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各向异性矩形板自由振动的一般解析解法*

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摘要: 根据各向异性矩形薄板自由振动横向位移函数的微分方程建立了一般性的解析解. 该一般解包括三角函数和双曲线函数组成的解, 它能满足4 个边为任意边界条件的问题. 还有代数多项式和双正弦级数解, 它能满足 4 个角的边界条件问题. 因此, 这一解析解可用于精确地求解具有任意边界条件的各向异性矩形卞的振动问题. 解中的积分常数可由 4 边和 4 角的边界条件来确定. 由此得出的齐次线性代数方程系数矩阵行列式等于零可以求得各阶固有频率及其振型, 以四边平夹的对称角铺设复合材料迭层板为例进行了计算和讨论.

关 键 词: 各向异性板; 自由振动; 一般解析解法; 频率; 振型

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对称迭层角铺设复合材料板为各向异性板, 已广泛应用于航空、航天、土木和化学工程等领域, 其振动问题分析具有重要的工程实际意义. 目前, 可以用各种解析的和数值的方法来研究不同边界的各向异性矩形板的振动问题[1~9]. 黄炎等人[10,11]采用解析法求解了各向同性的和正交异性的矩形板的振动问题, 首先求出偏微分方程所有类型的特解, 然后选取一个能满足任意不同边界的一般解. 本文仍采用这一方法来研究各向异性矩形板的振动问题以得出其一般解析解.

1 微分方程的解

各向异性矩形薄板(如图 1) 自由振动横向位移函数的微分方程为[12]

$$D_{11}\frac{\partial^4 w}{\partial x^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}+D_{22}\frac{\partial^4 w}{\partial y^4}-\rho\omega^2 w=0, \tag{1}$$

式中 w 为板的挠度, D_{11}、D_{12}、D_{16}、D_{22}、D_{26}、D_{66} 为挠曲刚度系数, ω 为固有圆频率, ρ 为单位面积质量. 取

$$w=e^{i\alpha x}e^{i\alpha' y}\text{ 或 }w=e^{-i\alpha x}e^{-i\alpha' y}, \tag{2}$$

式中 $i=\sqrt{-1}$, $\alpha=m\pi/a$, $m=1,2,3,\cdots$ 将上式代入(1) 式, 并令 $\rho\omega^2=D_{11}(M\pi/a)^4$, 可得

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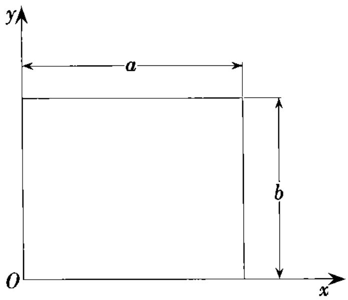


图 1 板的坐标系

$$D_{22}\alpha'^4 + 4D_{26}\alpha\alpha'^3 + 2(D_{12} + 2D_{66})\alpha^2\alpha'^2 + 4D_{16}\alpha^3\alpha' + D_{11}\alpha^4(1 - M^4/m^4) = 0. \tag{3}$$

当 $m > M$, $\alpha' = \alpha_1 \pm i\alpha_2$ 和 $\alpha_3 \pm i\alpha_4$, 可得

$$w = e^{\pm i(\alpha x + \alpha_1 y)} e^{\pm \alpha_2 y} \text{或} w = e^{\pm i(\alpha x + \alpha_3 y)} e^{\pm \alpha_4 y}. \tag{4}$$

上列的解可改用三角函数和双曲线函数表示为

$$w = [A\sin(\alpha x + \alpha_1 y) + B\cos(\alpha x + \alpha_1 y)](C\sinh\alpha_2 y + D\cosh\alpha_2 y) + [E\sin(\alpha x + \alpha_3 y) + F\cos(\alpha x + \alpha_3 y)](G\sinh\alpha_4 y + H\cosh\alpha_4 y), \tag{5}$$

当 $m < M$, $\alpha' = \alpha_1 \pm i\alpha_2$, α_3 和 α_4. 此时可得

$$w = e^{\pm i(\alpha x + \alpha_3 y)} \text{或} w = e^{\pm(\alpha x + \alpha_4 y)}. \tag{6}$$

同样, 可设

$$w = e^{i\beta y} e^{i\beta' x} \text{或} w = e^{-i\beta y} e^{-i\beta' x}. \tag{7}$$

式中 $\beta = n\pi/b$, $n = 1, 2, 3, \cdots$ 和 $\rho\omega^2 = D_{22}(N\pi/b)^4$, 可得另一类与以上相似的解. 此外, 当 $\rho\omega^2 = 0$ 时, 可得代数多项式解[13]

$$\begin{aligned} w = {} & a_{00}(1-\zeta)(1-\eta) + a_{01}(1-\zeta)\eta + a_{10}\zeta(1-\eta) + a_{11}\zeta\eta + \\ & a_{02}[(1-\zeta)(2\eta - 3\eta^2 + \eta^3) + t_2(\zeta - \zeta^2)(\eta - \eta^2)] + \\ & a_{20}[(1-\eta)(2\zeta - 3\zeta^2 + \zeta^3) + t_1(\zeta - \zeta^2)(\eta - \eta^2)] + \\ & a_{12}[\zeta(2\eta - 3\eta^2 + \eta^3) - t_2(\zeta - \zeta^2)(\eta - \eta^2)] + a_{21}[\eta(2\zeta - 3\zeta^2 + \zeta^3) - \\ & t_1(\zeta - \zeta^2)(\eta - \eta^2)] + a_{03}[(1-\zeta)(\eta - \eta^3) - t_2(\zeta - \zeta^2)(\eta - \eta^2)] + \\ & a_{30}[(1-\eta)(\zeta - \zeta^3) - t_1(\zeta - \zeta^2)(\eta - \eta^2)] + a_{13}[\zeta(\eta - \eta^3) + \\ & t_2(\zeta - \zeta^2)(\eta - \eta^2)] + a_{31}[\eta(\zeta - \zeta^3) + t_1(\zeta - \zeta^2)(\eta - \eta^2)], \end{aligned} \tag{8}$$

式中 $\zeta = \dfrac{x}{a}$, $\eta = \dfrac{y}{b}$, $t_1 = \dfrac{3D_{16}b}{(D_{12} + 2D_{66})a}$, $t_2 = \dfrac{3D_{26}a}{(D_{12} + 2D_{66})b}$.

上式也可简单地写成 $w = \sum_i \sum_j a_{ij} E_{ij}$, 式中 $i = 0, 1, 2, 3; j = 0, 1$ 和 $i = 0, 1; j = 0, 1, 2, 3$. 为满足(1)式, 令

$$w_1 = \sum_i \sum_j a_{ij}\left(E_{ij} + \sum_m \sum_n A_{mnij}\sin\alpha x \sin\beta y\right). \tag{9}$$

将上式代入(1)式并令

$$E_{ij} = \sum_m \sum_n B_{mnij}\sin\alpha x \sin\beta y, \tag{10}$$

$$\cos\gamma x \cos\delta y = \sum_m \sum_n F_{mn}\sin\alpha x \sin\beta y, \tag{11}$$

式中

$$\gamma = k\pi/a, \quad \delta = l\pi/b, \qquad k, l = 1, 2, 3, \cdots,$$

$$B_{mnij} = \frac{4}{ab}\int_0^a\int_0^b E_{ij}\sin\alpha x \sin\beta y \, dx\, dy, \quad F_{mn} = \frac{16mn}{\pi^2(m^2 - k^2)(n^2 - l^2)},$$

仅当 $m \pm k$ 和 $n \pm l$ 均为奇数时, 否则 $F_{mn} = 0$. 由此可得

$$A_{mnij}[D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 + \rho\omega^2] - \sum_k \sum_l A_{klij}(4D_{16}\gamma^3\delta + 4D_{26}\gamma\delta^3)F_{mn} - \rho\omega^2 B_{mnij} = 0. \tag{12}$$

2 一般解的建立

求解满足 4 边和 4 角为任意边界条件的各向异性矩形板自由振动问题的一般解, 本文建议取下列形式:

$$
\begin{aligned}
w = & \sum_{m} \frac{A_m \sin[\alpha(a-x)+\alpha_1(b-y)]\sinh\alpha_2 y + B_m \sin(\alpha x+\alpha_1 y)\sinh\alpha_2(b-y)}{\sinh\alpha_2 b} + \\
& \sum_{m<M}\Big\{ C_m\big\{\sin[\alpha(a-x)+\alpha_3(b-y)] + \sin(\alpha x+\alpha_4 y)\big\} + \\
& D_m\big\{\sin[\alpha(a-x)+\alpha_4(b-y)] + \sin(\alpha x+\alpha_3 y)\big\}\Big\} + \\
& \sum_{m>M} \frac{C_m \sin[\alpha(a-x)+\alpha_3(b-y)]\sinh\alpha_4 y + D_m \sin(\alpha x+\alpha_3 y)\sinh\alpha_4(b-y)}{\sinh\alpha_4 b} + \\
& \sum_{n} \frac{E_n \sin[\beta(b-y)+\beta_1(a-x)]\sinh\beta_2 x + F_n \sin(\beta y+\beta_1 x)\sinh\beta_2(a-x)}{\sinh\beta_2 a} + \\
& \sum_{n<N}\Big\{ G_n\big\{\sin[\beta(b-y)+\beta_3(a-x)] + \sin(\beta y+\beta_4 x)\big\} + \\
& H_n\big\{\sin[\beta(b-y)+\beta_4(a-x)] + \sin(\beta y+\beta_3 x)\big\}\Big\} + \\
& \sum_{n>N} \frac{G_n \sin[\beta(b-y)+\beta_3(a-x)]\sinh\beta_4 x + H_n \sin(\beta y+\beta_3 x)\sinh\beta_4(a-x)}{\sinh\beta_4 a} + w_1.
\end{aligned}
\tag{13}
$$

在(13)式中共有 $4m+4n+12$ 个积分常数, 其中第 1 部分能满足 $y=0$ 和 $y=b$ 两个边界的边界条件; 第 2 部分能满足 $x=0$ 和 $x=a$ 两个边的边界条件; 第 3 部分能满足 4 个角的边界条件. 对于每个边来说, 有两个边界条件: 即挠度或等效剪力; 斜度或弯矩. 将边界条件方程式中的非正弦函数均展成正弦级数, 根据正弦级数的正交性可得 $4m+4n$ 个方程式. 对于每个角则有 3 个边界条件: 即挠度或反力; 沿角两边的斜度或弯矩, 故又有 12 个方程式. 令所有方程式积分常数的系数矩阵行列式等于零可以求得振动的固有频率及其振型. 关于非正弦函数展成正弦级数的公式可参看文献[14].

如果仅研究一块单独的板的振动问题, 根据变分原理, 则角点的斜度或弯矩可以不一定满足, 故可令

$$
a_{02}=a_{20}=a_{12}=a_{21}=a_{03}=a_{30}=a_{13}=a_{31}=0 \tag{14}
$$

且

$$
B_{mn00}=\frac{4}{mn\pi^2},\quad B_{mn01}=-\frac{4\cos n\pi}{mn\pi^2},\quad B_{mn10}=-\frac{4\cos m\pi}{mn\pi^2},\quad B_{mn11}=\frac{4\cos m\pi\cos n\pi}{mn\pi^2}. \tag{15}
$$

3 算　　例

以四边平夹的板为例, 4 边和 4 角的边界条件为

$$
(w)_{x=0}=0,\ (w)_{x=a}=0,\ (w)_{y=0}=0,\ (w)_{y=b}=0, \tag{16}
$$

$$
\left(\frac{\partial w}{\partial x}\right)_{x=0}=0,\ \left(\frac{\partial w}{\partial x}\right)_{x=a}=0,\ \left(\frac{\partial w}{\partial y}\right)_{y=0}=0,\ \left(\frac{\partial w}{\partial y}\right)_{y=b}=0, \tag{17}
$$

$$
w_{(0,0)}=0,\ w_{(a,b)}=0,\ w_{(0,b)}=0,\ w_{(a,0)}=0. \tag{18}
$$

如果利用变形的对称或反对称条件将使求解振动问题更加简化. 对于各向异性矩形板来说[13], 两对边的变形与板的中心为对称或反对称时, 则两对边和两对角的边界条件是相同的, 即方程(16)、(17)和(18)的第 2 式和第 4 式可不应用, 且

$$
B_m=\pm A_m,\ D_m=\pm C_m,\ F_n=\pm E_n,\ H_n=\pm G_n,\ a_{11}=\pm a_{00},\ a_{10}=\pm a_{01}. \tag{19}
$$

式中正负号同时书写时, 上号为对称振型, 下号为反对称振型.

将(9)式代入(13)式,然后代入以上各式,首先由(18)式的前2式可得

$$a_{00} = -\sum_{m<M} C_m \cos m\pi(\sin\alpha_3 b \pm \sin\alpha_4 b) - \sum_{n<N} G_n \cos n\pi(\sin\beta_3 a \pm \sin\beta_4 a), \quad (20)$$

$$a_{01} = -\sum_{m<M} C_m(\sin\alpha_4 b \pm \sin\alpha_3 b) - \sum_{n<N} G_n(\sin\beta_3 a \pm \sin\beta_4 a). \quad (21)$$

由(16)式和(17)式的第1式,并将其中非正弦函数均展成正弦级数可得

$$\sum_m A_m \frac{4\beta}{b}\frac{\alpha_1\alpha_2(\cos m\pi \mid \cos n\pi)}{(\beta^2-\alpha_1^2+\alpha_2^2)^2+4\alpha_1^2\alpha_2^2}\left(\frac{\cos\alpha_1 b}{\sinh\alpha_2 b} - \cos n\pi\coth\alpha_2 b\right) +$$
$$\sum_{m<M} C_m \frac{2\beta}{b}(\cos m\pi \mid \cos n\pi)\left(\frac{\cos\alpha_3 b}{\beta^2-\alpha_3^2} \pm \frac{\sin\alpha_4 b}{\beta^2-\alpha_4^2}\right) +$$
$$\sum_{m>M} C_m \frac{4\beta}{b}\frac{\alpha_3\alpha_4(\cos m\pi \mid \cos n\pi)}{(\beta^2-\alpha_3^2+\alpha_4^2)^2+4\alpha_3^2\alpha_4^2}\left(\frac{\cos\alpha_3 b}{\sinh\alpha_4 b} - \cos n\pi\coth\alpha_4 b\right) \pm$$
$$E_n + \begin{cases} G_n[1 \pm 1 - \cos n\pi(\cos\beta_3 a \pm \cos\beta_4 a)] + & (n<N); \\ \pm G_n + & (n>N) \end{cases}$$
$$\sum_{k<N} G_k \cos k\pi(\sin\gamma_3 a \pm \sin\gamma_4 a)K_n + a_{00}\frac{2}{n\pi} - a_{01}\frac{2\cos n\pi}{n\pi} = 0, \quad (22)$$

$$\sum_m A_m \frac{2}{b}\frac{\alpha\beta(\cos m\pi \mid \cos n\pi)}{(\beta^2-\alpha_1^2+\alpha_2^2)^2+4\alpha_1^2\alpha_2^2}\left[2\alpha_1\alpha_2\frac{\sin\alpha_1 b}{\sinh\alpha_2 b} + \cos n\pi(\beta^2-\alpha_1^2+\alpha_2^2)\right] -$$
$$\sum_{m<M} C_m \frac{2\alpha\beta}{b}(\cos m\pi \pm \cos n\pi)\left(\frac{\cos\alpha_3 b - \cos n\pi}{\beta^2-\alpha_3^2} \pm \frac{\cos\alpha_4 b - \cos n\pi}{\beta^2-\alpha_4^2}\right) +$$
$$\sum_{m>M} C_m \frac{2}{b}\frac{\alpha\beta(\cos m\pi \mid \cos n\pi)}{(\beta^2-\alpha_3^2+\alpha_4^2)^2+4\alpha_3^2\alpha_4^2}\left[2\alpha_3\alpha_4\frac{\sin\alpha_3 b}{\sinh\alpha_4 b} + \cos n\pi(\beta^2-\alpha_3^2+\alpha_4^2)\right] -$$
$$E_n\beta_2\left(\cos n\pi\frac{\cos\beta_1 a}{\sinh\beta_2 a} + \coth\beta_2 a\right) + \sum_k E_k K_n\left(\gamma_2\cos k\pi\frac{\sin\gamma_1 a}{\sinh\gamma_2 a} \pm \gamma_1\right) -$$
$$\begin{cases} G_n\cos n\pi(\beta_3\sin\beta_3 a \pm \beta_4\sin\beta_4 a) - & (n<N) \\ G_n\beta_4\left(\cos n\pi\frac{\cos\beta_3 a}{\sinh\beta_4 a} \pm \coth\beta_4 a\right) - & (n>N) \end{cases}$$
$$\sum_{k<N} G_k K_n[\cos k\pi(\gamma_3\cos\gamma_3 a \pm \gamma_4\cos\gamma_4 a) \mid \gamma_3 - \gamma_4] +$$
$$\sum_{k>N} G_k K_n\left(\gamma_4\cos k\pi\frac{\sin\gamma_3 a}{\sinh\gamma_4 a} \pm \gamma_3\right) - (a_{00} \mid a_{01})\frac{2(1 \pm \cos n\pi)}{n\pi a} +$$
$$\sum_m [a_{00}(A_{mn00} \pm A_{mn11}) + a_{01}(A_{mn01} \pm A_{mn10})]\alpha = 0, \quad (23)$$

式中$K_n = 4n/(\pi n^2 - \pi k^2)$,当$n \pm k$为奇数时,否则$K_n = 0$,$\gamma = k\pi/b$,$k = 1, 2, 3, \cdots$,$\gamma_{1,2,3,4}$相当于$\beta_{1,2,3,4}$.

采用石墨/环氧制成的对称迭层角铺设45°/-45°/-45°/45°复合材料迭层正方形板,Poisson比为$\mu_{12} = 0.25$,弹性模量为$G_{12} = 0.5E_2$,$E_1 = 40E_2$,故有$D_{22} = D_{11}$,$D_{12} = 0.9082 D_{11}$,$D_{16} = D_{26} = 0.6724 D_{11}$,$D_{66} = 0.9341 D_{11}$以及$a = b$,此时变形将对称或反对称于对角线$x = y$,故又有$E_n = \pm A_n$,$G_n = \pm C_n$.(16)式和(17)式的第3式将不应用.此外,还可将以上各式中的N,k,$\gamma_{1,2,3,4}$分别改成M,m,$\alpha_{1,2,3,4}$.仅由以上4个方程式即可求得M值和振型.取m和n由1至2,4,6,8时求得各种类型的一阶频率见表1.当m和n取8项求得各种类型的前6阶频率和一阶振型的等高线分别见表2和图2.

$$M_{11}:\ B_m = A_m,\ D_m = C_m,\ F_n = E_n = A_n,\ H_n = G_n = C_n, \quad (24)$$

$$M_{12}:\ B_m = -A_m,\ D_m = -C_m,\ F_n = -E_n = -A_n,\ H_n = -G_n = -C_n, \quad (25)$$

$$M_{21}:\ B_m = -A_m,\ D_m = -C_m,\ F_n = -E_n = A_n,\ H_n = -G_n = C_n, \tag{26}$$

$$M_{22}:\ B_m = A_m,\ D_m = C_m,\ F_n = E_n = -A_n,\ H_n = G_n = -C_n. \tag{27}$$

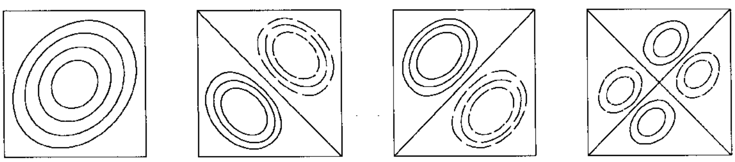
由表 1 可以看出: M 值的收敛性是良好的. 由图2可以看出所有振型对 $x = y$ 和 $x + y = a$ 两对角线对称或反对称而对 $x = a/2$ 和 $y = b/2$ 不对称. 这是因为本算例的材料恰好是以正方形对角线 $x = y$ 和 $x + y = a$ 为主轴的正交异性板的缘故. 这些结果和文献[15] 是一致的. 由表2 可以看出各阶 M_{21} 的值均大于 M_{12} 相应的值. 这是因为沿对角线 $x = y$ 的弯曲刚度大于沿 $x + y = a$ 的缘故.

表 1 各种类型的一阶频率的收敛性

$m,\ n$	2	4	6	8
M_{11}	2.268	2.286	2.287	2.287
M_{12}	2.439	2.501	2.506	2.507
M_{21}	2.722	2.775	2.758	2.748
M_{22}	3.624	3.560	3.560	3.599

表 2 各种类型的前 6 阶频率的 M 值

	(1)	(2)	(3)	(4)	(5)	(6)
M_{11}	2.287	3.445	4.085	4.725	5.385	6.368
M_{12}	2.507	3.550	4.381	4.946	5.493	6.914
M_{21}	2.748	3.825	4.670	5.339	6.354	7.119
M_{22}	3.599	4.468	5.339	6.289	7.593	7.806



M_{11}　　M_{12}　　M_{21}　　M_{22}

图 2 各种类型的一阶振型等高线图

4 结　　论

本文精确地建立了一般解析解. 可用以求解任意边界各向异性矩形板的振动问题. 这种解也能用于数块板组成的板结构的振动问题. 其中对每块板除边界条件外, 还有相连的两块板的连续性条件. 如需求角点的弯矩, 则必需加上(9) 式中所有的项. 本文的理论分析简单, 计算方法容易, 便于工程应用.

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Free Vibration of Anisotropic Rectangular Plates by General Analytical Method

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Abstract: According to the differential equation for transverse displacement function of anisotropic rectangular thin plates in free vibration, a general analytical solution was established. This general solution composed of the composite solutions made by trigonometric function and hyperbolic function, can satisfy the problem of arbitrary boundary conditions along four edges. The algebraic polynomial with double sine series solutions can also satisfy the problem of boundary conditions at four corners. Consequently, this general solution can be used to solve the vibration problem of anisotropic rectangular plates with arbitrary boundaries accurately. The integral constants can be determined by boundary conditions of four edges and four corners. Each natural frequency and vibration mode can be solved by the determinate of coefficient matrix from the homogeneous linear algebraic equations equal to zero. For example, a composite symmetric angle ply laminated plate with four edges clamped was calculated and discussed.

Key words: anisotropic plate; free vibration; general analytical method; frequency; mode shape