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# 圆柱形容器中竖直激励表面波的毛细影响<sup>\*</sup>

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(戴世强推荐)

摘要: 在竖直振动的圆柱形容器中, 利用理想流体中两时间尺度奇摄动展开法, 研究了包括表面张力影响的自由面单一表面驻波的运动。通过求解势流方程, 获得了一个包含三阶非线性项、外激励及表面张力影响的非线性振幅方程。结果表明当驱动频率较低时, 表面张力对表面波模式选择不重要; 然而, 当驱动频率较高时, 表面张力的影响是不可忽略的。说明表面张力具有使得自由面返回到平衡位置的作用。另外, 由于考虑了表面张力的影响, 使得理论结果比无表面张力时更加接近先前的实验结果。

关键词: 垂直强迫振动; 非线性振幅方程; 表面张力; 表面波模式

中图分类号: O353.2 文献标识码: A

## 引 言

自上世纪 80 年代以来, 人们对垂直激励表面波的初始不稳定性<sup>[1]</sup>、二次不稳定性和向时空混沌的转变<sup>[2,3]</sup>、以及湍流方面<sup>[4]</sup>进行了大量的研究。鄂学全等人<sup>[5~7]</sup>在垂直振动的圆柱形容器中进行了表面波流谱流动显示和实验研究, 获得了非常精美的表面波流谱的实验照片, 且大部分的模式先前从未报道。最近, 针对文献<sup>[5~7]</sup>的实验结果, 菅永军等人<sup>[8~10]</sup>在理想流体中对文献<sup>[5~7]</sup>的实验结果进行了理论分析。他们利用两时间尺度的奇异摄动展开法, 获得了没有考虑表面张力时的非线性表面波振幅方程和二阶自由面的解析表达式。尽管理论等值线的形状上与实验结果非常吻合, 但驱动频率与实验结果差别较大。

本文从理想流体的欧拉方程出发, 考虑了表面张力的影响, 利用奇异摄动法获得了类似于文献<sup>[8]</sup>的非线性振幅方程。这两个振幅方程的差别在于后者的振幅方程的系数包括了表面张力的影响。最后对有表面张力和无表面张力时的理论计算结果进行了比较, 使得理论计算结果比无表面张力时更接近实验测量结果。

## 1 控制方程及无量纲化

考虑圆柱形容器中有表面张力时的垂直激励表面波问题。物理模型为文献<sup>[8]</sup>中图 1 所

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示, 参数的意义与文献[8]中的完全一致。假设流体是理想、不可压缩, 且运动是无旋的, 则存在速度势函数  $\phi(r, \theta, z, t)$ , 满足如下的控制方程:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \quad 0 < r < R, -h \leq z \leq \eta(r, \theta, t) \quad (1)$$

在自由面  $z = \eta(r, \theta, t)$  上, 动力学边界条件和运动学边界条件分别满足:

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial \phi}{\partial \theta} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 \right] + (g - \ddot{z}_0) \eta - \frac{\Gamma}{\rho} \left( \frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r} \frac{\partial \eta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \eta}{\partial \theta^2} \right) = 0, \quad (2)$$

$$\frac{\partial \eta}{\partial t} + \frac{1}{r^2} \frac{\partial \phi}{\partial \theta} \frac{\partial \eta}{\partial \theta} + \frac{\partial \phi}{\partial r} \frac{\partial \eta}{\partial r} - \frac{\partial \phi}{\partial z} = 0, \quad (3)$$

其中常数  $\Gamma$  和  $\rho$  分别为表面张力系数和流体的密度, 表面张力的影响由自由面边界条件引入。另外, 由于忽略了粘性效应, 在刚性容器的侧壁和底部满足法向速度为零, 即:

$$\partial \phi / \partial r = 0, \quad r = R, \quad (4)$$

$$\partial \phi / \partial z = 0, \quad z = -h \quad (5)$$

以容器半径  $R$  作为长度尺度, 将相关的独立变量和未知函数进行无量纲化, 令:

$$\begin{cases} z^* = z/R, r^* = r/R, \eta^* = \eta/R, t^* = t/\sqrt{R/g}, \phi^* = \phi/(R\sqrt{gR}), \\ \omega_0^* = \omega_0/\sqrt{g/R}, \varepsilon^{*2} = 4A\omega_0^2/g, A^* = A/R, \Gamma^*/\rho^* = (gR^2)\Gamma/\rho, \end{cases} \quad (6)$$

上式中的星号表示无量纲变量, 并在下面的推导过程中将其省略。代式(6)到式(1)~(5), 将式(2)和式(3)在平衡位置  $z = 0$  展开为 Taylor 级数。若忽略 4 阶小量  $O(\varepsilon^4)$ , 可得如下的控制方程:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0; \quad (7)$$

自由面  $z = 0$  的边界条件为

$$\begin{aligned} \frac{\partial \phi}{\partial t} + \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial \phi}{\partial \theta} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 \right] + [1 + \varepsilon^2 \cos(2\omega_0 t)] \eta - \\ \frac{\Gamma}{\rho} \left( \frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r} \frac{\partial \eta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \eta}{\partial \theta^2} \right) + \frac{\partial^2 \phi}{\partial t \partial z} \eta + \\ \left( \frac{\partial \phi}{\partial r} \frac{\partial^2 \phi}{\partial r \partial z} + \frac{1}{r^2} \frac{\partial \phi}{\partial \theta} \frac{\partial^2 \phi}{\partial \theta \partial z} + \frac{\partial \phi}{\partial z} \frac{\partial^2 \phi}{\partial z^2} \right) \eta + \frac{1}{2} \frac{\partial^3 \phi}{\partial t \partial z^2} \eta^2 = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial \eta}{\partial t} - \frac{\partial \phi}{\partial z} + \frac{\partial \phi}{\partial r} \frac{\partial \eta}{\partial r} + \frac{1}{r^2} \frac{\partial \phi}{\partial \theta} \frac{\partial \eta}{\partial \theta} - \frac{\partial^2 \phi}{\partial z^2} \eta + \frac{\partial^2 \phi}{\partial r \partial z} \eta \frac{\partial \eta}{\partial r} + \\ \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta \partial z} \eta \frac{\partial \eta}{\partial \theta} - \frac{1}{2} \frac{\partial^3 \phi}{\partial z^3} \eta^2 = 0; \end{aligned} \quad (9)$$

侧壁和底部的边界条件为

$$\partial \phi / \partial r = 0, \quad r = 1, \quad (10)$$

$$\partial \phi / \partial z = 0, \quad z = -h/R \quad (11)$$

对式(7)~(11)进行两时间尺度摄动展开, 引入一个慢变时间尺度  $\tau$ , 并设  $\tau = \varepsilon^2 t$ , 则有:

$$\partial / \partial t = \partial / \partial t + \varepsilon^2 \partial / \partial \tau + \dots; \quad (12)$$

并将速度势  $\phi(r, \theta, z, t)$  和自由面位移  $\eta(r, \theta, t)$  展为如下  $\varepsilon$  的幂级数形式:

$$\begin{cases} \phi(r, \theta, z, t, \tau) = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \dots, \\ \eta(r, \theta, t, \tau) = \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \varepsilon^3 \eta_3 + \dots \end{cases} \quad (13)$$

将式(12)和式(13)代到无量纲方程(7)~(11)中,通过比较方程两边小参数 $\varepsilon^i$ 的系数,可以给出各阶近似方程和相应的定解条件。

## 2 一阶近似解

一阶问题的控制方程和边界条件为:

$$\frac{\partial^2 \phi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_1}{\partial \theta^2} + \frac{\partial^2 \phi_1}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0, \quad (14)$$

$$\frac{\partial \phi_1}{\partial t} + \eta_1 - \frac{\Gamma}{\rho} \left[ \frac{\partial^2 \eta_1}{\partial r^2} + \frac{1}{r} \frac{\partial \eta_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \eta_1}{\partial \theta^2} \right] = 0, \quad z = 0, \quad (15)$$

$$\partial \eta_1 / \partial t - \partial \phi_1 / \partial z = 0, \quad z = 0, \quad (16)$$

$$\partial \phi_1 / \partial r = 0, \quad r = 1, \quad (17)$$

$$\partial \phi_1 / \partial z = 0, \quad z = -h/R. \quad (18)$$

令 $\phi_1 = (\Phi_1(r, z, \tau)e^{i\Omega\tau} + \Phi_1(r, z, \tau)e^{-i\Omega\tau})\cos m\theta$ , 这里 $\Phi_1$ 表示 $\phi_1$ 的复共轭,  $\Omega$ 为表面波的自然频率。将 $\phi_1$ 代到式(14)~(18),合并式(14)、(15),得 $\Phi_1$ 满足的控制方程和边界条件:

$$\frac{\partial^2 \Phi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi_1}{\partial r} - \frac{m^2}{r^2} \Phi_1 + \frac{\partial^2 \Phi_1}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0, \quad (19)$$

$$\frac{\partial \Phi_1}{\partial z} - \Omega^2 \Phi_1 + \frac{\Gamma}{\rho} \frac{\partial^3 \Phi_1}{\partial z^3} = 0, \quad z = 0, \quad (20)$$

$$\partial \Phi_1 / \partial r = 0, \quad r = 1, \quad (21)$$

$$\partial \Phi_1 / \partial z = 0, \quad z = -h/R. \quad (22)$$

令 $\Phi_1 = p(\tau)R(r)Z(z)$ ,代 $\Phi_1$ 到式(19)~(22),利用分离变量法,可得一阶速度势及自由面位移解:

$$\phi_1 = J_m(\lambda r) \cosh[\lambda(z + h/R)] \cdot [p(\tau)e^{i\Omega\tau} + p(\tau)e^{-i\Omega\tau}] \cos m\theta, \quad (23)$$

$$\eta_1 = -\frac{i\lambda}{\Omega} J_m(\lambda r) \sinh\left[\frac{\lambda h}{R}\right] [p(\tau)e^{i\Omega\tau} + p(\tau)e^{-i\Omega\tau}] \cos m\theta. \quad (24)$$

上式出现的函数及符号的含义与文献[8]中的完全一致,并有下列的色散关系:

$$\Omega^2 = \lambda_{mn} \tanh(\lambda_{mn} h/R) (1 + (\Gamma/\rho) \lambda_{mn}^2) = \Omega_{mn}^2. \quad (25)$$

## 3 二阶近似解

二阶问题的控制方程和边界条件为:

$$\frac{\partial^2 \phi_2}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_2}{\partial \theta^2} + \frac{\partial^2 \phi_2}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0, \quad (26)$$

$$\begin{aligned} \frac{\partial \phi_2}{\partial t} + \eta_2 - \frac{\Gamma}{\rho} \left[ \frac{\partial^2 \eta_2}{\partial r^2} + \frac{1}{r} \frac{\partial \eta_2}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \eta_2}{\partial \theta^2} \right] = \\ - \frac{\partial^2 \phi_1}{\partial t \partial z} \eta_1 - \frac{1}{2} \left[ \left( \frac{\partial \phi_1}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial \phi_1}{\partial \theta} \right)^2 + \left( \frac{\partial \phi_1}{\partial z} \right)^2 \right], \quad z = 0, \end{aligned} \quad (27)$$

$$\frac{\partial \eta_2}{\partial t} - \frac{\partial \phi_2}{\partial z} = - \frac{\partial \phi_1}{\partial r} \frac{\partial \eta_1}{\partial r} - \frac{1}{r^2} \frac{\partial \phi_1}{\partial \theta} \frac{\partial \eta_1}{\partial \theta} + \frac{\partial^2 \phi_1}{\partial z^2} \eta_1, \quad z = 0, \quad (28)$$

$$\partial \phi_2 / \partial r = 0, \quad r = 1, \quad (29)$$

$$\partial \phi_2 / \partial z = 0, \quad z = -h/R. \quad (30)$$

合并边界条件(27)和(28),消去 $\eta_2$ ,利用一阶解的表达式(23)和(24),假设二阶速度势解 $\phi_2$ 有如下的形式:

$$\phi_2 = B_{m, G_1}(r, z) \cos 2m\theta (p^2(\tau) e^{2i\Omega\tau} + p^2(\tau) e^{-2i\Omega\tau}), \quad (31)$$

这里  $B_{m, G_1}(r, z)$  为  $r$  和  $z$  的函数。将式(31) 代到式(26) ~ (30) 中, 则函数  $B_{m, G_1}(r, z)$  满足:

$$\frac{\partial^2 B}{\partial r^2} + \frac{1}{r} \frac{\partial B}{\partial r} - \frac{4m^2}{r^2} B + \frac{\partial^2 B}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0, \quad (32)$$

$$\frac{\partial B}{\partial z} - 4\Omega^2 B + \frac{\Gamma}{\rho} \frac{\partial^3 B}{\partial z^3} = G_l(r), \quad z = 0, \quad (33)$$

$$\partial B / \partial r = 0, \quad r = 1, \quad (34)$$

$$\partial B / \partial z = 0, \quad z = -h/R. \quad (35)$$

通过求解方程(32) ~ (35) 可得  $B_{m, G_1}(r, z)$  的表达式, 将其代入式(31), 最后得二阶解:

$$\phi_2(r, \theta, z, t, \tau) = [X_1(r, z) + X_2(r, z) \cos(2m\theta)] (p^2(\tau) e^{2i\Omega\tau} - p^2(\tau) e^{-2i\Omega\tau}), \quad (36)$$

$$\eta_2(r, \theta, t, \tau) = [Y_1(r) + Y_2(r) \cos(2m\theta)] (p^2(\tau) e^{2i\Omega\tau} + p^2(\tau) e^{-2i\Omega\tau}). \quad (37)$$

其中  $X_1(r, z)$ 、 $X_2(r, z)$  和  $Y_1(r)$ 、 $Y_2(r)$  的详细表达式在文献[10] 中给出。

上面求得了二阶速度势和自由面位移解, 但慢变复振幅  $p(\tau)$  仍然不能确定, 故需进一步考虑三阶问题的控制方程和边界条件, 利用三阶问题满足的相容性条件来确定  $p(\tau)$  所满足的振幅方程。

#### 4 三阶近似方程及振幅方程

三阶问题的控制方程和边界条件为:

$$\frac{\partial^2 \phi_3}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_3}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_3}{\partial \theta^2} + \frac{\partial^2 \phi_3}{\partial z^2} = 0, \quad 0 < r < 1, -\frac{h}{R} < z < 0, \quad (38)$$

$$\begin{aligned} \frac{\partial \phi_3}{\partial t} + \eta_3 - \frac{\Gamma}{\rho} \left[ \frac{\partial^2 \eta_3}{\partial r^2} + \frac{1}{r} \frac{\partial \eta_3}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \eta_3}{\partial \theta^2} \right] &= \frac{1}{2} \frac{\partial^3 \phi_1}{\partial z^2 \partial t} \eta_1^2 - \frac{\partial \phi_1}{\partial \tau} - \frac{\partial^2 \phi_2}{\partial t \partial z} \eta_1 - \\ &\frac{\partial^2 \phi_1}{\partial t \partial z} \eta_2 - \frac{\partial \phi_1}{\partial r} \frac{\partial \phi_2}{\partial r} - \frac{\partial \phi_1}{\partial r} \frac{\partial^2 \phi_1}{\partial r \partial z} \eta_1 - \frac{\partial \phi_1}{\partial z} \frac{\partial \phi_2}{\partial z} - \frac{1}{r^2} \left[ \frac{\partial \phi_1}{\partial \theta} \frac{\partial \phi_2}{\partial \theta} + \frac{\partial \phi_1}{\partial \theta} \frac{\partial^2 \phi_1}{\partial \theta \partial z} \eta_1 \right] - \\ &\frac{\partial \phi_1}{\partial z} \frac{\partial^2 \phi_1}{\partial z^2} \eta_1 - \frac{1}{r^2} \left[ \frac{\partial \phi_1}{\partial \theta} \frac{\partial \phi_2}{\partial \theta} + \frac{\partial \phi_1}{\partial \theta} \frac{\partial^2 \phi_1}{\partial \theta \partial z} \eta_1 \right] - \eta_1 \cos(2\omega_0 t), \quad z = 0, \end{aligned} \quad (39)$$

$$\begin{aligned} \frac{\partial \phi_3}{\partial z} - \frac{\partial \eta_3}{\partial t} &= -\frac{\partial^2 \phi_2}{\partial z^2} \eta_1 - \frac{\partial^2 \phi_1}{\partial z^2} \eta_2 - \frac{1}{2} \frac{\partial^3 \phi_1}{\partial z^3} \eta_1^2 + \frac{\partial \eta_1}{\partial \tau} + \frac{\partial \eta_2}{\partial r} \frac{\partial \phi_1}{\partial r} + \\ &\frac{\partial \eta_1}{\partial r} \frac{\partial \phi_2}{\partial r} + \frac{\partial \eta_1}{\partial r} \eta_1 \frac{\partial^2 \phi_1}{\partial r \partial z} + \frac{1}{r^2} \left[ \frac{\partial \eta_1}{\partial \theta} \frac{\partial \phi_2}{\partial \theta} + \frac{\partial \eta_2}{\partial \theta} \frac{\partial \phi_1}{\partial \theta} + \frac{\partial \eta_1}{\partial \theta} \frac{\partial^2 \phi_1}{\partial \theta \partial z} \eta_1 \right], \quad z = 0, \end{aligned} \quad (40)$$

$$\partial \phi_3 / \partial r = 0, \quad r = 1, \quad (41)$$

$$\partial \phi_3 / \partial z = 0, \quad z = -h/R. \quad (42)$$

利用与文献[8] 类似的方法, 经过复杂的数学推导, 得到自由表面波慢变时间振幅方程:

$$dp_1(\tau) / d\tau = M_1 p_2(\tau) [p_1^2(\tau) + p_2^2(\tau)] + M_2 [p_1(\tau) \sin(2\sigma\tau) - p_2(\tau) \cos(2\sigma\tau)], \quad (43)$$

$$dp_2(\tau) / d\tau = -M_1 p_1(\tau) [p_1^2(\tau) + p_2^2(\tau)] - M_2 [p_1(\tau) \cos(2\sigma\tau) + p_2(\tau) \sin(2\sigma\tau)]. \quad (44)$$

上式中的各参数及其详细表达式在文献[10] 中给出。

## 5 计算结果

### 5.1 表面张力对表面波模式选择的影响

为了描述表面张力对自然频率和模式选择的影响,利用色散关系(25),给出了图1所示的在有表面张力和无表面张力的情况下表面波频率随波数的变化规律。其中参数的选择与文献[5~7]一致。从图1中可以看出,无论有表面张力还是无表面张力,表面波的频率随波数的增加而增加。当驱动的频率较低时(如低于10 Hz),表面张力对模式选择的影响可以忽略不计;然而,当驱动频率较高时(如高于10 Hz),表面张力的影响是非常重要的。

文献[8,9]已将在没有考虑表面张力影响的理想流体中所得的结果和实验流谱模式进行了比较。尽管他们得到的理论等高线和实验非常吻合,但竖直驱动频率有较大的差别。如对(8,4)和(9,6)模式来说,理论得到的驱动频率分别为16.71 Hz及19.66 Hz,但实验中的驱动频率分别为50 Hz及52 Hz。考虑到表面张力的影响后,理论得到的驱动频率分别为21.10 Hz及28.67 Hz,使得目前的理论结果更加接近实验测量。

### 5.2 与实验的比较

图2给出了在不同驱动频率下,目前的理论结果和文献[6~8]的实验流谱等值线比较图(其中左图是理论计算结果,右图为实验结果)。在图2中左图的等高线中,实线和虚线分别代表此时的表面波高出或低于静水面。从图中可以看出,考虑了表面张力的影响后,理论结果进一步接近实验结果。然而,理论和实验的驱动频率仍然差别较大。其原因可能是接触线、模式竞争和粘性等的影响。其中弱粘性对模式选择的影响将在另文发表。

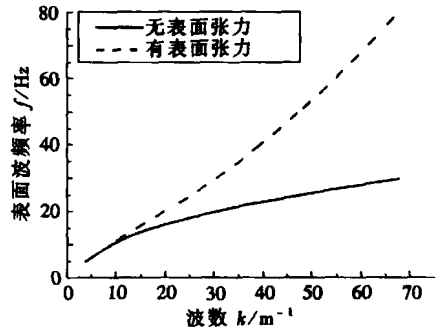
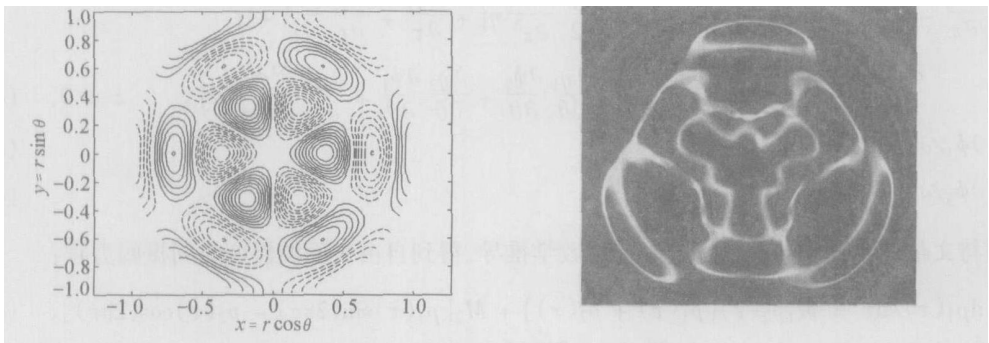


图1 有量纲的表面波频率随波数的变化  
(流体深度  $h = 1.0$  cm, 容器半径  $R = 7.5$  cm, 表面张力系数  $\Gamma = 0.0727$  N/m, 流体密度  $\rho = 10^3$  kg/m<sup>2</sup>)

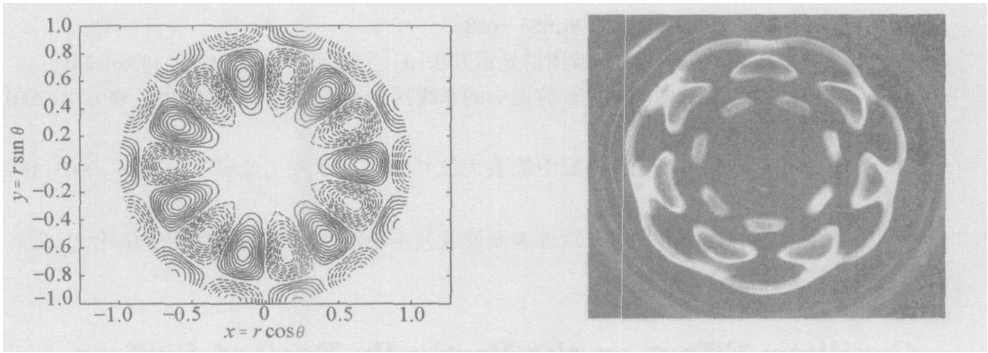
## 6 结 论

通过以上的分析,可得如下的结论:

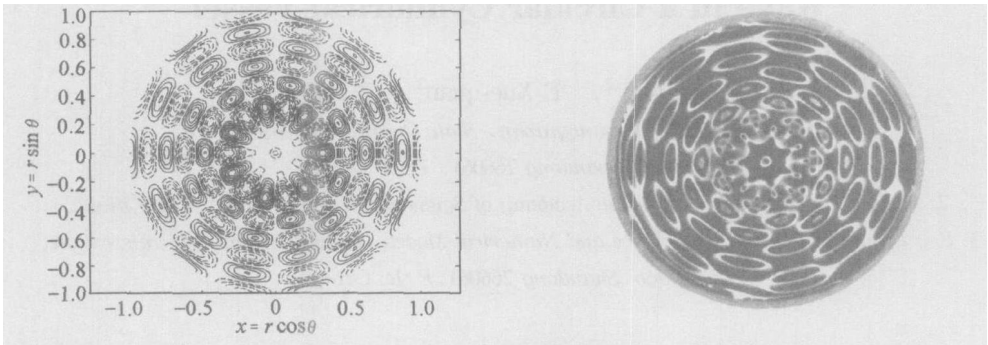


(a) (3,3) 模式, (左) 驱动频率  $f = 12.63$  Hz, (右) 驱动频率  $f = 20$  Hz

- I. 两时间尺度展开法对于求解具有表面张力的竖直激励表面波问题是有效的;
- II. 非线性振幅方程(43)、(44)及自由面位移表达式(13)可以用来描述垂直激励表面波



(b) (7,2) 模式, (左) 驱动频率  $f = 14.01$  Hz (右) 驱动频率  $f = 50$  Hz



(c) (7,6) 模式, (左) 驱动频率  $f = 26.02$  Hz (右) 驱动频率  $f = 52$  Hz

图 2 表面波模式的理论等高线和实验等高线的比较

(流体深度  $h = 1.0$  cm, 容器半径  $R = 7.5$  cm, 驱动振幅  $A = 11.4 \mu\text{m}$ ,  
表面张力  $\Gamma = 0.0727$  N/m, 流体密度  $\rho = 10^3$  kg/m<sup>2</sup>)

的运动;

III 当波数较小时, 表面张力对模式选择的影响不是很重要; 但当波数较大时, 表面张力的影响非常重要;

IV. 表面张力具有使得自由面恢复到平衡位置的作用。也就是说, 对于激发的同一个表面波模式, 在有表面张力时要比无表面张力时需要更高的驱动频率。

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## Capillary Effect on the Vertically Excited Surface Wave in a Circular Cylindrical Vessel

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**Abstract:** In a vertically oscillating circular cylindrical container, singular perturbation theory of two-time scale expansions was developed in inviscid fluids to investigate the motion of single free surface standing wave including the effect of surface tension. A nonlinear slowly varying amplitude equation, which incorporates cubic nonlinear term, external excitation and the influence of surface tension, was derived from potential flow equation. The results show that when forced frequency is lower, the effect of surface tension on mode selection of surface wave is not important. However, when forced frequency is higher, the surface tension can not be neglected. This proved that the surface tension causes free surface returning to equilibrium location. In addition, due to considering the effect of surface tension, the theoretical result much more approaches to experimental results than that of no surface tension.

**Key words:** vertically forced oscillation; nonlinear amplitude equation; surface tension; surface wave mode