应用数学和力学编委会编
重庆出版社出版

弹性地基上正交各向异性变厚度圆薄板的大挠度问题

王嘉新[1]　刘　杰[2]

（刘人怀推荐，1995年1月25日收到）

摘　要

本文推出了均布载荷下弹性基地上的正交各向异性变厚度圆薄板大挠度问题的基本方程。利用修正迭代法获得了该问题的二阶近似解。

关键词　变厚度圆薄板　大挠度　修正迭代法

一、引　　言

弹性基地上的圆薄板大挠度问题，是一个具有重要理论意义和实用价值的研究课题。近年来，许多学者对此问题进行了研究[1~3]。本文利用叶开沅、刘人怀在1965年提出的修正迭代法[4~5]求解了均布载荷下弹性地基上正交各向异性变厚度圆薄板的大挠度问题。在不同的ε和泊松比$\mu_{r\theta}$及λ下，本文给出了无量纲载荷和无量纲挠度的关系曲线，作为本文特例，给出了与文[1]和文[6]相一致的结果。

二、基本方程和边界条件

1．平衡方程

如图1所示，假设q为横向分布载荷，w，θ为圆板的挠度和转角，K^*为地基系数，T_r，T_θ为径向和周向薄膜力，M_r，M_θ为径向和周向弯矩，Q_r为横向剪力。由于轴对称性，$T_{r\theta}=T_{\theta r}$，$M_{r\theta}=0$。由$\Sigma R=0$，得

$$T_\theta=r\frac{dT_r}{dr}+T_r \tag{2.1}$$

及

$$rT_r\frac{dw}{dr}+rQ_r+\frac{qr^2}{2}-K\int wrdr=0 \tag{2.2}$$

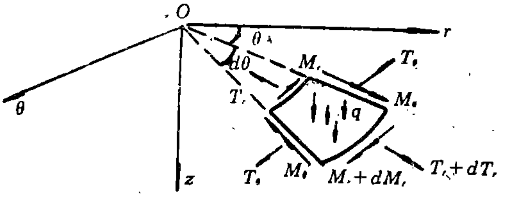

图　1

考虑环向力矩的平衡，得

1　中南工业大学，长沙 410083；2　株州工学院，株州 412000。

$$rQ_r=\frac{d}{dr}(rM_r)-M_\theta \tag{2.3}$$

对于轴对称正交各向异性变厚度圆薄板有：

$$\left.\begin{aligned} M_r&=-D_r\left(\frac{d^2w}{dr^2}+\frac{\mu_{\theta r}}{r}\ \frac{dw}{dr}\right)\\ M_\theta&=-D_\theta\left(\mu_{r\theta}\frac{d^2w}{dr^2}+\frac{1}{r}\ \frac{dw}{dr}\right)\end{aligned}\right\} \tag{2.4}$$

其中 $\mu_{r\theta}$，$\mu_{\theta r}$为 Poisson 比，D_r，D_θ为圆板的径向和周向抗弯刚度，等于

$D_r=E_rh^3(r)/12(1-\mu_{r\theta}\mu_{\theta r})$，$D_\theta=E_\theta h^3(r)/12(1-\mu_{r\theta}\mu_{\theta r})$，

这里$h(r)$为圆板的厚度，E_r，E_θ为沿径向和周向的弹性模量。

将式(2.4)代入式(2.3)，再代入式(2.2)，并利用$\lambda^2=E_r/E_\theta=\mu_{\theta r}/\mu_{r\theta}$，便得

$$D_r\left(r\frac{d^3w}{dr^3}+\frac{d^2w}{dr^2}-\frac{\lambda^2}{r}\ \frac{dw}{dr}\right)+\frac{dD_r}{dr}\left(r\frac{d^2w}{dr^2}+\mu_{\theta r}\frac{dw}{dr}\right)$$
$$=rT_r\ \frac{dw}{dr}+\frac{qr^2}{2}-K\int wrdr \tag{2.5}$$

2．协调方程

极坐标下轴对称圆板大挠度问题的Green应变为

$$\varepsilon_r=\frac{du_r}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2,\quad \varepsilon_\theta=\frac{u_r}{r},\quad \gamma_{r\theta}=0$$

其中 ε_r，ε_θ，$\gamma_{r\theta}$为板中面的应变分量，u_r，u_θ为中面在径向和周向的位移分量。

协调方程为

$$\varepsilon_r=\frac{d}{dr}(\varepsilon_\theta r)+\frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{2.6}$$

对于正交各向异性圆板，有

$$T_r=\frac{E_rh(r)}{1-\mu_{r\theta}\mu_{\theta r}}\varepsilon_r+\frac{\mu_{r\theta}E_\theta h(r)}{1-\mu_{r\theta}\mu_{\theta r}}\varepsilon_\theta$$
$$T_\theta=\frac{\mu_{\theta r}E_rh(r)}{1-\mu_{r\theta}\mu_{\theta r}}\varepsilon_r+\frac{E_\theta h(r)}{1-\mu_{r\theta}\mu_{\theta r}}\varepsilon_\theta$$

由上两式解出ε_r和ε_θ代入式(2·6)，并利用式(2·1)以及关系式$\mu_{r\theta}/E_r=\mu_{\theta r}/E_\theta$和$\lambda^2=\mu_{\theta r}/\mu_{r\theta}=E_\theta/E_r$可得

$$r\frac{d^2}{dr^2}(rT_r)+\frac{d}{dr}(rT_r)-\lambda^2T_r=\frac{r}{h(r)}\ \frac{dh(r)}{dr}\ \frac{d}{dr}(rT_r)$$
$$-\frac{\mu_{\theta r}}{h(r)}\ \frac{dh(r)}{dr}rT_r-\frac{1}{2}E_\theta h(r)\left(\frac{dw}{dr}\right)^2 \tag{2.7}$$

方程(2·5)和(2·7)就是弹性地基上受均布载荷作用的正交各向异性变厚度圆薄板大挠度问题的基本方程组。

为了简便，本文只讨论固定夹紧的周边条件：

$$\left.\begin{aligned}&当r=a时，\quad w=\frac{dw}{dr}=0\\&\qquad u_r=0\ 即\ \frac{d}{dr}(rT_r)-\mu_{\theta r}T_r=0\\&当r=0时，\quad \frac{dw}{dr}和T_r有限\end{aligned}\right\}\tag{2.8}$$

其中 a为圆板的半径。

将板厚度的变化规律表为

$$h(r)=h_0\left(1+\varepsilon_1\frac{r}{a}+\varepsilon_2\frac{r^2}{a^2}+\varepsilon_3\frac{r^3}{a^3}+\cdots\right)$$

式中 h_0为板中心处的厚度，ε_1，ε_2，ε_3，…为变厚度参数。为了简便，我们仅考虑线变厚度的情形，即 $h(r)=h_0\left(1+\varepsilon\frac{r}{a}\right)\quad |\varepsilon|<1$。

因此，本问题归结为在边界条件(2.8)下求解方程组(2.5)、(2.7)。

三、解　　法

我们利用修正迭代法求解上述方程组。为求解方便，引进无量纲参量：

$$\rho=\frac{r}{a},\quad x=\frac{w}{h_0},\quad Q=\frac{a^4q}{2D_0h_0},\quad S=\frac{\rho a^2}{D_0}T_r,$$

$$R=\frac{Ka^4}{D_0},\quad \eta=\frac{dx}{d\rho}$$

其中 $D_0=E_rh_0^3/12(1-\mu_{r\theta}\mu_{\theta r})$。

将式(2.5)、(2.7)化为

$$(1+\varepsilon\rho)^3\mathscr{L}(\rho^\lambda\eta)=Q\rho^2+S\eta-3\varepsilon(1+\varepsilon\rho)^2\left(\rho\frac{d\eta}{d\rho}+\mu_{\theta r}\eta\right)-R\int x\rho d\rho\tag{3.1}$$

$$\mathscr{L}(\rho^\lambda S)=\frac{\varepsilon}{1+\varepsilon\rho}\left(\rho\frac{dS}{d\rho}-\mu_{\theta r}S\right)-6(\lambda^2-\mu_{\theta r}^2)(1+\varepsilon\rho)\eta^2\tag{3.2}$$

边界条件(2.8)为

$$\left.\begin{aligned}&\rho=1,\quad x=\eta=0,\quad \frac{dS}{d\rho}-\mu_{\theta r}\frac{S}{\rho}=0\\&\rho=0,\quad \eta和S有限\end{aligned}\right\}\tag{3.3}$$

其中 $\mathscr{L}(\quad)=\rho^\lambda\frac{d}{d\rho}\rho^{-(2\lambda-1)}\frac{d}{d\rho}(\quad)$

修正迭代法的求解过程如下：

$$(1+\varepsilon\rho)^3\mathscr{L}(\rho^\lambda\eta_1)=Q\rho^2$$

$$\mathscr{L}(\rho^\lambda S_1)=-6(\lambda^2-\mu_{\theta r}^2)(1+\varepsilon\rho)\eta_1^2$$

$$\mathscr{L}(\rho^\lambda\eta_2)=Q\rho^2+S_1\eta_1-3\varepsilon(1+\varepsilon\rho^2)\left(\rho\frac{d\eta_1}{d\rho}+\mu_{\theta r}\eta_1\right)$$

$$-\varepsilon\rho(3+3\varepsilon\rho+\varepsilon^2\rho^2)\mathscr{L}(\rho^\lambda\eta_1)-R\int x_1\rho d\rho$$

$$\mathscr{L}(\rho^\lambda S_2)=\frac{\varepsilon}{1+\varepsilon}\left(\rho\frac{dS_1}{d\rho}-\mu_{\theta r}S_1\right)-6(\lambda^2-\mu_{\theta r}^2)(1+\varepsilon\rho)\eta_2^2$$

……

下标1，2表示迭代的阶数．在一阶迭代中，我们有如下方程：

$$(1+\varepsilon\rho)^3\mathscr{L}(\rho^\lambda\eta_1)=Q\rho^2 \tag{3.4}$$

$$\mathscr{L}(\rho^\lambda S_1)=-6(\lambda^2-\mu_{\theta r}^2)(1+\varepsilon\rho)\eta_1^2 \tag{3.5}$$

以及相应的边界条件：

$$\left.\begin{array}{l}\rho=1\text{时，}x_1=\eta_1=0,\ \dfrac{dS_1}{d\rho}-\mu_{\theta r}\dfrac{S_1}{\rho}=0\\ \rho=0\text{时，}\eta_1,\ S_1\text{有限}\end{array}\right\} \tag{3.6}$$

解得

$$\eta_1=Q\left(\frac{1}{q-\lambda^2}\rho^3-\frac{3\varepsilon}{16-\lambda^2}\rho^4+H_0\rho^\lambda\right) \tag{3.7}$$

$$x_1=Q\left[\frac{1}{4(9-\lambda^2)}\rho^4-\frac{3\varepsilon}{5(16-\lambda^2)}\rho^5+\frac{H_0}{\lambda+1}\rho^{\lambda+1}+H\right] \tag{3.8}$$

当$\lambda^2\neq 9$，$\lambda^2\neq 16$且$\lambda>0$时，令$\rho=0$，$x_1=x_0$，则

$$Q=\frac{x_0}{H}$$

于是

$$\eta_1=\frac{x_0}{H}\left(\frac{1}{9-\lambda^2}\rho^3-\frac{3\varepsilon}{16-\lambda^2}\rho^4+H_0\rho^\lambda\right) \tag{3.9}$$

$$x_1=\frac{x_0}{H}\left[\frac{1}{4(9-\lambda^2)}\rho^4-\frac{3\varepsilon}{5(16-\lambda^2)}\rho^5+\frac{H_0}{\lambda+1}\rho^{\lambda+1}+H\right] \tag{3.10}$$

式中 $H_0=\dfrac{3\varepsilon}{16-\lambda^2}-\dfrac{1}{9-\lambda^2}$

$$H=\frac{3\varepsilon}{5(16-\lambda^2)}-\frac{1}{4(9-\lambda^2)}-\frac{H_0}{\lambda+1}$$

将式(3·9)代入方程(3·5)的右端，并利用式(3·6)解得

$$\begin{aligned}S_1=-\frac{6(\lambda^2-\mu_{\theta r}^2)}{H^2}x_0^2\Bigg[&\frac{H_0}{4(9-\lambda^2)(\lambda+2)}\rho^{\lambda+4}+\frac{3H_0\varepsilon(2\lambda^2-11)}{5(2\lambda+5)(9-\lambda^2)(16-\lambda^2)}\rho^{\lambda+5}\\&-\frac{H_0\varepsilon^2}{2(\lambda+3)(16-\lambda^2)}\rho^{\lambda+6}+\frac{1}{(49-\lambda^2)(9-\lambda^2)^2}\rho^7\\&+\frac{\varepsilon(5\lambda^2-38)}{(64-\lambda^2)(9-\lambda^2)^2(16-\lambda^2)}\rho^8-\frac{3\varepsilon^2(\lambda^2+5)}{(81-\lambda^2)(16-\lambda^2)^2(9-\lambda^2)}\rho^9\\&+\frac{9\varepsilon^2}{(100-\lambda^2)(16-\lambda^2)^2}\rho^{10}+\frac{H_0^2}{(3\lambda+1)(\lambda+1)}\rho^{2\lambda+1}\\&+\frac{H_0^2\varepsilon}{(\lambda+2)(3\lambda+2)}\rho^{2\lambda+2}+H_1\rho^{\lambda+2}\Bigg]\end{aligned}$$

式中 $$\begin{aligned}H_1=\frac{1}{\mu_{\theta r}-\lambda-2}\Bigg[&\frac{H_0(\lambda+4-\mu_{\theta r})}{4(\lambda+2)(9-\lambda^2)}+\frac{3H_0\varepsilon(2\lambda^2-11)(\lambda+5-\mu_{\theta r})}{5(2\lambda+5)(9-\lambda^2)(16-\lambda^2)}\\&+\frac{H_0\varepsilon^2(\mu_{\theta r}-\lambda-6)}{2(\lambda+3)(16-\lambda^2)}+\frac{7-\mu_{\theta r}}{(49-\lambda^2)(9-\lambda^2)^2}\end{aligned}$$

$$+\frac{\varepsilon(5\lambda^2-38)(8-\mu_{\theta r})}{(64-\lambda^2)(9-\lambda^2)^2(16-\lambda^2)}+\frac{3\varepsilon^2(\lambda^2+5)(\mu_{\theta r}-9)}{(81-\lambda^2)(16-\lambda^2)(9-\lambda^2)}$$

$$+\frac{9\varepsilon^2(10-\mu_{\theta r})}{(100-\lambda^2)(16-\lambda^2)^2}+\frac{H_0^2(2\lambda+1-\mu_{\theta r})}{(3\lambda+1)(\lambda+1)}+\frac{H_0^2\varepsilon(2\lambda+2\mu_{\theta r})}{(\lambda+2)(3\lambda+2)}\Big]$$

在二阶迭代中，我们有如下方程：

$$\mathscr{L}(\rho^\lambda\eta_2)=Q\rho^2+S_1\eta_1-3\varepsilon(1+\varepsilon\rho)^2\left(\rho\frac{d\eta_1}{d\rho}+\mu_{\theta r}\eta_1\right)$$

$$-\varepsilon\rho(3+3\varepsilon\rho+\varepsilon^2\rho^2)\mathscr{L}(\rho^\lambda\eta_1)-R\int x_1\rho d\rho \tag{3.11}$$

$$\mathscr{L}(\rho^\lambda S_2)=\frac{\varepsilon}{1+\varepsilon\rho}\left(\rho\frac{dS_1}{d\rho}-\mu_{\theta r}S_1\right)-6(\lambda^2-\mu_{\theta r}^2)(1+\varepsilon\rho)\eta_2^2 \tag{3.12}$$

以及相应的边界条件：

$$\left.\begin{aligned}&\rho=1\text{ 时，}x_2=\eta_2=0,\quad \frac{dS_2}{d\rho}-\mu_{\theta r}\frac{S_2}{\rho}=0\\&\rho=0\text{时，}\eta_2,\ S_2\text{有限}\end{aligned}\right\} \tag{3.13}$$

解得

$$x_2=\frac{-6(\lambda^2-\mu_{\theta r}^2)}{H^3}x_0^3\Big(\frac{C_0}{16(\lambda+4)(\lambda+9)}\rho^{\lambda+8}+\frac{C_1}{9(2\lambda+9)(\lambda+10)}\rho^{\lambda+10}$$

$$-\frac{C_2}{20(\lambda+5)(\lambda+11)}\rho^{\lambda+11}+\frac{C_3}{11(2\lambda+11)(\lambda+12)}\rho^{\lambda+12}$$

$$+\frac{C_4}{12(121-\lambda^2)}\rho^{12}+\frac{C_5}{13(144-\lambda^2)}\rho^{13}-\frac{C_6}{14(169-\lambda^2)}\rho^{14}$$

$$+\frac{C_7}{15(196-\lambda^2)}\rho^{15}-\frac{C_8}{16(225-\lambda^2)}\rho^{16}+\frac{C_9}{2(\lambda+5)(3\lambda+6)(\lambda+3)}\rho^{2\lambda+6}$$

$$+\frac{C_{10}}{3(\lambda+6)(\lambda+2)(2\lambda+7)}\rho^{2\lambda+7}-\frac{C_{11}}{2(\lambda+4)(\lambda+7)(3\lambda+7)}\rho^{2\lambda+8}$$

$$-\frac{C_{12}}{7(2\lambda+7)(\lambda+8)}\rho^{\lambda+8}+\frac{C_{13}}{12(\lambda+1)(2\lambda+1)(\lambda+1)}\rho^{3\lambda+3}$$

$$+\frac{C_{14}}{(2\lambda+3)(4\lambda+3)(3\lambda+4)}\rho^{3\lambda+4}+\frac{H_1H_0}{6(\lambda+3)(\lambda+1)(\lambda+2)}\rho^{2\lambda+4}$$

$$+\frac{H_1}{12(\lambda+3)(\lambda+7)}\rho^{\lambda+7}\Big)-\frac{3x_0}{H}\Big[\frac{\varepsilon(B_0+1)}{5(16-\lambda^2)}\rho^5$$

$$+\frac{RH}{24(9-\lambda^2)}\rho^4-\frac{\varepsilon(2\varepsilon+B_1)}{6(25-\lambda^2)}\rho^6-\frac{\varepsilon(8\varepsilon^2+3B_2)}{21(36-\lambda^2)\rho}\rho^7$$

$$-\frac{72\varepsilon(9-\lambda^2)(\varepsilon^3+B_3)-R}{576(49-\lambda^2)}\rho^8-\frac{R\varepsilon}{315(16-\lambda^2)(64^2-\lambda^2)}\rho^9$$

$$+\frac{H_0\varepsilon(\lambda+\mu_{\theta r})}{(2\lambda+1)(\lambda+2)}\rho^{\lambda+2}+\frac{H_0\varepsilon^2(\lambda+\mu_{\theta r})}{2(\lambda+1)(\lambda+3)}\rho^{\lambda+3}$$

$$+\frac{H_0R}{24(\lambda+1)(\lambda+2)(\lambda+3)(\lambda+5)}\rho^{\lambda+5}\Big]+\frac{Q}{(3-\lambda)(2\lambda+3)(\lambda+4)}\rho^{\lambda+4}$$

$$+\frac{D_0}{\lambda+1}\rho^{\lambda+1}+\mathscr{L}_0 \tag{3.14}$$

其中

$$B_0=\frac{3+\mu_{\theta r}}{9-\lambda^2}$$

$$B_1=\frac{3\varepsilon(4+\mu_{\theta r})}{16-\lambda^2}-\frac{2\varepsilon(3+\mu_{\theta r})}{9-\lambda^2}$$

$$B_2=\frac{6\varepsilon^2(4+\mu_{\theta r})}{16-\lambda^2}-\frac{(3+\mu_{\theta r})\varepsilon^2}{9-\lambda^2}$$

$$C_0=\frac{H_0}{(9-\lambda^2)^2}\left[\frac{1}{4(\lambda+2)}+\frac{1}{49-\lambda^2}\right]$$

$$C_1=\frac{3H_0\varepsilon(2\lambda^2-11)}{5(2\lambda+5)(9-\lambda^2)^2(16-\lambda^2)}-\frac{3\varepsilon H_0}{4(16-\lambda^2)(9-\lambda^2)(\lambda+2)}$$
$$+\frac{H_0\varepsilon(5\lambda^2-38)}{(64-\lambda^2)(9-\lambda^2)(16-\lambda^2)}$$

$$C_2=\frac{H_0\varepsilon^2}{2(\lambda+3)(9-\lambda^2)(16-\lambda^2)}+\frac{9H_0\varepsilon^2(2\lambda^2-11)}{5(2\lambda+5)(9-\varepsilon^2)(16-\varepsilon^2)^2}$$
$$+\frac{3\varepsilon^2(\lambda^2+5)H_0}{(81-\lambda^2)(16-\lambda^2)^2(9-\lambda^2)}$$

$$C_3=\frac{3H_0\varepsilon^3}{(16-\lambda^2)^2}\left[\frac{1}{2(\lambda+3)}+\frac{3}{100-\lambda^2}\right]$$

$$C_4=\frac{1}{(49-\lambda^2)(9-\lambda^2)^3}$$

$$C_5=\frac{\varepsilon(5\lambda^2-38)}{(64-\lambda^2)(9-\lambda^2)^3(16-\lambda^2)}-\frac{3\varepsilon}{(49-\lambda^2)(9-\lambda^2)^2(16-\lambda^2)}$$

$$C_6=\frac{3\varepsilon^2(\lambda^2+5)}{(81-\lambda^2)(16-\lambda^2)^2(9-\lambda^2)^3}+\frac{3\varepsilon^2(5\lambda^2-38)}{(64-\lambda^2)(9-\lambda^2)^2(16-\lambda^2)^2}$$

$$C_7=\frac{9\varepsilon^2}{(100-\lambda^2)(16-\lambda^2)^2(9-\lambda^2)}+\frac{9\varepsilon^2(\lambda^2+5)}{(81-\lambda^2)(16-\lambda^2)^3(9-\lambda^2)}$$

$$C_8=\frac{27\varepsilon^4}{(100-\lambda^2)(16-\lambda^2)^3}$$

$$C_9=\frac{H_0^2}{(3\lambda+1)(\lambda+1)(9-\lambda^2)}+\frac{H_0^2}{4(9-\lambda^2)^2(\lambda+2)}$$

$$C_{10}=\frac{H_0^2\varepsilon}{(\lambda+2)(3\lambda+2)(9-\lambda^2)}-\frac{3\varepsilon H_0^2}{(3\lambda+1)(\lambda+1)(16-\lambda^2)}$$
$$+\frac{3H_0^2\varepsilon(2\lambda-11)}{5(2\lambda+5)(9-\lambda^2)(16-\lambda^2)}$$

$$C_{11}=\frac{3\varepsilon_0^2H_0^2}{(\lambda+2)(3\lambda+2)(16-\lambda^2)}+\frac{H_0^2\varepsilon^2}{2(\lambda+3))(16-\lambda^2)}$$

$$C_{12}=\frac{3\varepsilon H_1}{16-\lambda^2}$$

$$C_{13}=\frac{H_0^3}{(3\lambda+1)(\lambda+1)}$$

$$C_{14}=\frac{H_0^3\varepsilon}{(\lambda+2)(3\lambda+2)}$$

$$\mathscr{L}_0=-\alpha_0x_0^3-\beta_0x_0+\frac{3Q}{(3-\lambda)(2\lambda+3)(\lambda+1)(\lambda+4)}$$

$$\alpha_0=\frac{6(\lambda-\mu_{\theta r}^2)}{H^3}\Big[\frac{C_0}{2(\lambda+1)(\lambda+4)(\lambda+9)}+\frac{C_1}{(2\lambda+9)(\lambda+1)(\lambda+10)}$$

$$-\frac{C_2}{2(\lambda+1)(\lambda+5)(\lambda+11)}+\frac{C_3}{(\lambda+1)(\lambda+12)(2\lambda+11)}$$

$$+\frac{C_4}{12(\lambda+1)(11+\lambda)}+\frac{C_5}{13(\lambda+1)(\lambda+12)}-\frac{C_6}{14(\lambda+1)(\lambda+13)}$$

$$+\frac{C_7}{15(\lambda+1)(\lambda+14)}-\frac{C_8}{16(\lambda+1)(\lambda+15)}$$

$$+\frac{C_9}{(\lambda+1)(2\lambda+6)(3\lambda+5)}+\frac{C_{10}}{3(\lambda+1)(\lambda+2)(2\lambda+7)}$$

$$-\frac{C_{11}}{(3\lambda+7)(2\lambda+8)(\lambda+1)}-\frac{C_{12}}{(\lambda+1)(\lambda+8)(2\lambda+7)}$$

$$+\frac{C_{13}}{6(\lambda+1)^2(2\lambda+1)}+\frac{C_{14}}{(4\lambda+3)(3\lambda+4)(\lambda+1)}$$

$$+\frac{H_1H_0}{6(\lambda+1)^2(\lambda+2)}+\frac{H_1}{2(\lambda+1)(\lambda+3)(\lambda+7)}\Big]$$

$$\beta_0=\frac{3}{H}\Big[\frac{(B_0+1)\varepsilon}{5(\lambda+1)(\lambda+4)}+\frac{RH}{24(\lambda+1)(\lambda+3)}-\frac{\varepsilon(2\varepsilon+B_1)}{6(\lambda+1)(\lambda+5)}$$

$$-\frac{\varepsilon(8\varepsilon^2+3B_2)}{21(6+\lambda)(\lambda+1)}-\frac{72\varepsilon(9-\lambda^2)(\varepsilon^3+B_3)-R}{576(\lambda+1)(\lambda+7)(9-\lambda^2)}$$

$$-\frac{R\varepsilon}{315(\lambda+1)(\lambda+8)(16-\lambda^2)}+\frac{H_0\varepsilon(\lambda+\mu_{\theta r})}{(2\lambda+1)(\lambda+1)(\lambda+2)}$$

$$+\frac{H_0\varepsilon^2(\lambda+\mu_{\theta r})}{(\lambda+1)^2(\lambda+3)}+\frac{H_0R}{6(\lambda+1)^2(\lambda+2)(\lambda+3)(\lambda+5)}$$

式(3.14)近似地表示了弹性地基上均布载荷下正交各向异性变厚度圆薄板大挠度问题的无量纲挠度。

令$\rho=0$时，$x_2=x_0$，则由式(3.14)导出无量纲中心挠度x_0与无量纲载荷Q的关系式：

$$Q=\frac{1}{3}(\alpha x_0^3+\beta x_0) \tag{3·15}$$

其中 $\alpha=(3-\lambda)(2\lambda+3)(\lambda+1)(\lambda+4)\alpha_0$

$\beta=(3-\lambda)(2\lambda+3)(\lambda+1)(\lambda+4)(\beta_0+1)$

当$R=100$时，对于不同的ε，λ 和 $\mu_{\theta r}$，无量纲中心挠度与无量纲载荷的关系曲线，如图

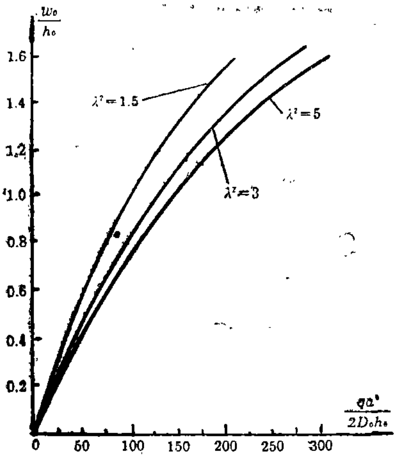

图2 在$R=100$，$\varepsilon=0.01$，$\mu_{r\theta}=0.25$下，不同的λ时无量纲载荷与无量纲挠度关系曲线

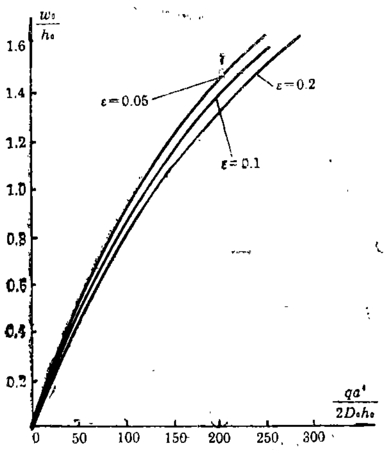

图3 在$R=100$，$\lambda^2=3$，$\mu_{r\theta}=0.25$下，不同的ε时无量纲荷载与无量纲挠度关系曲线

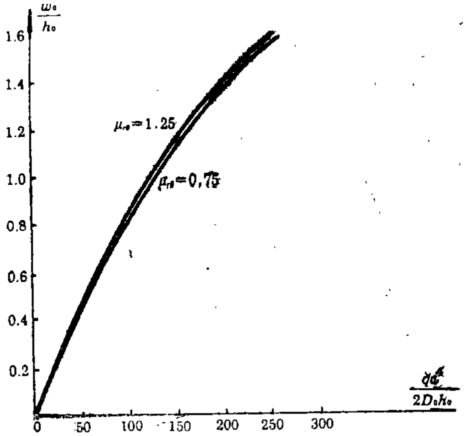

图4 $R=100$，$\varepsilon=0.1$，$\lambda^2=3$下，不同的$\mu_{r\theta}$时无量纲载荷与无量纲挠度关系曲线

2、图3和图4所示。

四、讨 论

如令式(3.14)中的$R=0$，$\lambda=1$，$\mu_{\theta r}=\mu$及$\varepsilon=0$，则得到各向同性等厚度均布载荷作用下圆薄板的非线性弯曲结果[6]，即：

$$\frac{qa^4}{D_0h_0}=34.9208x_0^3+64x_0 \tag{4.1}$$

如令式(3.14)中的$\varepsilon=0$，$\lambda=1$，$\mu_{\theta r}=\mu=0.3$，则当$R=100$时，得无量纲载荷与无量纲中心挠度关系曲线方程为

$$\frac{q_0a^4}{Eh_0^4}=11.7114x_0+3.1979x_0^3 \tag{4.2}$$

参 考 文 献

[1] Chia Chuenyuan, *Nonlinear Analysls of Plates*, Printed and Bound in the United States of America (1980).

[2] Y. Nath, Large amplitude response of circular plates foundations, *Int. J. Nonlinear Mech.*, 17(4) (1982), 285—296.

[3] R. Baton, Stress in circular plate on elastic foundation. *J. Eng. Mech. Div. Proc. ASCE.*, 98 (EMB) (1972), 629—640.

[4] 叶开沅、刘人怀等，在对称线布载荷作用下的圆底扁薄球壳的非线性稳定问题，科学通报，(2) (1965), 142.

[5] 刘人怀，在内边缘均布力矩作用下中心开孔圆底扁球壳的非线性稳定问题，科学通报，(3) (1965), 253.

[6] Yeh Kaiyuan and Wang Xinzhi, Modified iteration method in the problem of large deflection of thin circnlor plates with non-uniform thickness, *Proc. of the International Conference on Nonlinear Mechanics*, Shanghai, Editor-in-Chief Chien Weizang, Science Press (1985), 398—403.

Large Deflection Problem of Thin Orthotropic Circular Plate on Elastic Foundation with Variable Thickness under Uniform Pressure

Wang Jiaxin

(*Centra-Sounth University of Technology, Changsha* 410083, *P. R. China*)

Liu Jie

(*Zhuzhou Engineering Institute, Zhuzhou, Hu'nan* 412000 *P. R. China*)

Abstract

Basicequations for large deflection theory of thin orthotropic circular plate on elastic foundation with variable thickness under uniform pressure are derived in this paper. The modified iteration method is adopted to solve the large deflection problem of thin orthotropic circular plate on elastic foundation with variable thickness under uniform pressure. If $\varepsilon=0$, $R=100$ and $R=200$, $\mu=0.3$, $\lambda=1$, then the result derived from the solution in this paper agrees statisfactorily with the result given by Galerkin's method[1] for solving large deflection problem of thin circular plate with constant thickness on elastic foundation under uniform pressure.

Key words variable thickness, thin circular plate, large deflection, modified iteration method