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非线性非完整空间变质量体的一种运动方程

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摘　要

引入非线性非完整空间的约束超曲面的基矢量和密歇尔斯基方程点乘，作为非线性非完整系统变质量体的基本动力学方程。它简明、运算简便，而且由它可导出Чаплыгин, Nielsou, Appell, Mac-Millan等已有的方程，不必附加关于虚位移的Appell-Четаев定义或牛青萍定义。本方程与D′Alembert-Lagrange微分变分原理也相容。

关键词　非线性非完整约束　变质量体系统　约束超曲面基矢量　Мещерский方程点乘

N个质点组成的可变质量系统，其位形由n个广义坐标q_s确定，受到g个一阶非线性非完整约束，

$$f_\beta(q_s,\dot q_s;\ t)=0 \qquad (\beta=1,2,\cdots,g;\ s=1,2,\cdots,n) \tag{1}$$

设g个广义速度$\dot q_{\varepsilon+\beta}$可用$\varepsilon$个独立的广义速度$\dot q_\sigma$表示出来，即

$$\dot q_{\varepsilon+\beta}=\dot q_{\varepsilon+\beta}(q_s,\dot q_\sigma;\ t) \qquad (\sigma=1,2,\cdots,\varepsilon) \tag{2}$$

则通常由变质量力学系统的D′Alembert-Lagrange原理

$$\sum_{i=1}^{N}\{\boldsymbol F_i+\boldsymbol R_i-m_i\ddot{\boldsymbol r}_i\}\cdot\delta\boldsymbol r_i=0$$

附加虚位移的Appell-Четаев定义

$$\delta q_{\varepsilon+\beta}=\sum_{\sigma=1}^{\varepsilon}\frac{\partial\dot q_{\varepsilon+\beta}}{\partial\dot q_\sigma}\delta q_\sigma$$

可导出非线性非完整系统的广义Чаплыгин, Nielson, Appell等方程[1]，式中$\boldsymbol R_i=\dot m_i\boldsymbol u_i$为反推力，$\boldsymbol u_i$为从质点分离出来（或并入）的微粒相对质点的速度。但对Appell-Четаев定义有争议[2,3,4,5]，且是额外加的，对第二类（伺服）约束也不适用[6]。因此，我们另找途径，采用几何图象建立简明的动力学方程，并可由此导出已有的微分运动方程。

令$\tilde{\dot{\boldsymbol r}}_i$为第$i$个质点速度$\dot{\boldsymbol r}_i$中的$\dot q_{\varepsilon+\beta}$已用(2)式消去，即

$$\tilde{\dot{\boldsymbol r}}_i(q_s,\dot q_\sigma;\ t)=\dot{\boldsymbol r}_i(q_s,\dot q_\sigma,\dot q_{\varepsilon+\beta}(q_s,\dot q_\sigma;t);t) \tag{3}$$

$$(i=1,2,\cdots,N;\ s=1,2,\cdots,n;\ \sigma=1,2,\cdots,\varepsilon;\ \beta=1,2,\cdots,g;\ \varepsilon=n-g)$$

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则(3)式为一阶非线性非完整（第一类）约束超曲面的参数方程．因而，我们可引入一阶非线性非完整空间（即速度空间或切空间）的约束超曲面的基矢量为 $\partial\tilde{\boldsymbol{r}}_i/\partial\dot{q}_\sigma$．约束允许的空间可看成为$\partial\tilde{\boldsymbol{r}}_i/\partial\dot{q}_1$，$\partial\tilde{\boldsymbol{r}}_i/\partial\dot{q}_2,\cdots,\partial\tilde{\boldsymbol{r}}_i/\partial\dot{q}_\varepsilon$的生成子空间（即约束超曲面）．在这子空间的体系可看成是自由的，密歇尔斯基（Мещерский）方程的力是给定力，不含约束力，因后者与约束曲面垂直．对于不与约束曲面垂直的第二类（伺服）约束力可分解为与约束曲面垂直的和平行的部分，平行部分的约束力可归入给定力．将（第一类）约束超曲面的基矢量$\partial\tilde{\boldsymbol{r}}_i/\partial\dot{q}_\sigma$和密歇尔斯基方程点乘：

$$\sum_{i=1}^{N}\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma}\cdot(\boldsymbol{F}_i+\boldsymbol{R}_i-m_i\ddot{\boldsymbol{r}}_i)=0\qquad(\sigma=1,2,\cdots,\varepsilon)\tag{4}$$

作为非线性非完整系统变质量体的基本运动方程式．由此可导出广义的 Mac-Millan, Nielson, Appell, Чаплыгин, Boltzman-Hamel, Volterra等方程．

系统的动能及其微商为：

$$T=\frac{1}{2}\sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\dot{\boldsymbol{r}}_i,\quad \dot{T}=\sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\ddot{\boldsymbol{r}}_i+\frac{1}{2}\sum_{i=1}^{N}\dot{m}_i\dot{\boldsymbol{r}}_i\dot{\boldsymbol{r}}_i\tag{5}$$

设$\tilde{T}$和$\tilde{\dot{T}}$各为T和$\dot{T}$中的$\dot{q}_{\varepsilon+\beta}$已用(2)式消去，即

$$\tilde{T}(q_s,\dot{q}_\sigma;t)=T(q_s,\dot{q}_\sigma,\dot{q}_{\varepsilon+\beta}(q_s,\dot{q}_\sigma;t);t)\tag{6}$$

$$\tilde{\dot{T}}(q_s,\dot{q}_\sigma,\ddot{q}_\sigma;t)=\dot{T}(q_s,\dot{q}_\sigma,\dot{q}_{\varepsilon+\beta}(q_s,\dot{q}_\sigma;t),\ddot{q}_\sigma,\ddot{q}_{\varepsilon+\beta}(q_s,\dot{q}_\sigma,\ddot{q}_\sigma;t);t),\tag{7}$$

则得

$$\frac{d}{dt}\frac{\partial\tilde{T}}{\partial\dot{q}_\sigma}-\frac{\partial\tilde{T}}{\partial q_\sigma}=\sum_{i=1}^{N}m_i\ddot{\boldsymbol{r}}_i\cdot\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma}+\sum_{i=1}^{N}m_i\tilde{\boldsymbol{r}}_i\cdot\left(\frac{d}{dt}\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma}-\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial q_\sigma}\right)+\tilde{P}_\sigma$$

将上式代入(4)式，便得广义的Mac-Millan方程：

$$\frac{d}{dt}\frac{\partial\tilde{T}}{\partial\dot{q}_\sigma}-\frac{\partial\tilde{T}}{\partial q_\sigma}=(\tilde{Q}_F)_\sigma+(\tilde{Q}_R)_\sigma+\sum_{i=1}^{N}m_i\tilde{\boldsymbol{r}}_i\cdot\left(\frac{d}{dt}\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma}-\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial q_\sigma}\right)+\tilde{P}_\sigma\tag{8}$$

式中

$$(\tilde{Q}_F)_\sigma=\sum_{i=1}^{N}\boldsymbol{F}_i\cdot\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma},\quad(\tilde{Q}_R)_\sigma=\sum_{i=1}^{N}\boldsymbol{R}_i\cdot\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma},\quad\tilde{P}_\sigma=\sum_{i=1}^{N}\dot{m}_i\tilde{\boldsymbol{r}}_i\cdot\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial\dot{q}_\sigma}\tag{9}$$

将

$$\tilde{\dot{T}}=\sum_{i=1}^{N}m_i\tilde{\dot{\boldsymbol{r}}}_i\cdot\tilde{\ddot{\boldsymbol{r}}}_i+\frac{1}{2}\sum_{i=1}^{N}\dot{m}_i\tilde{\boldsymbol{r}}_i\tilde{\boldsymbol{r}}_i$$

对$\ddot{q}_\sigma$求偏微商后代入(4)式，并由

$$\frac{\partial\tilde{T}}{\partial q_\sigma}=\sum_{i=1}^{N}m_i\tilde{\boldsymbol{r}}_i\cdot\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial q_\sigma}$$

便得Nielson型方程：

$$\frac{\partial\tilde{\dot{T}}}{\partial\dot{q}_\sigma}-2\frac{\partial\tilde{T}}{\partial q_\sigma}=(\tilde{Q}_F)_\sigma+(\tilde{Q}_R)_\sigma+\sum_{i=1}^{N}m_i\tilde{\boldsymbol{r}}_i\cdot\left(\frac{\partial\tilde{\dot{\boldsymbol{r}}}_i}{\partial\dot{q}_\sigma}-2\frac{\partial\tilde{\boldsymbol{r}}_i}{\partial q_\sigma}\right)+\tilde{P}_\sigma\tag{10}$$

由(6)式得：

$$\frac{\partial \dot{T}}{\partial q_\sigma}=\frac{\partial T}{\partial q_\sigma}+\sum_{\beta=1}^{g}\frac{\partial T}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial q_\sigma} \tag{11}$$

由(7)式得

$$\frac{\partial \tilde{\dot{T}}}{\partial \dot{q}_\sigma}=\frac{\partial \dot{T}}{\partial \dot{q}_\sigma}+\sum_{\beta=1}^{g}\frac{\partial \dot{T}}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}+\sum_{\beta=1}^{g}\frac{\partial \dot{T}}{\partial \ddot{q}_{\varepsilon+\beta}}\frac{\partial \ddot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}$$

由(5)式得

$$\begin{cases}\dfrac{\partial \dot{T}}{\partial \dot{q}_\sigma}=\sum\limits_{i=1}^{N}m_i\ddot{\boldsymbol{r}}_i\cdot\dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}+\sum\limits_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\dfrac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}+\sum\limits_{i=1}^{N}\dot{m}_i\dot{\boldsymbol{r}}_i\cdot\dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}\\[2ex] \dfrac{\partial \dot{T}}{\partial \dot{q}_{\varepsilon+\beta}}=\sum\limits_{i=1}^{N}m_i\ddot{\boldsymbol{r}}_i\cdot\dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}+\sum\limits_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\dfrac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}+\sum\limits_{i=1}^{N}\dot{m}_i\dot{\boldsymbol{r}}_i\cdot\dfrac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}\end{cases}$$

代入前式，并利用(4)式，得

$$\begin{aligned}\frac{\partial \tilde{\dot{T}}}{\partial \dot{q}_\sigma}&=\sum_{i=1}^{N}m_i\ddot{\boldsymbol{r}}_i\cdot\left(\frac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}+\sum_{\beta=1}^{g}\frac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}\right)+\sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\left(\frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}+\sum_{\beta=1}^{g}\frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}\right)\\&\quad+\sum_{\beta=1}^{g}\frac{\partial \dot{T}}{\partial \ddot{q}_{\varepsilon+\beta}}\frac{\partial \ddot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}+\sum_{i=1}^{N}\dot{m}_i\dot{\boldsymbol{r}}_i\cdot\left(\frac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}+\sum_{\beta=1}^{g}\frac{\partial \dot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}\right)\\&=(\tilde{Q}_{\mathrm{F}})_\sigma+(\tilde{Q}_{\mathrm{R}})_\sigma+2\left(\frac{\partial T}{\partial q_\sigma}+\sum_{\beta=1}^{g}\frac{\partial T}{\partial q_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}\right)+\sum_{\beta=1}^{g}\frac{\partial T}{\partial \dot{q}_{\varepsilon+\beta}}\frac{\partial \ddot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}+\tilde{P}_\sigma\end{aligned} \tag{12}$$

已利用

$$\frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}=2\frac{\partial \dot{\boldsymbol{r}}_i}{\partial q_\sigma},\quad \frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}=2\frac{\partial \dot{\boldsymbol{r}}_i}{\partial q_{\varepsilon+\beta}},\quad \sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_\sigma}=2\sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\frac{\partial \dot{\boldsymbol{r}}_i}{\partial q_\sigma}=2\frac{\partial T}{\partial q_\sigma}$$

$$\sum_{i=1}^{N}m_i\dot{\boldsymbol{r}}_i\cdot\frac{\partial \ddot{\boldsymbol{r}}_i}{\partial \dot{q}_{\varepsilon+\beta}}=2\frac{\partial T}{\partial q_{\varepsilon+\beta}}\text{和}\frac{\partial \dot{T}}{\partial \ddot{q}_{\varepsilon+\beta}}=\frac{\partial T}{\partial \dot{q}_{\varepsilon+\beta}}$$

由(11)和(12)式，得Nielsen方程的另一形式：

$$\begin{aligned}\frac{\partial \tilde{\dot{T}}}{\partial \dot{q}_\sigma}-2\frac{\partial \tilde{T}}{\partial q_\sigma}&=(\tilde{Q}_{\mathrm{F}})_\sigma+(\tilde{Q}_{\mathrm{R}})_\sigma+\sum_{\beta=1}^{g}\frac{\partial T}{\partial \dot{q}_{\varepsilon+\beta}}\left(\frac{\partial \ddot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}-2\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial q_\sigma}\right)\\&\quad+2\sum_{\beta=1}^{g}\frac{\partial T}{\partial q_{\varepsilon+\beta}}\frac{\partial \dot{q}_{\varepsilon+\beta}}{\partial \dot{q}_\sigma}+\tilde{P}_\sigma\end{aligned} \tag{13}$$

因 $\sum\limits_{i=1}^{N}m_i\tilde{\ddot{\boldsymbol{r}}}_i\cdot\dfrac{\partial \tilde{\dot{\boldsymbol{r}}}_i}{\partial \dot{q}_\sigma}=\sum\limits_{i=1}^{N}m_i\tilde{\ddot{\boldsymbol{r}}}_i\cdot\dfrac{\partial \tilde{\ddot{\boldsymbol{r}}}_i}{\partial \ddot{q}_\sigma}=\dfrac{\partial \tilde{S}}{\partial \ddot{q}_\sigma}$

将此式代入(4)式，便得Appell方程：

$$\frac{\partial \tilde{S}}{\partial \ddot{q}_\sigma}=(\tilde{Q}_{\mathrm{F}})_\sigma+(\tilde{Q}_{\mathrm{R}})_\sigma \tag{14}$$

式中 $s=\frac{1}{2}\sum_{i=1}^{N}m_i\ddot{\boldsymbol{r}}_i\cdot\ddot{\boldsymbol{r}}_i$，$\tilde{s}$为$s$已借助约束方程(2)消去$\dot{q}_{\varepsilon+\beta}$而得的表达式．

还可由(4)式导出Чаплыгин方程，Boltzman-Hamel方程，Volterra方程等等，以上都未附加虚位移的Appell-Четаев定义或牛青萍定义．

若力学系统受到g个m阶非线性非完整约束

$$f_\beta(q_s,\dot{q}_s,\cdots,\overset{(m)}{q}_s;t)=0 \qquad (s=1,2,\cdots,n;\beta=1,2,\cdots,g) \tag{15}$$

且g个m阶广义速度$\overset{(m)}{q}_{\varepsilon+\beta}$可用$\varepsilon$个彼此独立的$m$阶广义速度$\overset{(m)}{q}_\sigma$表示出来，即

$$\overset{(m)}{q}_{\varepsilon+\beta}=\overset{(m)}{q}_{\varepsilon+\beta}(q_s,\dot{q}_s,\cdots,\overset{(m-1)}{q}_s,\ \overset{(m)}{q}_\sigma;t) \qquad (\sigma=1,2,\cdots,\varepsilon) \tag{16}$$

令$\overset{(\tilde{m})}{\boldsymbol{r}}_i$为$\overset{(m)}{\boldsymbol{r}}_i$中的$\overset{(m)}{q}_{\varepsilon+\beta}$已用(16)消去，即

$$\overset{(\tilde{m})}{\boldsymbol{r}}_i(q_s,\dot{q}_s,\cdots,\overset{(m-1)}{q}_s,\overset{(m)}{q}_\sigma;t)=\overset{(m)}{\boldsymbol{r}}_i(q_s,\dot{q}_s,\cdots,\overset{(m-1)}{q}_s,\ \overset{(m)}{q}_\sigma,\ \overset{(m)}{q}_{\varepsilon+\beta}(q_s,\dot{q}_s,\cdots,\overset{(m-1)}{q}_s,\ \overset{(m)}{q}_\sigma;t);t) \tag{17}$$

则(17)式为m阶非线性非完整约束超曲面的参数方程．m阶非线性非完整约束超曲面的基矢量为$\partial\overset{(\tilde{m})}{\boldsymbol{r}}_i/\partial\overset{(m)}{q}_\sigma$，它与密歇尔斯基方程点乘：

$$\sum_{i=1}^{N}\frac{\partial\overset{(\tilde{m})}{\boldsymbol{r}}_i}{\partial\overset{(m)}{q}_\sigma}\cdot(\boldsymbol{F}_i+\boldsymbol{R}_i-m_i\ddot{\boldsymbol{r}}_i)=0 \tag{18}$$

可作为受m阶非线性非完整约束的变质量力学系统的基本动力学方程．可引入广义力$(\tilde{Q}\mathrm{F})_\sigma$和广义反推力$(\tilde{Q}\mathrm{R})_\sigma$：

$$(\tilde{Q}\mathrm{F})_\sigma=\sum_{i=1}^{N}\frac{\partial\overset{(\tilde{m})}{\boldsymbol{r}}_i}{\partial\overset{(m)}{q}_\sigma}\cdot\boldsymbol{F}_i,\quad(\tilde{Q}\mathrm{R})_\sigma=\sum_{i=1}^{N}\frac{\partial\overset{(\tilde{m})}{\boldsymbol{r}}_i}{\partial\overset{(m)}{q}_\sigma}\cdot\boldsymbol{R}_i \tag{19}$$

当$m=1$时，(18)式成为(4)式；$m=0$时，得完整约束空间的基本动力学方程：

$$\sum_{i=1}^{N}\frac{\partial\boldsymbol{r}_i}{\partial q_\sigma}\cdot(\boldsymbol{F}_i+\boldsymbol{R}_i-m_i\ddot{\boldsymbol{r}}_i)=0 \tag{20}$$

如将(18)式改为用m阶准速度$\overset{(m)}{\pi}_\sigma$表示，则得

$$\sum_{i=1}^{N}\frac{\partial\overset{(\tilde{m})}{\boldsymbol{r}}_i}{\partial\overset{(m)}{\pi}_\sigma}\cdot(\boldsymbol{F}_i+\boldsymbol{R}_i-m_i\ddot{\boldsymbol{r}}_i)=0 \tag{21}$$

当$\overset{(m)}{\pi}_\sigma=\overset{(m)}{q}_\sigma$时，即得用$m$阶广义速度表示的(18)式．

现证明(4)式与微分变分原理是一致的．因(3)式是约束超曲面的参数方程，参数为t，q_s，而自变量为$\dot{q}_\sigma$，于是有变分

$$\delta t=\delta q_s=0,\ \text{而}\delta\dot{q}_\sigma\neq0 \tag{22}$$

所以

$$\delta\boldsymbol{r}_i=\sum_{s=1}^{n}\frac{\partial\boldsymbol{r}_i}{\partial q_s}\delta q_s=0 \tag{23}$$

$$\delta\dot{\tilde{\boldsymbol{r}}}_i=\sum_{\sigma=1}^{\varepsilon}\frac{\partial\dot{\tilde{\boldsymbol{r}}}_i}{\partial\dot{q}_\sigma}\delta\dot{q}_\sigma+\sum_{s=1}^{n}\frac{\partial\dot{\tilde{\boldsymbol{r}}}_i}{\partial q_s}\delta q_s=\sum_{\sigma=1}^{\varepsilon}\frac{\partial\dot{\tilde{\boldsymbol{r}}}_i}{\partial\dot{q}_\sigma}\delta\dot{q}_\sigma \tag{24}$$

将(4)式乘以$\delta \dot{q}_\sigma$，并对σ求和，利用(24)式，便得一阶的微分变分原理

$$\sum_{i=1}^{N} \delta \dot{\boldsymbol{r}}_i \cdot (\boldsymbol{F}_i + \boldsymbol{R}_i - m_i \ddot{\boldsymbol{r}}_i) = 0 \tag{25}$$

反之，由(25)和(24)式，并注意$\delta \dot{q}_\sigma$彼此独立，便得(4)式．

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The Equation of Motion for the System of the Variable Mass in the Non-Linear Non-Holonomic Space

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Abstract

The dot product of bases vectors on the super-surface of constraints of the non-linear non-holonomic space and Mesherskii equations may act as the equations of fundamental dynamics of mechanical system for the variable mass. These are very simple and convenient for computation. From these known equations, the equations of Chaplygin, Nielson, Appell, Mac-Millan etc., are derived, it is unnecessary to introduce the definition of Appell-Chetaev or Niu Qinping for the virtual displacement. These are compatible with the D'Alembert-Lagrange's principle.

Key words the non-linear non-holonomic constraints, the system of the variable mass, dot product, bases vectors on supersurface of constraints, Misherskii equation