$可以假定为

$$
w_k=w_k^{(o)}+w_k^{(i)}+w_k^{(I)},\quad f_k=f_k^{(o)}+f_k^{(i)}+f_k^{(I)}\tag{3.2}
$$

式中 $w_k^{(o)}$, $f_k^{(o)}$为外部正则解，$w_k^{(i)}$, $f_k^{(i)}$和$w_k^{(I)}$, $f_k^{(I)}$分别为$x=0$端和$x=\pi$端内部边界层解. 边界层变量ξ, ζ被定义为

$$\xi=\frac{x}{\sqrt{\varepsilon}},\ \zeta=\frac{\pi-x}{\sqrt{\varepsilon}}$$

由于对称性，仅给出 $x=0$端的边界层解．将上式及式(3.1)和式(3.2)代入到式(2.5)，得前屈曲、分支屈曲、初始后屈曲正则方程和边界层方程及其相应的解如下：

1　前屈曲

前屈曲正则方程为

$$\left.\begin{aligned}&L_h[f_0^{(o)}]+\varepsilon e_1\frac{d^4w_0^{(o)}}{dx^4}+\frac{d^2w_0^{(o)}}{dx^2}=0\\&\varepsilon^2h_1\frac{d^4w_0^{(o)}}{dx^4}-\varepsilon L_q[f_0^{(o)}]-\frac{\partial^2f_0^{(o)}}{\partial x^2}-\beta^2\frac{\partial^2f_0^{(o)}}{\partial y^2}\frac{d^2w_0^{(o)}}{dx^2}=0\end{aligned}\right\}\tag{3.3}$$

前屈曲边界层方程为

$$\left.\begin{aligned}&L_h^{(i)}[f_0^{(i)}]+\varepsilon e_1\frac{d^4w_0^{(i)}}{d\xi^4}+\varepsilon\frac{d^2w_0^{(i)}}{d\xi^2}=0\\&\varepsilon h_1\frac{d^4w_2^{(i)}}{dx^4}-L_q^{(i)}[f_0^{(i)}]-\frac{\partial^2f_0^{(i)}}{\partial\xi^2}-\beta^2\Big[\frac{\partial^2f_2^{(i)}}{\partial y^2}\frac{d^2w_0^{(i)}}{d\xi^2}\\&\quad+\frac{\partial^2f_0^{(o)}(\sqrt{\varepsilon}\,\xi,y)}{\partial y^2}\frac{d^2w_0^{(i)}}{d\xi^2}+\frac{\partial^2f_0^{(o)}}{\partial y^2}\frac{d^2w_0^{(o)}(\sqrt{\varepsilon}\,\xi)}{d\xi^2}\Big]=0\end{aligned}\right\}\tag{3.4}$$

前屈曲解为

$$\left.\begin{aligned}&w_0^{(o)}=2\lambda_pv_{xy}\varepsilon,\ f_0^{(o)}=-\frac{y^2}{\beta^2}\lambda_p\varepsilon\\&w_0^{(i)}=-2\lambda_pv_{xy}\varepsilon\Big(\cos\phi\xi+\frac{\alpha}{\phi}\sin\phi\xi\Big)\exp[-\alpha\xi]+\cdots\\&f_0^{(i)}=2\lambda_pv_{xy}\varepsilon^2\Big\{\Big[\frac{3\alpha^2-\phi^2}{s_1(\phi^2+\alpha^2)^2}-\frac{e_1}{s_1}\Big]\cos\phi\xi\\&\qquad-\frac{\alpha}{\phi}\Big[\frac{3\phi^2-\alpha^2}{s_1(\phi^2+\alpha^2)^2}-\frac{e_1}{s_1}\Big]\sin\phi\xi\Big\}\exp[-\alpha\xi]+\cdots\end{aligned}\right\}\tag{3.5}$$

式中

$$\phi=\sqrt{\frac{\sqrt{h_1s_1+e_1^2}+(\lambda_ps_1+e_1)}{2(h_1s_1+e_1^2)}},\quad\alpha=\sqrt{\frac{\sqrt{h_1s_1+e_1^2}-(\lambda_ps_1+e_1)}{2(h_1s_1+e_1^2)}}$$

前屈曲单位端部缩短为

$$\delta_{0p}=\lambda_ps_1-\frac{4v_{xy}^2}{\pi s_1}\frac{\alpha}{\phi^2+\alpha^2}\lambda_p\varepsilon^{\frac{1}{2}}+\frac{v_{xy}^2}{2\pi}\frac{\phi^2+\alpha^2}{\alpha}\lambda_p^2\varepsilon^{\frac{1}{2}}+\cdots\tag{3.6}$$

2　分支屈曲

分支屈曲正则方程

$$\left.\begin{aligned}&L_1(w_1^{(o)},f_1^{(o)})=L_h[f_1^{(o)}]+\varepsilon L_q[w_1^{(o)}]+\varepsilon\frac{\partial^2w_1^{(o)}}{dx^2}=0\\&L_2(w_1^{(o)},f_1^{(o)})=\varepsilon L_d[w_1^{(o)}]-L_q[f_1^{(o)}]-\frac{\partial^2f_1^{(o)}}{\partial x^2}+2\lambda_p\varepsilon\frac{\partial^2w_1^{(o)}}{\partial x^2}=0\end{aligned}\right\}\tag{3.7}$$

分支屈曲边界层方程

$$\left.\begin{aligned}&L_1^{(i)}(w_1^{(i)},f_1^{(i)})=L_h^{(i)}[f_1^{(i)}]+\varepsilon L_q^{(i)}[w_1^{(i)}]+\varepsilon\frac{\partial^2 w_1^{(i)}}{\partial\xi^2}+\varepsilon^{\frac{1}{2}}\frac{\partial^2 w_1^{(i)}}{\partial y^2}\frac{d^2 w_0^{(i)}}{d\xi^2}=0\\&L_2^{(i)}(w_1^{(i)},f_1^{(i)})=\varepsilon L_h^{(i)}[w_1^{(i)}]-L_q^{(i)}[f_1^{(i)}]-\frac{\partial^2 f_1^{(i)}}{\partial\xi^2}\\&-\varepsilon^{-\frac{1}{2}}\gamma^2\left[\frac{\partial^2 f_1^{(i)}}{\partial y^2}\frac{d^2 w_0^{(i)}}{d\xi^2}+\frac{d^2 f_0^{(i)}}{dy^2}\frac{\partial^2 w_1^{(i)}}{\partial\xi^2}\right]+2\lambda_p\varepsilon\frac{\partial^2 w_1^{(i)}}{\partial\xi^2}\Bigg]=0\end{aligned}\right\}\tag{3.8}$$

分支屈曲方程和其后的后屈曲方程中，由于纵向加筋圆柱壳周向波型参数$\eta\beta$与ε有关，为了得到分支屈曲及各级后屈曲关于ε的渐近展开式，需要引入以下周向坐标转换

$$y=\frac{\beta}{\gamma}\varepsilon^{-\frac{1}{4}}\bar{y}$$

其中 γ为仅与纵向加筋有关的常数．为方便起见，符号中的上划线被省略．正则分支解为

$$\left.\begin{aligned}&w_1^{(o)}=2C\varepsilon\sin mx\sin y\\&f_1^{(o)}=2C\left[\varepsilon^2\left(\frac{m^2}{s_4\gamma^4}-\frac{e_4}{s_4}\right)-\varepsilon^{\frac{5}{2}}\frac{2m^2}{s_4\gamma^2}\left(\frac{s_2}{s_4}\frac{m^2}{\gamma^4}-\frac{e_4}{s_4}s_2+e_2\right)+\cdots\right]\sin mx\sin y\end{aligned}\right\}\tag{3.9}$$

正则分支屈曲载荷为

$$\begin{aligned}\lambda_{pc}^{(o)}=&\frac{1}{2}\left[\frac{m^2}{s_4\gamma^4}+\frac{\gamma^4}{m^2}\left(h_4+\frac{e_4^2}{s_4}\right)-2\frac{e_4}{s_4}\right]+\varepsilon^{\frac{1}{2}}\left[\left(h_2+\frac{e_2e_4}{s_4}\right)\gamma^2-\frac{e_2m^2}{s_4\gamma^2}\right.\\&\left.-\frac{m^2-e_4\gamma^4}{s_4\gamma^2}-\left(\frac{s_2m^2}{s_4\gamma^4}-\frac{e_4s_2}{s_4}+e_2\right)\right]+\cdots\end{aligned}\tag{3.10}$$

分支屈曲边界层解为

$$\left.\begin{aligned}&w_1^{(i)}=-2C\varepsilon^{\frac{3}{2}}\frac{m}{\phi}\sin\phi\xi\exp[-\alpha\xi]\sin y\\&f_1^{(i)}=2C\varepsilon^{\frac{5}{2}}\frac{m}{\phi}\left\{\frac{2\alpha\phi}{s_1(\phi^2+\alpha^2)^2}\cos\phi\xi\right.\\&\left.-\left[\frac{\phi^2-\alpha^2}{s_1(\phi^2+\alpha^2)^2}-\frac{e_1}{s_1}\right]\sin\phi\xi\right\}\exp[-\alpha\xi]\sin y+\cdots\end{aligned}\right\}\tag{3.11}$$

上述分支屈曲边界层解不能满足高阶边界层分支方程，为此应用Galerkin法

$$\int_0^{2\pi}\int_0^{\pi}[L_1(w_1,\ f_1)f_1-L_2(w_1,\ f_1)w_1]dxdy=0$$

对正则分支屈曲载荷进行修正，得计及边界层影响的分支屈曲载荷

$$\lambda_{pc}=\frac{1}{1+\varepsilon\dfrac{v_{xy}\gamma^2}{2\pi s_1\alpha(\phi^2+\alpha^2)}}\left\{\begin{aligned}&\lambda_{pc}^{(o)}+\varepsilon\frac{\gamma^2}{\pi\alpha}\left[\left(h_2+2\frac{e_1e_2}{s_1}-\frac{e_1^2s_2}{s_1^2}\right)\right.\\&\left.-2\frac{e_2-\dfrac{e_1s_2}{s_1}}{s_1(\phi^2+\alpha^2)}+\frac{s^2}{s_1^2}\frac{3\alpha^2-\phi^2}{s_1^2(\phi^2+\alpha^2)^2}\right)\end{aligned}\right\}\tag{3.12}$$

3 后屈曲

后屈曲分析需要补充如下可解性条件：

$$\int_0^{2\pi}\int_0^{\pi}[L_1(w_k,\ f_k)f_1-L_2(w_k,\ f_k)w_1]dxdy=0\qquad(k\geqslant 2)\tag{3.13}$$

二阶后屈曲正则和边界层解为

$$\left.\begin{aligned} w_2^{(o)}&=\varepsilon^{\frac{3}{2}}\left[\frac{1}{2}C^2\gamma^2(1-\cos 2mx)-\frac{C^2m^2}{4\left(h_4+\frac{e_4^2}{s_4}\right)\gamma^2}\left(\frac{m^2}{s_4\gamma^4}-\frac{3}{2}\frac{e_4}{s_4}\right)\cos 2y\right]\\ &+\varepsilon^2\frac{C^2m^4}{2\left(h_4+\frac{e_4^2}{s_4}\right)s_4\gamma^4}\left(\frac{s_1m^2}{s_4\gamma^4}-\frac{e_4s_2}{s_4^2}+e^2\right)\cos 2y+\cdots\\ f_2^{(o)}&=\varepsilon^{\frac{5}{2}}\left\{C^2\gamma^2\left(\lambda_{po}+\frac{m^2}{s_4\gamma^4}-\frac{e_4}{s_4}\right)\cos 2mx+\left[-\frac{C^2e_4m^2}{4\left(h_4+\frac{e_4^2}{s_4}\right)s_4\gamma^2}\left(\frac{m^2}{s_4\gamma^4}-\frac{3}{2}\frac{e_4}{s_4}\right)\right.\right.\\ &\left.\left.+\frac{C^2m^2}{8s_4\gamma^4}\right]\cos 2y\right\}+\varepsilon^3\left\{\frac{2C^2m^2}{s_4}\left(\frac{s_1m^2}{s_4\gamma^4}-\frac{e_4s_2}{s_4}+e_2\right)\cos 2mx\right.\\ &\left.-\frac{C^2e_4m^4}{2\left(h_4+\frac{e_4^2}{s_4}\right)s_4^2\gamma^4}\left(\frac{s_2m^2}{s_4\gamma^4}-\frac{e_4s_2}{s_4^2}+c_2\right)\cos 2y\right\}+\cdots \end{aligned}\right\} \tag{3.14}$$

$$\left.\begin{aligned} w_2^{(i)}&=\varepsilon^{\frac{3}{2}}\frac{C^2m^2}{4\left(h_4+\frac{e_4^2}{s_4}\right)\gamma^2}\left(\frac{m^2}{s_4\gamma^4}-\frac{3}{2}\frac{e_4}{s_4}\right)\left(\cos\phi\xi+\frac{\alpha}{\phi}\sin\phi\xi\right)\exp[-\alpha\xi]\cos 2y+\cdots\\ f_2^{(i)}&=-\varepsilon^{\frac{5}{2}}\frac{C^2m^2}{4\left(h_4+\frac{e_4^2}{s_4}\right)\gamma^2}\left(\frac{m^2}{s_4\gamma^4}-\frac{3}{2}\frac{e_4}{s_4}\right)\left\{\left[\frac{3\alpha^2-\phi^2}{s_1(\phi^2+\alpha^2)^2}-\frac{e_1}{s_1}\right]\cos\phi\xi\right.\\ &\left.-\frac{\alpha}{\phi}\left[\frac{3\phi^2-\alpha^2}{s_1(\phi^2+\alpha^2)^2}-\frac{e_1}{s_1}\right]\sin\phi\xi\right\}\exp[-\alpha\xi]\cos 2y+\cdots \end{aligned}\right\} \tag{3.15}$$

壳体的承载能力为

$$\left.\begin{aligned} \frac{\lambda_p}{\lambda_{pc}}&=1-\eta^2\frac{C^2\gamma^2}{\lambda_{pc}}\left\{\varepsilon\left[\left(\lambda_{pc}+\frac{15}{8}\frac{m^2}{s_4\gamma^4}-2\frac{e_4}{s_4}\right)\right.\right.\\ &\left.\left.+\frac{m^2}{4\left(h_4+\frac{e_4^2}{s_4}\right)\gamma^2}\left(\frac{2m^2}{s_4\gamma^4}-\frac{3e_4}{s_4}\right)\left(\frac{m^2}{s_4\gamma^4}-\frac{3e_4}{2s_4}\right)\right]+\cdots\right\}+\cdots \end{aligned}\right\} \tag{3.16}$$

壳体的单位端部缩短为

$$\begin{aligned} \delta_p&=\delta_{op}+(\lambda_p-\lambda_{pc})\left(s_4+\frac{2v_{xy}^2}{\pi}\frac{\phi^2+\alpha^2}{\alpha}\lambda_{pc}\varepsilon^{\frac{1}{2}}\right.\\ &\left.-\frac{4v_{xy}^2}{\pi s_1}\frac{\alpha}{\phi^2+\alpha^2}\lambda_{pc}\varepsilon^{\frac{1}{2}}\right)+\eta^2\frac{C^2m^2\varepsilon}{4}\left(1+\frac{1}{\alpha}\varepsilon^{\frac{1}{2}}\right) \end{aligned} \tag{3.17}$$

根据Yamaki[4]的方法可以计及初始几何缺陷对陷曲载荷的影响，具体推导见文[12]．无因次化初始几何缺陷具有如下形式：

$$w^*=2C\varepsilon\mu_{11}\sin mx\sin ny+2C\varepsilon\mu_{20}\cos 2mx+2C\varepsilon_{02}\cos 2ny \tag{3.18}$$

四、数值算例

本文对AS-2纵向加筋圆柱壳进行了比较详细的计算和分析．Arbocz[10]和Simitses[13]

对该壳采用不同的边界条件和初始几何缺陷形式给出了专门讨论．AS-2 壳几何和物理参数见表1．

表 1　　AS-2壳几何和物理参数

t	1.96596×10^{-2}cm	(0.00774in)
L	13.97cm	(5.5in)
R	10.16cm	(4.0in)
d_S	0.803402cm	(0.3163in)
e_S	-3.36804×10^{-2}cm	(-0.01326in^2)
A_S	7.98708×10^{-3}cm^2	(0.00124in^2)
I_S	1.50384×10^{-3}cm^4	(0.3613×10^{-7}in^4)
J_S	4.94483×10^{-6}cm^4	(0.1188×10^{-6}in^4)
E	6.89472×10^{-6}N/cm^2	(1.0×10^7psi)
ν	0.3	

试验给出的屈曲载荷为226.3N/cm．本文摄动公式给出的外部正则解（经典分支屈曲载荷）为228.98N/cm，与Arbocz[11]的计算结果一致；考虑边界层影响（含前屈曲非线性和边界条件）给出的分支屈曲载荷为 23.800N/cm；考虑测量的初始几何缺陷 $\mu_{11}=0.024$，$\mu_{20}=-0.005$和$\mu_{02}=0.0$给出的屈曲载荷为226.46N/cm．Arbocz取CC-4边界条件和30个缺陷模式给出了他的最好计算结果243.80N/cm，取更多的缺陷模式和B-样条缺陷拟合对计算结果没有明显改善．这说明 Arbocz 所取的CC-4边界条件与试验条件有差距，而且对于以一个不对称屈曲模式为主的纵向加筋圆柱壳，取多个缺陷模式并非合适．Simitses取CC-3边界条件和三种不同的缺陷模型，给出了与试验吻合的计算结果，但在他的计算中缺陷模型与实际的屈曲模式有出入，缺陷模型为何如此选取缺乏理论依据．本文在计及与屈曲模式一致的测量的初始几何缺陷后给出的屈曲载荷与试验结果非常吻合，这说明了本文理论的合理性．图 1 显示了屈曲前三种加载情况下的前屈曲变形，随着加载的深入，前屈曲变形越来越明显．图 2 显示了AS-2壳的缺陷敏感性，正的$\mu_{11}=0.024$和负的μ_{20}，μ_{02}的缺陷组合使屈曲载荷下降最大，但 μ_{20}，μ_{02} 的影响较小．图 3 和图 4 显示了边界层对屈曲载荷的影响及缺陷敏感性因子随着Batdorf参数Z的变化曲线，Batdorf参数升高，边界层影响及缺陷敏感性因子降低．以上这些结果与已有的试验结果完全一致．

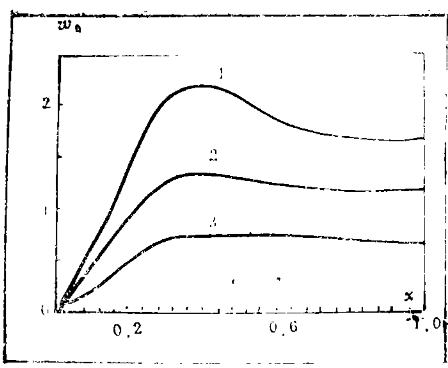

图 1

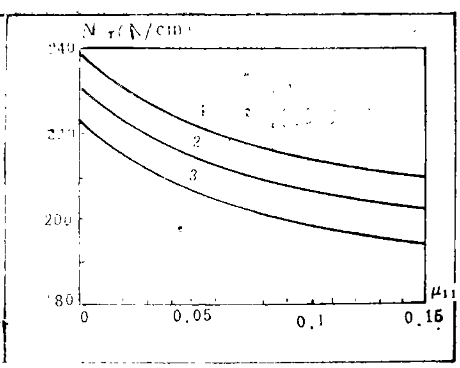

图 2

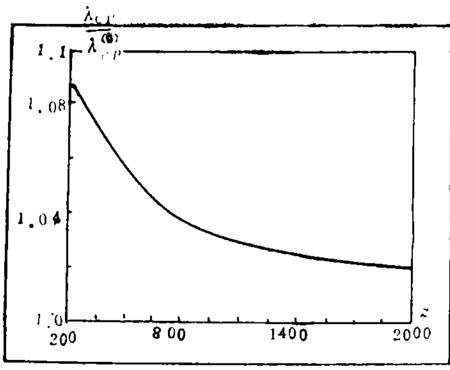

图 3

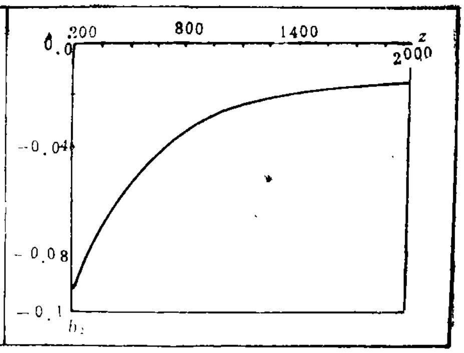

图 4

五、结 论

本文提出了轴压加筋圆柱壳 Koiter-边界层奇异摄动法，将Koiter理论和奇异摄动理论中的边界层法相结合求解轴压加筋壳无因次化非线性边界层型 Karman-Donnell方程。从文中的理论公式可以直接给出如下结论：

(1) 由(3.10)式，纵向外加筋($e_1<0$，$e_2<0$和$e_4=0$)比纵向内加筋具有更高的屈曲载荷；

(2) 由(3.12)式，边界层影响关于小参数ε为一阶量；

(3) 由(3.16)式，缺陷敏感性关于小参数ε为一阶量。

对于具有比较详细试验数据的 AS-2纵向加筋圆柱壳，本文给出了与试验值非常吻合的计算结果。可以预计的是由于径厚比较小的轴压圆柱壳首先在边界附近发生弹塑性皱曲，卸载之后再发生不对称分支屈曲[14]，本文的理论对这类问题的分析具有良好的前景。

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Koiter-Boundary Layer Singular Perturbtion Method for Axial Compressed Stiffened Cylindrical Shells

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Abstract

The double singularities induced by bifurcation point and boundary layer in non-demensionalized nonlinear boundary-layer-type Karman-Donnell equations for axially compressed stiffened cylindrical shells can be treated by Koiter-boundary layer singular perturbation method in this paper. based on the analysis of AS-2 shell, it is demonstrated that the method has high computing efficiency and accuracy, and some new conclusions can be directly drawn from the perturbation formulas.

Key words singular perturbation theory, stiffened cylindrical shells, buckling and postbuckling