拟线性常微分方程组边值问题的奇摄动*

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摘　　要

本文研究拟线性常微分方程组边值问题

$$x'=f(t,x,y,\varepsilon),\ \ \varepsilon y''=g(t,x,y,\varepsilon)y'+h(t,x,y,\varepsilon),$$
$$x(0,\varepsilon)=A(\varepsilon),\ \ y(0,\varepsilon)=B(\varepsilon),y(1,\varepsilon)=C(\varepsilon)$$

的奇摄动．其中 x，f，y，h，A，B和C均属于R^n和g是对角矩阵．在适当的假设下，利用对角化技巧和微分不等式理论获得了解的存在和它的按分量逐个一致有效的估计．

关键词　拟线性系统　奇摄动边值问题　对角化和微分不等式　渐近展开

一、引　　言

本文研究含小参数$\varepsilon>0$的拟线性奇摄动边值问题

$$x'=f(t,x,y,\varepsilon),\ \ x(0,\varepsilon)=A(\varepsilon) \tag{1.1}$$

$$\varepsilon y''=g(t,x,y,\varepsilon)y'+h(t,x,y,\varepsilon),\ \ y(0,\varepsilon)=B(\varepsilon),\ \ y(1,\varepsilon)=C(\varepsilon) \tag{1.2}$$

其中 $x=(x^1,x^2,\cdots,x^i,\cdots,x^n)$，$f=(f^1,\cdots,f^i,\cdots,f^n)$，$y=(y^1,\cdots,y^i,\cdots,y^n)$，$h=(h^1,\cdots,h^i,\cdots,h^n)$，$A=(A^1,\cdots,A^i,\cdots,A^n)$，$B=(B^1,\cdots,B^i,\cdots,B^n)$，$C=(C^1,\cdots,C^i,\cdots,C^n)$属于$R^n$，$g$是对角矩阵，即$g=\mathrm{diag}\{g^1,\cdots,g^i,\cdots,g^n\}$，这里$g^i$是充分光滑的函数．在适当的假设下利用边界层校正法和对角化技巧我们构造边值问题(1.1)，(1.2)的任意阶的形式渐近解．然后，应用微分不等式理论我们证明了(1.1)，(1.2)解的存在性和它的按分量逐个一致有效的渐近估计．

与(1.1)，(1.2)相应的退化问题如下：

$$\left.\begin{aligned}&x'=f(t,x,y,0),\ \ x(0,0)=A(0)\\&0=g(t,x,y,0)y'+h(t,x,y,0),\ \ y(0,0)=B(0)\end{aligned}\right\} \tag{1.3}$$

假设：

(i)　退化问题(1.3)有一个解 $(X_0,Y_0)=(X_0(t),Y_0(t))\in C^{(N+1)}[0,1]\times C^{(N+2)}[0,1]$，使得

$$g(t,X_0(t),Y_0(t),\ 0)>0 \tag{1.4}$$

同时，对于满足$0\leqslant\theta\leqslant\|Y_0(1)-y(0,0)\|$的所有$\theta+Y_0(1)$，内积

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$$\theta^T\int_0^\theta g(1,X_0(1),\ Y_0(1)+s,0)ds>0 \tag{1.5}$$

这里$\|\cdot\|$表示欧氏模和T表示转置;

(ii) $A(\varepsilon)$, $B(\varepsilon)$, $C(\varepsilon)\in C^\infty([0,1])$;

(iii) f, g, h和它关于x, y的偏导数在D上是充分光滑, $D=[0,1]\times[u,v]\times[\alpha,\beta]\times[0,\varepsilon_0]$.而且存在两个正的常数$\delta_i$和$\sigma_i(i=1,2,\cdots,n)$, 使得在$D$上$f^i_x\leqslant-\delta_i<0$, $g^i_y y_N^{i\prime}+h^i_y\geqslant\sigma_i>0$, 其中$f^i$和$h^i$分别是向量$f$和$h$的分量; g^i是g的第i个元素; u, v, α, β是下界和上界函数, 它们在下节中定义.

从条件(iii)我们知道, 若

$$x(t,\varepsilon)\sim\sum_{i=0}^\infty\varepsilon^i x_i(t),\ y(t,\varepsilon)\sim\sum_{i=0}^\infty\varepsilon^i y_i(t)$$

则

$$f(t,x,y,\varepsilon)\sim f[\cdot]+\sum_{s=0}^\infty[f_x[\cdot]\cdot x_s(t)+f_y[\cdot]y_s(t)+f_{s-1}(t)]\varepsilon^s,$$

$$g(t,x,y,\varepsilon)\sim g[\cdot]+\sum_{s=0}^\infty[g_x[\cdot]x_s(t)+g_y[\cdot]y_s(t)+g_{s-1}(t)]\varepsilon^s,$$

$$h(t,x,y,\varepsilon)\sim h[\cdot]+\sum_{s=0}^\infty[h_x[\cdot]x_s(t)+h_y[\cdot]y_s(t)+h_{s-1}(t)]\varepsilon^s.$$

其中

$$[\cdot]=(t,x_0(t),y_0(t),0);$$
$$f_{s-1}(t)=f_{s-1}(t,x_0(t),\cdots,x_{s-1}(t);\ y_0(t),\cdots,y_{s-1}(t));$$
$$g_{s-1}(t)=g_{s-1}(t,x_0(t),\cdots,x_{s-1}(t);\ y_0(t),\cdots,y_{s-1}(t));$$
$$h_{s-1}(t)=h_{s-1}(t,x_0(t),\cdots,x_{s-1}(t);\ y_0(t),\cdots,y_{s-1}(t)).$$

从条件(ii)我们得到

$$A(\varepsilon)\sim\sum_{s=0}^\infty A_s\varepsilon^s,\ A_s=\frac{1}{s!}\frac{d^sA(\varepsilon)}{d\varepsilon^s}\bigg|_{\varepsilon=0};\ B(\varepsilon)\sim\sum_{s=0}^\infty B_s\varepsilon^s,$$

$$B_s=\frac{1}{s!}\frac{d^sB(\varepsilon)}{d\varepsilon^s}\bigg|_{\varepsilon=0},\ C(\varepsilon)\sim\sum_{s=0}^\infty\varepsilon^sC_s,\ C_s=\frac{1}{s!}\frac{d^sC(\varepsilon)}{d\varepsilon^s}\bigg|_{\varepsilon=0}$$

二、预 备 定 理

我们需要下面的对角化技巧和微分不等式理论(参看[1～4]):

(a) 研究系统

$$\frac{dx}{dt}=A(t)x+B(t)y+f(t) \tag{2.1}$$

$$\frac{dy}{dt}=C(t)x+D(t)y+g(t) \tag{2.2}$$

其中 x，y分别为m维和n维实的（或复的）向量值函数，A，B，C，D，f和g是实（或复）矩阵值或向量值函数且有适当的可共存的阶。这些数据函数都假定为充分光滑的。

系统(2.1)，(2.2)能够简单地用线性变换

$$\begin{pmatrix} x \\ y \end{pmatrix}=H(t)\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \tag{2.3}$$

转化为下列的对角线的形式

$$\frac{d}{dt}\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}=\begin{pmatrix} A-BT & 0 \\ 0 & D+TB \end{pmatrix}\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}+\begin{pmatrix} I_m+ST & S \\ T & I_n \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix} \tag{2.4}$$

其中非奇异的$(m+n)\times(m+n)$矩阵$H=H(t)$为

$$H(t)=\begin{pmatrix} I_m & \underset{m\times n}{-S(t)} \\ \underset{n\times m}{-T(t)} & I_n+T(t)S(t) \end{pmatrix} \tag{2.5}$$

而$S=S(t)$和$T=T(t)$是矩阵值函数，满足下列微分方程

$$\frac{dT}{dt}=TBT+DT-TA-C \tag{2.6}$$

$$\frac{dS}{dt}=[A-BT]S-S[D+TB]-B \tag{2.7}$$

我们注意到，(2.5)的变换矩阵H是非奇异的（事实上是单位模的，参看文[5]），所以是可逆的：

$$H^{-1}(t)=\begin{pmatrix} I_m+ST & S \\ T & I_n \end{pmatrix}$$

众所周知，若数据函数A，B，C，D是光滑的则解T和S一般地也是适当光滑的。

(b) 现在我们研究下面的边值问题：

$$y'=f(t,y,z),\quad y(a)=A \tag{2.8}$$

$$z''=g(t,y,z,z'),\quad z(a)=B,\quad z(b)=C \tag{2.9}$$

其中 $y=(y_1,\cdots,y_i,\cdots,y_m)$，$f=(f^1,\cdots,f^i,\cdots,f^m)$，$A=(A^1,\cdots,A^i,\cdots,A^m)$ 属于R^m和$z=(z_1,\cdots,z_i,\cdots,z_n)$，$g=(g^1,\cdots,g^i,\cdots,g^n)$，$B=(B^1,\cdots,B^i,\cdots,B^n)$，$C=(C^1,\cdots,C^i,\cdots,C^n)$属于$R^n$。

假设函数f，g分别在区域$D_1=[a,b]\times[u,v]\times[\alpha,\beta]$和$D_1\times R^n$中是连续的。这里的下界和上界函数$(u,v)=(u_1,\cdots,u_i,\cdots,u_m;\ v_1,\cdots,v_i,\cdots,v_m)$，$(\alpha,\beta)=(\alpha_1,\cdots,\alpha_i,\cdots,\alpha_n;\ \beta_1,\cdots,\beta_i,\cdots,\beta_n)$分别是$C^{(1)}([a,b])$和$C^{(2)}([a,b])$类，满足

$$u\leqslant v,\quad u(a)\leqslant A\leqslant v(a)$$

$$u_i'(t)\leqslant f^i(t,\bar{u}_i(t),z),\quad v_i'(t)\geqslant f^i(t,\bar{v}_i(t),z),\quad t\in[a,b],\quad z\in[\alpha,\beta]\quad (i=1,2,\cdots,m)$$

其中

$$\bar{u}_i=(y_1,\cdots,y_{i-1},u_i,y_{i+1},\cdots,y_m);$$

$$\bar{v}_i=(y_1,\cdots,y_{i-1},v_i,y_{i+1},\cdots,y_m)\qquad (y_j\in[u_j,v_j];\ j\neq i)$$

和

$$\alpha\leqslant\beta,\ \alpha(a)\leqslant B\leqslant\beta(a),\ \alpha(b)\leqslant C\leqslant\beta(b)$$

$$\alpha_i''(t)\geqslant g^i(t,y,\bar{\alpha}_i(t),\bar{\alpha}_i'(t)),\ \beta_i''(t)\leqslant g^i(t,y,\bar{\beta}_i(t),\bar{\beta}_i'(t)),$$

$$(t\in(a,b),\ y\in[u,v];\ i=1,2,\cdots,n).$$

其中
$$\bar{\alpha}_i=(z_1,\cdots,z_{i-1},\alpha_i,z_{i+1},\cdots,z_n),$$

$$\bar{\alpha}_i'=(z_1',\cdots,z_{i-1}',\alpha_i',z_{i+1}',\cdots,z_n'),$$

$$\bar{\beta}_i=(z_1,\cdots,z_{i-1},\beta_i,z_{i+1},\cdots,z_n),$$

$$\bar{\beta}_i'=(z_1',\cdots,z_{i-1}',\beta_i',z_{i+1}',\cdots,z_n'),$$

$$(z_j\in[\alpha_j,\beta_j],\ z_j'\in[\alpha_j',\beta_j'],j\neq i).$$

此外，仍假设g满足Nagumo条件，则边值问题(2.8)，(2.9)有解$(y(t);\ z(t))=(y_1(t),\cdots,y_i(t),\cdots,y_m(t);\ z_1(t),\cdots,z_i(t),\cdots,z_n(t))$，满足

$$u_i(t)\leqslant y_i(t)\leqslant v_i(t),\ \alpha_i(t)\leqslant z_i(t)\leqslant\beta_i(t)$$

三 构造形式渐近解

我们假设系统(1.1)，(1.2)有下列的形式渐近解：

$$x(t,\varepsilon)=X(t,\varepsilon)+\varepsilon p(\tau,\varepsilon)\tag{3.1}$$

$$y(t,\varepsilon)=Y(t,\varepsilon)+q(\tau,\varepsilon)\tag{3.2}$$

其中 $\tau=(1-t)/\varepsilon$，$0\leqslant\tau<+\infty$，$p(\tau,\varepsilon)$，$q(\tau,\varepsilon)\to0$，当$\varepsilon\to\infty$时.

外解 $(X(t,\varepsilon),\ Y(t,\varepsilon))$ 满足 $(X(t,0),\ Y(t,0))=(X_0(t),\ Y_0(t))$，而 $(X(t,\varepsilon),\ Y(t,\varepsilon),\ p(\tau,\varepsilon),\ q(\tau,\varepsilon))\sim\left(\sum_{s=0}^{\infty}X_s(t)\varepsilon^s,\ \sum_{s=0}^{\infty}Y_s(t)\varepsilon^s,\ \sum_{s=0}^{\infty}p_s(\tau)\varepsilon^s,\ \sum_{s=0}^{\infty}q_s(\tau)\varepsilon^s\right)$.

把(3.1)，(3.2)代入(1.1)，(1.2)，并顾及边界层函数在$t=0$是超无穷小量可略去不计，我们就得到外解和内解的方程和初值条件如下：

$$X'(t,\varepsilon)=f(t,X,Y,\varepsilon),\ X(0,\varepsilon)=A(\varepsilon)\tag{3.3}$$

$$\varepsilon Y''(t,\varepsilon)=g(t,X,Y,\varepsilon)Y'+h(t,X,Y,\varepsilon)\tag{3.4}$$

$$Y(0,\varepsilon)=B(\varepsilon),\ Y(1,\varepsilon)=C(\varepsilon)\tag{3.5}$$

$$\begin{aligned}(-1)\frac{dp}{d\tau}=&f(1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon)+\varepsilon p(\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon)+q(\tau,\varepsilon),\varepsilon)\\&-f(1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon),\varepsilon)\end{aligned}\tag{3.6}$$

$$p(+\infty,\varepsilon)=0\tag{3.7}$$

$$\begin{aligned}\frac{d^2q}{d\tau^2}=&-g[1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon)+\varepsilon p(\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon)\\&+q(\tau,\varepsilon),\varepsilon]q_\tau'(\tau,\varepsilon)+\varepsilon\{g[1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon)\\&+\varepsilon p(\tau,\varepsilon),\ Y(1-\varepsilon\tau,\varepsilon)+q(\tau,\varepsilon),\varepsilon]Y_s'(1-\varepsilon\tau,\varepsilon)\\&+h(1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon)+\varepsilon p(\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon)+q(\tau,\varepsilon),\varepsilon)\\&-g[1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon),\varepsilon]Y_s'(1-\varepsilon\tau,\varepsilon)\\&-h[1-\varepsilon\tau,X(1-\varepsilon\tau,\varepsilon),Y(1-\varepsilon\tau,\varepsilon),\varepsilon]\}\end{aligned}\tag{3.8}$$

$$q(0,\varepsilon)=C(\varepsilon)-Y(1,\varepsilon),q(+\infty)=0\tag{3.9}$$

把(3.3)～(3.9)按ε的次幂展开，比较等式两端同次幂的系数，得

$$X_0'=f(t,X_0,Y_0,0),\ X_0(0)=A(0)\tag{3.10}$$

$$0=g(t,X_0,Y_0,0)Y_0'+h(t,X_0,Y_0,0),\ Y_0(0)=B(0) \tag{3.11}$$

$$X_s'=f_x[\cdot]X_s(t)+f_y[\cdot]y_s(t)+f_{s-1}(t),\ X_s(0)=A_s \tag{3.12$_s$}$$

$$Y_s'=-g^{-1}[\cdot]\{g_x[\cdot]Y_0'(t)+h_x[\cdot]\}X_s(t)-g^{-1}[\cdot]\{g_y[\cdot]Y_0'(t)+h_y[\cdot]\}Y_s(t)+g^{-1}[\cdot]E_{s-1}(t),\ Y_s(0)=B_s \tag{3.13$_s$}$$

$$\left.\begin{aligned}&\frac{d^2q_0}{d\tau^2}=g(1,X_0(1),Y_0(1)+q_0(\tau),0)\frac{dq_0}{d\tau}\\&q_0(0)=C_0-Y_0(1),\ q_0(+\infty)=0\end{aligned}\right\} \tag{3.14}$$

$$\left.\begin{aligned}\frac{dp_0}{d\tau}&=-f(1,X_0(1),Y_0(1)+q_0(\tau),0)+f(1,X_0(1),Y_0(1),0)\\&=-f_y(1,X_0(1),Y_0(1)+\theta q_0(\tau),0)q_0(\tau)\qquad(0<\theta<1)\\p_0(+\infty)&=0\end{aligned}\right\} \tag{3.15}$$

$$\left.\begin{aligned}&\frac{d^2q_s}{d\tau^2}=g(1,X_0(1),Y_0(1)+q_0(\tau),0)\frac{dq_s}{d\tau}+\tilde{g}_{s-1}(\tau)\\&q_s(0)=C_s-Y_s(1),\ q_s(+\infty)=0\end{aligned}\right\} \tag{3.16$_s$}$$

$$\frac{dp_s}{d\tau}=-f_x(1,X_0(1),Y_0(1),0)p_{s-1}(\tau)-f_y(1,X_0(1),Y_0(1)+q_0(\tau),0)q_s(\tau)+\tilde{f}_{s-1}(\tau),\ p_s(+\infty)=0 \tag{3.17$_s$}$$

其中 $\tilde{f}_{s-1}(\tau)$和$\tilde{g}_{s-1}(\tau)$是不含常数项的$p_0(\tau),\cdots,p_{s-1}(\tau)$; $q_0(\tau),\cdots,q_{s-1}(\tau)$的多项式，它们的系数都是$\tau$的多项式。$E_{s-1}(t)$是$X_0(t),\cdots,X_{s-1}(t),Y_0(t),\cdots,Y_{s-1}(t)$的已知函数。

问题(3.10)和(3.11)的解也就是退化问题(1.3)的解，我们假定它是已知的。再由条件(i)和(iii)知道，问题(3.12)$_s$和(3.13)$_s$右端的向量函数和系数的矩阵函数都是适当光滑的。因此，根据第二节的预备定理(a)的结果，可把(3.12)$_s$和(3.13)$_s$化为如同形式(2.4)的对角线系统，再根据一阶常微分方程组初值问题解的存在唯一性定理，就可分别解出$Y_s(t)$和$X_s(t)$ $(s=1,2,\cdots)$。

由于假设条件(i)中的g满足稳定性条件(1.4)和边界层稳定性条件(1.5)，因此，根据文[6]或[7]的结果可得(3.14)的解$q_0(t)$存在且有估计式 $q_0(\tau)=O(\exp[-k\tau])$，其中$k$是一正的常数，$\tau\geqslant0$。

积分(3.15)，我们得

$$p_0(\tau)=-\int_{-\infty}^{\tau}f_y(1,X_0(1),Y_0(1)+\theta q_0(s),0)q_0(s)ds$$

利用假设条件(iii)和$q_0(\tau)$的性质，我们得

$$\|p_0(\tau)\|\leqslant C_1\exp[-k\tau]\qquad(\tau\geqslant0)$$

由$\tilde{g}_{s-1}(\tau)$的结构知，$\|\tilde{g}_{s-1}(\tau)\|\leqslant\tilde{C}_{s-1}\exp[-k\tau]$ $(\tau\geqslant0)$。同样地，根据文[6]或[7]的结果可得(3.16)$_s$的解存在且满足$\|q_s(\tau)\|\leqslant\tilde{C}_s\exp[-k\tau]$ $(\tau\geqslant0)$。

从(3.17)$_s$我们得

$$p_s(\tau)=-\int_{-\infty}^{\tau}\{f_x(1,X_0(1),Y_0(1),0)p_{s-1}(\bar{s})+f_y(1,X_0(1),Y_0(1)+q_0(\bar{s}),0)q_s(\bar{s})-\tilde{f}_{s-1}(\bar{s})\}d\bar{s}$$

由假设条件(iii)和 $p_{s-1}(\tau)$ 与 $\tilde{f}_{s-1}(\tau)$ 的构造以及 $q_s(\tau)$ 的性质知，$p_s(\tau)$ 存在且有估计式 $\|p_s(\tau)\|\leqslant C_s\mathrm{epx}[-k\tau]$，$\tau\geqslant0$，其中$C_s,\tilde{C}_s(s=1,2,\cdots)$ 都是某一正常数.

四、主要结果

定理1 在假设条件(i)~(iii)下，对于充分小的ε，边值问题(1.1)，(1.2)存在解$(x(t,\varepsilon),\ y(t,\varepsilon))$且满足不等式

$$|x^i(t,\varepsilon)-x_N^i(t,\varepsilon)|\leqslant M_i\varepsilon^{N+1}$$
$$|y^i(t,\varepsilon)-y_N^i(t,\varepsilon)|\leqslant M_i\varepsilon^{N+1}\qquad(0\leqslant t\leqslant 1)$$

其中 x^i，y^i和x_N^i，y_N^i分别是$x(t,\varepsilon)$，$y(t,\varepsilon)$和$x_N(t,\varepsilon)$，$y_N(t,\varepsilon)$的第i个分量．$y_N(t,\varepsilon)$和$x_N(t,\varepsilon)$将在下面证明中给出．$M_i(i=1,2,\cdots,n)$是某个与ε无关的正的常数．

证明 我们定义

$$x_N(t,\varepsilon)=\sum_{s=0}^{N}\left[X_s(t)+\varepsilon p_s\left(\frac{1-t}{\varepsilon}\right)\right]\varepsilon^s$$

$$y_N(t,\varepsilon)=\sum_{s=0}^{N}\left[Y_s(t)+q_s\left(\frac{1-t}{\varepsilon}\right)\right]\varepsilon^s$$

令$R_N=x(t,\varepsilon)-x_N(t,\varepsilon)$，$Q_N(t,\varepsilon)=y(t,\varepsilon)-y_N(t,\varepsilon)$，则$R_N$，$Q_N$满足边值问题

$$\left.\begin{aligned}R_N'=&f(t,x_N+R_N,y_N+Q_N,\varepsilon)-f(t,x_N,y_N,\varepsilon)\\&+f(t,x_N,y_N,\varepsilon)-y_N'\end{aligned}\right\}$$
$$R_N(0,\varepsilon)=A(\varepsilon)-x_N(0,\varepsilon)\tag{4.1}$$
$$\left.\begin{aligned}\varepsilon Q_N''=&g(t,x_N+R_N,y_N+Q_N,\varepsilon)(y_N'+Q_N')+h(t,x_N+R_N,y_N+Q_N,\varepsilon)\\&-g(t,x_N,y_N,\varepsilon)y_N'-h(t,x_N,y_N,\varepsilon)\\&+g(t,x_N,y_N,\varepsilon)y_N'+h(t,x_N,y_N,\varepsilon)-\varepsilon y_N''\end{aligned}\right\}\tag{4.2}$$
$$Q_N(0,\varepsilon)=B(\varepsilon)-y_N(0,\varepsilon),\ Q_N(1,\varepsilon)=C(\varepsilon)-y_N(1,\varepsilon)\tag{4.3}$$

从定义我们易知存在正常数向量$M=(M_1,\cdots,M_i,\cdots,M_n)$，使得

$$-M\varepsilon^{N+1}\leqslant x_N'(t,\varepsilon)-f(t,x_N,y_N,\varepsilon)\leqslant M\varepsilon^{N+1}\tag{4.4}$$
$$-M\varepsilon^{N+1}\leqslant\varepsilon y_N''(t,\varepsilon)-g(t,x_N,y_N,\varepsilon)y_N'-h(t,x_N,y_N,\varepsilon)\leqslant M\varepsilon^{N+1}\tag{4.5}$$
$$-M\varepsilon^{N+1}\leqslant A(\varepsilon)-x_N(0,\varepsilon)\leqslant M\varepsilon^{N+1}\tag{4.6}$$
$$-M\varepsilon^{N+1}\leqslant B(\varepsilon)-y_N(0,\varepsilon)\leqslant M\varepsilon^{N+1}\tag{4.7}$$
$$-M\varepsilon^{N+1}\leqslant C(\varepsilon)-y_N(1,\varepsilon)\leqslant M\varepsilon^{N+1}\tag{4.8}$$

现在我们再构造下界和上界函数：

$$u(t,\varepsilon)=x_N(t,\varepsilon)-r\varepsilon^{N+1},\ v(t,\varepsilon)=x_N(t,\varepsilon)+r\varepsilon^{N+1}\tag{4.9}$$
$$\alpha(t,\varepsilon)=y_N(t,\varepsilon)-r\varepsilon^{N+1},\ \beta(t,\varepsilon)=y_N(t,\varepsilon)+r\varepsilon^{N+1}\tag{4.10}$$

其中 $r=(r_1,\cdots,r_i,\cdots,r_n)$是一个正的常数向量，它将在稍后选取，例如，我们选取$r_i\geqslant\max\{M_i,M_i/\delta_i,M_i/\sigma_i\}$．

显然，$u(t,\varepsilon)\leqslant v(t,\varepsilon)$，$\alpha(t,\varepsilon)\leqslant\beta(t,\varepsilon)$，和当$r\geqslant M$时，我们有$u(0,\varepsilon)\leqslant A(\varepsilon)\leqslant v(0,\varepsilon)$，$\alpha(0,\varepsilon)\leqslant B(\varepsilon)\leqslant\beta(0,\varepsilon)$，$\alpha(1,\varepsilon)\leqslant C(\varepsilon)\leqslant\beta(1,\varepsilon)$．

最后，我们检验 $u_i'(t,\varepsilon)\leqslant f^i(t,\bar{u}_i,y,\varepsilon)$；

$v_i'(t,\varepsilon)\geqslant f^i(t,\bar{v}_i,y,\varepsilon)$，

$\varepsilon\alpha_i''\geqslant g^i(t,x,\bar{\alpha}_i',\varepsilon)\bar{\alpha}_i'+h^i(t,x,_i,\bar{\alpha}\varepsilon)$；

$$\varepsilon\beta_i''\leqslant g^i(t,x,\bar\beta_i,\varepsilon)\bar\beta_i'+h^i(t,x,\bar\beta_i,\varepsilon).$$

事实上，从(4.4)～(4.10)和假设条件(iii)，我们有

$$\begin{aligned}u_i'(t,\varepsilon)-f^i(t,\bar u_i,y,\varepsilon)&=(x_N^i)'-f^i(t,\overline{x_N^i-r_i\varepsilon^{N+1}},y,\varepsilon)\\&=(x_N^i)'-f^i(t,\bar x_N^i,y,\varepsilon)-f^i(t,\overline{x_N^i-r_i\varepsilon^{N+1}},y,\varepsilon)\\&\quad-f^i(t,\bar x_N^i,y,\varepsilon)\leqslant M_i\varepsilon^{N+1}-f_x^i\{\cdot\}[-r_i\varepsilon^{N+1}]\\&\leqslant M_i\varepsilon^{N+1}-\delta_ir_i\varepsilon^{N+1}\leqslant 0.\end{aligned}$$

只要选取$r_i\geqslant M_i/\delta_i$和

$$\begin{aligned}&\varepsilon\alpha_i''-g^i(t,x,\bar\alpha_i,\varepsilon)\alpha_i'-h(t,x,\bar\alpha_i,\varepsilon)\\&=\varepsilon(y_N^i)''-g^i(t,x,\overline{y_N^i-r_i\varepsilon^{N+1}},\varepsilon)y_N^{i\prime}-h^i(t,x,\overline{y_N^i-r_i\varepsilon^{N+1}},\varepsilon)\\&=\varepsilon(y_N^i)''-g^i(t,x,\bar y_N^i,\varepsilon)y_N^{i\prime}-[g^i(t,x,\overline{y_N^i-r_i\varepsilon^{N+1}},\varepsilon)y_N^{i\prime}\\&\quad-g^i(t,x,\bar y_N^i,\varepsilon)y_N^{i\prime}]-h^i(t,x,\bar y_N^i,\varepsilon)-[h^i(t,x,\overline{y_N^i-r_i\varepsilon^{N+1}},\varepsilon)\\&\quad-h^i(t,x,\bar y_N^i,\varepsilon)]\geqslant -M_i\varepsilon^{N+1}-[g_y^i[\cdot\cdot]y_N^{i\prime}+h_y^i[\cdot\cdot]](-r_i\varepsilon^{N+1})\\&\geqslant -M_i\varepsilon^{N+1}+\sigma_i\varepsilon^{N+1}\geqslant 0\quad(\text{当}r_i\geqslant M_i/\sigma_i\text{时}).\end{aligned}$$

同样地，我们能够检验其他两个不等式.

其中 u_i，v_i，α_i，β_i，r_i，f^i，g^i和h^i分别是u,v，α，β，r，f，g和h的分量；$\bar u=(x^1,\cdots,x^{i-1},u_i,x^{i+1},\cdots,x^n)$；$\cdots$；$\overline{y_N^i-r_i\varepsilon^{N+1}}=(y_N^1,\cdots,y_N^{i-1},y_N^i-r_i\varepsilon^{N+1},y_N^{i+1},\cdots,y_N^n)$；$x\in[u,v]$，$y\in[\alpha,\beta]$，$x_j\in[u_j,v_j]$，$y_j\in[\alpha_j,\beta_j]$，$(j\neq i)$；$\{\cdot\}=(t,\overline{x_N^i-\theta_ir_i\varepsilon^{N+1}},y,\varepsilon)$；$[\cdot\cdot]=(t,x,\overline{y_N^i-\theta_ir_i\varepsilon^{N+1}},\varepsilon)$ $(0<\theta_i<1)$. 因此，从第二节预备定理(b)的结果我们知道，问题(1.1)，(1.2)存在解$(x,y)\in C^{(1)}[0,1]\times C^{(2)}[0,1]$，满足

$$u_i\leqslant x^i(t,\varepsilon)\leqslant v_i,\quad \alpha_i\leqslant y^i(t,\varepsilon)\leqslant\beta_i$$

也就是在[0,1]中

$$|x^i(t,\varepsilon)-x_N^i(t,\varepsilon)|\leqslant M_i\varepsilon^{N+1},$$
$$|y^i(t,\varepsilon)-y_N^i(t,\varepsilon)|\leqslant M_i\varepsilon^{N+1},\qquad (i=1,2,\cdots,n)$$

注 我们也可以研究退化问题

$$x'=f(t,x,y,0),\quad x(0,0)=A(0)$$
$$0=g(t,x,y,0)y'+h(t,x,y,0),\quad y(1,0)=C(0)$$

这时稳定性条件(1.4)和边界层稳定性条件(1.5)将以

$$g(t,\ X_0(t),\ Y_0(t),\ 0)<0$$

和

$$\zeta^T\int_0^\zeta g(0,\ X_0(0),\ Y_0(0)+s,\ 0)dS<0$$

来代替．(1.1)，(1.2)的解的非一致收敛性将选择在点$t=0$的近旁，伸长变量将取$\tau=t/\varepsilon$.

参 考 文 献

[1] Chang, K. W., Singular perturbation of a general boundary value problem, *SIAM J. Math. Anal.*, 3, 3 (1972), 520—526.

[2] 林宗池，微分不等式理论和对角化技巧在奇摄动问题中的应用，第二届全国近代数学与力学讨论会文集，中国力学会，上海 (1987, 12), 37—39.

[3] Chang, K. W. and F. A. Howes, 《非线性奇异摄动现象的理论和应用》，林宗池等译，福建科技出版社 (1989).

[4] Howes, F. A., Differntial inequalities of higher order and the asymptotic

solution of nonlinear boundary value problems, *SIAM. J. Math. Anal.*, **13**, 1 (1982), 61—80.

[5] Smith, D. R., *Singular Perturbation Theory*, An introduction with applications, Cambridge University Press, Cambridge (1985).

[6] Howes, F. A. and R. E. O'Malley, Jr., Singular perturbation of semilinear second order systems, O. D. E. and P. D. E., 131—150. *Lecture Notes in Math.*, Springer, 827.

[7] Lin Zong-chi, The higher order approximation of solution of quasilinear second order systems for singular perturbation, *Chin. Ann.*, **8B**, 3 (1987), 357—363.

Singular Perturbation of Boundary Value Problem of Systems for Quasilinear Ordinary Differential Equations

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Abstract

In this paper, we study the singular perturbation of boundary value problem of systems for quasilinear ordinary differential equations: $x'=f(t,x,y,\varepsilon)$, $\varepsilon y''=g(t,x,y,\varepsilon)y'+h(t,x,y,\varepsilon)$, $x(0,\varepsilon)=A(\varepsilon)$, $y(0,\varepsilon)=B(\varepsilon)$, $y(1,\varepsilon)=C(\varepsilon)$, where x, f, y, h, A, B and C belong to R^n and g is a diagonal matrix. Under the appropriate assumptions, using the technique of diagonalization and theory of differential inequalities we obtain the existence of solution and its componentwise uniformly valid asymptotic estimation.

Key words quasilinear systems, singularly perturbed boundary value problem, diagonalization and differential inequality, asymptotic expansion