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地球引力场对卫星有摄运动的一种计算方法

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摘　　要

本文应用 Delaunay 变量，从理论力学的哈密顿方程出发，通过正则变换求解了地球引力摄动对卫星运动轨道的影响，导出卫星位置和速度随时间的变化关系．

关键词　地球引力场　哈密顿方程　摄动　正则变换

一、引　　言

人造地球卫星的轨道计算是空间技术的重要部分，随着我国空间事业的不断发展，对轨道计算的要求也日益提高．实际的人造卫星运动不是简单的两体问题，首先，地球就不是一个均匀的球体，引力势不能简单表示成

$$U=GM/r$$

而且不能用一个简单的函数来表达实际的地球引力场．同时卫星运动中还要受到地球大气的阻力，太阳辐射压力，太阳和月球对卫星的引力等等，这些力称之为摄动力．由于摄动力的作用，使得卫星的运动非常复杂，其轨道并不简单是一个圆锥曲线．也就不能用一个解析函数给出卫星的位置和速度随时间的变化关系．

本文应用Delaunay变量，从理论力学的哈密顿方程出发，通过正则变换求解了地球引力摄动对卫星运动轨道的影响，导出了卫星位置和速度随时间的变化关系．这种方法推导简单，易于进一步向高阶理论推导，并给出了6个根数的一阶、二阶长期摄动和一阶周期摄动．

二、卫星运动方程

设地球质量为M，赤道面为xy平面，β为纬度，K为高斯常量，$\mu=K^2M$，则地球引力势可写成

$$U=\frac{\mu}{r}+\frac{\mu C_2}{r^3}(1-3\sin^2\beta)+\frac{\mu C_4}{r^5}\left(1-10\sin^2\beta+\frac{35}{3}\sin^4\beta+\cdots\right) \tag{2.1}$$

得卫星运动方程

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$$\frac{d^2x}{dt^2}=\frac{\partial U}{\partial x},\quad \frac{d^2y}{dt^2}=\frac{\partial U}{\partial y},\quad \frac{d^2z}{dt^2}=\frac{\partial U}{\partial z} \tag{2.2}$$

令i为轨道面倾角，g为近地点辐角，f为真近点角，则

$$\sin\beta=\sin i\sin(g+f),\quad 2\sin^2\beta=\sin^2 i[1-\cos(2g+2f)]$$

代入(2.1)式写出摄动函数

$$R=\frac{\mu C_2}{a^3}\left[\left(-\frac{1}{2}+\frac{3}{2}\cos^2 i\right)\frac{a^3}{r^3}+\left(\frac{3}{2}-\frac{3}{2}\cos^2 i\right)\frac{a^3}{r^3}\cos(2g+2f)\right] \tag{2.3}$$

令a，e分别为瞬时轨道的半长轴和偏心率，取Delaunay变量

$L=(\mu a)^{1/2}$，　$l=M$（平近点角）

$G=L(1-e^2)^{1/2}$，$g=\omega$（近地点辐角）

$N=G\cos i$，　$n=\Omega$（升交点赤经）

将这6个变量的正则方程写为

$$\left.\begin{aligned}\frac{dL}{dt}&=\frac{\partial H}{\partial l},\quad \frac{dl}{dt}=-\frac{\partial H}{\partial L}\\ \frac{dG}{dt}&=\frac{\partial H}{\partial g},\quad \frac{dg}{dt}=-\frac{\partial H}{\partial G}\\ \frac{dN}{dt}&=\frac{\partial H}{\partial n},\quad \frac{dn}{dt}=-\frac{\partial H}{\partial N}\end{aligned}\right\} \tag{2.4}$$

式中H为哈密顿函数，即

$$H=\frac{\mu^2}{2L^2}+\frac{\mu^4C_2}{L^6}\left[\left(-\frac{1}{2}+\frac{3}{2}\ \frac{N^2}{G^3}\right)\frac{a^3}{r^3}\right.$$
$$\left.+\left(\frac{3}{2}-\frac{3}{2}\ \frac{N^2}{G^2}\right)\frac{a^3}{r^3}\cos(2f+2g)\ \right]+\cdots \tag{2.5}$$

应用级数展开

$$\frac{a^3}{r^3}=\frac{L^3}{G^3}+\sum_{j=1}^{\infty}\ 2P_j\cos jl=\frac{L^3}{G^3}+\sigma_1$$

$$\frac{a^3}{r^3}\cos(2f+2g)=\sum_{j=-\infty}^{\infty}\ Q_j\cos(2f+2g)=\sigma_2$$

系数P、Q为e的函数，H中不含t，且H中不包括变量n，σ_2中系数Q_0为零．一般地讲，如$q>(p-2)\geqslant 0$，p，q为正整数，$(a^p/r^p)\cos qf$对l的平均值为零．

三、方程的求解方法

考虑找一种正则变换，将变量L，G，N，l，g，n换成新变量L'，G'，N'，l'，g'，n'求解方程确定一个母函数S 使得

$$\left.\begin{aligned}L&=\frac{\partial S}{\partial l'},\quad G=\frac{\partial S}{\partial g'},\quad N=\frac{\partial S}{\partial n'}\\ l'&=\frac{\partial S}{\partial L'},\quad g'=\frac{\partial S}{\partial G'},\quad n'=\frac{\partial S}{\partial N'}\end{aligned}\right\} \tag{3.1}$$

新的变量方程

$$\left.\begin{aligned}\frac{dL'}{dt}=\frac{\partial H^*}{\partial l'}, \quad \frac{dl'}{dt}=-\frac{\partial H^*}{\partial L'}\\ \frac{dG'}{dt}=\frac{\partial H^*}{\partial g'}, \quad \frac{dg'}{dt}=-\frac{\partial H^*}{\partial G'}\\ \frac{dN'}{dt}=\frac{\partial H^*}{\partial n'}, \quad \frac{dn'}{dt}=-\frac{\partial H^*}{\partial N'}\end{aligned}\right\} \tag{3.2}$$

$$H^*(L', G', N')=H(L, G, N, l, g, —) \tag{3.3}$$

如果H^*仅仅为L'，G'，N'的函数，则问题就全部解决了，从方程(3.2)式可见此时L'，G'，N'都为常量，而l'，g'，n'则为时间的线性函数。

现在的问题是寻找函数S，使得l'不再出现在H^*中，而g'保留存在，这时L'，N'将为常量，则问题简化为

$$\frac{dG'}{dt}=\frac{\partial H^*}{\partial g'}, \quad \frac{dg'}{dt}=-\frac{\partial H^*}{\partial G'} \tag{3.4}$$

$$\frac{dl'}{at}=\frac{\partial H^*}{\partial L'}, \quad \frac{dn'}{dt}=-\frac{\partial H^*}{\partial N'} \tag{3.5}$$

四、方程的一阶解

应用Zeipel变换来确定母函数，过程中将哈密顿函数按小量G_2展开

$$H=H_0+H_1 \tag{4.1}$$

H_0仅为L的函数

$$S=S_0+S_1+S_2+\cdots \tag{4.2}$$

$$H^*=H_0^*+H_1^*+H_2^*+\cdots \tag{4.3}$$

选取 $S_0=L'l+G'g+N'n$

要求新的哈密顿量不显含l'和n'，现在来确定S和H^*，由于哈密顿量不显含时间，所以

$$H(L, G, N, l, g, —)=H^*(L, G, N, —, g', —)$$

展开

$$H_0\left(\frac{\partial S}{\partial l}\right)+H_1\left(\frac{\partial S}{\partial l'}, \frac{\partial S}{\partial g'}, \frac{\partial S}{\partial n'}, l, g, —\right)$$

$$=H_0^*+H_1^*\left(L', G', N', —, \frac{\partial S}{\partial G'}, —\right)+H_2\left(L', G', N', —, \frac{\partial S}{\partial G'}, —\right) \tag{4.4}$$

应用泰勒展开

$$H_0(L')+\frac{\partial H_0}{\partial L'}\frac{\partial S_1}{\partial l}+\frac{\partial H_0}{\partial L'}\frac{\partial S_2}{\partial l}+\frac{1}{2}\frac{\partial^2 H}{\partial L'^2}\left(\frac{\partial S_1}{\partial l}\right)^2$$

$$+H_1(L',G',N',l,g,—)+\frac{\partial H_1}{\partial L'}\frac{\partial S_1}{\partial l}+\frac{\partial H_1}{\partial G'}\frac{\partial S_1}{\partial g}$$

$$=H_0^*+H_1^*(L',G',N',—,g,—)+\frac{\partial H^*}{\partial g}\frac{\partial S_1}{\partial G'}+H_2^*(L',G',N',—,g,—) \tag{4.5}$$

比较两边C_2的同次幂，则有

零阶 $H_0(L')=H_0^*(L')$

一阶 $\frac{\partial H_0}{\partial L'}\frac{\partial S_1}{\partial l}+H_1=H_1^*$

二阶 $$\frac{\partial H_0}{\partial L'}\frac{\partial S_2}{\partial l}+\frac{1}{2}\frac{\partial^2 H}{\partial L'^2}\left(\frac{\partial S_1}{\partial l}\right)^2+\frac{\partial H_1}{\partial L'}\frac{\partial S_1}{\partial l}+\frac{\partial H_1}{\partial G'}\frac{\partial S_1}{\partial g}$$

$$=H_2^*+\frac{\partial H_2^*}{\partial g}\frac{\partial S_1}{\partial G} \tag{4.6}$$

故

$$H_0^*=H_0(L')=\mu^2/2L'^2 \tag{4.7}$$

将H_1分成两部分：$H_1=H_{1S}+H_{1P}$

因H_{1S}不含l，H_{2P}含l（为短周期项），$(\partial S_1/\partial l)$也为含$l$的短周期项，比较则有

$$H_1^*=H_{1S}=\frac{\mu^4 C_1}{L'^3G'^3}A,\quad A=-\frac{1}{2}+\frac{3}{2}\frac{N^2}{G^2}$$

$$\frac{\partial S_1}{\partial l}=\frac{\mu^2 C_2}{L'^3}(A\sigma_1+B\sigma_2),\quad B=\frac{3}{2}-\frac{3}{2}\frac{N^2}{G^2}$$

积分得

$$S_1=\frac{\mu^2C_2}{L'^3}\left[A\sum_{j=-\infty}^{\infty}\frac{2}{j}P_j\sin jl+B\sum_{j=1}^{\infty}\frac{1}{j}Q_j\sin(2g+jl)\right] \tag{4.8}$$

由于仅仅是S_1对L'，G'，N'，l，g，n的偏微分，因而并不要求有积分常量

$$\int\sigma_1 dl=\int\left(\frac{a^3}{r^3}-\frac{L^3}{G^3}\right)dl=\frac{L^3}{G^3}[f-l+e\sin f]$$

$$\int\sigma_2 dl=\int\frac{a^3}{r^3}\cos(2g+2f)dl=\frac{L^3}{G^3}\left[\frac{1}{2}\sin(2g+2f)+\frac{e}{2}\sin(2g+f)\right.$$
$$\left.+\frac{e}{6}\sin(2g+3f)\right]$$

所以

$$S_1=\frac{\mu^2C_2}{G'^3}\left\{A(f-l+e\sin f)+B\left[\frac{1}{2}\sin(2g+2f)+\frac{e}{2}\sin(2g+f)\right.\right.$$
$$\left.\left.+\frac{e}{6}\sin(2g+3f)\right]\right\} \tag{4.9}$$

$$\frac{\partial S_1}{\partial g}=\frac{\mu^2C_2}{G'^3}B\left[\cos(2g+2f)+e\cos(2g+f)+\frac{e}{3}\cos(2g+3f)\right] \tag{4.10}$$

引入 $$\lambda_2=\mu^2C_2/L'^4 \tag{4.11}$$

则有

$$L=L'+\frac{\partial S_1}{\partial l}=L'\left\{1+\lambda_2\left[\left(-\frac{1}{2}+\frac{3}{2}\frac{N^2}{G'^2}\right)\left(\frac{a^3}{r^3}-\frac{L'^3}{G'^3}\right)\right.\right.$$
$$\left.\left.+\left(\frac{3}{2}-\frac{3}{2}\frac{N^2}{G'^2}\right)\frac{a^3}{r^3}\cos(2g+2f)\right]\right\}$$

$$G=G'+\frac{\partial S_1}{\partial g}=G'\left\{1+\lambda_2\frac{L'^4}{G'^4}\left(\frac{3}{2}-\frac{3}{2}\frac{N^2}{G'^2}\right)\right[\cos(2g+2f)$$

$$+e\cos(2g+f)+\frac{e}{3}\cos(2g+3f)\Big]\Big\}$$

$$N=N'$$

$$\frac{\partial S_1}{\partial e}=\frac{\mu^2C_2}{L'^3}\Big\{A\Big[(1+e\cos f)\frac{\partial f}{\partial e}+\sin f\Big]+B\Big[\cos(2g+2f)\cdot(1+e\cos f)\frac{\partial f}{\partial e}+\frac{1}{2}\sin(2g+f)+\frac{1}{6}\sin(2g+3f)\Big]\Big\} \tag{4.12}$$

应用

$$\cos(2f+2g)+\frac{e}{2}\cos(2g+f)+\frac{e}{2}\cos(2g+f)+\frac{e}{2}\cos(2g+3f)$$

$$=\cos(2g+2f)(1+e\cos f)(1+e\cos f)\frac{\partial f}{\partial e}$$

$$=\Big(\frac{a^2}{r^2}\frac{G^2}{L^2}+\frac{a}{r}\Big)\sin f$$

则有

$$\frac{\partial S_1}{\partial e}=\frac{\mu^2C_2}{L'^3}\Big\{A\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+1\Big)\sin f+\frac{1}{2}B\Big[\Big(-\frac{a^2G'^2}{r^2L'^2}-\frac{a}{r}+1\Big)\cdot\sin(2g+f)+\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+\frac{1}{3}\Big)\sin(2g+3f)\Big]\Big\} \tag{4.13}$$

应用函数关系

$$\frac{\partial\psi}{\partial L}=\frac{1}{e}\frac{\partial\psi}{\partial e}\frac{G^2}{L^3},\qquad \frac{\partial\psi}{\partial G}=-\frac{1}{e}\frac{\partial\psi}{\partial e}\frac{G}{L^2} \tag{4.14}$$

则可得

$$l=l'-\Big(\frac{\partial S_1}{\partial L'}\Big)=l'-\frac{\lambda_2}{e}\frac{L'}{G'}\Big\{\Big(-\frac{1}{2}+\frac{3}{2}\frac{N^2}{G'^2}\Big)\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+1\Big)\cdot\sin f+\Big(\frac{3}{4}-\frac{3}{4}\frac{N^2}{G'^2}\Big)\Big[\Big(-\frac{a^2}{r^2}\frac{G'^2}{L'^2}-\frac{a}{r}+1\Big)\sin(2g+f)+\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+\frac{1}{3}\Big)\sin(2g+3f)\Big]\Big\} \tag{4.15}$$

$$g=g'-\Big(\frac{\partial S_1}{\partial G'}\Big)=g'-\frac{\lambda_2}{e}\frac{L'^2}{G'^2}\Big\{\Big(-\frac{1}{2}+\frac{3}{2}\frac{N^2}{G'^2}\Big)\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+1\Big)\cdot\sin f+\Big(\frac{3}{4}-\frac{3}{4}\frac{N^2}{G'^2}\Big)\Big[\Big(-\frac{a^2}{r^2}\frac{G'^2}{L'^2}-\frac{a}{r}+1\Big)\sin(2g+f)+\Big(\frac{a^2}{r^2}\frac{G'^2}{L'^2}+\frac{a}{r}+\frac{1}{3}\Big)\sin(2g+3f)\Big]\Big\}+\lambda_2\frac{L'^4}{G'^4}\Big\{\Big(-\frac{3}{2}+\frac{15}{2}\frac{N^2}{G'^2}\Big)(f-l+e\sin f)+\Big(\frac{9}{2}-\frac{15}{2}\frac{N^2}{G'^2}\Big)\Big[\frac{1}{2}\sin(2g+2f)+\frac{e}{2}\sin(2g+f)+\frac{e}{6}\sin(2g+3f)\Big]\Big\} \tag{4.16}$$

$$n=n'-\frac{\partial S_1}{\partial N}=n'-3\lambda_2\frac{L'^4}{G'^4}\Big[f-l+e\sin f-\frac{1}{2}\sin(2g+2f)-\frac{e}{2}\sin(2g+f)-\frac{e}{6}\sin(2g+3f)\Big]\frac{N}{G'} \tag{4.17}$$

在计算卫星的位置时，常常l和g不分开。由于$(1-e^2)^{-1}-(1-e^2)^{-1/2}$是可化成$e^2$的因子，因而在分母中虽出现$e$的因子，并不作为除数。即其结果并不因$e\to0$而出现奇点。

五、方程的二阶解

令

$$\rho_2=\frac{L'^3}{G'^3}\left[\cos(2g+2f)+e\cos(2g+f)+\frac{e}{3}\cos(2g+f)\right]$$

所以

$$\frac{\partial S_1}{\partial g}=\frac{\mu^2C_2}{L'^3}B\rho_2 \tag{5.1}$$

定义

$$\tau_1=\frac{1}{e}\ \frac{\partial\sigma_1}{\partial e}=\frac{3}{e}\ \frac{a^4}{r^4}\cos f-3\ \frac{L^5}{G^5} \tag{5.2}$$

$$\tau_2=\frac{1}{e}\ \frac{\partial\sigma_2}{\partial e}=\frac{1}{e}\left[\frac{1}{2}\ \frac{a^4}{r^4}-\frac{a^3}{r^3}\ \frac{L^2}{G^2}\right]\cos(2g+f)+\frac{1}{e}\left[\frac{5}{2}\ \frac{a^4}{r^4}\right.$$

$$\left.+\frac{a^3}{r^3}\ \frac{L^2}{G^2}\right]\cos(2g+3f) \tag{5.3}$$

容易得到

$$\frac{\partial H_{1P}}{\partial L'}=\frac{\mu^2C_2}{L'^7}\left[-6(A\sigma_1+B\sigma_2)+\frac{G'^2}{L'^2}(A\tau_1+B\tau_2)\right]$$

$$\frac{\partial H_{1P}}{\partial G'}=\frac{\mu^2C_2}{L'^7}\left[\frac{G'}{L'}(A\tau_1+B\tau_2)+\frac{3L'^2}{G'}\ \frac{N^2}{G'^2}(\sigma_1-\sigma_2)\right]$$

比较(4.6)式两端则有

$$H_2^*=\left[\frac{1}{2}\ \frac{\partial^2H_0}{\partial L'^2}\left(\frac{\partial S_1}{\partial l}\right)^2+\frac{\partial H_{1P}}{\partial L'}\ \frac{\partial S_1}{\partial l}+\frac{\partial H_{1S}}{\partial G'}\ \frac{\partial S_1}{\partial g}\right] \tag{5.4}$$

下标S表示了不含l的部分，容易计算出

$$\frac{1}{2}\ \frac{\partial^2H_0}{\partial L'^2}\left(\frac{\partial S_1}{\partial l^2}\right)=\frac{3}{2}\ \frac{\mu^6C_2^2}{L'^{10}}(A\sigma_1+B\sigma_2)^2$$

$$\frac{\partial H_{1P}}{\partial L'}\ \frac{\partial S_1}{\partial l}=\frac{\mu^6C_2^2}{L'^{10}}\left[-6(A\sigma_1+B\sigma_2)^2+\frac{G'^2}{L'^2}(A\sigma_1+B\sigma_2)(A\tau_1+B\tau_2)\right]$$

$$\frac{\partial H_{1P}}{\partial G'}\ \frac{\partial S_1}{\partial g}=\frac{\mu^6C_{10}^2}{L'^{10}}\left[3\frac{L'}{G'}\ \frac{N^2}{G'^2}B(-\sigma_1\rho_2+\sigma_2\rho_2)-\frac{G'}{L'}(AB\tau_1+B^2\tau_2)\rho_2\right]$$

$$\frac{\partial H_{1S}}{\partial G'}\ \frac{\partial S_1}{\partial g}=\frac{\mu^6C_2^2}{L'^{10}}\left[-3\frac{L'^4}{G'^4}\left(AB+B\frac{N^4}{G'^2}\right)\rho_2\right]$$

应用

$$\frac{e}{\pi}\int_0^{\pi}\cos f\,dl=-e^2$$

$$\frac{e}{\pi}\int_0^{\pi}\cos 2f\,dl=\frac{1}{e^2}\left[2\frac{G^3}{L^3}-3\frac{G^2}{L^2}+1\right]$$

$$\frac{e}{\pi}\int_0^{\pi}\cos 3f\,dl=-\frac{4}{e^2}\left[2\frac{L^3}{G^3}+3\frac{G^2}{L^2}+1\right]+3e^2$$

则有

$$H_2^*=\frac{\mu^6C_2^2}{L'^{10}}\left[\frac{15L'^5}{32G'^5}\left(1-\frac{18}{5}\ \frac{N^2}{G'^2}+\frac{N^4}{G'^4}\right)+\frac{3}{8}\ \frac{L'^6}{G'^6}\left(1-6\frac{N^2}{G'^2}+9\frac{N^4}{G'^4}\right)\right.$$

$$-\frac{15}{32}\frac{L'^7}{G'^7}\left(1-2\frac{N^2}{G'^2}-7\frac{N^4}{G'^4}\right)\Big]+\frac{\mu^6C_2^2}{L'^{10}}\left[-\frac{3}{16}\left(\frac{L'^5}{G'^5}-\frac{L'^7}{G'^7}\right)\right.$$

$$\left.\times\left(1-16\frac{N^2}{G'^2}+15\frac{N^4}{G'^4}\right)\right]\cos 2g' \qquad (5.5)$$

这里g已换成了g'，前面有因子C_2^2，到此为止求出的新哈密顿量为

$$H^*=H_0^*(L')+H_1^*(L',\ G',\ N')+H_2^*(L',\ G',\ N',\ g)$$

$$=-\frac{\mu^2}{2L'^2}+\frac{\mu^4C_2}{L'^3G'^3}\left(-\frac{1}{2}+\frac{3}{2}\ \frac{N^2}{G'^2}\right)+H_2^* \qquad (5.6)$$

引入新的待定母函数S^*

$$S^*=L''l'+G''g'+N''n'+S_1^*(L'',\ G'',\ N'',\ g') \qquad (5.7)$$

应用 $H^*=H^{**}$

有 $$H_0^*+H_1^*+\frac{\partial H_1^*}{\partial G''}\ \frac{\partial S_1^*}{\partial g'}+H_{2S}^*+H_{2P}^*=H_0^{**}+H_1^{**}+H_2^{**}$$

得

$$H_0^*=H_0^{**},\ \ H_1^*=H_1^{**},\ \ \frac{\partial H_1^*}{\partial G''}\frac{\partial S_1^*}{\partial g'}+H_{2P}^*=0,\ \ H_{2S}^*=H_2^{**}$$

由于

$$\frac{\partial H_1^*}{\partial G''}=\frac{3}{2}\ \frac{\mu^4C_2}{L'^3G''^4}\left(1-5\frac{N^2}{G''^2}\right)$$

则

$$\frac{\partial S_1^*}{\partial g'}=G''\lambda_2\left[\frac{1}{8}\left(\frac{L'^2}{G''^2}-\frac{L'^4}{G''^4}\right)\left(1-16\frac{N^2}{G''^2}+15\frac{N^4}{G''^4}\right)\right]\left(1-5\frac{N^2}{G''^2}\right)^{-1}\cos 2g' \qquad (5.8)$$

代入 $G'=G''+\partial S_1^*/\partial g'$

有

$$G'=G''\left\{1+\lambda_2\left(\frac{L'^2}{G''^2}-\frac{L'^4}{G''^4}\right)\left[\frac{1}{8}\left(1-11\frac{N^2}{G''^2}\right)-5\frac{N^4}{G''^4}\left(1-5\frac{N^2}{G''^2}\right)^{-1}\right]\cos 2g''\right\} \qquad (5.9)$$

所以

$$S_1^*=G''\lambda_2\left(\frac{L'^2}{G''^2}-\frac{L'^4}{G''^4}\right)\left[\frac{1}{16}\left(1-11\frac{N^2}{G''^2}\right)-\frac{5}{2}\ \frac{N^4}{G''^4}\left(1-5\frac{N^2}{G''^2}\right)^{-1}\right]\sin 2g'' \qquad (5.10)$$

$$l'=l''-\frac{\partial S^*}{\partial L'}=l''+\lambda_2\ \frac{L'}{G''}\left[\frac{1}{8}\left(1-11\frac{N^2}{G''^2}\right)-5\frac{N^4}{G''^4}\left(1-5\frac{N^2}{G''^2}\right)^{-1}\right]\sin 2g'' \qquad (5.11)$$

$$g'=g''-\frac{\partial S^*}{\partial G''}=g''+\lambda_2\left[\frac{1}{16}\ \frac{L'^2}{G''^2}\left(1-33\frac{N^2}{G''^2}\right)-\frac{3}{16}\ \frac{L'^4}{G''^4}\left(1-\frac{55}{3}\ \frac{N^2}{G''^2}\right)\right.$$

$$+\left(-\frac{25}{2}\ \frac{L'^2}{G''^2}+\frac{35}{2}\ \frac{L'^4}{G''^4}\right)\frac{N^4}{G''^4}\left(1-5\frac{N^2}{G''^2}\right)^{-1}$$

$$\left.-25\left(\frac{L'^2}{G''^2}-\frac{L'^4}{G''^4}\right)\frac{N^6}{G''^6}\left(1-5\frac{N^2}{G''^2}\right)^{-2}\right]\sin 2g'' \qquad (5.12)$$

$$n'=n''-\frac{\partial S_1^*}{\partial N}=n''+\lambda_2\left(\frac{L'^2}{G''^2}-\frac{L'^4}{G''^4}\right)\left[\frac{11}{8}\ \frac{N}{G''}+10\frac{N^3}{G''^3}\left(1-5\frac{N^2}{G''^2}\right)\right.$$

$$\left.+25\frac{N^5}{G''^5}\left(1-5\frac{N^2}{G''^2}\right)^{-2}\right]\sin 2g'' \qquad (5.13)$$

这里g'已转换为g''。

哈密顿量仅仅作为L''，G''，N的函数

$$H^{**}=\frac{\mu^2}{2L'^2}+\frac{\mu^4C_2}{L'^3G''^3}\left(-\frac{1}{2}+\frac{3}{2}\frac{N^2}{G''^2}\right)+H_2^{**} \tag{5.14}$$

定义 $h_0=\mu^2/L'^3$

则有

$$\frac{dl''}{dt}=-\frac{\partial H^{**}}{\partial L'}=h_0\left\{1+3\lambda_2\frac{L'^3}{G''^3}\left(-\frac{1}{2}+\frac{3}{2}+\frac{N^2}{G''^2}\right)+\lambda_2^2\left[\frac{75}{32}\frac{L'^5}{G''^5}+\frac{3}{2}\frac{L'^6}{G''^6}\right.\right.$$
$$-\frac{45}{32}\frac{L'^7}{G''^7}+\left(-\frac{135}{16}\frac{L'^5}{G''^5}-9\frac{L'^6}{G''^6}+\frac{45}{16}\frac{L'^7}{G''^7}\right)\frac{N^2}{G''^2}$$
$$\left.\left.+\left(\frac{75}{32}\frac{L'^5}{G''^5}+\frac{27}{2}\frac{L'^6}{G''^6}+\frac{315}{32}\frac{L'^7}{G''^7}\right)\frac{N^4}{G''^4}\right]\right\} \tag{5.15}$$

$$\frac{dg''}{dt}=-\frac{\partial H^{**}}{\partial G''}=h_0\left\{3\lambda_2\frac{L'^4}{G''^4}\left(-\frac{1}{2}+\frac{5}{2}+\frac{N^2}{G''^2}\right)+\lambda_2^2\left[\frac{75}{32}\frac{L'^6}{G''^6}\right.\right.$$
$$+\frac{9}{4}\frac{L'^7}{G''^7}-\frac{105}{32}\frac{L'^8}{G''^8}+\left(-\frac{189}{16}\frac{L'^6}{G''^6}-18\frac{L'^7}{G''^7}+\frac{135}{16}\frac{L'^8}{G''^8}\right)\frac{N^2}{G''^2}$$
$$\left.\left.+\left(\frac{135}{32}\frac{L'^6}{G''^6}+\frac{135}{4}\frac{L'^7}{G''^7}+\frac{1155}{32}\frac{L'^8}{G''^8}\right)\frac{N^4}{G''^4}\right]\right\} \tag{5.16}$$

$$\frac{dn''}{dt}=-\frac{\partial H^{**}}{\partial N}=h_0\left\{-3\lambda_2\frac{L'^4}{G''^4}\frac{N}{G''}+\lambda_2^2\left[\left(\frac{27}{8}\frac{L'^6}{G''^6}+\frac{9}{2}\frac{L'^7}{G''^7}-\frac{15}{8}\frac{L'^8}{G''^8}\right)\frac{N}{G''}\right.\right.$$
$$\left.\left.+\left(-\frac{15}{8}\frac{L'^6}{G''^6}-\frac{27}{2}\frac{L'^7}{G''^7}-\frac{105}{8}\frac{L'^8}{G''^8}\right)\frac{N^3}{G''^3}\right]\right\} \tag{5.17}$$

以上推导给出了6个根数的一、二阶长期摄动．这与用平根数迭代法导出的结果相一致．

参 考 文 献

[1] W. H. Goodyear, Completely general closed form solution for coordinates and partial derivatives of two-body problem, *The Astronomical Journal*, 70(3) (1965), 189—193.

[2] R. H. Gooding, Orbit determination of a man-made earth satellite, *Royal Aircraft Establishment TR*, 74164 (1974), 125—145.

A Calculation Method to the Perturbation of a Satellite Caused by the Gravitation Field of the Earth

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Abstract

In this paper, with the application of the Delauney variables, according to the Hamilton equations, the influence on the perturbation of a satellite exerted by the gravitational force of the earth through canonical transformation has been found out. As a result, the equation about how the position and velocity of the satellite vary with time is deduced.

Key words gravitation field, Hamilton equation, perturbation, canonical transformation