应用数学数力学，第13卷第7期（1962年7月）
Applied Mathematics and Mechanics

应用数学和力学编委会编
重庆出版社出版

有初始几何缺陷的一般壳体的非线性应变分量公式*

李丽娟　梅占馨　万　虹　刘　锋

（西安冶金建筑学院，1991年7月9日收到）

摘　要

本文从非线性三维连续介质的应变分量公式出发，导出具有初始几何缺陷一般薄壳的非线性应变分量公式，在推导过程中没有局限于任何一种特定的壳体，因此公式具有一般性。这组公式可以为研究有初始几何缺陷的壳体几何非线性问题提供应变几何学理论基础。

关键词　初始几何缺陷　壳体　非线性　应变

一、引　　言

实际工程中的壳体结构由于制造和施工偏差，经常会使按某种理想曲面设计的壳体具有不同程度的初始几何缺陷，较严重的会造成产品报废或形成结构倒塌的主要原因[1]。

至今，人们对有初始几何缺陷壳体的线性问题已作了不少研究工作，如陈叔陶[2]的小参数法；卢文达、高世桥[3]的摄动法；Han，Gould[4]及Kato Shiro[5]的有限元法等等。对壳体非线性问题的研究相对少些[6,7]，对具有初始几何缺陷壳体的非线性问题的研究就更少[8]。本文提出的有初始及何缺陷一般壳体的非线性应变分量公式，是为求解有初始几何缺陷的各种壳体几何非线性问题提供应变几何学理论基础。与通常采用的方法不同，本文不是从壳体中曲面变形理论出发，而是从非线性连续介质三维问题的几何方程出发，视初始几何缺陷为初始位移，导出了具有初始几何缺陷一般壳体非线性应变位移关系式。作为验证，由这组公式的退化情形可以得出文献[8]有初始挠度的板与扁壳的应变分量公式，可以得出文献[6,9,10,12]中完善壳的线性与非线性应变分量公式。本文公式所指的壳体初始几何缺陷可以是有限变形量级。

二、三维连续介质的非线性几何关系

把初始几何缺陷视为初始位移。初始位移和变形位移都属大位移。由文献[9]T.L，列

* 薛大为推荐。

式，$t+\Delta t$时刻的应变张量可以写成：

$$
{}^{t+\Delta t}_{0}\varepsilon^{*}_{ij}=\frac{1}{2}({}^{t}_{0}U_{i,j}+{}^{t}_{0}U_{j,i}+{}^{t}_{0}U_{k,j}{}^{t}_{0}U_{k,i}+{}_{0}U_{i,j}+{}_{0}U_{j,i}+{}^{t}_{0}U_{k,i}\ {}_{0}U_{k,j})
$$

$$
+\frac{1}{2}({}^{t}_{0}U_{k,j}\ {}_{0}U_{k,i}+{}_{0}U_{k,i}\ {}_{0}U_{k,j}) \tag{2.1}
$$

设$t=0$为初始时刻，则${}^{0}_{0}U_{i}$，${}_{0}U_{i}(i=x,y,z)$分别为初始位移和变形位移，于是由式(2.1)可以得出应变张量公式：

$$
{}^{\Delta t}_{0}\varepsilon^{*}_{ij}=\frac{1}{2}({}^{0}_{0}U_{i,j}+{}^{0}_{0}U_{j,i}+{}^{0}_{0}U_{k,j}{}^{0}_{0}U_{k,i}+{}_{0}U_{i,j}+{}_{0}U_{j,i}+{}^{0}_{0}U_{k,i}\ {}_{0}U_{k,j})
$$

$$
+\frac{1}{2}({}^{0}_{0}U_{k,j}\ {}_{0}U_{k,i}+{}_{0}U_{k,i}\ {}_{0}U_{k,j})
$$

考虑到初始位移是指离开理想形状的初偏离，所以不产生初应变，故：

$$
\frac{1}{2}({}^{0}_{0}U_{i,j}+{}^{0}_{0}U_{j,i}+{}^{0}_{0}U_{k,j}\ {}^{0}_{0}U_{k,i})=0
$$

所以应变张量 $\{{}^{\Delta t}_{0}\varepsilon^{*}_{ij}\}$由下列三部分组成：

$$
\{{}^{\Delta t}_{0}\varepsilon^{*}_{ij}\}=L(\mathbf{V})+NL(\mathbf{V})+IL(\mathbf{V}_0,\mathbf{V})
$$

其中L，NL，IL分别为线性项，非线性项，耦合项。

将符号${}^{\Delta t}_{0}\varepsilon^{*}_{ij}$简写成$\varepsilon_{ij}$并将变形位移分量符号${}_{0}U_{x}$，${}_{0}U_{y}$，${}_{0}U_{z}$分别改用$U,V,W$，初始位移分量改用$U_0$，$V_0$，$W_0$，引进记号：

$$
\left.\begin{aligned}
&e_{xx}=\frac{\partial U}{\partial x},\quad e_{yy}=\frac{\partial V}{\partial y},\quad e_{zz}=\frac{\partial W}{\partial z}\\
&e_{xy}=\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x},\quad e_{xz}=\frac{\partial U}{\partial z}+\frac{\partial W}{\partial x},\quad e_{yz}=\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\\
&2\omega_x=\frac{\partial W}{\partial y}-\frac{\partial V}{\partial z},\quad 2\omega_y=\frac{\partial U}{\partial z}-\frac{\partial W}{\partial x},\quad 2\omega_z=\frac{\partial V}{\partial x}-\frac{\partial U}{\partial y}
\end{aligned}\right\} \tag{2.2}
$$

则有初始几何缺陷的应变分量可以写成下列形式：

$$
\varepsilon_{xx}=e_{xx}+\frac{1}{2}\left[e_{xx}^{2}+\left(\frac{1}{2}e_{xy}+\omega_z\right)^2+\left(\frac{1}{2}e_{xz}-\omega_y\right)^2\right]+e_{xx}e_{0xx}
$$

$$
+\left(\frac{1}{2}e_{0xy}+\omega_{0z}\right)\left(\frac{1}{2}e_{xy}+\omega_z\right)+\left(\frac{1}{2}e_{0xz}-\omega_{0y}\right)\left(\frac{1}{2}e_{xz}-\omega_y\right)
$$

$$
\varepsilon_{yy}=e_{yy}+\frac{1}{2}\left[e_{yy}^{2}+\left(\frac{1}{2}e_{xy}-\omega_z\right)^2+\left(\frac{1}{2}e_{yz}+\omega_x\right)^2\right]+e_{yy}e_{0yy}
$$

$$
+\left(\frac{1}{2}e_{0xy}-\omega_{0z}\right)\left(\frac{1}{2}e_{xy}-\omega_z\right)+\left(\frac{1}{2}e_{0yz}+\omega_{0x}\right)\left(\frac{1}{2}e_{yz}+\omega_x\right)
$$

$$
\left.\begin{aligned}
\varepsilon_{zz}=&e_{zz}+\frac{1}{2}\left[e_{zz}^{2}+\left(\frac{1}{2}e_{xz}+\omega_y\right)^2+\left(\frac{1}{2}e_{yz}-\omega_x\right)^2\right]+e_{zz}e_{0zz}\\
&+\left(\frac{1}{2}e_{0xz}-\omega_{0y}\right)\left(\frac{1}{2}e_{xz}+\omega_y\right)+\left(\frac{1}{2}e_{0yz}-\omega_{0x}\right)\left(\frac{1}{2}e_{yz}-\omega_x\right)\\
\gamma_{xy}=&e_{xy}+e_{xx}\left(\frac{1}{2}e_{xy}-\omega_z\right)+e_{yy}\left(\frac{1}{2}e_{xy}+\omega_z\right)+\left(\frac{1}{2}e_{xz}-\omega_y\right)\left(\frac{1}{2}e_{yz}+\omega_x\right)\\
&+e_{0xx}\left(\frac{1}{2}e_{xy}-\omega_z\right)+e_{yy}\left(\frac{1}{2}e_{0xy}+\omega_{0z}\right)+\left(\frac{1}{2}e_{0xz}-\omega_{0y}\right)\left(\frac{1}{2}e_{yz}+\omega_x\right)\\
&+e_{xx}\left(\frac{1}{2}e_{0xy}-\omega_{0z}\right)+e_{0yy}\left(\frac{1}{2}e_{xy}+\omega_z\right)+\left(\frac{1}{2}e_{xz}-\omega_y\right)\left(\frac{1}{2}e_{0yz}+\omega_{0x}\right)\\
\gamma_{yz}=&e_{yz}+e_{yy}\left(\frac{1}{2}e_{yz}-\omega_x\right)+e_{zz}\left(\frac{1}{2}e_{yz}+\omega_x\right)+\left(\frac{1}{2}e_{xy}-\omega_z\right)\left(\frac{1}{2}e_{xz}+\omega_y\right)\\
&+e_{0yy}\left(\frac{1}{2}e_{yz}-\omega_x\right)+e_{zz}\left(\frac{1}{2}e_{0yz}+\omega_{0x}\right)+\left(\frac{1}{2}e_{0xy}-\omega_{0z}\right)\left(\frac{1}{2}e_{xz}+\omega_y\right)\\
&+e_{yy}\left(\frac{1}{2}e_{0yz}-\omega_{0x}\right)+e_{0zz}\left(\frac{1}{2}e_{yz}+\omega_x\right)+\left(\frac{1}{2}e_{xy}-\omega_z\right)\left(\frac{1}{2}e_{0xz}+\omega_{0y}\right)\\
\gamma_{zx}=&e_{xz}+e_{xx}\left(\frac{1}{2}e_{xz}+\omega_y\right)+e_{zz}\left(\frac{1}{2}e_{xz}-\omega_y\right)+\left(\frac{1}{2}e_{xy}+\omega_z\right)\left(\frac{1}{2}e_{yz}-\omega_x\right)\\
&+e_{0xx}\left(\frac{1}{2}e_{xz}+\omega_y\right)+e_{zz}\left(\frac{1}{2}e_{0xz}-\omega_{0y}\right)+\left(\frac{1}{2}e_{0xy}+\omega_{0z}\right)\left(\frac{1}{2}e_{yz}-\omega_x\right)\\
&+e_{xx}\left(\frac{1}{2}e_{0xz}+\omega_{0y}\right)+e_{0zz}\left(\frac{1}{2}e_{xz}-\omega_y\right)+\left(\frac{1}{2}e_{xy}+\omega_z\right)\left(\frac{1}{2}e_{0yz}-\omega_{0x}\right)
\end{aligned}\right\}\tag{2.3}
$$

式中e_{0xx},…, $2\omega_{0z}$的表示式可由式(2.2)中的变形位移U, V, W分别换成初位 移U_0, V_0, W_0得到。参数e_{xx},…, $2\omega_z$在正交曲线坐标系中相应于下式：

$$
\left.\begin{aligned}
e_{11}&=\frac{1}{H_1}\frac{\partial U_\alpha}{\partial\alpha}+\frac{1}{H_1H_2}\frac{\partial H_1}{\partial\beta}U_\beta+\frac{1}{H_1H_3}\frac{\partial H_1}{\partial\gamma}U_\gamma\\
e_{22}&=\frac{1}{H_2}\frac{\partial U_\beta}{\partial\beta}+\frac{1}{H_2H_3}\frac{\partial H_2}{\partial\gamma}U_\gamma+\frac{1}{H_1H_2}\frac{\partial H_2}{\partial\alpha}U_\alpha\\
e_{33}&=\frac{1}{H_3}\frac{\partial U_\gamma}{\partial\gamma}+\frac{1}{H_1H_3}\frac{\partial H_3}{\partial\alpha}U_\alpha+\frac{1}{H_2H_3}\frac{\partial H_3}{\partial\beta}U_\beta\\
e_{13}&=\frac{H_1}{H_3}\frac{\partial}{\partial\gamma}\left(\frac{U_\alpha}{H_1}\right)+\frac{H_3}{H_1}\frac{\partial}{\partial\alpha}\left(\frac{U_\gamma}{H_3}\right)\\
e_{23}&=\frac{H_3}{H_2}\frac{\partial}{\partial\beta}\left(\frac{U_\gamma}{H_3}\right)+\frac{H_2}{H_3}\frac{\partial}{\partial\gamma}\left(\frac{U_\beta}{H_2}\right)\\
e_{12}&=\frac{H_2}{H_1}\frac{\partial}{\partial\alpha}\left(\frac{U_\beta}{H_2}\right)+\frac{H_1}{H_2}\frac{\partial}{\partial\beta}\left(\frac{U_\alpha}{H_1}\right)\\
2\omega_1&=\frac{1}{H_2H_3}\left[\frac{\partial}{\partial\beta}(H_3U_\gamma)-\frac{\partial}{\partial\gamma}(H_2U_\beta)\right]\\
2\omega_2&=\frac{1}{H_1H_3}\left[\frac{\partial}{\partial\gamma}(H_1U_\alpha)-\frac{\partial}{\partial\alpha}(H_3U_\gamma)\right]\\
2\omega_3&=\frac{1}{H_1H_2}\left[\frac{\partial}{\partial\alpha}(H_2U_\beta)-\frac{\partial}{\partial\beta}(H_1U_\alpha)\right]
\end{aligned}\right\}\tag{2.4}
$$

其中H_1，H_2，和H_3为拉梅系数；U_α，U_β，U_γ分别为曲线坐标系(α,β,γ)中任意一点的变形位移在三个坐标线切线方向的投影．定义式$e_{011},\cdots,2\omega_{03}$则由式(2.4)中位移分量$U_\alpha,U_\beta$，$U_\gamma$用初位移$U_{0\alpha}$，$U_{0\beta}$，$U_{0\gamma}$取代后得出．这使我们立即能写出在任意正交曲线坐标系中非线性应变分量的一般表示式：

$$\left.\begin{aligned}
\varepsilon_{11}=&e_{11}+\frac{1}{2}\left[e_{11}^2+\left(\frac{1}{2}e_{12}+\omega_3\right)^2+\left(\frac{1}{2}e_{13}-\omega_2\right)^2\right]+e_{11}e_{011}\\
&+\left(\frac{1}{2}e_{012}+\omega_{03}\right)\left(\frac{1}{2}e_{12}+\omega_3\right)+\left(\frac{1}{2}e_{013}-\omega_{02}\right)\left(\frac{1}{2}e_{13}-\omega_2\right)\\
\varepsilon_{22}=&e_{22}+\frac{1}{2}\left[e_{22}^2+\left(\frac{1}{2}e_{12}-\omega_3\right)^2+\left(\frac{1}{2}e_{23}+\omega_1\right)^2\right]+e_{22}e_{022}\\
&+\left(\frac{1}{2}e_{012}-\omega_{03}\right)\left(\frac{1}{2}e_{12}-\omega_3\right)+\left(\frac{1}{2}e_{023}+\omega_{01}\right)\left(\frac{1}{2}e_{23}+\omega_1\right)\\
\varepsilon_{33}=&e_{33}+\frac{1}{2}\left[e_{33}^2+\left(\frac{1}{2}e_{13}+\omega_2\right)^2+\left(\frac{1}{2}e_{23}-\omega_1\right)^2\right]+e_{33}e_{033}\\
&+\left(\frac{1}{2}e_{013}+\omega_{02}\right)\left(\frac{1}{2}e_{13}+\omega_2\right)+\left(\frac{1}{2}e_{23}-\omega_{01}\right)\left(\frac{1}{2}e_{23}-\omega_1\right)\\
\gamma_{12}=&e_{12}+e_{11}\left(\frac{1}{2}e_{12}-\omega_3\right)+e_{22}\left(\frac{1}{2}e_{12}+\omega_3\right)+\left(\frac{1}{2}e_{13}-\omega_2\right)\left(\frac{1}{2}e_{23}+\omega_1\right)\\
&+e_{011}\left(\frac{1}{2}e_{12}-\omega_3\right)+e_{22}\left(\frac{1}{2}e_{012}+\omega_{03}\right)+\left(\frac{1}{2}e_{013}+\omega_{02}\right)\left(\frac{1}{2}e_{23}+\omega_1\right)\\
&+e_{11}\left(\frac{1}{2}e_{012}-\omega_{03}\right)+e_{022}\left(\frac{1}{2}e_{12}+\omega_3\right)+\left(\frac{1}{2}e_{13}-\omega_2\right)\left(\frac{1}{2}e_{023}+\omega_{01}\right)\\
\gamma_{23}=&e_{23}+e_{22}\left(\frac{1}{2}e_{23}-\omega_1\right)+e_{33}\left(\frac{1}{2}e_{23}+\omega_1\right)+\left(\frac{1}{2}e_{12}-\omega_3\right)\left(\frac{1}{2}e_{13}+\omega_2\right)\\
&+e_{022}\left(\frac{1}{2}e_{23}-\omega_1\right)+e_{33}\left(\frac{1}{2}e_{023}+\omega_{01}\right)+\left(\frac{1}{2}e_{012}-\omega_{03}\right)\left(\frac{1}{2}e_{13}+\omega_2\right)\\
&+e_{22}\left(\frac{1}{2}e_{023}-\omega_{01}\right)+e_{033}\left(\frac{1}{2}e_{23}+\omega_1\right)+\left(\frac{1}{2}e_{12}-\omega_3\right)\left(\frac{1}{2}e_{013}+\omega_{02}\right)\\
\gamma_{31}=&e_{13}+e_{11}\left(\frac{1}{2}e_{13}+\omega_2\right)+e_{33}\left(\frac{1}{2}e_{13}-\omega_2\right)+\left(\frac{1}{2}e_{12}+\omega_3\right)\left(\frac{1}{2}e_{23}-\omega_1\right)\\
&+e_{011}\left(\frac{1}{2}e_{13}+\omega_2\right)+e_{33}\left(\frac{1}{2}e_{013}-\omega_{02}\right)+\left(\frac{1}{2}e_{012}+\omega_{03}\right)\left(\frac{1}{2}e_{23}-\omega_1\right)\\
&+e_{11}\left(\frac{1}{2}e_{013}+\omega_{02}\right)+e_{033}\left(\frac{1}{2}e_{13}-\omega_2\right)+\left(\frac{1}{2}e_{12}+\omega_3\right)\left(\frac{1}{2}e_{023}-\omega_{01}\right)
\end{aligned}\right\}\quad(2.5)$$

三、有初始几何缺陷的一般壳体的非线性应变分量公式

在壳体理论中，取α_1，α_2成高斯曲线坐标，ζ为垂直于中面的直线坐标，且α_1，α_2，ζ正交．曲面的梅拉系数有如下关系：

$$H_1=A_1\left(1+\frac{\zeta}{R_1}\right),\qquad H_2=A_2\left(1+\frac{\zeta}{R_2}\right),\qquad H_3=1 \tag{3.1}$$

将(3.1)式代入(2.4)式将有:

$$
\left.\begin{aligned}
e_{11}&=\frac{1}{1+\zeta/R_1}\left(\frac{1}{A_1}\frac{\partial U^\zeta}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}V^\zeta+\frac{W^\zeta}{R_1}\right)\\
e_{22}&=\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_2}\frac{\partial V^\zeta}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}U^\zeta+\frac{W^\zeta}{R_2}\right),\ e_{33}=\frac{\partial W^\zeta}{\partial\zeta}\\
e_{12}&=\frac{1}{1+\zeta/R_1}\left(\frac{1}{A_1}\frac{\partial V^\zeta}{\partial\alpha_1}-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}U^\zeta\right)\\
&\quad+\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_2}\frac{\partial U^\zeta}{\partial\alpha_2}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}V^\zeta\right)\\
e_{13}&=\frac{\partial U^\zeta}{\partial\zeta}+\frac{1}{1+\zeta/R_1}\left(\frac{1}{A_1}\frac{\partial W^\zeta}{\partial\alpha_1}-\frac{U^\zeta}{R_1}\right)\\
e_{23}&=\frac{\partial V^\zeta}{\partial\zeta}+\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_2}\frac{\partial W^\zeta}{\partial\alpha_2}-\frac{V^\zeta}{R_2}\right)\\
2\omega_1&=-\frac{\partial V^\zeta}{\partial\zeta}+\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_2}\frac{\partial W^\zeta}{\partial\alpha_2}-\frac{V^\zeta}{R_2}\right)\\
2\omega_2&=\frac{\partial U^\zeta}{\partial\zeta}-\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_1}\frac{\partial W^\zeta}{\partial\alpha_1}-\frac{U^\zeta}{R_1}\right)\\
2\omega_3&=\frac{1}{1+\zeta/R_1}\left(\frac{1}{A_1}\frac{\partial V^\zeta}{\partial\alpha_1}-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}U^\zeta\right)\\
&\quad-\frac{1}{1+\zeta/R_2}\left(\frac{1}{A_2}\frac{\partial U^\zeta}{\partial\alpha_2}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}V^\zeta\right)
\end{aligned}\right\}\tag{3.2}
$$

此处U^ζ,V^ζ,W^ζ是壳体中任意一点在坐标线α_1,α_2,ζ方向的变形位移。e_{011},…,$2\omega_{03}$可由壳体任意一点的初位移替换式(3.2)中变形位移后得出。

这样,由式(2.5)即可写出在曲线坐标系$(\alpha_1,\alpha_2,\zeta)$中壳体内任意一点应变分量的表示式。

取壳体变形位移U^ζ, V^ζ, W^ζ为:

$$
U^\zeta=U+\zeta v,\quad V^\zeta=V+\zeta\psi,\quad W^\zeta=W+\zeta\chi \tag{3.3}
$$

这里U, V, W是壳体中面的变形位移。U, V, W, v, ψ, χ只是α_1, α_2的函数。

利用薄壳理论的直法线假定与厚度不变如定,即:$\gamma_{13}=0$, $\gamma_{23}=0$, $\varepsilon_{33}=0$,取$1+\zeta/R_1\approx 1$, $1+\zeta/R_2\approx1$,假设初位移与ζ无关,令:

$$
\left.\begin{aligned}
{}^*e_{11}&=\frac{1}{A_1}\frac{\partial U}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}V+\frac{W}{R_1}\\
{}^*e_{22}&=\frac{1}{A_2}\frac{\partial V}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}U+\frac{W}{R_2}\\
{}^*e_{12}&=\frac{1}{A_1}\frac{\partial V}{\partial\alpha_1}-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}U,\quad {}^*e_{13}=\frac{1}{A_1}\frac{\partial W}{\partial\alpha_1}-\frac{U}{R_1}\\
{}^*e_{21}&=\frac{1}{A_2}\frac{\partial U}{\partial\alpha_2}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}V,\quad {}^*e_{23}=\frac{1}{A_2}\frac{\partial W}{\partial\alpha_2}-\frac{V}{R_2}
\end{aligned}\right\}\tag{3.4}
$$

${}^*e_{011}$,…,${}^*e_{023}$的表示式可由(3.4)式中的位移用相应的初位移取代后得到。则可得出:

$$\left.\begin{aligned}&v(1+{}^*e_{11}^{023}+{}^*e_{011})+\psi({}^*e_{12}+{}^*e_{012})+\chi({}^*e_{13}+{}^*e_{013})+{}^*e_{13}=0\\&\psi(1+{}^*e_{22}+{}^*e_{022})+v({}^*e_{21}+{}^*e_{021})+\chi({}^*e_{23}+{}^*e_{023})+{}^*e_{23}=0\\&v^2+\psi^2+(1+\chi)^2=1\end{aligned}\right\}\tag{3.5}$$

在小应变情况下解得：

$$\left.\begin{aligned}&v=-({}^*e_{13}+{}^*e_{013})(1+{}^*e_{22}+{}^*e_{022})+({}^*e_{23}+{}^*e_{023})({}^*e_{12}+{}^*e_{012})\\&\psi=-({}^*e_{23}+{}^*e_{023})(1+{}^*e_{11}+{}^*e_{011})+({}^*e_{13}+{}^*e_{013})({}^*e_{21}+{}^*e_{021})\\&\chi=({}^*e_{11}+{}^*e_{011})+({}^*e_{22}+{}^*e_{022})+({}^*e_{11}+{}^*e_{011})({}^*e_{22}+{}^*e_{022})\\&\quad-({}^*e_{12}+{}^*e_{012})({}^*e_{21}+{}^*e_{021})\end{aligned}\right\}\tag{3.6}$$

$$\begin{aligned}&\kappa_{11}=\frac{1}{A_1}\frac{\partial v}{\partial\alpha_1}+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}\psi+\frac{\chi}{R_1}\\&\kappa_{22}=\frac{1}{A_2}\frac{\partial\psi}{\partial\alpha_2}+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}v+\frac{\chi}{R_2}\\&\kappa_{12}=\frac{1}{A_1}\frac{\partial\psi}{\partial\alpha_1}-\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}v,\quad\kappa_{13}=\frac{1}{A_1}\frac{\partial\chi}{\partial\alpha_1}-\frac{v}{R_1}\\&\kappa_{21}=\frac{1}{A_2}\frac{\partial v}{\partial\alpha_2}-\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}\psi,\quad\kappa_{23}=\frac{1}{A_2}\frac{\partial\chi}{\partial\alpha_2}-\frac{\psi}{R_2}\end{aligned}\tag{3.7}$$

将式(3.2)代回(2.5)式中的第一、二、四式，并在$\varepsilon_{11},\varepsilon_{22},\gamma_{12}$的表示式中用式(3.3)将中面位移引入，忽略$\zeta$的二次项，考虑式(3.4)与(3.7)得有初始几何缺陷壳体内任意一点应变为：

$$\begin{aligned}\varepsilon_{11}=&{}^*e_{11}+\frac{1}{2}({}^*e_{11}^2+{}^*e_{12}^2+{}^*e_{13}^2)+{}^*e_{11}{}^*e_{011}+{}^*e_{12}{}^*e_{012}+{}^*e_{13}{}^*e_{013}\\&+\zeta[(1+{}^*e_{11})\kappa_{11}+{}^*e_{12}\kappa_{12}+{}^*e_{13}\kappa_{13}+{}^*e_{011}\kappa_{11}+{}^*e_{012}\kappa_{12}+{}^*e_{013}\kappa_{13}]\\\varepsilon_{22}=&{}^*e_{22}+\frac{1}{2}({}^*e_{21}^2+{}^*e_{22}^2+{}^*e_{23}^2)+{}^*e_{21}{}^*e_{021}+{}^*e_{22}{}^*e_{022}+{}^*e_{23}{}^*e_{023}\\&+\zeta[(1+{}^*e_{22})\kappa_{22}+{}^*e_{21}\kappa_{21}+{}^*e_{23}\kappa_{23}+{}^*e_{022}\kappa_{22}+{}^*e_{021}\kappa_{21}+{}^*e_{023}\kappa_{23}]\\\gamma_{12}=&{}^*e_{12}+{}^*e_{21}+{}^*e_{11}{}^*e_{21}+{}^*e_{22}{}^*e_{12}+{}^*e_{13}{}^*e_{23}+{}^*e_{011}{}^*e_{21}+{}^*e_{012}{}^*e_{22}\\&+{}^*e_{013}{}^*e_{23}+{}^*e_{021}{}^*e_{11}+{}^*e_{022}{}^*e_{12}+{}^*e_{023}{}^*e_{13}\\&+\zeta[(1+{}^*e_{22})\kappa_{12}+(1+{}^*e_{11})\kappa_{21}+{}^*e_{21}\kappa_{11}+{}^*e_{12}\kappa_{22}+{}^*e_{23}\kappa_{13}+{}^*e_{13}\kappa_{32}]\\&+\zeta[{}^*e_{021}\kappa_{11}+{}^*e_{022}\kappa_{12}+e^*{}_{023}\kappa_{13}+{}^*e_{011}\kappa_{21}+{}^*e_{012}\kappa_{22}+{}^*e_{013}\kappa_{23}]\end{aligned}\tag{3.8}$$

因此，有初始几何缺陷壳体内任意一点的应变就可由中面变形及初位移表示出来。

作为验证，由这组公式的退化情形可以得出文献[8]有初始挠度的板与扁壳的应变分量公式，可以得出文献[6,9,10,12]中完善壳的线性与非线性应变分量公式。

参考文献

[1] Kemp, K. O. and J. G. A. Groll, Report of the committee of inquiry into the collapse of the cooling tower at Ardeer Nylcn Works, Ayrshire on Thursday 27th, September 1973, Engineering Services Department, Imperial Chemical House, Millbank, London SWIP 3JF.

[2] 陈叔陶，中面接近规则曲面的薄壳的基本方程，力学学报，7(3) (1964).

[3] 卢文达、高世桥，旋转壳局部几何缺陷对其系统固有频率的影响，上海力学，(3)(1987).

[4] Han, K. J and P. L. Gould, Shell of revolution with local deviations, *Int.*

J. Num. Meth. Eng., 20 (1984), 305—313.

[5] Shiro, Kato and Yoshitura Yokoo, Effect of geometric imperfections on stress distributions in cooling tower, *Eng. Struct.*, 2(7) (1980).

[6] Dennis, S. T. and A. N. Palazotto, Large displacement and rotational formulation for laminated shells including parabolic transverse shear, *Int. J. Non-Linear Mechanics*, 25(1) (1990), 83—85.

[7] 陶启坤，壳体的非线性应变分量，应用数学和力学，3(4) (1982).

[8] A. C. 沃耳蜜耳，《柔韧板与柔韧壳》，科学出版社 (1959)

[9] B. B. 诺沃日洛夫，《薄壳论》，科学出版社 (1963)

[10] 北京大学固体力学教研室，《旋转壳的应力分析》，水力电力出版社 (1979)

[11] Bathe, K. J., *Finite Element Procedures in Engineering Analysis*, Prentice-Hall Inc., Englewood Cliffs, N. J.(1982)

[12] B. B. 诺沃日洛夫，《非线性弹性力学基础》，科学出版社 (1958).

Nonlinear Strain Components of General Shells with Initial Geometric Imperfections

Li Li-juan, Mei Zhan-xin, Wan Hong, Liu Feng

(Xi'an Institute of Metallurgy and Construction Engineering, Xi'an)

Abstract

On the basis of nonlinear strain component formulations of three-dimensional continuum, this paper has derived the nonlinear strain component formulations of shells with initial geometric imperfections. The derivation is not confined to a special shell, therefore they possess general properties. These formulations provide the theoretical basis of the strain analysis for geometric nonlinear problems of shells with initial geometric imperfections.

Key words initial geometric imperfections, shells, nonlinear, strain