应用数学和力学, 第 25 卷 第 4 期(2004 年 4 月)
Applied Mathematics and Mechanics

应用数学和力学编委会编
重 庆 出 版 社 出 版

文章编号: 1000_0887(2004) 04_0433_08

一类可再生资源系统的最优动态平衡收获*

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(我刊原编委林宗池推荐)

摘要: 研究一类可再生资源系统的最优利用问题· 首先, 引进一个新的效用函数, 它依赖于收获努力度和资源量, 由此导出最优控制问题· 其次证明该控制问题最优解的存在性· 然后, 利用无穷区间上控制问题的最大值原理, 得到一个非线性的四维最优系统· 通过对上述系统正平衡解的详细分析, 借助 Hopf 分支定理证明了极限环的存在性· 之后考虑中心流形上的简化系统, 分析极限环的稳定性· 最后, 解释所得结果的生物经济学意义·

关 键 词: 可再生资源; 最优收获; 稳定性; 分支; 极限环
中图分类号: O175.1 文献标识码: A

引 言

利用数学模型研究可再生资源的最优收获问题, 已有不少文献[1~9]· 常见的研究策略可分为三类: 最优静态均衡收获[3], 最优反馈收获[2,4], 最优动态平衡收获[6~9]· 每种策略都有其优缺点· 遗憾的是, 无论采用何种策略, 寻求最优控制率都是一项极其困难的任务·

本文结构如下: 在第 1 节里, 通过拓展文献[8]和[9]中的效用函数, 建立一个新的资源管理模型, 并讨论最优解的存在性· 接下来在第 2 节和第 3 节中研究最优系统的极限环的存在性和稳定性· 最后, 得出结论并解释所得结果的生物经济学意义·

1 模型及最优解的存在性

考虑控制问题

$$\max_u \int_0^\infty \left[\frac{m}{2}x^2 + \alpha y - \frac{1}{2}eu^2\right] e^{-\delta t}\,dt, \tag{1}$$

$$\dot{x} = x(1 - x) - y,\ x(0) = x_0, \qquad 0 < x_0 < 1, \tag{2}$$

$$\dot{y} = u(t),\ y(0) = y_0 \geqslant 0, \qquad u(t) \in (-c, c),\ 0 < c < \infty, \tag{3}$$

$$x(t) \geqslant 0,\ y(t) \geqslant 0, \tag{4}$$

* 收稿日期: 2002_05_03; 修订日期: 2003_09_09
基金项目: 国家自然科学基金资助项目(19971066)
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其中 $x(t)$ 表示 t 时刻的资源量, $y(t)$ 为人对资源的收获率, $u(t)$ 是收获调节函数. 瞬时效用函数为 $(1/2)x^2 + ay$, $(1/2)eu^2$ 代表调节代价. δ 为贴现率, $0 < \delta < 1$. m, e, a 均为正常数.

本文假定: $m < 2a\delta$.

命题 1　控制问题(1)~(4)至少存在一个解.

证明　我们应用文献[10]中的定理 6.10.

显然, 控制集 $U = [-c, c]$ 是紧的. 其次, 对每一个容许控制 $u(t)$, 目标泛函于上有界. 这是因为: 由方程(3)可知 $y(t) < ct + y_0$, 从而

$$\int_0^{\infty}\left[\frac{m}{2}x^2 + ay - \frac{1}{2}eu^2\right]\mathrm{e}^{-\delta t}\mathrm{d}t <$$
$$\int_0^{\infty}\left(\frac{m}{2} + act + ay_0\right)\mathrm{e}^{-\delta t}\mathrm{d}t =$$
$$\frac{ay_0 + \dfrac{m}{2}}{\delta} + \frac{ac}{\delta^2}.$$

第三, 集

$$N(x, y, U, t) = \left\{(n_1, n_2, n_3) = \left(\left[\frac{m}{2}x^2 + ay - \frac{1}{2}eu^2\right]e^{-\delta t} + \theta,\ x(1-x) - y,\ u\right): u \in U,\ \theta \leqslant 0\right\}$$

的凸性可由函数

$$f := \left[\frac{m}{2}x^2 + ay - \frac{1}{2}eu^2\right]\mathrm{e}^{-\delta t}$$

关于 u 的凸性导出.

第四, 下列不等式成立:

$$[|x(1-x) - y|^2 + u^2]^{1/2} \leqslant [(|x| + |y|)^2 + u^2]^{1/2} \leqslant$$
$$[2(|x|^2 + |y|^2) + c^2]^{1/2} \leqslant$$
$$\sqrt{2}(x^2 + y^2)^{1/2} + c.$$

最后, 上述定理的其它条件显然满足, 因为此处不存在混合约束条件. 定理证毕.

2　最优性条件和极限环的存在性

上列控制问题的现值拉格朗日函数为

$$L = \frac{m}{2}x^2 + ay - \frac{1}{2}eu^2 + \lambda_1 x(1-x) - \lambda_1 y + \lambda_2 u + \mu_1 x + \mu_2 y.$$

利用文[11]中的最大值原理, 可得

$$\frac{\partial L}{\partial u} = -eu + \lambda_2 = 0,\quad \mu_1 x = 0,\quad \mu_2 y = 0. \tag{5}$$

结合(3)和(5), 导出

$$\dot y = \lambda_2 / e. \tag{6}$$

协态变量方程组为

$$\dot\lambda_1 = \delta\lambda_1 - \frac{\partial L}{\partial x} = (\delta - 1)\lambda_1 - mx + 2x\lambda_1 - \mu_1, \tag{7}$$

$$\dot\lambda_2 = \delta\lambda_2 - \frac{\partial L}{\partial y} = \delta\lambda_2 + \lambda_1 - a - \mu_2. \tag{8}$$

如果系统(1)及(6)~(8)有平衡态$(x, y, \lambda_1, \lambda_2)$, $x > 0$, $y > 0$, 则方程(5)告诉我们

$$\mu_1 = \mu_2 = 0. \tag{9}$$

容易求得平衡态为$E(x, y, \lambda_1, \lambda_2)$, 其中

$$0 < x = a(1-\delta)/(2a-m) < 1, y = x(1-x) > 0, \lambda_1 = a, \quad \lambda_2 = 0.$$

下面,我们的分析将局限于平衡态的邻域,因此,可假设方程(9)满足.

系统(1)及(6)~(8)在平衡态E处的Jacobi阵为

$$\boldsymbol{J} = \begin{pmatrix} \frac{2a\delta - m}{2a-m} & -1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{e} \\ 2a-m & 0 & \frac{m(1-\delta)}{2a-m} & 0 \\ 0 & 0 & 1 & \delta \end{pmatrix}, \tag{10}$$

不难导出$\boldsymbol{J}$的特征方程为

$$s^4 - 2\delta s^3 + \left[\delta^2 + \frac{m(1-\delta)(2a\delta - m)}{(2a-m)^2}\right] s^2 - \frac{m\delta(1-\delta)(2a\delta-m)}{(2a-m)^2} s + \frac{2a-m}{e} = 0. \tag{11}$$

方程(11)的所有特征根是(见文[8]定理1)

$$s = \frac{\delta}{2} \pm \sqrt{\left(\frac{\delta}{2}\right)^2 - \frac{K}{2} \pm \frac{1}{2}\sqrt{K^2 - 4|\boldsymbol{J}|}}, \tag{12}$$

其中

$$K = \frac{m(1-\delta)(2a\delta-m)}{(2a-m)^2} > 0, \ |\boldsymbol{J}| = \det(\boldsymbol{J}) = \frac{2a-m}{e} > 0. \tag{13}$$

为了求得s的实部表达式, 令$s = p + q\mathrm{i}$. 由(12)可得

$$\left(p + q\mathrm{i} - \frac{\delta}{2}\right)^2 = \left(\frac{\delta}{2}\right)^2 - \frac{K}{2} \pm \frac{1}{2}\sqrt{K^2 - 4|\boldsymbol{J}|}.$$

比较上述方程中实部和虚部系数, 我们得到

$$p^2 - q^2 - p\delta = -\frac{K}{2}, \tag{14}$$

$$-q^2(2p-\delta)^2 = \frac{1}{4}(K^2 - 4|\boldsymbol{J}|). \tag{15}$$

利用(14)和(15)消去q^2项,得出

$$p^4 - 2\delta p^3 + \frac{5\delta^2 + 2K}{4}p^2 - \frac{\delta^3 + 2\delta K}{4} + \frac{1}{16}(K^2 - 4|\boldsymbol{J}| + 2\delta^2 K) = 0. \tag{16}$$

方程(16)的根为(见[8])

$$p = \frac{\delta}{2} \pm \frac{1}{4}\sqrt{2\delta^2 - 4K \pm 2\sqrt{\delta^4 - 4\delta^2 K + 16|\boldsymbol{J}|}}. \tag{17}$$

当且仅当

$$K^2 + 2\delta^2 K - 4|\boldsymbol{J}| = 0, \tag{18}$$

方程(11)有唯一一对纯虚根

$$\pm \omega\mathrm{i}, \quad \omega = \sqrt{K/2}, \tag{19}$$

$\boldsymbol{J}$的其它特征根为$\delta \pm \omega\mathrm{i}$.

如果方程(18)满足,则

$$p = \frac{\delta}{2} - \frac{1}{4}\sqrt{2\delta^2 - 4K + 2\sqrt{\delta^4 - 4\delta^2 K + 16|\boldsymbol{J}|}} = 0.$$

然而，由方程(13)知

$$\frac{\mathrm{d}p}{\mathrm{d}e} = -\frac{2(\mathrm{d}|\boldsymbol{J}|/\mathrm{d}e)}{\sqrt{2\delta^2 - 4K + 2\sqrt{\delta^4 - 4\delta^2 K + 16|\boldsymbol{J}|}}\sqrt{\delta^4 - 4\delta^2 K + 16|\boldsymbol{J}|}} > 0.$$

据Hopf分支定理，我们断言系统(1)及(6)～(8)当 $0 < e - e_0 < < 1$ 时存在极限环，这里 e_0 可由(18)求出得

$$e_0 = \frac{4(2a - m)^5}{m(1-\delta)(2a\delta - m)[m(1-\delta)(2a\delta - m) + 2\delta^2(2a - m)^2]}. \tag{20}$$

3 极限环的稳定性

$\boldsymbol{J}$ 相应于特征值 $\pm\omega\mathrm{i}$ 的特征向量为

$$\beta_{1,2} = \begin{pmatrix} 1 \\ v_1 \mp \mathrm{i}\omega \\ e\omega[-\delta\omega - v_1\omega \pm \mathrm{i}(\omega^2 - \delta v_1)] \\ e\omega(\omega \pm \mathrm{i}v_1) \end{pmatrix}, \quad v_1 = \frac{2a\delta - m}{2a - m}. \tag{21}$$

相应于 $\delta \pm \omega\mathrm{i}$ 的特征向量为

$$\beta_{3,4} = \begin{pmatrix} 1 \\ v_1 - \delta \mp \mathrm{i}\omega \\ e\omega[2\delta\omega - v_1\omega \pm \mathrm{i}(\omega^2 + \delta v_1 - \delta^2)] \\ e[\delta v_1 - \delta^2 + \omega^2 \pm \mathrm{i}(\omega v_1 - 2\delta\omega)] \end{pmatrix}.$$

令

$$r_1 = x - x,\ r_2 = y - y,\ r_3 = \lambda_1 - \lambda_1,\ r_4 = \lambda_2 - \lambda_2,$$

系统(1)及(6)～(8)化为

$$\dot{\boldsymbol{r}} = \boldsymbol{J}\boldsymbol{r} + \boldsymbol{\phi}(\boldsymbol{r}), \tag{22}$$

其中 $\boldsymbol{r} = (r_1, r_2, r_3, r_4)^{\mathrm{T}}, \quad \boldsymbol{\phi}(\boldsymbol{r}) = (r_1^2, 0, 2r_1 r_3, 0)^{\mathrm{T}}.$

令 $\boldsymbol{B} := (\beta_1, \beta_2, \beta_3, \beta_4)$, $\boldsymbol{D} := \mathrm{diag}(\mathrm{i}\omega, -\mathrm{i}\omega, \delta + \mathrm{i}\omega, \delta - \mathrm{i}\omega)$. 考虑变换

$$\boldsymbol{B}^{-1}:\ R^4 \to \{\boldsymbol{z} = (z_1, \bar{z}_1, z_2, \bar{z}_2) : z_i \in \mathbf{C}\},$$

$$\boldsymbol{r} \to \boldsymbol{z} = \boldsymbol{B}^{-1}\boldsymbol{r},$$

其中 $\bar{z}_i$ 表示 z_i 的复共轭. 则方程(21)变为

$$\dot{\boldsymbol{z}} = \boldsymbol{B}^{-1}\dot{\boldsymbol{r}} = \boldsymbol{B}^{-1}\boldsymbol{J}\boldsymbol{r} + \boldsymbol{B}^{-1}\boldsymbol{\phi}(\boldsymbol{r}) = (\boldsymbol{B}^{-1}\boldsymbol{J}\boldsymbol{B})\boldsymbol{r} + \boldsymbol{B}^{-1}\boldsymbol{\phi}(\boldsymbol{B}\boldsymbol{z}) =$$
$$\boldsymbol{D}\boldsymbol{z} + \boldsymbol{B}^{-1}\boldsymbol{\phi}(\boldsymbol{B}\boldsymbol{z}). \tag{23}$$

记矩阵 $\boldsymbol{B}$ 的逆为

$$\boldsymbol{B}^{-1} = |\boldsymbol{B}|^{-1}\begin{pmatrix} B_{11} & \cdots & B_{31} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ B_{13} & \cdots & B_{331} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix},$$

其中 B_{ij} 表示元素 b_{ij} 在 $\boldsymbol{B}$ 中的代数余子式.

不难得到

$$\boldsymbol{B}\boldsymbol{z} = (F_1, F_2, F_3, F_4)^{\mathrm{T}}, \quad F_1 = z_1 + \bar{z}_1 + z_2 + \bar{z}_2,$$

$$F_3 = e\omega[-\delta\omega - v_1\omega + \mathrm{i}(\omega^2 - v_1\delta)]z_1 + e\omega[-\delta\omega - v_1\omega - \mathrm{i}(\omega^2 - v_1\delta)]z_1 + e\omega[2\omega\delta - \omega v_1 + \mathrm{i}(\delta v_1 - \delta^2 + \omega^2)]z_2 + e\omega[2\omega\delta - \omega v_1 - \mathrm{i}(\delta v_1 - \delta^2 + \omega^2)]z_2\cdot$$

再由(22)可知

$$\dot{z}_1 = \mathrm{i}\omega z_1 + |\boldsymbol{B}|^{-1}(-B_{11}F_1^2 + 2B_{13}F_1F_3) := \mathrm{i}\omega z_1 + |\boldsymbol{B}|^{-1}(f_1z_1^2 + f_2z_1z_1 + f_3z_1z_2 + f_4z_1z_2 + f_5z_1z_2 + f_6z_1z_2 + \cdots), \tag{24}$$

$$\dot{z}_2 = (\delta + \mathrm{i}\omega)z_2 + |\boldsymbol{B}|^{-1}(-B_{13}F_1^2 + 2B_{33}F_1F_3) + \cdots := (\delta + \mathrm{i}\omega)z_2 + |\boldsymbol{B}|^{-1}(g_1z_1^2 + g_2z_1z_1 + g_3z_1^2) + \cdots, \tag{25}$$

其中

$$f_1 = -B_{11} + 2B_{31}e\omega[-\delta\omega - v_1\omega + \mathrm{i}(\omega^2 - v_1\delta)], \tag{26}$$

$$f_2 = -2B_{11} + 4B_{31}e\omega[-\delta\omega - v_1\omega], \tag{27}$$

$$f_3 = -2B_{11} + 2B_{31}e\omega[\delta\omega - 2v_1\omega + \mathrm{i}(2\omega^2 - \delta^2)], \tag{28}$$

$$f_4 = -2B_{11} + 2B_{31}e\omega[\delta\omega - 2v_1\omega + \mathrm{i}(-2\delta v_1 + \delta^2)], \tag{29}$$

$$f_5 = -2B_{11} + 2B_{31}e\omega[\delta\omega - 2v_1\omega + \mathrm{i}(2\delta v_1 - \delta^2)], \tag{30}$$

$$f_6 = -2B_{11} + 2B_{31}e\omega[\delta\omega - 2v_1\omega + \mathrm{i}(-2\omega^2 + \delta^2)], \tag{31}$$

$$g_1 = -B_{13} + 2B_{33}e\omega[-\delta\omega - v_1\omega + \mathrm{i}(\omega^2 - v_1\delta)], \tag{32}$$

$$g_2 = -2B_{13} + 4B_{33}e\omega[-\delta\omega - v_1\omega], \tag{33}$$

$$g_3 = -B_{13} + 2B_{33}e\omega[-\delta\omega - v_1\omega + \mathrm{i}(-\omega^2 + v_1\delta)]\cdot \tag{34}$$

在中心流形上, 令

$$z_2 = h(z_1, z_1) = h_1z_1^2 + h_2z_1z_1 + h_3z_1^2 + \cdots, \tag{35}$$

$$\dot{z}_2 = \frac{\partial h}{\partial z_1}\dot{z}_1 + \frac{\partial h}{\partial z_1}\dot{z}_1\cdot \tag{36}$$

结合(23)~(24)与(34)~(35), 我们得到

$$(\delta + \mathrm{i}\omega)(h_1z_1^2 + h_2z_1z_1 + h_3z_1^2) + g_1z_1^2 + g_2z_1z_1 + g_3z_1^2 + \cdots = (2h_1z_1 + h_2z_1)(\mathrm{i}z_1\omega + \cdots) + (h_2z_1 + 2h_3z_1)(-\mathrm{i}\omega z_1 + \cdots)\cdot$$

比较 z_1、z_1 的系数, 并利用(31)~(33)推出

$$h_1 = -\frac{g_1(\delta + \mathrm{i}\omega)}{\delta^2 + \omega^2}, \tag{37}$$

$$h_2 = -\frac{g_2(\delta - \mathrm{i}\omega)}{\delta^2 + \omega^2}, \tag{38}$$

$$h_3 = -\frac{g_3(\delta - 3\mathrm{i}\omega)}{\delta^2 + 9\omega^2}\cdot \tag{39}$$

当 $H := \mathrm{Re}(\mathrm{i}f_1f_2/\omega + f_3h_2 + f_4h_2 + f_5h_1 + f_6h_3) < 0$ 时, 极限环稳定•

结合等式(26)~(31)、(37)~(39)、(13)、(19)~(21), 并使用 Mapple7, 我们得出如下表达式

$$H = -4\,096(-2a + m)^{12}\delta^3(4a^2\delta^2 + 4\delta ma - 8ma\delta^2 - 2m^2 + 2\delta m^2 + \delta^2m^2) \times (2m^5\delta^5 - 74m^4\delta^5 + 344m^3a^2\delta^5 + 208m^2a^3\delta^5 - 400ma^4\delta^5 + 64a^5\delta^5 + 27m^5\delta^4 - 48m^4a\delta^4 - 134m^3a^2\delta^4 + 384m^2a^3\delta^4 + 240ma^4\delta^4 - 42m^5\delta^3 + 744m^4a\delta^3 + 1\,056m^3a^2\delta^3 - 432m^2a^3\delta^3 - 82m^5\delta^2 - 848m^4a\delta^2 -$$

$$136m^3a^2\delta^2+147m^5\delta+246m^4a\delta-54m^5)/[m^2(2a\delta-m)^2(\delta-1)^2\times(-4a\delta+m+m\delta)^4(-2a\delta-m+2m\delta)^5(8a^2\delta^2-26ma\delta^2+2m^2\delta^2+18ma\delta-9m^2+9m^2\delta)]\cdot$$

令 $m=a\theta\delta, 0<\theta<2$, 则上述等式变为

$$H=-4\,096a^4(\theta\delta-2)^{12}(4+4\theta-8\theta\delta-2\theta^2+2\delta\theta^2+\theta^2\delta^2)\times(2\theta^5\delta^5-74\theta^4\delta^4+344\theta^3\delta^3+208\theta^2\delta^2-400\theta\delta+64+27\theta^5\delta^4-48\theta^4\delta^3-1\,344\theta^3\delta^2+384\delta\theta^2+240\theta-42\theta^5\delta^3+744\theta^4\delta^2+1\,056\theta^3\delta-432\theta^2-82\theta^5\delta^2-848\theta^4\delta-136\theta^3+147\theta^5\delta+246\theta^4-54\theta^5)/[\delta^5\theta^2(\theta-2)^2(\delta-1)^2(\theta+\theta\delta-4)^4(2\theta\delta-\theta-2)^5\times(8-26\theta\delta+2\theta^2\delta^2+18\theta-9\theta^2+9\delta\theta^2)]:=\left\{-4\,096a^4(\theta\delta-2)^{12}/[\delta^5\theta^2(\theta-2)^2(\delta-1)^2(\theta-4+\theta\delta)^4\times(2\theta\delta-2-\theta)^5]\right\}F\cdot$$

显然, 当 $0<\delta<1/2$ 时, $\mathrm{sgn}(H)=\mathrm{sgn}(F)\cdot$

如果 $0<\theta<2,\ 0<\delta<1/2$, 则易知

$$R:=8-26\theta\delta+2\theta^2\delta^2+18\theta-9\theta^2+9\theta^2\delta>0,$$

$$P_1:=4+4\theta-8\theta\delta-2\theta^2+2\delta\theta^2+\theta^2\delta^2>0\cdot$$

因此, $\mathrm{sgn}(H)=\mathrm{sgn}(P_2)$, 其中

$$P_2:=2\theta^5\delta^5-74\theta^4\delta^4+344\theta^3\delta^3+208\theta^2\delta^2-400\theta\delta+64+27\theta^5\delta^4-48\theta^4\delta^3-1\,344\theta^3\delta^2+384\delta\theta^2+240\theta-42\theta^5\delta^3+744\theta^4\delta^2+1\,056\theta^3\delta-432\theta^2-82\theta^5\delta^2-848\theta^4\delta-136\theta^3+147\theta^5\delta+246\theta^4-54\theta^5\cdot$$

然而, 很难再对 P_2 的符号作理论分析· 为此, 我们在区域 $\{0<\theta<2, 0<\delta<0.01\}$ 和 $\{0.8<\theta<2, 0<\delta<0.01\}$ 上作出 P_2 的图形(见图 1 和图 2)· 从图上可以清楚地看到, 当 $0.8<\theta<2, 0<\delta<0.01$ 时, $P_2<0$, 即 $H<0\cdot$

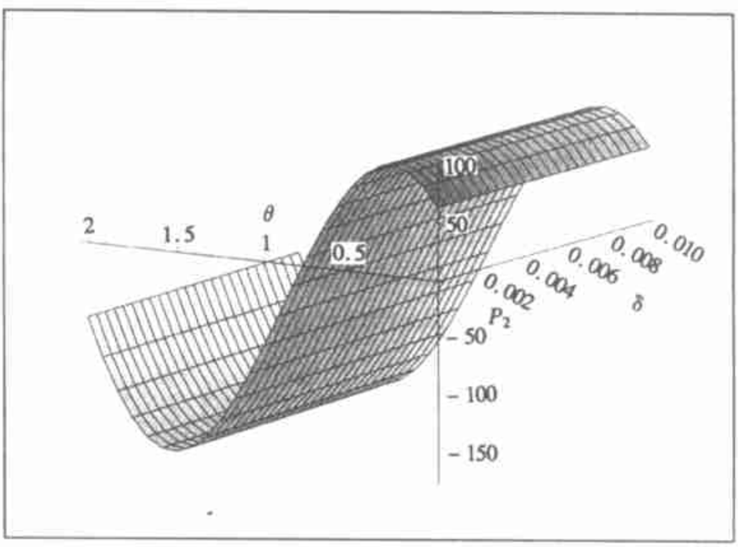

图 1　P_2 的值分布

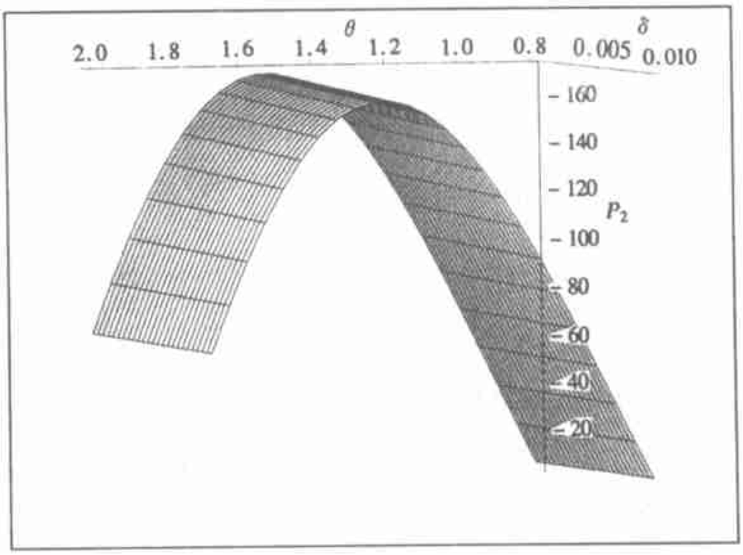

图2　P_2的符号

4　结论及其生物经济学意义

综合第2节和第3节的结果,得到

命题2　当$0<\delta<0.01, 0.8a\delta<m<2a\delta, 0<e-e_0<<1$时,最优控制问题(1)~(4)存在稳定的极限环•

命题2意味着:在适当的条件下,最优收获率和资源量都随时间周期性变化•　这样的收获策略保证资源量处于稳定的动态平衡,亦即这种平衡不会被小扰动所破坏,同时,人们的经济利益得以最大化,并且该最优平衡将持续下去•

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Optimal Dynamical Balance Harvesting for a Class of Renewable Resources System

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Abstract: An optimal utilization problem for a class of renewable resources system is investigated. Firstly, a control problem was proposed by introducing a new utility function which depends on the harvesting effort and the stock of resources. Secondly, the existence of optimal solution for the problem was discussed. Then, using a maximum principle for infinite horizon problem, a nonlinear four_dimensional differential equations system was attained. After a detailed analysis of the unique positive equilibrium solution, the existence of limit cycles for the system is demonstrated. Next a reduced system on the central manifold is carefully derived, which assures the stability of limit cycles. Finally significance of the results in bioeconomics is explained.

Key words: renewable resource; optimal harvesting; stability; bifurcation; limit cycle