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均布载荷作用下各向异性固支梁的解析解*

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(我刊编委 皓江来稿)

摘要: 针对均布载荷作用下的各向异性梁在两端固支条件下的平面应力问题, 给出了一个求解应力和位移解析解的方法• 该方法构造了一个含待定系数的应力函数, 通过 Airy 应力函数解法, 给出了含待定系数的应力和位移通式• 对固支端边界条件采用两种处理办法• 利用应力和位移边界条件, 确定应力函数中的待定系数, 得到了应力和位移的解析表达式• 结果表明, 该解析解与有限元数值结果相比, 两者较为吻合• 该解析解是对弹性理论中相关经典例题的补充•

关 键 词: 固支梁; 各向异性; 应力函数; 解析解

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引 言

梁的平面应力问题是一个非常经典的弹性力学问题, 也是在实际工程中经常碰到的问题• Timoshenko 和 Goodier[1] 曾用应力函数法研究了各向同性弹性梁的拉伸、纯弯曲、受横力作用悬臂梁的弯曲、均布载荷作用简支梁的弯曲和其它连续载荷作用梁的弯曲问题• Lekhniskii[2] 曾研究了各向异性梁的平面应力问题, 包括拉伸、剪切、纯弯曲、受横力作用的悬臂梁、受均布载荷和线性分布载荷作用的简支梁和悬臂梁• 江爱民和丁皓江[3] 研究了正交各向异性悬臂梁上下表面受法向和切向分布载荷作用的问题• 对两端固支各向同性梁受均布载荷问题, Gere 和 Timoshenko[4] 用 Euler_Bernoulli 梁理论给出了梁的挠曲线方程和应力解, Ahmed 等(1996) [5] 用差分法给出了两端固支深梁的弹性力学数值解• 最近, 丁皓江等[6] 用应力函数法给出了各向同性的两端固支梁受均布载荷作用的解析解• 对两端固支各向异性梁在均匀载荷作用下的解析解, Lekhniskii[1] 专著中没有研究这个问题, 也没有查到有关文献在这方面的报道• 本文用应力函数法研究了受均布载荷作用下各向异性固支梁的平面应力问题, 给出了应力和位移公式, 退化到各向同性情形时与已有结果完全一致•

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1　基 本 方 程

考虑 xy 面内的平面应力问题，各向异性弹性体本构方程为

$$\begin{cases}\partial u/\partial x = s_{11}\sigma_x + s_{12}\sigma_y + s_{16}\tau_{xy},\\ \partial v/\partial y = s_{12}\sigma_x + s_{22}\sigma_y + s_{26}\tau_{xy},\\ \partial u/\partial y + \partial v/\partial x = s_{16}\sigma_x + s_{26}\sigma_y + s_{66}\tau_{xy},\end{cases} \tag{1}$$

式中 u、v、σ_x、σ_y 和 τ_{xy} 分别为位移分量和应力分量，s_{ij} 为弹性柔度常数· 用Airy应力函数 ϕ 表示的应力公式为

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2},\quad \sigma_y = \frac{\partial^2\phi}{\partial x^2},\quad \tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y}\cdot \tag{2}$$

式中应力函数 ϕ 满足如下协调方程

$$s_{22}\frac{\partial^4\phi}{\partial x^4} - 2s_{26}\frac{\partial^4\phi}{\partial x^3\partial y} + (2s_{12} + s_{66})\frac{\partial^4\phi}{\partial x^2\partial y^2} - 2s_{16}\frac{\partial^4\phi}{\partial x\partial y^3} + s_{11}\frac{\partial^4\phi}{\partial y^4} = 0\cdot \tag{3}$$

2　固支梁的解析解

图1所示的受均布载荷作用的固支梁，梁的跨度为 l，高为 h，宽度为1·

取应力函数

$$\phi = Ay^5 + Bxy^4 + Cy^4 + Dx^2y^3 + Exy^3 + Fy^3 + Gxy^2 + Hy^2 + Ix^2y + Jxy + Kx^2, \tag{4}$$

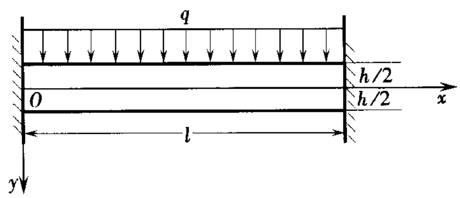

图1　受均布载荷作用的固支梁

式中 A、B、C、D、E、F、G、H、I、J 和 K 为11个待定常数· 应力函数 ϕ 要满足协调方程(3)，则待定系数 A、B、C、D、E 必须满足如下关系式

$$10s_{11}A - 4s_{16}B + (2s_{12} + s_{66})D = 0,\ s_{11}B - s_{16}D = 0,\ 2s_{11}C - s_{16}E = 0\cdot \tag{5}$$

将式(4)代入式(2)，得应力表达式

$$\begin{cases}\sigma_x = 20Ay^3 + 12Bxy^2 + 12Cy^2 + 6Dx^2y + 6Exy + 6Fy + 2Gx + 2H,\\ \sigma_y = 2Dy^3 + 2Iy + 2K,\\ \tau_{xy} = -4By^3 - 6Dxy^2 - 3Ey^2 - 2Gy - 2Ix - J\cdot\end{cases} \tag{6}$$

将式(6)代入式(1)并作积分，得到下列位移表达式

$$\begin{aligned}u = {} & [5s_{16}A - (s_{12} + s_{66})B + s_{26}D]y^4 + [(20s_{11}A - 4s_{16}B + 2s_{12}D)x + 4s_{16}C - \\ & (s_{12} + s_{66})E]y^3 + [3(2s_{11}B - s_{16}D)x^2 + 3(4s_{11}C - s_{16}E)x + 3s_{16}F - \\ & (s_{12} + s_{66})G + 2s_{26}I]y^2 + [2s_{11}Dx^3 + 3s_{11}Ex^2 + (6s_{11}F + 2s_{12}I - \\ & 2s_{16}G)x]y + (s_{11}G - s_{16}I)x^2 + (2s_{11}H - s_{16}J + 2s_{12}K)x + \omega y + u_0,\end{aligned} \tag{7a}$$

$$\begin{aligned}v = {} & (5s_{12}A - 0.5s_{22}D - s_{26}B)y^4 + [2(2s_{12}B - s_{26}D)x + 4s_{12}C - s_{26}E]y^3 + \\ & [3s_{12}(Dx^2 + Ex + F) - s_{26}G + s_{22}I]y^2 + [2(s_{12}G - s_{26}I)x + 2s_{12}H - \\ & s_{26}J + 2s_{22}K]y - 0.5s_{11}Dx^4 - s_{11}Ex^3 - [3s_{11}F - 2s_{16}G + (s_{12} + s_{66})I]x^2 + \\ & (s_{16}H + 2s_{26}K - s_{66}J)x - \omega y + v_0,\end{aligned} \tag{7b}$$

式中 u_0、v_0 和 ω 是积分常数，表示刚体位移· 因此位移分量中包含14个待定常数·

Timoshenko 和 Goodier[1] 对于梁的固支边界条件给出两种形式· 本文的研究中同样采用这

两种形式，第 1 种形式的边界条件为：1) $y = h/2,\ \sigma_y = 0$; 2) $y = -h/2,\ \sigma_y = -q$; 3) $y = \pm h/2,\ \tau_{xy} = 0$; 4) $x = 0,\ y = 0$ 点和 $x = l,\ y = 0$ 点，$u = v = 0,\ \partial v/\partial x = 0$；第 2 种形式的边界条件只将前面边界条件中 $x = y = 0$ 和 $x = l,\ y = 0$ 处的 $\partial v/\partial x = 0$ 改为 $\partial u/\partial y = 0$•

将应力表达式(6)和位移表达式(7)代入边界条件，可得到 11 个方程，加上式(5)的 3 个方程，共 14 个方程，正好可以联立求解 14 个待定常数• 对于第 1 种边界条件，得到应力分量和位移分量表达式是

$$\sigma_x = -(6x^2 - 6xl + l^2)\frac{qy}{h^3} + \frac{s_{11}s_{66} + 2s_{11}s_{12} - 4s_{16}^2}{2s_{11}^2h^3}qy(4y^2 - h^2) + \frac{s_{16}}{2s_{11}h^3}q(l - 2x)(12y^2 - h^2) + \frac{s_{12}}{2s_{11}h}q(h - y) - \frac{s_{66}}{s_{11}h}qy, \tag{8a}$$

$$\sigma_y = -\frac{q(y + h)(h - 2y)^2}{2h^3},\quad \tau_{xy} = \frac{q}{4h^3}(4y^2 - h^2)\left[4\frac{s_{16}}{s_{11}}y + 3(2x - l)\right]; \tag{8b, c}$$

$$\begin{aligned}u = {} & \frac{3s_{11}s_{16}s_{66} - 4s_{16}^3 - 2s_{26}s_{11}^2 + 4s_{11}s_{12}s_{16}}{2s_{11}^2h^3}qy^4 + \frac{(s_{11}s_{66} + s_{11}s_{12} - 2s_{16}^2)(2x - l)}{s_{11}h^3}qy^3 + \\ & \left(-3\frac{x^2s_{16}}{h^3} + \frac{3xls_{16}}{h^3} - \frac{l^2s_{16}}{2h^3} - \frac{5s_{12}s_{16} + 5s_{16}s_{66}}{4s_{11}h} + \frac{3s_{26}}{2h} + \frac{s_{16}^3}{s_{11}^2h}\right)qy^2 - \\ & \left[\frac{2s_{11}x^3}{h^3} - \frac{3s_{11}x^2l}{h^3} + \left(\frac{3s_{66}}{2h} - \frac{s_{16}^2}{s_{11}h} + \frac{s_{11}l^2}{h^3}\right)x - \right. \\ & \left.\left(\frac{3s_{66}}{4h} - \frac{s_{16}^2}{2s_{11}h}\right)l + \frac{s_{26}q}{2} - \frac{s_{12}s_{16}}{2s_{11}}\right]qy + \frac{qs_{16}x(l - x)}{4h},\end{aligned} \tag{9a}$$

$$\begin{aligned}v = {} & \frac{2s_{12}^2s_{11} + 2s_{26}s_{16}s_{11} - 4s_{12}s_{16}^2 + s_{11}s_{12}s_{66} - s_{22}s_{11}^2}{2s_{11}^2h^3}qy^4 + \\ & \frac{(2s_{12}s_{16} - s_{11}s_{26})(l - 2x)}{s_{11}h^3}qy^3 - \\ & \left[\frac{3s_{12}x^2}{h^3} - \frac{3s_{12}xl}{h^3} + \frac{s_{12}l^2}{2h^3} + \frac{s_{26}s_{16}}{2hs_{11}} - \frac{s_{12}s_{16}^2}{hs_{11}^2} + \frac{3s_{12}^2}{4hs_{11}} + \frac{3s_{12}s_{66}}{4hs_{11}} - \frac{3s_{22}}{4h}\right]qy^2 + \\ & \left(\frac{2s_{12}s_{16} - 3s_{11}s_{26}}{2hs_{11}}x + \frac{3s_{11}s_{26} - 2s_{12}s_{16}}{4hs_{11}}l + \frac{s_{12}^2 - s_{11}s_{22}}{2s_{11}}\right)qy + \frac{qs_{11}(l - x)^2x^2}{2h^3}.\end{aligned} \tag{9b}$$

对于第 2 种边界条件，可得到应力和位移分量的表达式分别为

$$\begin{aligned}\sigma_x = {} & -(6x^2 - 6xl + l^2)\frac{qy}{h^3} + \frac{(s_{11}s_{66} + 2s_{11}s_{12} - 4s_{16}^2)}{s_{11}^2h^3}2qy^3 + \\ & \frac{6s_{16}}{s_{11}h^3}\left[l - 2x + \frac{2s_{11}s_{26} - 2s_{12}s_{16}}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}h^3\right]qy^2 - \\ & \frac{12x(s_{12}s_{16} - s_{11}s_{26})}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}qy - \frac{6(s_{11}s_{26} - s_{12}s_{16})qly}{2s_{11}^2l^2 + 3s_{66}h^2s_{11} - 3s_{16}^2h^2} - \frac{3s_{11}s_{12} - 2s_{16}^2}{2s_{11}^2h}qy + \\ & \frac{s_{16}q}{2s_{11}h}(2x - l) + \frac{2s_{11}^2s_{12}l^2 + 3s_{11}s_{12}s_{66}h^2 - 3s_{11}s_{16}s_{26}h^2}{2s_{11}(2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2)}q,\end{aligned} \tag{10a}$$

$$\sigma_y = -\frac{q(y + h)(h - 2y)^2}{2h^3}, \tag{10b}$$

$$\tau_{xy} = \frac{(4y^2 - h^2)q}{4h^3}\left[3(2x - l) + 4\frac{s_{16}}{s_{11}}y - \frac{6(s_{11}s_{26} - s_{12}s_{16})h^3}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}\right]; \tag{10c}$$

$$u = \frac{qy^4(3s_{11}s_{16}s_{66} - 4s_{16}^3 - 2s_{26}s_{11}^2 + 4s_{11}s_{12}s_{16})}{2s_{11}^2h^3} +$$

$$\frac{(s_{11}s_{12} - 2s_{16}^2 + s_{11}s_{66})}{s_{11}h^3}\left[2x - l - \frac{2(s_{11}s_{26} - s_{12}s_{16})h^3}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}\right]qy^3 -$$

$$\left[\frac{3s_{16}}{h^3}x^2 - \frac{3s_{16}l}{h^3}x - \frac{6s_{16}(s_{11}s_{26} - s_{12}s_{16})}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}x + \right.$$

$$\left.\frac{s_{16}l^2}{2h^3} - \frac{6s_{11}^2s_{26} + s_{16}(2s_{16}^2 - 5s_{11}s_{12} - 2s_{11}s_{66})}{4s_{11}^2h} + \frac{3s_{16}(s_{11}s_{26} - s_{12}s_{16})l}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}\right]qy^2 +$$

$$\frac{qs_{11}}{h^3}(l - x)(2x - l)xy + \frac{6qs_{11}(l - x)xy}{2s_{11}^2l^2 - 3s_{16}^2h^2 + 3s_{11}s_{66}h^2}(s_{12}s_{16} - s_{26}s_{11}) + \frac{qs_{16}}{4h}x(l - x), \tag{11a}$$

$$v = \frac{(2s_{11}s_{12}^2 - s_{11}^2s_{22} + s_{11}s_{12}s_{66} + 2s_{11}s_{26}s_{16} - 4s_{12}s_{16}^2)}{2s_{11}^2h^3}qy^4 + \frac{(s_{11}s_{26} - 2s_{12}s_{16})}{s_{11}h^3} \times$$

$$\left(2x - l + \frac{2(s_{12}s_{16} - s_{11}s_{26})h^3}{2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2}\right)qy^3 -$$

$$3s_{12}\left[\frac{x - l}{h^3} - \frac{2(s_{11}s_{26} - s_{12}s_{16})}{2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2}\right]qxy^2 +$$

$$\left[\frac{2s_{12}s_{16}^2 - 3s_{11}s_{12}^2 + 3s_{11}^2s_{22} - 2s_{11}s_{16}s_{26}}{4s_{11}^2h} - \frac{3s_{12}(s_{11}s_{26} - s_{12}s_{16})l}{2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2} - \frac{s_{12}l^2}{2h^3}\right]qy^2 +$$

$$\left(\frac{s_{12}s_{16}}{s_{11}} - \frac{3s_{26}}{2}\right)\frac{qxy}{h} + \left[\frac{s_{12}^2 - s_{11}s_{22}}{2s_{11}} + \frac{3(s_{11}s_{26} - 2s_{12}s_{16})l}{4s_{11}h} + \right.$$

$$\left.\frac{3(s_{11}s_{26} - s_{12}s_{16})^2h^2}{2s_{11}(2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2)}\right]qy -$$

$$\frac{s_{11}qx^3(l - x)}{2h^3} + \frac{s_{11}qx^2(l - x)}{2h^3(2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2)}[2l^3s_{11}^2 +$$

$$3(s_{11}s_{66} - s_{16}^2)h^2l + 4(s_{11}s_{26} - s_{12}s_{16})h^3] +$$

$$\{qx(l - x)[2s_{11}^2(3s_{11}s_{66} - 2s_{16}^2)l^2 + 4s_{11}^2(s_{12}s_{16} - s_{11}s_{26})hl + 3(3s_{11}s_{66} -$$

$$2s_{16}^2)(s_{11}s_{66} - s_{16}^2)h^2]\} \Big\backslash \{4s_{11}h(2s_{11}^2l^2 + 3s_{11}s_{66}h^2 - 3s_{16}^2h^2)\}. \tag{11b}$$

如果令 $s_{11} = 1/E_0$, $s_{12} = -\mu/E_0$, $s_{66} = 2(1+\mu)/E_0$, $s_{16} = s_{26} = 0$, 其中 E_0 是弹性模量, μ 是泊松比, 即退化为各向同性情形, 则式(8) ~ (11) 与文献[6] 的结果完全一样。 各向异性梁 σ_x 中出现的y^2 项, τ_{xy} 中出现的y^3 项, u 中出现的y^4 和 y^2 项和 v 中出现的y^3 项, 在各向同性或正交各向异性梁中均是没有的。

3 数值算例

下面给出解析解的一个数值算例, 并与有限元数值结果作了对比。 计算一跨度为 10 m、高度为 1 m、单位宽度的两端固支各向异性梁, 受 $q = 10^7$ N/m 均布载荷作用的结果。 采用的材料常数如表 1 所示。 有限元法结果由 MSC. Nastran 程序计算给出, 采用的固支端边界条件为: 1) $x = 0$, $l - h/2 \leqslant y \leqslant h/2$, $u = v = 0$; 2) $y = h/2$, $0 \leqslant x \leqslant l$, $\sigma_y = \tau_{xy} = 0$; 3) $y = -h/2$, $0 \leqslant x \leqslant l$, $\sigma_y = 10^7$ Pa, $\tau_{xy} = 0$。 所采用的有限单元为正方形单元(Quad4), 单元边长为0.1 h, 共计 1 000 个单元。

表 1 材料常数 $m^2 \cdot N^{-1}$

s_{11}	s_{12}	s_{16}	s_{22}	s_{26}	s_{66}
11.162×10^{-12}	-4.557×10^{-12}	1.847×10^{-12}	11.970×10^{-12}	2.171×10^{-12}	33.778×10^{-12}

图2 是固支梁在 $y = 0$ 处的位移分量 v 的曲线(即中心线挠度), 图3 是 $y = -h/2$ 处的位移分量 u 的曲线, 图中 BC1 和 BC2 分别表示在第 1 种和第 2 种边界条件下解析解的结果, FEM 表示有限元的结果•

由解析解表达式及其数值结果可知, 中心线挠度为 x 的4 次曲线; $y = -h/2$ 处的位移分量 u 则为 x 的3 次曲线• 从图2 和图3 中可以看出, 位移分量的有限元结果位于两种边界条件下的结果之间, 而更接近于第 2 种边界条件下的结果• 这在物理上正好显示出固支边界条件的不同提法将给梁以不同的约束• 第 1 种边界条件给予梁的约束较强, 而第 2 种则较弱• 从物理上看第 2 种边界条件更接近有限元法所提的约束, 这里的两个解析解是实际问题的两个重要近似•

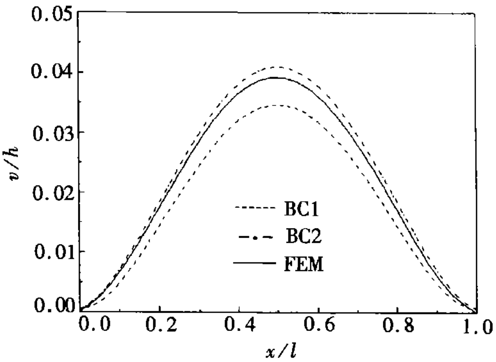

图2 在 $y = 0$ 处的位移分量 v/h

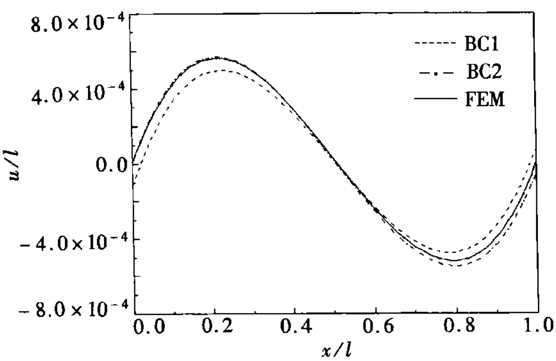

图3 在 $y = -h/2$ 截面的位移分量 u/l

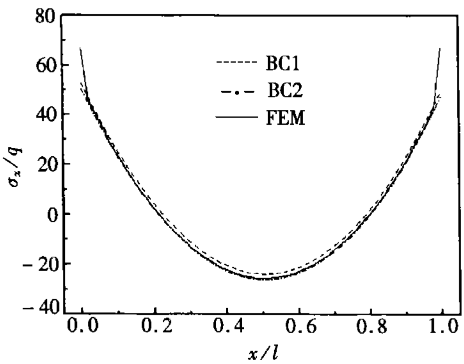

图4 在 $y = -h/2$ 处无量纲应力分量 σ_x/q 分布

图 4 是 $y = -h/2$ 处无量纲应力分量 σ_x/q 的分布曲线• 从图 4 也可以发现, 除距离端部极小的区域外, σ_x/q 的有限元解也均落在两种边界条件下的结果之间, 且有限元解也更接近于第 2 种边界条件下的结果• 而在距离端部极小的区域内, σ_x/q 的有限元解快速增大, 这与应力集中有关•

从位移分量的表达式可得, 在第 1 种边界条件下, 梁中 $y = -h/2$ 截面上位移分量 u 为零的点位于 $x = 5.0897$ m, 在第 2 种边界条件下位于 $x = 5.0866$ m, 并非梁的中点• 而在 $y = 0$ 截面上的位移分量 v, 第 1 种边界条件下的极大值位于梁的中点 $x = l/2$ 处, 而第 2 种边界条件下的极大值位于 $x = 5.0012$ m 处, 不在梁的中点• 从应力分量 σ_x 的表达式可知, 在第 1 种边界条件下, $y = -h/2$ 截面处的应力极值位于 $x = 5.0551$ m 处, 第 2 种边界条件下为 $x = 5.0564$ m 处, 也并非梁的中点• 这些现象与各向异性材料常数中的 s_{16} 和 s_{26} 有关, 如果 $s_{16} = 0$ 和 $s_{26} = 0$, 即材料为正交各向异性材料, 则梁 $y = -h/2$ 截面上位移 u 为零的点, $y = 0$ 截面上位移 v 的极值点和 $y = -h/2$ 应力极值都在梁的中点 $x = 5$ m 处•

4 结 论

本文研究了各向异性固支梁在均布载荷作用下的解析解,该解析解是对弹性理论中相关经典例题的补充• 得到的解析解与有限元数值解相比较,结果吻合很好• 两种边界条件下的解析解限定了有限元数值解的范围• 这个解析解既有理论意义,又有实际工程应用价值•

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Analytical Solution for Fixed_Fixed Anisotropic Beam Subjected to Uniform Load

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Abstract: The analytical solutions of the stresses and displacements were obtained for fixed_fixed anisotropic beams subjected to uniform load. A stress function involving unknown coefficients was constructed, and the general expressions of stress and displacement were obtained by means of Airy stress function method. Two types of the description for the fixed end boundary condition were considered. The introduced unknown coefficients in stress function were determined by using the boundary conditions. The analytical solutions for stresses and displacements were finally obtained. Numerical tests show that the analytical solutions agree with the FEM results. The analytical solution supplies a classical example for the elasticity theory.

Key words: fixed_fixed beam; anisotropy; stress function; analytical solution